Do we live on a brane ?

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ISI, February 29, 2008

Based on

1. S. Pal and S. Kar, Class. Quant. Grav. 25, 045003 (2008)
2. S. Pal, Phys. Rev. D 74, 124019 (2006)
3. S. Pal, Phys. Rev. D 74, 024005 (2006)
4. S. Pal and S. Kar, Class. Quant. Grav. 23, 2571 (2006)
5. S. Pal, S. Bharadwaj and S. Kar, Phys. Lett. B 609, 194 (2005)
6. S. Pal, Phys. Teacher 47, 144 (2005)
7. R. Koley, S. Pal and S. Kar, Am. J. Phys. 71, 1037 (2003)
8. S. Pal and S. Kar, Phys. Lett. B submitted
9. R. Koley and S. Pal, Phys. Rev. D submitted
10. R. Koley and S. Pal, Phys. Lett. B submitted

The warm-up

- Braneworlds physics: what, why and how?
- What happens to SM fields and gravity?
- Braneworld gravity and cosmology
- Solutions to some long-standing problems
- Observational and experimental tests
- Open issues

Why braneworld physics?

Motivations from particle physics

$$\underbrace{\text{Gravitational} \qquad \underbrace{\underbrace{\text{Electromagnetic} \qquad \text{Weak}}_{\text{Electroweak}} \qquad \text{Strong}}_{\text{Standard Model}}}$$

$$\Downarrow$$

???

String Theory, LQG and many more... No definite answer till date
String Theory needs energy $\sim 10^{19}$ GeV to be tested $\Longrightarrow$ **Impossible !**
Search for a low-energy effective theory that will have some imprints of String Theory, which may be observable at particle colliders and from astrophysical/ cosmological observations

Can we have a "smoking gun"?

$$\Downarrow$$

$\boxed{\textbf{Braneworld physics}}$

Motivations from gravitational physics

General Relativity : gravity described by curvature of spacetime

$$\textbf{Geometry} \equiv \textbf{Matter}$$

General Relativity is a very successful theory of gravity, but

- Singularities: big bang, black holes, collapse
- Not tested inside sub-millimeter scales
- Issues in macroscopic domain: dark matter, dark energy...

Can the theory of gravity be modified with appropriate macroscopic and microscopic limits?

$$\Downarrow$$

$$\boxed{\textbf{Braneworld physics}}$$

What is a brane?

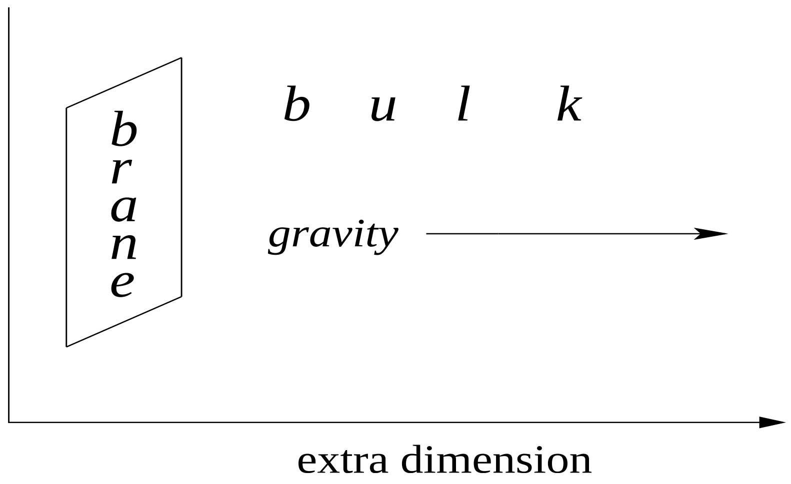
How many dimensions to our universe?

Newtonian mechanics : 3 space dimensions

Relativistic mechanics : 3 space + 1 time = 4 dimensions

(Super)string theory : 4 usual + 6 compact = 10 dimensions

What if at least 1 extra dimension is large? $\Rightarrow$ Braneworlds

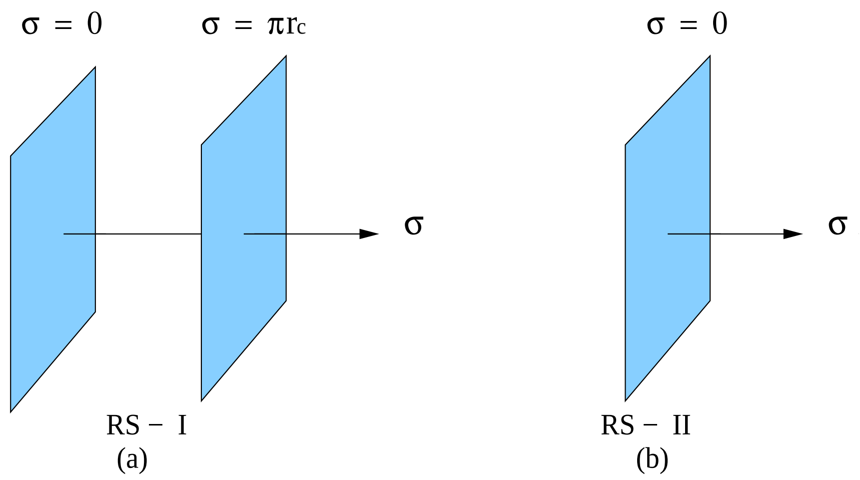


4D hypersurface we live on is called the **(mem)brane**

How is the braneworld?

$$ds_5^2 = e^{2f(\sigma)}\eta_{\mu\nu}dx^\mu dx^\nu + d\sigma^2$$

Solve $G_{AB} = -\Lambda_5\, g_{AB} + \kappa_5^2\, T_{AB}$ and obtain $f(\sigma)$



Simplest solution : $f(\sigma) = -kr_c|\sigma|$ L.Randall et.al. PRL(1999)

- RS-I : solves hierarchy problem $M_{Pl}^2 = \frac{M^3}{k}(1 - e^{-2kr_c\pi})$
- RS-II : localisation of gravity

Generalisations : bulk fields $\Longrightarrow$ thick brane models

What happens to SM fields and gravity?

Localisation of SM fields

* Write down KG/Dirac equation (in curved space) for 5D field
* Extract out the 4D part by separation of variables
* Check finiteness of the remainder (extra-dimensional part)

$\Rightarrow$ SM fields are confined to the brane ($\equiv$ open strings)

Modifications to Newton's law of gravity

Newtonian potential turn out to be

$$U(r) \simeq \frac{G_N \; m_1}{r} \left[1 + \frac{1}{k^2 r^2}\right]$$

$\Rightarrow$ Gravity propagates to the bulk ($\equiv$ closed strings)

The theory is consistent and in agreement with experiments

Effective Einstein equation on the brane

How is the 4D law of gravity modified?

* Bulk Einstein equation $G_{AB} = -\Lambda_5 \, g_{AB} + \kappa_5^2 \, T_{AB}$
* Gauss-Codacci projection mechanism
* Israel junction conditions
* Z_2 (mirror) symmetry

$$G_{\mu\nu} = \underbrace{-\Lambda g_{\mu\nu} + \kappa_4^2 T_{\mu\nu}}_{\Downarrow \atop \text{4D GR}} + \underbrace{\kappa_5^4 S_{\mu\nu}}_{\Downarrow \atop \text{Quadratic } T_{\mu\nu}} - \underbrace{\mathcal{E}_{\mu\nu}}_{\Downarrow \atop \text{Weyl term}} + \underbrace{\mathcal{F}_{\mu\nu}}_{\Downarrow \atop \text{Bulk matter}}$$

- **Does it have advantages over GR?**
- **How to obtain and interpret good old results?**
- **Observational probe of extra dimensions through this eqn?**

Applications

- Reissner-N$\ddot{o}$rdstrom black hole with 'tidal charge'

  N. Dadhich et.al. PLB(2000)

- Gravitational collapse with no-go theorem

  C. Germani et.al. PRD(2001)

- Chaotic inflation with new conditions of inflation

  R. Maartens et.al. PRD(2001)

- Inflation without inflaton on the brane

  Y. Himemoto et.al. PRD(2001)

Solution to dark matter problem

In Einsteinian framework, explain...

- Flat rotation curves of spiral galaxies
- Mismatch of mass profile with X-ray emission from clusters
- Large scale structure formation of the universe

... Need to introduce some invisible matter – **Dark Matter**

$$G_{\mu\nu} = \kappa_4^2 \left(T_{\mu\nu|\text{visible}} + \underbrace{\mathbf{T}_{\mu\nu|\textbf{dark}}}_{\substack{\downarrow \\ \text{has to be put by hand}}} \right)$$

But no direct non-gravitational evidence of dark matter so far

Can a new theory get rid of dark matter?

Clue: Change the geometry part instead of the matter part

Static, spherically symmetric metric in isotropic coordinates :

$$ds^2 = -(1 + 2\Phi)c^2 dt^2 + (1 - 2\Phi + 2\Psi)[dr^2 + r^2 d\Omega^2]$$

$$\underset{\text{Newtonian potential}}{\Downarrow} \qquad\qquad \underset{\text{Relativistic potential}}{\Downarrow}$$

Now, the modified Einstein equation is

$$G_{\mu\nu} = \underbrace{-\Lambda g_{\mu\nu}}_{\substack{\Downarrow \\ 0 \\ \text{(RS fine-tuning)}}} + \kappa_4^2 T_{\mu\nu} + \underbrace{\kappa_5^4 S_{\mu\nu}}_{\substack{\Downarrow \\ 0 \\ \text{(Weak field)}}} - \underbrace{\mathcal{E}_{\mu\nu}}_{\substack{\Downarrow \\ \text{Tracefree}}} + \underbrace{\mathcal{F}_{\mu\nu}}_{\substack{\Downarrow \\ 0 \\ \text{(No bulk matter)}}}$$

Hence, in the weak field limit

$$\boxed{G_{\mu\nu} = \kappa_4^2 T_{\mu\nu} - \mathcal{E}_{\mu\nu}}$$

Note that $\mathcal{E}_{\mu\nu}$ is traceless : Utilise this additional input

Take trace of both sides of EEE $\Longrightarrow$

$$\mathrm{Tr}\; G_{\mu\nu} = \mathrm{Tr}\; T_{\mu\nu}$$

With the above metric, this results in

$$\boxed{\nabla^2(\Phi - 2\Psi) = \frac{4\pi G}{c^2}\rho_v}$$

Thus,

$$\Psi = \frac{1}{2}\Phi - \frac{2\pi G}{c^2}(\nabla^2)^{-1}\rho_v$$

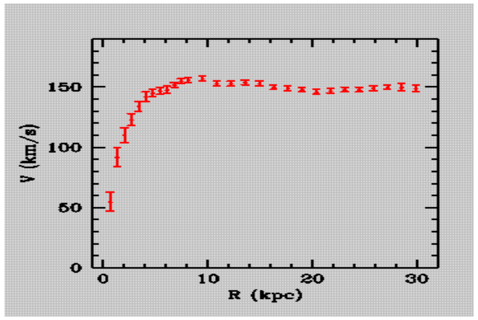
$\Downarrow$ Determined from observations

$\Downarrow$ Visible matter density

Consistent solution without any need for dark matter

SP, SB & SK, PLB 609, 194 (2005)

Application I : rotation curves of spiral galaxies



* Rotational velocity of HI cloud $v_c \approx 200 km/s$
* Visible matter as a disk with density profile $\rho_v = \rho_0 e^{-\gamma R'} \delta(z')$
* Obtain geodesic equation for HI cloud

$$\Phi'(r) = \frac{v_c^2(r)}{c^2}\frac{1}{r}$$

$$\boxed{\Phi(r) = \frac{v_c^2}{c^2}\left[\ln\left(\frac{r}{r_0}\right) - 1\right]}$$

$$\boxed{\Psi(r) = \frac{v_c^2}{2c^2}\left[\ln\left(\frac{r}{r_0}\right) - 1\right] - \left[\frac{4\pi^2 G\rho_0}{\gamma^2 c^2}\right]\frac{1}{r}}$$

SP, PT 47, 144 (2005)

Application II : X-ray emission from clusters

* Visible matter $\equiv$ intra-cluster gas $\Rightarrow \rho_v = \rho_g$
* Gas density :

$$\rho_g(r) = \rho_0[1 + (r/r_c)^2]^{-3\beta/2}$$

* Calculate for $r \gg r_c$ and $\beta = 2/3$
* Apply EM conservation for isothermal gas

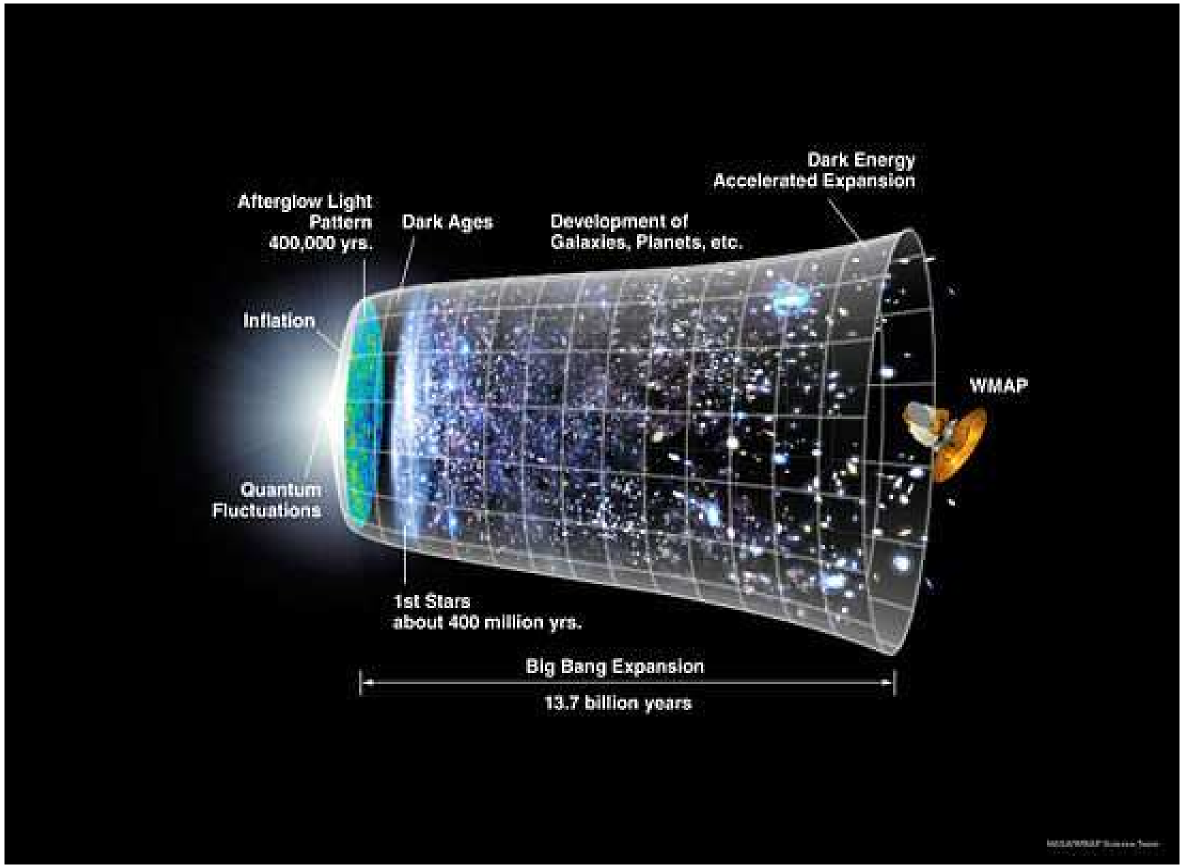
$$\Phi^{'}(r) = -\frac{kT}{\mu m_p c^2}\frac{d\ln\rho_g}{dr}$$

$$\boxed{\Phi = \frac{2kT}{\mu m_p c^2}\ln\frac{r}{r_c}}$$

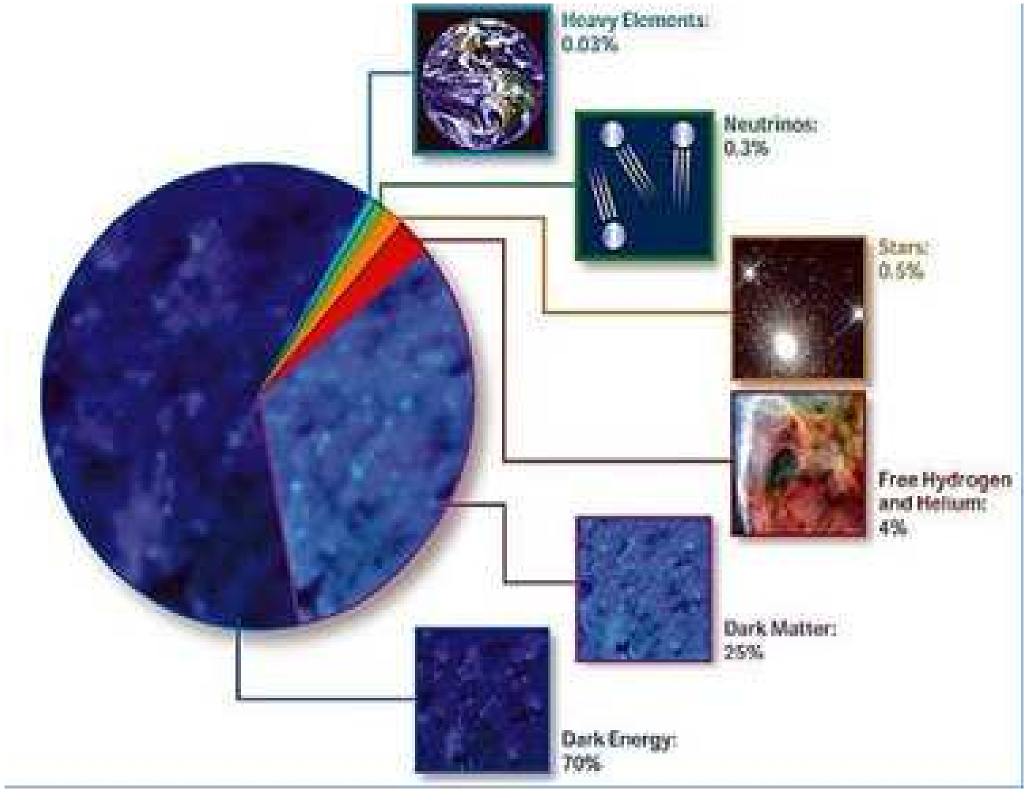
$$\boxed{\Psi = \left[\frac{kT}{\mu m_p c^2} - \frac{2\pi G\rho_o r_c^2}{c^2}\right]\ln\frac{r}{r_c}}$$

SP, SB & SK, PLB 609, 194 (2005)

Braneworld cosmology



The cosmic pie



Only $\sim 4\%$ of cosmic density based on standard cosmology is somewhat known to us !

$\boxed{\textbf{It really worths invoking modified cosmology theories}}$

The set-up

Brane : FRW $\Longleftrightarrow$ Bulk : Schwarzschild-AdS / Vaidya-AdS

	Brane metric	**Bulk source**	**Bulk metric**	**Comments**
1	Minkowski	$\Lambda_5 < 0$	warped, AdS_5	–
2	Minkowski	bulk field	warped	–
3	FRW	black hole (empty)	Schwarzschild-AdS_5	$m = 0 \Rightarrow 1$
4	FRW	radiative black hole(bulk field)	Vaidya-AdS_5	$m(v) = \mathrm{m} \Rightarrow 3$ $m(v) = 0 \Rightarrow 1$

Put $m = 0$ and get back warped (AdS) geometry in the bulk

This is a generalisation of RS II scenario

Brane cosmology: How to visualise?

Brane : FRW $\Longleftrightarrow$ Bulk : Schwarzschild-AdS / Vaidya-AdS

Schwarzschild-AdS bulk

$$dS_5^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma_3^2 \quad \text{with} \quad f(r) = k - \frac{\Lambda_5}{6}r^2 - \frac{m}{r^2}$$

$*$ For embedding : tangents $u^\mu = (\dot{t},\ \dot{r})$, normals $n^\mu = (\dot{r},\ -\frac{\sqrt{f+\dot{r}^2}}{f})$
$*$ Induced metric on the brane

$$\boxed{ds^2 = -d\tau^2 + r^2(\tau)d\Omega_3^2}$$

$*$ Identify $r(\tau)$ with the scale factor $a(\tau) \Longrightarrow$ **FRW**

Expanding 4D universe $\equiv$ Moving brane in the bulk

$\Uparrow$ Brane-based observer

$\Uparrow$ Bulk-based observer

J.Garriga et.al., PRD(2000)

Brane-based analysis: effective cosmology

The effective Einstein equation on the brane

$$G_{\mu\nu} = \underbrace{-\Lambda g_{\mu\nu} + \kappa_4^2 T_{\mu\nu}}_{\substack{\Downarrow \\ \text{4D GR}}} + \underbrace{\kappa_5^4 S_{\mu\nu}}_{\substack{\Downarrow \\ \text{Quadratic} \\ T_{\mu\nu}}} - \underbrace{\mathcal{E}_{\mu\nu}}_{\substack{\Downarrow \\ \text{Weyl} \\ \text{term}}} + \underbrace{\mathcal{F}_{\mu\nu}}_{\substack{\Downarrow \\ \text{Bulk} \\ \text{matter}}}$$

For convenience, express it in terms of an **effective EM tensor** as

$$\boxed{G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa_4^2 T_{\mu\nu}^{\text{eff}}}$$

For a **perfect fluid on the brane**, components of $T_{\mu\nu}^{\text{eff}}$

$$\begin{aligned}
\rho^{\text{eff}} &= \rho + \frac{\rho^2}{2\lambda_b} + \rho^* \\
p^{\text{eff}} &= p + \frac{\rho}{2\lambda_b}(\rho + 2p) + \frac{\rho^*}{3} \\
q_\mu^{\text{eff}} &= q_\mu^* \\
\pi_{\mu\nu}^{\text{eff}} &= \pi_{\mu\nu}^*
\end{aligned}$$

For a bulk compatible to FRW geometry on the brane, $q_\mu^{\text{eff}} = 0 = \pi_{\mu\nu}^{\text{eff}}$

Schwarzschild-AdS bulk, FRW brane

Empty bulk ($T_{AB}^{\text{bulk}} = 0$) $\Rightarrow \mathcal{F}_{\mu\nu} = 0$

Bianchi identity on the brane $\Rightarrow$ **conservation equations**

For brane matter/radiation :

$$\dot{\rho} + 3H(\rho + p) = 0$$

For Weyl fluid :

$$\dot{\rho}^* + 4H\rho^* = 0$$

$$\Longrightarrow \boxed{\rho^* = \frac{C}{a^4}}$$

$C \sim$ (constant) mass of the bulk Schwarzschild-AdS black hole

R.Maartens, LRR(2004)

Vaidya-AdS bulk, FRW brane

Radiative bulk ($T_{AB}^{\text{bulk}} = \psi \; q_A q_B$) $\Rightarrow \mathcal{F}_{\mu\nu} = \frac{2}{3}\kappa_5^2 \; \psi h_{\mu\nu}$

Bianchi identity on the brane $\Rightarrow$ **(non)conservation equations**

For brane matter/radiation :

$$\dot{\rho} + 3H(\rho + p) = -2\psi$$

For Weyl fluid :

$$\dot{\rho}^* + 4H\rho^* = 2\psi - \frac{2\kappa_5^2}{3\kappa_4^2}\left[\dot{\psi} + 3H\psi\right]$$

$$\Longrightarrow \boxed{\rho^* = \frac{C(\tau)}{a^4}}$$

$C(\tau) \sim$ (variable) mass of the bulk Vaidya-AdS black hole
Put $\psi = 0$ and get back Schwarzschild-AdS bulk scenario

Vaidya-AdS bulk is so far the most general scenario

Brane Friedmann equations

$$H^2 = \frac{\kappa_4^2}{3}\left[\rho + \frac{\rho^2}{2\lambda_b} + \frac{C(\tau)}{a^4}\right] + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\dot{H} = -\frac{\kappa_4^2}{2}\left[(\rho + p)\left(1 + \frac{\rho}{\lambda_b}\right) + \frac{4C(\tau)}{3a^4}\right] + \frac{k}{a^2} - \frac{\kappa_5^2}{3}\psi$$

Look too complicated !

Rewrite in terms of the effective perfect fluid $T_{\mu\nu}^{\text{eff}}$

$$H^2 = \frac{\kappa_4^2}{3}\rho^{\text{eff}} + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\dot{H} = -\frac{\kappa_4^2}{2}(\rho^{\text{eff}} + p^{\text{eff}}) + \frac{k}{a^2} - \frac{\kappa_5^2}{3}\psi$$

Now they look much familiar

As if there is an effective perfect fluid that governs cosmological dynamics

D.Langlois et.al., PRL(2002)

Just for the sake of completeness...

Effective equation of state

$$w^{\rm eff} = \frac{p^{\rm eff}}{\rho^{\rm eff}} = \frac{p + \frac{\rho}{2\lambda_b}(\rho + 2p) + \frac{\rho^*}{3}}{\rho + \frac{\rho^2}{2\lambda_b} + \rho^*}$$

Effective sound speed

$$c_{\rm eff}^2 = \frac{\dot{p}^{\rm eff}}{\dot{\rho}^{\rm eff}} = \left[c_s^2 + \frac{\rho + p}{\rho + \lambda_b} + \frac{4\rho^*}{9(\rho+p)(1+\rho/\lambda_b)}\right]\left[1 + \frac{4\rho^*}{3(\rho+p)(1+\rho/\lambda_b)}\right]^{-1}$$

Role of the quadratic term

Early universe :

$$\rho^2 \gg \lambda_b \Rightarrow \textbf{significant}$$

- Faster Hubble expansion implies inflation at a much faster rate than standard cosmology

R. Maartens et.al., PRD(2001)

- Inflation without 4D inflatonic field on the brane

Y. Himemoto et.al., PRD(2001)

Late time :

$$\rho^2 \ll \lambda_b > (100GeV)^4 \Rightarrow \textbf{negligible}$$

Role of the Weyl fluid

Evolution equation :

$$\dot{\rho}^* + 4H\rho^* = 2\psi - \frac{2\kappa_5^2}{3\kappa_4^2}\left[\dot{\psi} + 3H\psi\right]$$

For empty bulk :

$$\psi = 0 \Rightarrow \rho^* = \frac{C}{a^4} \Longrightarrow \textbf{dark radiation}$$

Nucleosynthesis data $\Rightarrow < 3\%$ of radiation energy density
Since radiation-like, its effect negligible at late times

J.D.Barrow et.al., PLB(2002)

For radiative bulk :

$$\psi \neq 0 \Rightarrow \rho^* = \frac{C(\tau)}{a^4} \Longrightarrow \text{no longer radiation-like !}$$

$$\Downarrow$$

$$\boxed{\textbf{may be significant at late times}}$$

Structure formation on the brane

SP, PRD 74, 024005 (2006)

"Newtonian" analysis of perturbations

Hydrodynamic equations in terms of effective perfect fluid

$$\frac{\partial \rho^{\rm eff}}{\partial t} + \vec{\nabla}.(\rho^{\rm eff}\ \vec{v}^{\rm eff}) = 0$$

$$\frac{\partial \vec{v}^{\rm eff}}{\partial t} + (\vec{v}^{\rm eff}.\vec{\nabla})\vec{v}^{\rm eff} = -\frac{\vec{\nabla} p^{\rm eff}}{\rho^{\rm eff}} - \vec{\nabla}\Phi^{\rm eff}$$

$$\nabla^2 \Phi^{\rm eff} = 4\pi G \rho^{\rm eff}$$

Consider **perturbations**

$$\rho^{\rm eff}(\vec{x},\ \tau) = \bar{\rho}^{\rm eff}(\tau)(1 + \delta^{\rm eff}(\vec{x},\ \tau))$$

$$\Phi^{\rm eff}(\vec{x},\ \tau) = \Phi_0^{\rm eff} + \phi^{\rm eff}$$

$*$ Use comoving coordinates $v^{\rm eff} = \dot{a}\ r + u^{\rm eff}$

$*$ Express in terms of Fourier transform

$*$ Apply $p^{\rm eff} = p(\rho^{\rm eff})$

Perturbation equations for Fourier modes $\delta_k^{\rm eff}(\tau)$

$$\frac{d^2\delta_k^{\rm eff}}{d\tau^2} + 2\frac{\dot{a}}{a}\frac{d\delta_k^{\rm eff}}{d\tau} - \left[4\pi G\bar{\rho}^{\rm eff} - \left(c_{\rm eff}^2 k/a\right)^2\right]\delta_k^{\rm eff} = 0$$

$$\rho^{\rm eff} = \rho + \underbrace{\frac{\rho^2}{2\lambda_b}}_{\Downarrow\atop 0} + \rho^*$$

$$\Rightarrow \boxed{\rho^{\rm eff} = \rho + \rho^*}$$

So, a two fluid system : baryonic matter + Weyl fluid

So, break the perturbation equation into :

- **Baryonic matter fluctuation**

$$\frac{d^2\delta_b}{d\tau^2} + 2\frac{\dot{a}}{a}\frac{d\delta_b}{d\tau} = 4\pi G\bar{\rho}_b\delta_b + 4\pi G\bar{\rho}^*\delta^*$$

- **Weyl fluid fluctuation**

$$\frac{d^2\delta^*}{d\tau^2} + 2\frac{\dot{a}}{a}\frac{d\delta^*}{d\tau} = 4\pi G\bar{\rho}^*\delta^* + 4\pi G\bar{\rho}_b\delta_b$$

For radiative bulk, evolution eqn for the Weyl fluid is, in general, governed by

$$\dot{\rho}^* + 4H\rho^* = \alpha H\rho^*$$

$$\Longrightarrow \rho^* = C_0 a^{-(4-\alpha)}$$

For $1 < \alpha < 4$, Weyl fluid can dominate over matter !

* Apply $\Omega_b \ll \Omega_*$
* Scale factor at late time $a(\tau) = \left(\frac{3}{2}H_0\tau\right)^{2/3(w^{\rm eff}+1)}$

And obtain a consistent solution for the fluctuation equations :

$$\boxed{\delta^*(z) = \delta^*(0)(1+z)^{-1}} \qquad \boxed{\delta_b(z) = \delta^*(z)\left(1 - \frac{1+z}{1+z_N}\right)}$$

So.....

- $z \ll z_N \Longrightarrow \delta_b \simeq \delta^*$
- $z \to z_N \Longrightarrow \delta_b \to 0$ but δ^* still finite

This is precisely what is required to explain structure formation

Weyl fluid mimics dark matter

Relativistic perturbations: a brief sketch

RK & SP, PRD Submitted

- Acts like a two fluid system
- Material fluid : $\rho^{(b)}$ & Geometric fluid : ρ^{*}
- Interacting and Exchanging energy between them

Comoving fractional gradients of density and expansion as in GR

$$\Delta_{\mu}^{(i)} = \frac{a}{\rho^{(i)}} D_{\mu}\rho^{(i)} \quad \text{and} \quad Z_{\mu} = aD_{\mu}\Theta$$

(Non)conservation equations can be written composedly as

$$\boxed{\dot{\rho}^{(i)} + \Theta(\rho^{(i)} + p^{(i)}) = I^{(i)}}$$

For Einstein-de Sitter brane universe $(\Omega^* = 1, \Omega_\Lambda = 0)$, the linearised evolution equations :

$$\dot{\Delta}_\mu^{(i)} = \left(3Hw^{(i)} - \frac{I^{(i)}}{\rho^{(i)}}\right)\Delta_\mu^{(i)} - (1+w^{(i)})Z_\mu - \frac{c_s^2 \;\; I^{(i)}}{\rho^{(i)}(1+w)}\Delta_\mu - \frac{3aHI_\mu^{(i)}}{\rho^{(i)}} + \frac{a}{\rho^{(i)}}D_\mu I^{(i)}$$

$$\dot{Z}_\mu + 2HZ_\mu = -\frac{\kappa^2}{2}\rho\Delta - \frac{c_s^2}{1+w}D_\mu D^\nu \Delta_\nu + \frac{\kappa_5^2 \psi}{1+w}c_s^2 \Delta_\mu - a\kappa_5^2 D_\mu \psi$$

Density perturbations are governed by the fluctuation of the **covariant projections**

$$\Delta^{(i)} = a \; D^\mu \Delta_\mu^{(i)} \quad \text{and} \quad Z = a \; D^\mu Z_\mu$$

Covariant density perturbation equations on the brane :

$$\dot{\Delta}^{(i)} = \left(3Hw^{(i)} - \frac{I^{(i)}}{\rho_{(i)}}\right)\Delta^{(i)} - (1+w^{(i)})Z - \frac{c_s^2 \;\; I^{(i)}}{\rho^{(i)}(1+w)}\Delta - \frac{3a^2 HD^\mu I_\mu^{(i)}}{\rho^{(i)}} + \frac{a^2}{\rho^{(i)}}D^2 I^{(i)}$$

$$\dot{Z} + 2HZ = -\frac{\kappa^2}{2}\rho\Delta - \frac{ac_s^2}{1+w}D^2\Delta + \frac{\kappa_5^2 \psi}{1+w}c_s^2 \Delta - a^2\kappa_5^2 D^2 \psi$$

If ψ is function of time only and the energy exchange is in equilibrium

$$\Rightarrow \boxed{\alpha = \frac{5}{2} \quad ; \quad \rho^* \propto \frac{1}{a^{3/2}}}$$

Hence .....

Scalar perturbation equation for matter on brane

$$\ddot{\Delta}^{(b)} + 2H\dot{\Delta}^{(b)} = \frac{\kappa^2}{2}\rho\Delta - \frac{c_s^2\kappa_5^2\psi}{1+w}\Delta + \frac{4H\psi}{\rho^{(b)}}\left(\Delta^{(b)} + \frac{c_s^2\Delta}{1+w}\right) + \left(\frac{2\psi}{\rho^{(b)}}\left(\Delta^{(b)} + \frac{c_s^2\Delta}{1+w}\right)\right)^{\cdot}$$

Scalar perturbation equation for Weyl fluid on brane

$$\ddot{\Delta}^* + 2H\dot{\Delta}^* = \frac{4}{3}\frac{\kappa^2}{2}\rho\Delta - \frac{c_s^2\Delta}{1+w}\left(\frac{7H\psi}{\rho^*} + \frac{4\kappa_5^2\psi}{3} + \frac{2\dot{\psi}}{\rho^*}\right) - \frac{c_s^2\dot{\Delta}}{1+w}\frac{2\psi}{\rho^*}$$
$$+ \ \Delta^*\left(2H^2 - \frac{\kappa^2}{2} - \frac{7H\psi}{\rho^*} - \frac{2\dot{\psi}}{\rho^*}\right) + \dot{\Delta}^*\left(H - \frac{2\psi}{\rho^*} - \frac{\kappa_5^2\psi}{3}\right)$$

With $\rho^{(b)} << \rho^*$ and $\Delta^{(b)} << \Delta^*$

$$\ddot{\Delta}^* + 2H\dot{\Delta}^* - \frac{\kappa^2}{2}\rho^*\Delta^* = -\frac{A_1}{t}\Delta^* + \left(\frac{B_1}{t} + \frac{C_1}{t^2}\right)\Delta^*$$

$$\Rightarrow \quad \ddot{\Delta}^* + \frac{A}{t}\dot{\Delta}^* - \left(\frac{B}{t} + \frac{C}{t^2}\right)\Delta^* = 0$$

Solution for Δ^* :

$$\boxed{\Delta^* \sim t^{\frac{1}{2}-\frac{A}{2}} BesselI\left[\sqrt{1-2A+A^2+4C}, 2\sqrt{B}\sqrt{t}\right]}$$

Solution for Weyl fluid fluctuation Δ^* has a growing mode

Question: Is it compatible with observations ?

Observational test I : Cosmology

In terms of dimensionless parameters

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \Omega_\rho = \frac{\kappa^2 \rho_0}{3H_0^2}, \Omega_* = \frac{2C_0}{a_0^{4-\alpha} H_0^2}, \Omega_\lambda = \frac{\kappa^2 \rho_0^2}{6\lambda H_0^2}; \Omega_{\mathrm{tot}} = \sum_i \Omega_i = 1$$

Friedmann equations:

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \Omega_\rho \frac{a_0^3}{a^3} + \Omega_* \frac{a_0^{4-\alpha}}{a^{4-\alpha}} + \Omega_\lambda \frac{a_0^6}{a^6}$$

Luminosity distance for FRW branes

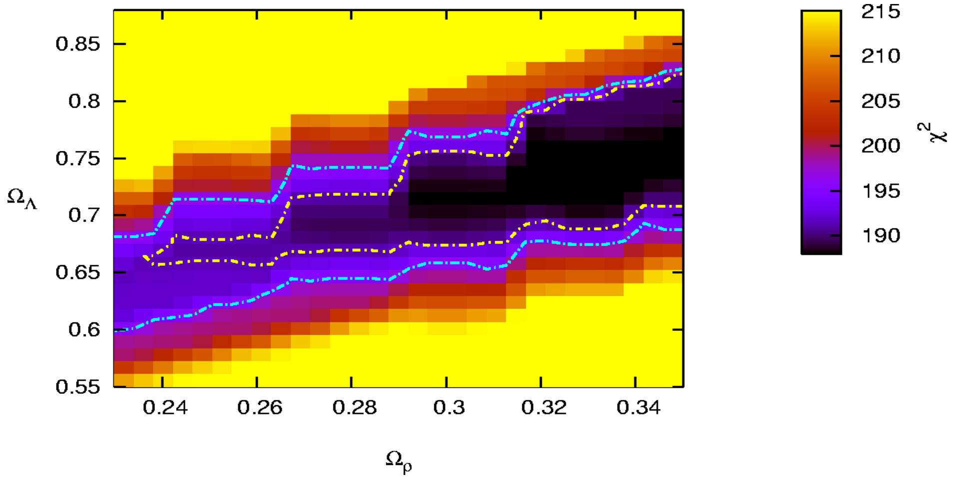
$$d_L(z) = \frac{(1+z) a_0}{H_0} \int_{a_{\mathrm{em}}}^{a_0} \frac{a\,da}{\left[\Omega_\Lambda a^6 + \Omega_\rho a_0^3 a^3 + \Omega_* a_0^{4-\alpha} a^{\alpha+2} + \Omega_\lambda a_0^6\right]^{1/2}}$$

For $\Omega_\lambda \to 0 \Longrightarrow$ $\boxed{d_L^{\Lambda\lambda *} = d_L^{\Lambda \mathrm{CDM}} + \Omega_* I_*}$

where I_* is a function having elliptic integrals of 1st and 2nd kind

Weyl fluid with $\alpha = 2 - 3$ is in nice agreement with SNe data

Results and Figure courtesy: L.A.Gergely et.al., 0709.0933[astro-ph]



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0.8
0.75
0.7
0.65
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0.55
Ω_Λ
0.24
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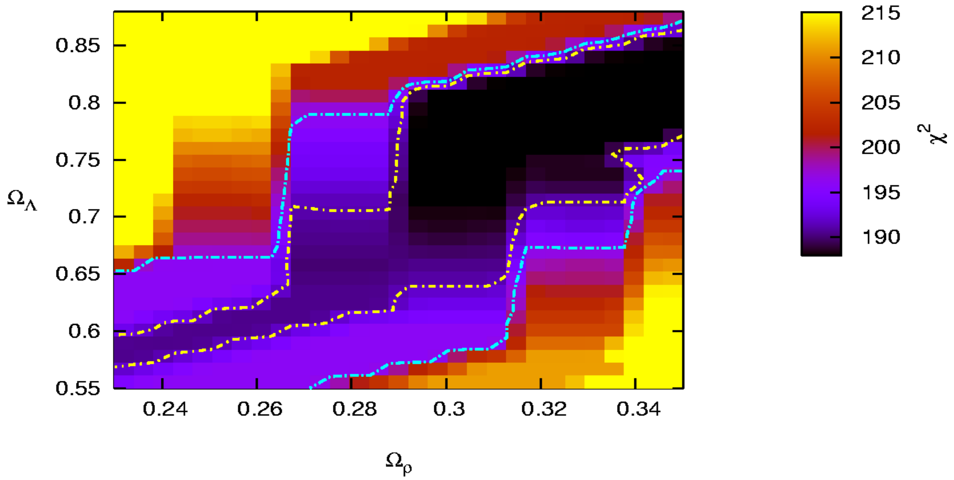
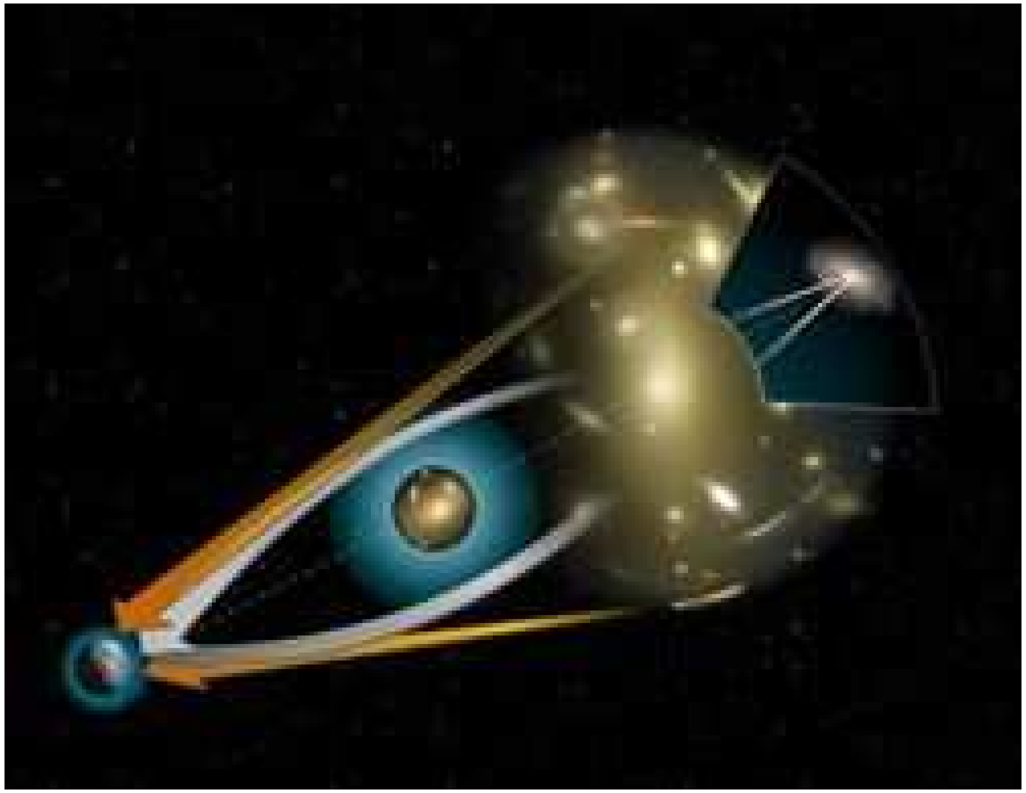
0.32
0.34
Ω_ρ
215
210
205
200
195
190
χ^2



0.85
0.8
0.75
0.7
0.65
0.6
0.55
Ω_Λ
0.24
0.26
0.28
0.3
0.32
0.34
Ω_ρ
215
210
205
200
195
190
χ^2

Observational test II : Gravitational lensing



$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + \left(1 - \frac{2\Phi - 2\Psi}{c^2}\right) d\overrightarrow{X}^2$$

Effective refractive index with the above spacetime

$$n = 1 + \frac{|2\Phi - \Psi|}{c^2} > 1$$

Time delay $\qquad \Delta t = \int_S^O \frac{2\Phi - \Psi}{c^3} dl$

$\Rightarrow$ Time delay is less than its GR value by

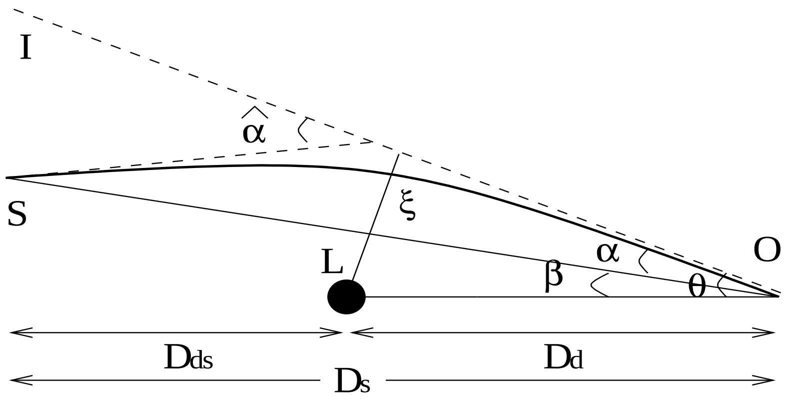
$$\Delta t_R - \Delta t = \frac{1}{c^3} \int_S^O |\Psi| dl$$

Bending angle $\hat{\alpha} = -\int_S^O \hat{\nabla}_\perp n = -\int_S^O \hat{\nabla}_\perp \left(1 - \frac{2\Phi - \Psi}{c^2}\right) dl$

$$\boxed{\hat{\alpha} = \hat{\alpha}_R - \frac{1}{c^2} \int_S^O \hat{\nabla}_\perp \Psi dl}$$

$\Rightarrow$ A light ray on the brane is deviated by a smaller amount

The lens equation (ray-tracing equation)



$$D_s \tan\beta = D_s \tan\theta - D_{ds}\left[\tan\theta + \tan(\hat{\alpha} - \theta)\right]$$

For small deflection angles, the lens equation reduces to

$$D_s\beta = D_s\theta - D_{ds}\hat{\alpha}$$

$$\Rightarrow \boxed{\overrightarrow{\beta} = \overrightarrow{\theta} - \overrightarrow{\alpha}(\theta)}$$

where the reduced deflection angle is defined as

$$\alpha = \frac{D_{ds}}{D_s}(\hat{\alpha}_R - \alpha_\Psi) < \alpha_R$$

This will give results quantitatively different from GR

Magnification : How a source is mapped onto the image?

Magnification is measured by the inverse of determinant of Jacobian of the lens-mapping matrix

$$\boxed{\mu = \det\mathcal{M} = (\det\mathcal{A})^{-1}}$$

It is customary to study magnification in terms of **lensing potential**

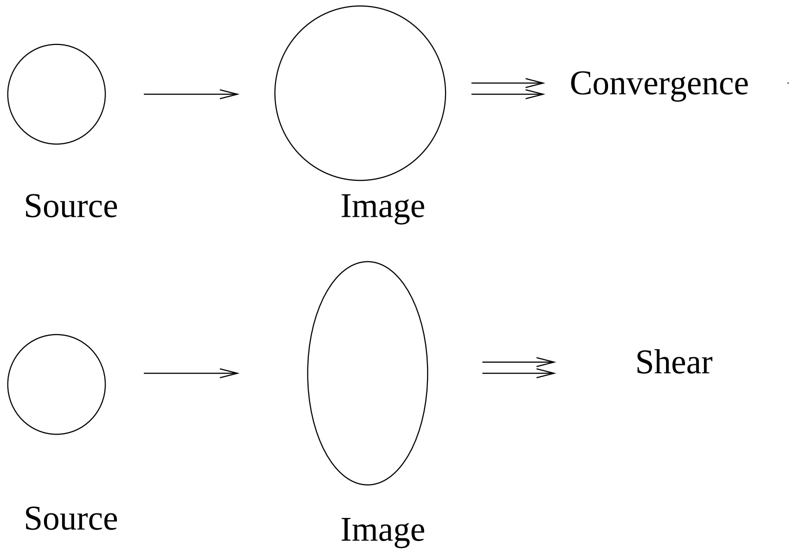
$$V(\theta) = \frac{D_{ds}}{D_d D_s} \int \frac{2\Phi - \Psi}{c^2}\, dl = V_R - V_\Psi$$

The **Jacobian matrix** is expressed in terms of the lensing potential by

$$\boxed{\mathcal{A} = \delta_{ij} - \frac{\partial^2}{\partial\theta_i \partial\theta_j}(V_R - V_\Psi)}$$

Define $\quad \frac{\partial^2 V}{\partial\theta_i \partial\theta_j} = V_{ij} \quad \Rightarrow \quad V_{ij} = V_{R_{ij}} - V_{\Psi_{ij}}$

Convergence and shear



Convergence (change in size) $\kappa = \frac{1}{2}(V_{11} + V_{22}) = \frac{1}{2}\mathrm{Tr}\mathrm{V}_{\mathrm{ij}}$

Shear (change in shape) $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$

$$\gamma_1 = \frac{1}{2}(V_{11} - V_{22}) = \gamma \cos 2\phi \qquad \gamma_2 = V_{12} = V_{21} = \gamma \sin 2\phi$$

In terms of convergence and shear, the Jacobian matrix is

$$\mathcal{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

Note : Lensing potential is different from GR. So, convergence and shear, and hence the Jacobian matrix will also differ from GR.

For spherically symmetric metric with Φ and Ψ :

$$
\begin{aligned}
\kappa &= \frac{1}{2}\mathrm{Tr}(\mathrm{V_{Rij}} - \mathrm{V_{\Psi ij}}) = \frac{\partial^2(\mathrm{V_R} - \mathrm{V_\Psi})}{\partial\theta^2} \\
\gamma_1 &= \frac{1}{2}\left[(V_{R11} - V_{\Psi 11}) - (V_{R22} - V_{\Psi 22})\right] = 0 \\
\gamma_2 &= V_{R12} - V_{\Psi 12} = V_{R21} - V_{\Psi 21} = \frac{\partial^2(V_R - V_\Psi)}{\partial\theta^2} \\
\gamma &= \gamma_2 = \frac{\partial^2(V_R - V_\Psi)}{\partial\theta^2}
\end{aligned}
$$

$$
\mathcal{A} = \underbrace{\begin{pmatrix} 1-\kappa_R-\gamma_{1R} & -\gamma_{2R} \\ -\gamma_{2R} & 1-\kappa_R+\gamma_{1R} \end{pmatrix}}_{\substack{\Downarrow \\ \text{Jacobian in GR}}} + \underbrace{\begin{pmatrix} \kappa_\Psi+\gamma_{1\Psi} & \gamma_{2\Psi} \\ \gamma_{2\Psi} & \kappa_\Psi-\gamma_{1\Psi} \end{pmatrix}}_{\substack{\Downarrow \\ \text{Braneworld modification}}}
$$

In braneworlds, magnification is different from GR

Quantitative estimation : Bending angle

$$\underbrace{\hat{\alpha}_R = 2\int_s^o \hat{\nabla}_\perp \Phi dl}_{\Downarrow \atop \text{In GR}} \qquad \underbrace{\hat{\alpha} = \int_s^o \hat{\nabla}_\perp (2\Phi - \Psi) dl}_{\Downarrow \atop \text{In braneworlds}}$$

For clusters

$$\boxed{\hat{\alpha}_R = \frac{4\pi kT}{\mu m_p c^2}} \qquad \boxed{\hat{\alpha} = \hat{\alpha_R}\left[\frac{3}{4} + \frac{\pi G \rho_0 r_c^2 \mu m_p}{2kT}\right]}$$

For galaxies

$$\boxed{\hat{\alpha}_R = \frac{2\pi v_c^2}{c^2}} \qquad \boxed{\hat{\alpha} = \hat{\alpha_R}\left[\frac{3}{4} - \frac{4\pi G \rho_0}{\gamma^2 v_c^2 b}\right]}$$

$\boxed{20 - 25\%\ \textbf{difference in the values of bending angle !}}$

Magnification	**Estimation**	**Comment**
For clusters		
Convergence	$\kappa = \kappa_R \left[\frac{3}{4} + \frac{\pi G \rho_0 r_c^2 \mu m_p}{2kT}\right]$	20% less change in size
Shear	$\gamma_1 = 0 = \gamma_{1R}$ $\gamma_2 = \gamma_{2R}\left[\frac{3}{4} + \frac{\pi G \rho_0 r_c^2 \mu m_p}{2kT}\right]$ $\gamma = \gamma_2$	γ_1 unchanged γ_2 less by 20% change in shape 20% less
For galaxies		
Convergence	$\kappa = \kappa_R \left[\frac{3}{4} - \frac{4\pi G \rho_0}{\gamma^2 v_c^2 b}\right]$	25% less change in size
Shear	$\gamma_1 = 0 = \gamma_{1R}$ $\gamma_2 = \gamma_{2R}\left[\frac{3}{4} - \frac{4\pi G \rho_0}{\gamma^2 v_c^2 b}\right]$ $\gamma = \gamma_2$	γ_1 unchanged γ_2 less by 25% change in shape 25% less

Present observational status: Lensing

- Present observational data reveal $25-30\%$ uncertainties in the observable quantities
- At present both braneworld and GR fare equally well in explaining those observations
- We need more accurate observational data for a conclusive remark

SP & SK, CQG 25, 045003 (2008)

“Seeing” extra dimensions

Particle physics sector

Search at collider LHC (CM energy ~ 4 TeV)

- Effects on Higgs mass under coupling with graviton/radion
- Corrections to SM cross-sections due to graviton exchange
- Finding out missing energy leaked into the bulk
- Production of mini black holes in laboratory

Gravitational sector

- Testing modified Newton’s law at sub-millimeter scales
- Cosmology from Supernova data
- Confronting brane cosmology with ΛCDM
- Precise measurement of gravitational lensing

Open issues

- Higher dimensional braneworld models and localisation
- Particle physics : LHC signatures

  $\longrightarrow$ **Any predictions related to LHC**
- Confront brane cosmology with observations : Calculate power spectrum etc and compare with ΛCDM
- Metric-based perturbations and allied phenomena
- Stability of bulk black hole under perturbations on Weyl fluid
- Expansion history of the universe : Studied to some extent in DGP, what about generalised RS II ?

  $\longrightarrow$ **Solve complicated Friedmann equations !**

Just to share with you ...

The idea came out of two papers by Randall and Sundrum in 1999

By Feb28, 2008 they received **3650** and **3337** citations respectively!

Data courtesy : SLAC SPIRES-HEP

Is this the beginning?