PUZZLES OF MODERN COSMOLOGY :
SOME (RANDOM) STEPS FORWARD

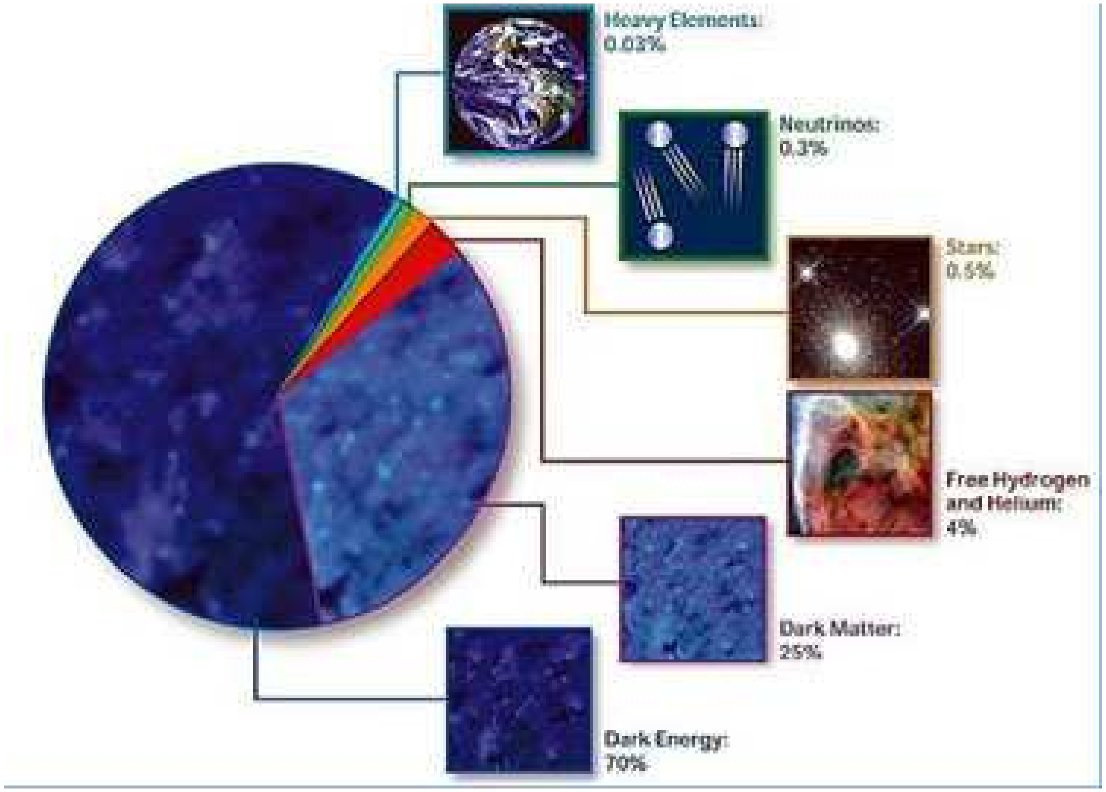
Supratik Pal

PAMU, Indian Statistical Institute Kolkata

and

PI, Universität Bonn

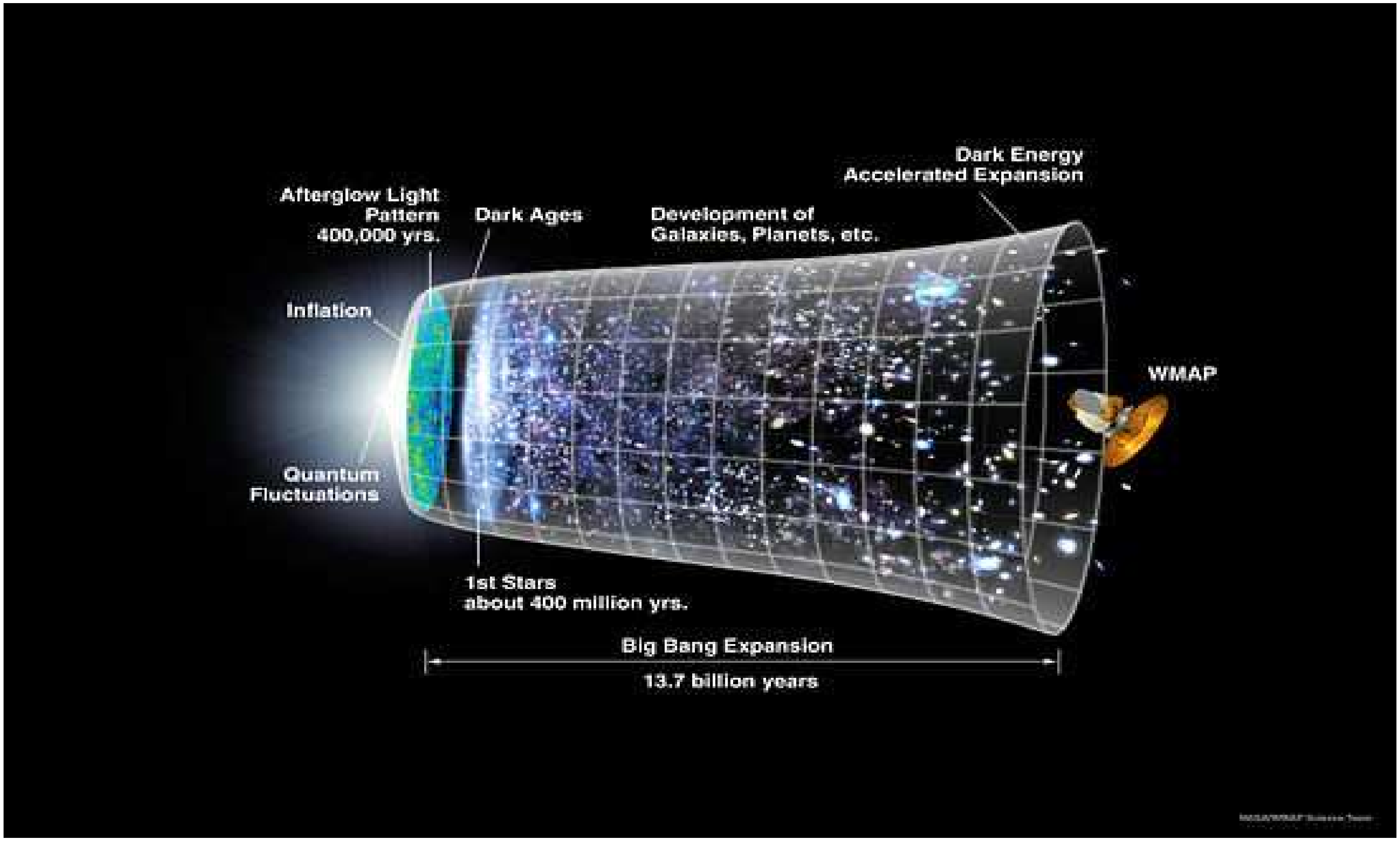
Warm-up... Why cosmology ?



Only $\sim 4\%$ of cosmic density is known to us conclusively

We are yet to know $\sim 96\%$ of the universe !

What do we know ?



How do we know ?

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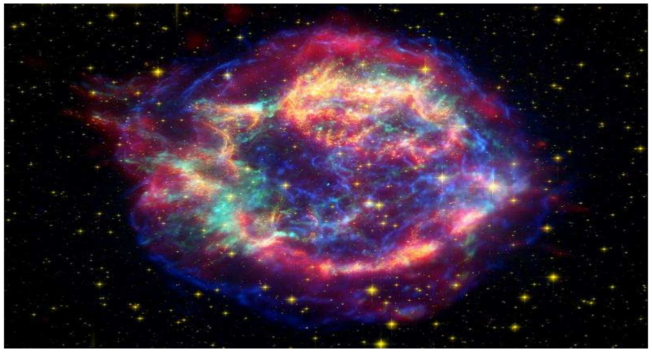
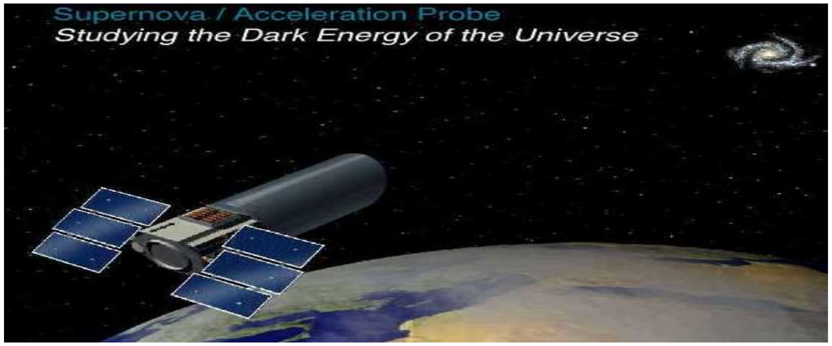
1. High-redshift Supernovae explosion





HST, SNLS, ESSENCE, HZT, GOODS... $\Longrightarrow$ **Dark energy**

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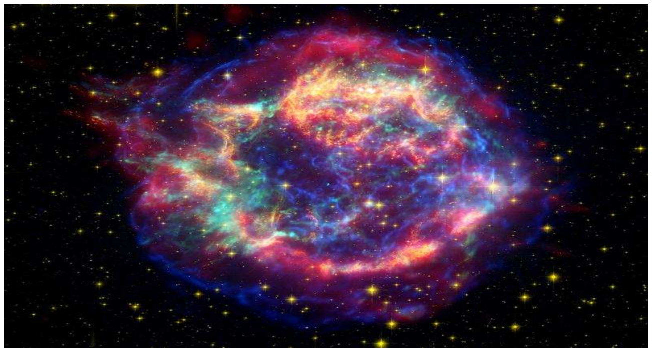

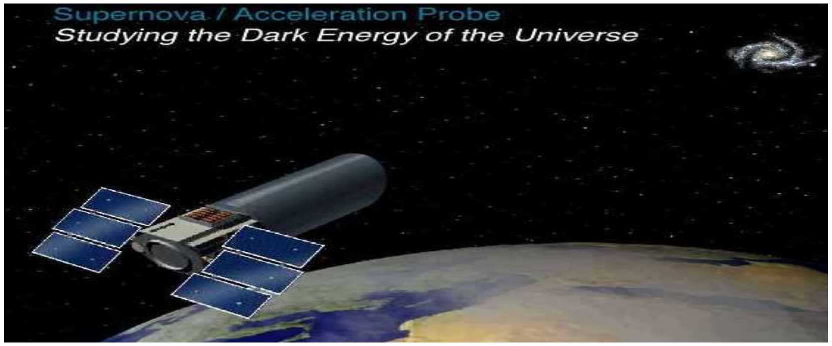

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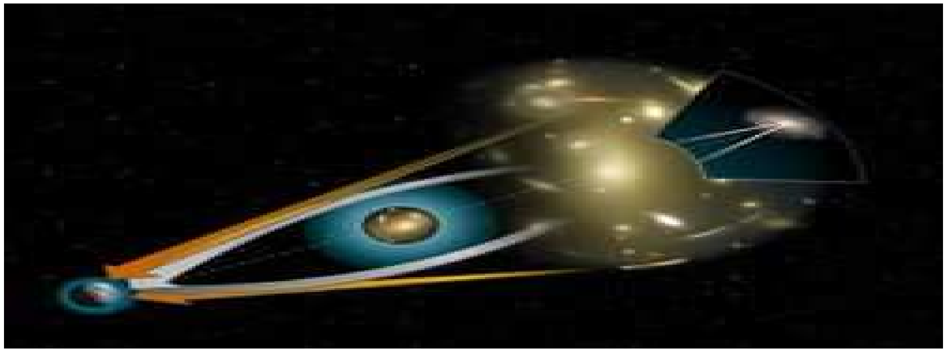
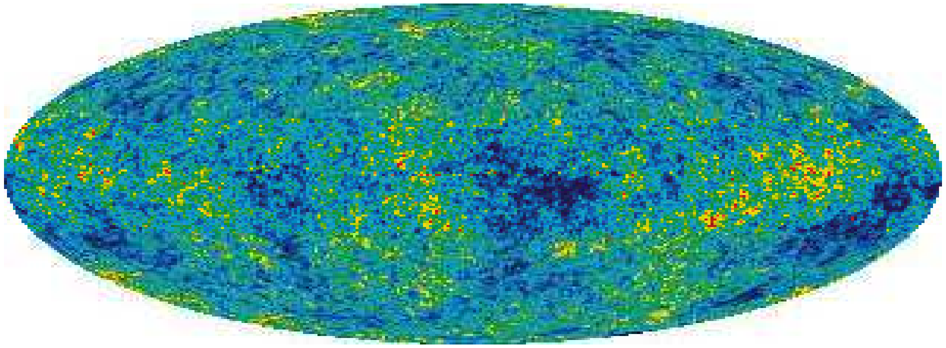
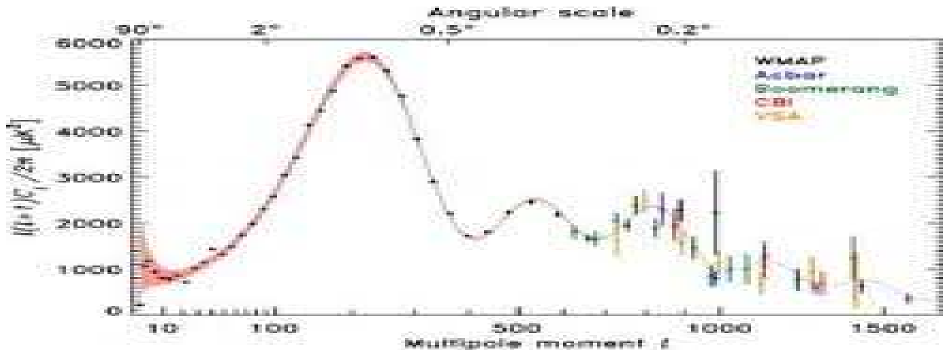
2. Gravitational lensing data



Cosmic Shear, Baryonic Acoustic Oscillations... $\Longrightarrow$ **Dark matter**

3. Cosmic Microwave Background sky





COBE, WMAP, PLANCK ..... $\Longrightarrow$ **Inflation, Baryonic matter, Dark matter, Dark energy, Gravitational waves...**

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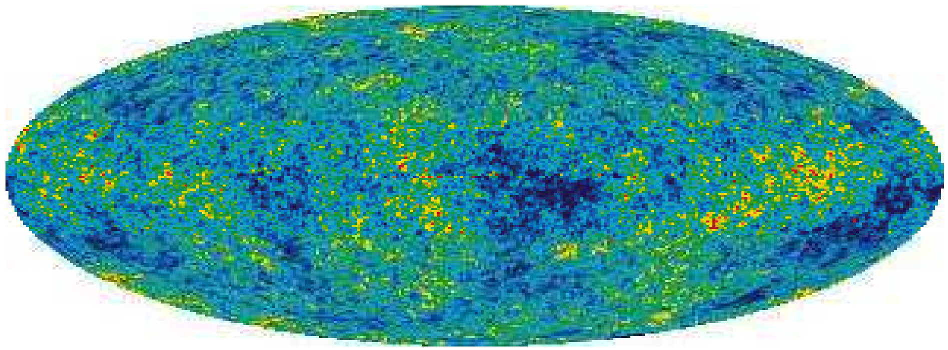

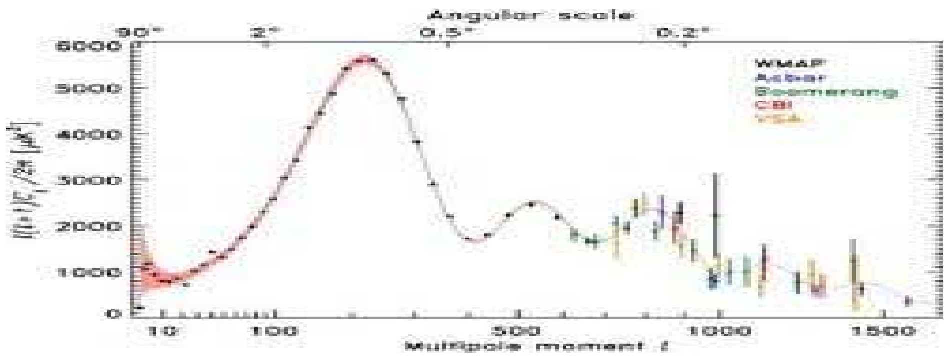

COBE, WMAP, PLANCK ..... $\Longrightarrow$ **Inflation, Baryonic matter, Dark matter, Dark energy, Gravitational waves...**

The observations open up so many avenues to venture

A brief history of “adventure”

Radiation-dominated era		
$\downarrow$	$\searrow$	**Deceleration**
Matter-dominated era	$\nearrow$	
$\downarrow$		
Present era	$\longrightarrow$	**Acceleration**

A brief history of “adventure”

Radiation-dominated era

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Matter-dominated era $\nearrow$

$\downarrow$

Present era $\longrightarrow$ **Acceleration**

Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Ordinary matter satisfying Strong Energy Condition $(\rho + 3p) \geq 0$ cannot supply the effective **“anti-gravity”** required for acceleration

Who governs this expansion history ?

Cosmological Constant

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

$$M = \frac{4}{3}\pi a^3(\rho + 3p) \Longrightarrow \quad \boxed{\ddot{a} = -\frac{GM}{a^2} + \frac{\Lambda}{3}a}$$

$\downarrow$ Effective mass

$\swarrow$ Attraction $\quad \searrow$ **Repulsion**

$\Lambda > 0$ guarantees repulsion, hence accelerated expansion

Late time solution

$$a(t) \propto \left[\sinh\left(\frac{3}{2}\sqrt{\frac{\Lambda_0}{3}}t\right)\right]^{2/3}$$

ΛCDM agrees fairly well with (some) late time observations

Puzzles of standard Big Bang Cosmology

- Homogeneity : Mismatch with CMB observations
- Flatness : Observations confirm spatially flat universe. Why?
- Horizon : Violates causality

but...

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Wayout: Super-fast expansion at the beginning $\Longrightarrow$ Inflation

Inflation with Λ

$$a(t) = \mathrm{Exp}[\sqrt{\frac{\Lambda_i}{3}}t]$$

Demands large cosmological constant $\Lambda_i \sim H_i^2$ during inflation $[H_i \sim 10^{15} GeV]$

but...

- Extreme fine-tuning to match present value $\rho_{\Lambda_0} \sim 10^{-47} GeV^4$.
  How to generate this from a large initial (inflationary) value?

Several fundamental puzzles confronting modern cosmology !

- Extreme fine-tuning to match present value $\rho_{\Lambda_0} \sim 10^{-47} GeV^4$. How to generate this from a large initial (inflationary) value?
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Mukhanov; Dodelson; Weinberg; Carroll; Turner; Smoot...

Several fundamental puzzles confronting modern cosmology !

Possible ansatz : Variable cosmological "constant"

Introduce a variable cosmological term in Friedmann equations, decaying with time from a large initial value

- Function of time : $\Lambda \propto t^{-\alpha}$
- Function of scale factor : $\Lambda \propto a^{-\beta}$
- Function of scale factor : $\Lambda \propto e^{-\beta a}$
- Function of temperature : $\Lambda \propto T^{-\alpha}$
- Function of Hubble parameter : $\Lambda \propto H^2$
- Function of several parameters : $\Lambda \propto H^2 + Ca^{-\beta}$

Bartolami; Gasperini; Overduin; Pavon; Lima...

Merely phenomenological models, each one has drawbacks

Dynamical models

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i (\rho_i + 3p_i)$$

$p_i < -\rho_i/3 \Rightarrow w_i < -1/3$ leads to acceleration $\Longleftrightarrow$ violates SEC

Guth; Linde; Starobinsky; Liddle; Lyth; Sahni...

Too many candidates with wide range of initial conditions

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Scalar field models

Lagrangian density $\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

EM tensor components $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$; $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

Choose the potential to be sufficiently steep so that $V^{''}V/V^{'2} \geq 1$

Scalar field rolls down the potential : “Tracker potential”

Includes Quintessence, Kessence, Chaplygin gas...

Guth; Linde; Starobinsky; Liddle; Lyth; Sahni...

Too many candidates with wide range of initial conditions

Modified gravity models

$*$ Einstein's theory is not directly tested in cosmological scales.

$*$ Modify the gravity sector rather than the matter sector.

- Brans-Dicke theory : G as a VEV of a geometric field, need vastly different values for solar system and cosmic scales
- f(R) gravity : R in the action replaced by f(R), [e.g. $R - \frac{\mu}{R}$], no unique choice
- DGP model : Extra dimensional effects at large scale, fails to produce small scale results

Brans; Lovelock; Odintsov; Polarski; Tsujikawa; Maartens...

Need tools to discriminate $\Longrightarrow$ Observational constraints

...Will come to this point in due course

Cosmology from effective theories

FRW $\Rightarrow$ Brane ; 5D Static, spherically symmetric metric $\Rightarrow$ Bulk

$$ds_5^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_3^2$$

Embedding mechanism $\Rightarrow$ Induced metric on the brane

$$ds_4^2 = -d\tau^2 + r^2(\tau)d\Omega_3^2$$

Identify $r(\tau)$ with the scale factor $a(\tau)$ $\Longrightarrow$ **FRW !**

Visser, PLB(2000); Sahni, JCAP(2003); Maartens, LRR(2004)

SP, PRD(2006),(2006),(2008); Mukherji, **SP**, MPLA(2010)

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Expanding 4D universe $\equiv$ Moving brane in the bulk

$\Uparrow$ $\Uparrow$

Brane-based observer Bulk-based observer

Accelerated universe is the manifestation of geodesic motion

Visser, PLB(2000); Sahni, JCAP(2003); Maartens, LRR(2004)

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Dark energy from effective theory

Das, Ghosh, van Holten, **SP**, JHEP(2009)

The general bulk action

$$S = m\int d\tau\left[\frac{1}{2e}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - \frac{e}{2} - \lambda g_{\mu\nu}\xi^\mu\dot{x}^\nu + \frac{e\lambda^2}{2}g_{\mu\nu}\xi^\mu\xi^\nu + \frac{e\beta\lambda^2}{2}\right]$$

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$\tau =$ worldline evolution parameter

$\xi^\mu =$ Killing vectors associated with symmetry of spacetime

$e(\tau) =$ worldline einbein to maintain reparametrization-invariance

$\lambda(\tau) =$ auxiliary worldline scalar variable

$\beta =$ a nonzero parameter of the theory

The action has been derived by Kaluza-Klein decomposition

Study geodesics in the background

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -\left(k - \frac{2M}{r^2} + \Lambda_5 r^2\right) dt^2 + \frac{dr^2}{k - \frac{2M}{r^2} + \Lambda_5 r^2} + r^2 d\Omega_3^2$$

Using the Killing vectors, radial geodesics look

$$\dot{r}^2 + V_{\rm eff}(r) = \varepsilon^2$$

Effective potential : $V_{\rm eff} = \left(k - \frac{2M}{r^2} + \Lambda_5 r^2\right)\left(1 + \frac{l^2}{r^2} + \frac{l^2}{\beta}\right)$

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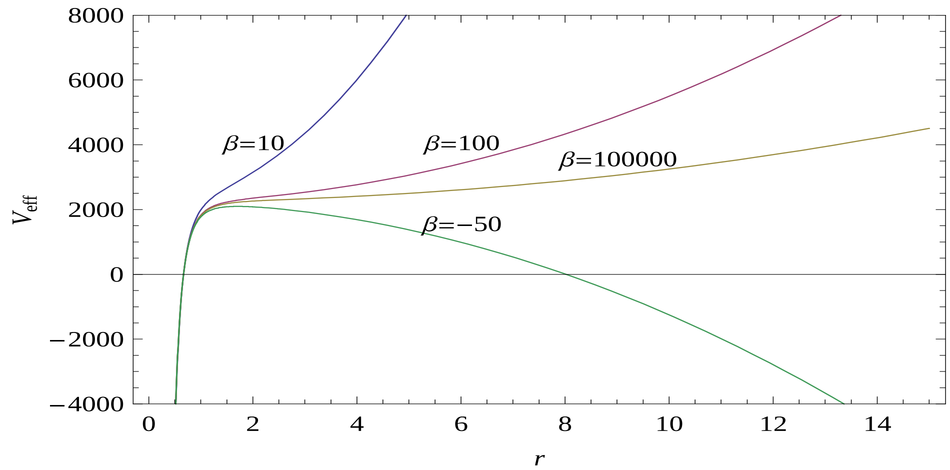

Repulsive force is generated for $\beta < 0 \Rightarrow$ Dark Energy?

Manifestation in 4D cosmology

Effectively gives rise to a metric function

$$F(r) = \left(k - \frac{2M}{r^2} + \Lambda_5 r^2\right)\left(\frac{\mu^2 r^2 + l^2}{r^2 + l^2}\right); \quad \mu^2 = 1 + \frac{l^2}{\beta}$$

Embed a 4D FRW metric into this bulk spacetime with $r(\tau) = a(\tau)$

$*$ Tangent $u^\mu \equiv \left(\frac{\sqrt{F(a)+\dot{a}^2}}{F(a)}, \dot{a}, 0, 0\right)$

$*$ Normal $n^\mu \equiv \left(-\frac{\dot{a}}{F(a)}, -\sqrt{F(a)+\dot{a}^2}, 0, 0\right)$

$*$ Extrinsic curvature $K_{ij} = \frac{\sqrt{F(a)+\dot{a}^2}}{a}\,\tilde{g}_{ij}$

$$K_{\tau\tau} = \frac{d}{da}\left(\sqrt{F(a)+\dot{a}^2}\right)$$

$*$ Junction conditions + Z_2 symmetry $\Longrightarrow$

$$\boxed{K_{\mu\nu} = -8\pi G\left(S_{\mu\nu} - \frac{1}{3}S\tilde{g}_{\mu\nu}\right)}$$

Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_4}{3}\rho + \frac{2M}{a^4} + \left[\alpha - \Lambda_5 + \frac{l^2(2M/a^2 - \Lambda_5 a^2)}{\beta(a^2+l^2)}\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_4}{3}(\rho + 3p) - \frac{2M}{a^4} + \left[\alpha - \Lambda_5 - \frac{l^2(\Lambda_5 a^4 + 2M + 2\Lambda_5 l^2 a^2)}{\beta(a^2+l^2)^2}\right]$$

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$$8\pi G_4/3 = 2\rho_0\,(8\pi G_5/3)^2$$

$$\alpha = (8\pi G_5 \rho_0/3)^2$$

$\rho_0 =$ brane tension

$\frac{M}{a^4} =$ dark radiation

$\leq .03\%$ of radiation density (Nucleosynthesis data)

The equations look a bit complicated but we can conveniently express them in terms of an effective energy-momentum tensor

Role of the repulsive force

Deceleration parameter

$$q = -\frac{\ddot{a}/a}{(\dot{a}/a)^2} \approx -\frac{l^2}{l^2+a^2} - \frac{M}{a-2M}$$

< 0 for $a > 2M$

$\Longrightarrow$ One may have accelerated expansion

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Approximate solution

$$\boxed{a(t) \approx M + \frac{1}{2}\left[e^{t/\sqrt{-\beta}} - M^2 e^{-t/\sqrt{-\beta}}\right]}$$

$\Longrightarrow$ **Accelerating solution !**

Behaves pretty close to ΛCDM, with $\beta^{-1} \approx \Lambda$

Das, Ghosh, van Holten, **SP**, JHEP(2009)

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Beyond solution: Observational aspects

Hundreds of models for expanding universe ....

How to appreciate a (few) model(s) and rule out the others?

Beyond solution: Observational aspects

Hundreds of models for expanding universe ....

How to appreciate a (few) model(s) and rule out the others?

- $*$ Galaxies are redshifting away $\Rightarrow$ expanding universe
- $*$ Directly measurable quantity is redshift z s.t. $a \propto \frac{1}{1+z}$
- $*$ Obtain Hubble parameter $H(t) = \frac{\dot{a}}{a}$ in terms of redshift
- $*$ Express observable quantities as functions of redshift
- $*$ Estimate how they vary with redshift
- $*$ Confront results with observational data

Observable quantities

Stringent constraints on dark energy models

Observable quantities

- Luminosity distance $d_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')} \Rightarrow \Omega_{DE} \approx 0.7$

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- Deceleration parameter $q(z) = \frac{-\ddot{a}/a}{(\dot{a}/a)^2} = \frac{H'(z)}{H(z)}(1+z) - 1 < 0$

Onset of recent acceleration $z \approx 0.6$

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- Statefinder parameters {r, s} $\Rightarrow$ dynamical dark energy vs Λ

$$r = \frac{\dddot{a}/a}{(\dot{a}/a)^3} = 1 + \left[\frac{H''}{H} + \left(\frac{H'}{H}\right)^2\right](1+z)^2 - 2\frac{H'}{H}(1+z)$$

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- Averaging over entire redshift $Om(z)$ and $\bar{q}(z) \Rightarrow \Lambda$ fails !

Stringent constraints on dark energy models

Observational aspects from our model

Das, Ghosh, van Holten, **SP**, JCAP(2010)submitted

Express Friedmann equations in terms of redshift s.t. $a \propto \frac{1}{1+z}$

Neglect all terms $\geq (1+z)^4$

Hubble parameter boils down to

$$H^2 = H_0^2 \left[\underbrace{\Omega_X}_{\text{Dark Energy}} \left(1 + b(1+z)^2\right) + \underbrace{\Omega_M}_{\text{Matter Sector}} (1+z)^3 \right]$$

Dark Energy Matter Sector

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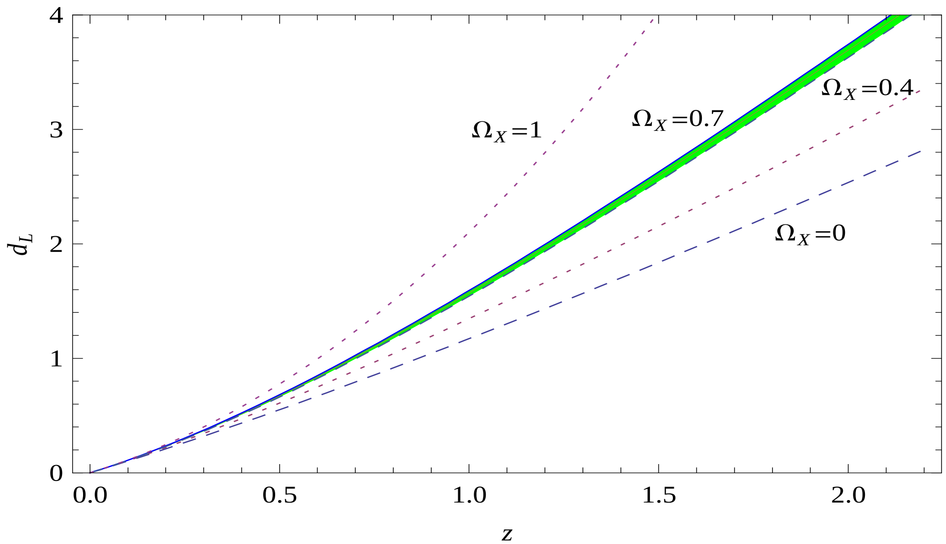
- $H_0 = 74.2 \pm 3.6$ km/s/Mpc : WMAP5/SHOES
- $\Omega_M = 8\pi G_4 \rho / 3H_0^2 = 0.28 \pm 0.08,\ 95\%$ CL ? : CMB/LSST
- $\Omega_X = (\alpha - \Lambda_5 \mu^2)/H_0^2 = 0.726 \pm 0.015,\ 95\%$ CL ? : WMAP5/SNIa
- $b = \Lambda_5 \beta (\mu^2 - 1)^2 / (\alpha - \Lambda_5 \mu^2)$ = ?

Luminosity distance: SNIa: $\Omega_X \approx 0.7$; CMB+LSST: $\Omega_M = 0.3$

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$$= \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_X(1+b(1+z')^2)+\Omega_M(1+z')^3}}$$



Observational data from all of the SNIa fall on **green** line.

$$\boxed{\Omega_M = 0.3 \quad ; \quad \Omega_X = 0.7 \quad ; \quad -0.07 \leq b < 0}$$

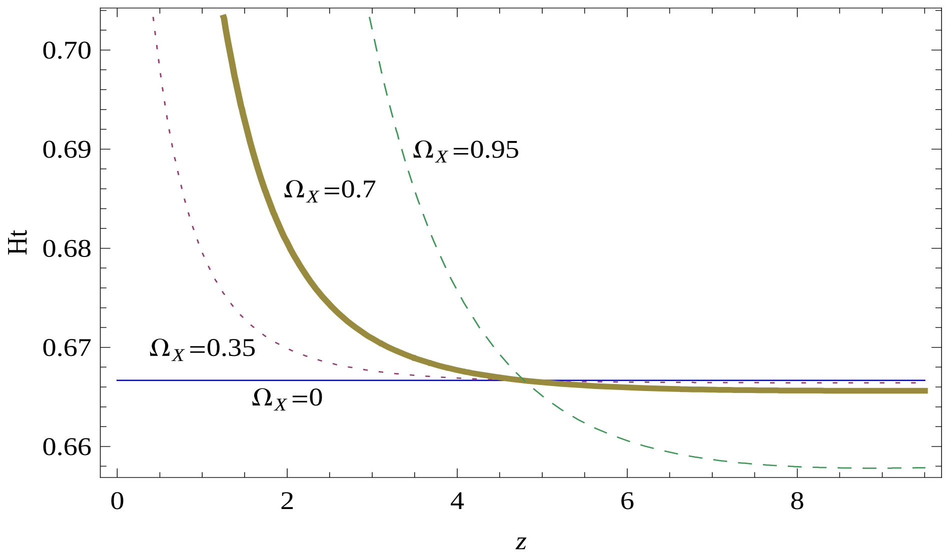
Matches observations for Dark Energy and Matter density

Age of the universe: Latest accepted value 13.7 ± 0.02 Gyr

Correct Dark Energy density results in correct calculation of age

$$t(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

$$= \frac{1}{H_0} \int_z^\infty \frac{dz'}{(1+z')\sqrt{\Omega_X(1+b(1+z')^2)+\Omega_M(1+z')^3}}$$

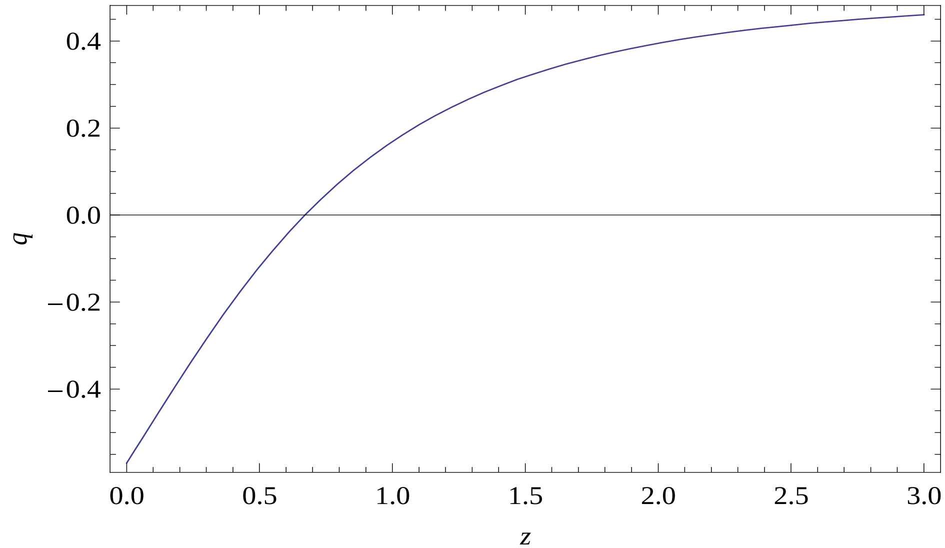


$\Omega_X = 0.7$ matches the plot obtained from observational data

Deceleration parameter: $q < 0$, onset of acceleration $z = 0.6$

$$q(z) = \frac{-\ddot{a}/a}{\dot{a}^2/a^2} = \frac{H'(z)}{H(z)}(1+z) - 1$$

$$= \frac{\Omega_M(1+z)^3 - 2\Omega_X}{2[\Omega_X(1+b(1+z)^2) + \Omega_M(1+z)^3]}$$



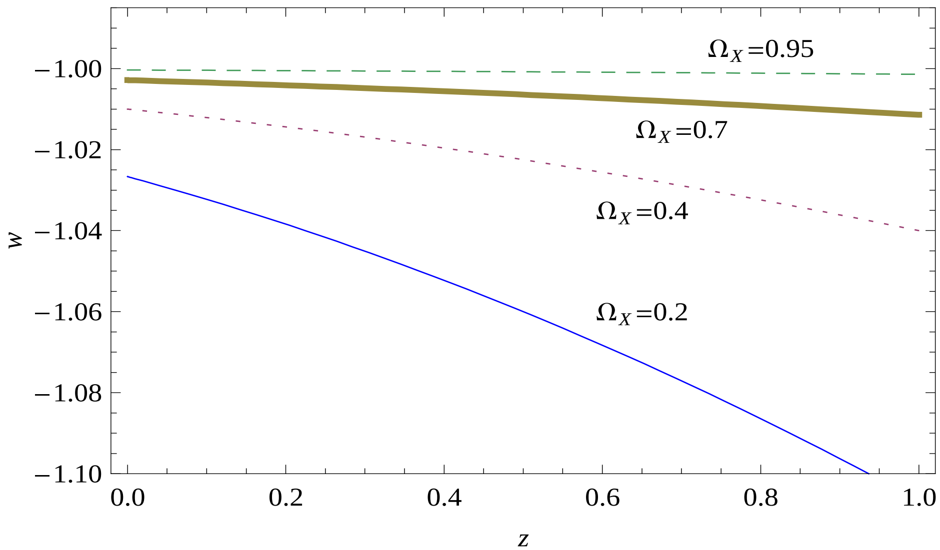
Plot for $\Omega_M = 0.3$; $\Omega_X = 0.7$; $b = -0.05$

$*$ $q < 0$ **at present**

$*$ **Onset of recent acceleration** $z \approx 0.6$ **confirmed**

Equation of state: SNIa Gold dataset: $-1.11 < w_X < -1$

$$w_X(z) = \frac{2q(z)-1}{3[1-\Omega_M(z)]} \approx -1 + \frac{2b(1+z)^2}{3}$$



* **Shows phantom behavior without any phantom field**

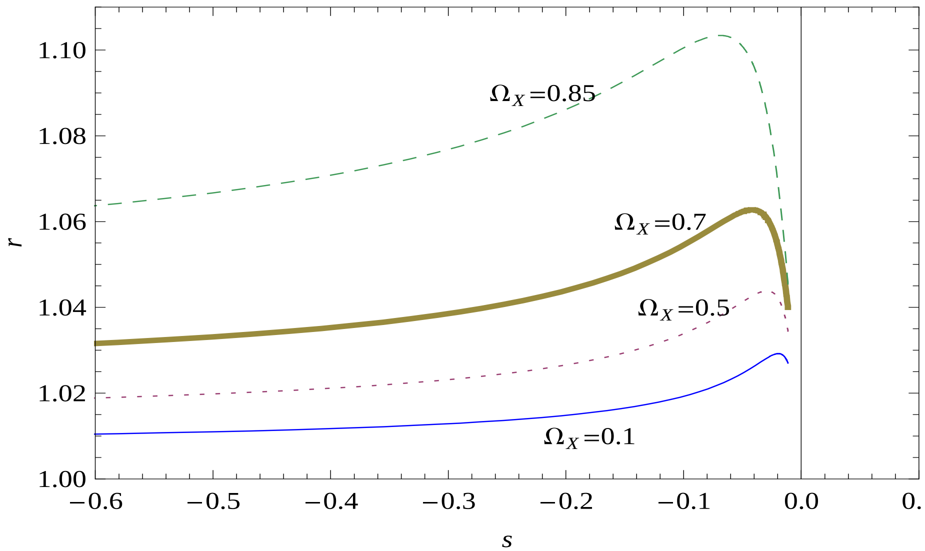
$$\boxed{-1.11 < w_X < -1 \Rightarrow -0.15 < b < 0 \;\; ; \;\; d_L \Rightarrow -0.07 \leq b < 0}$$

* **Will fit well with more precise observational data too**

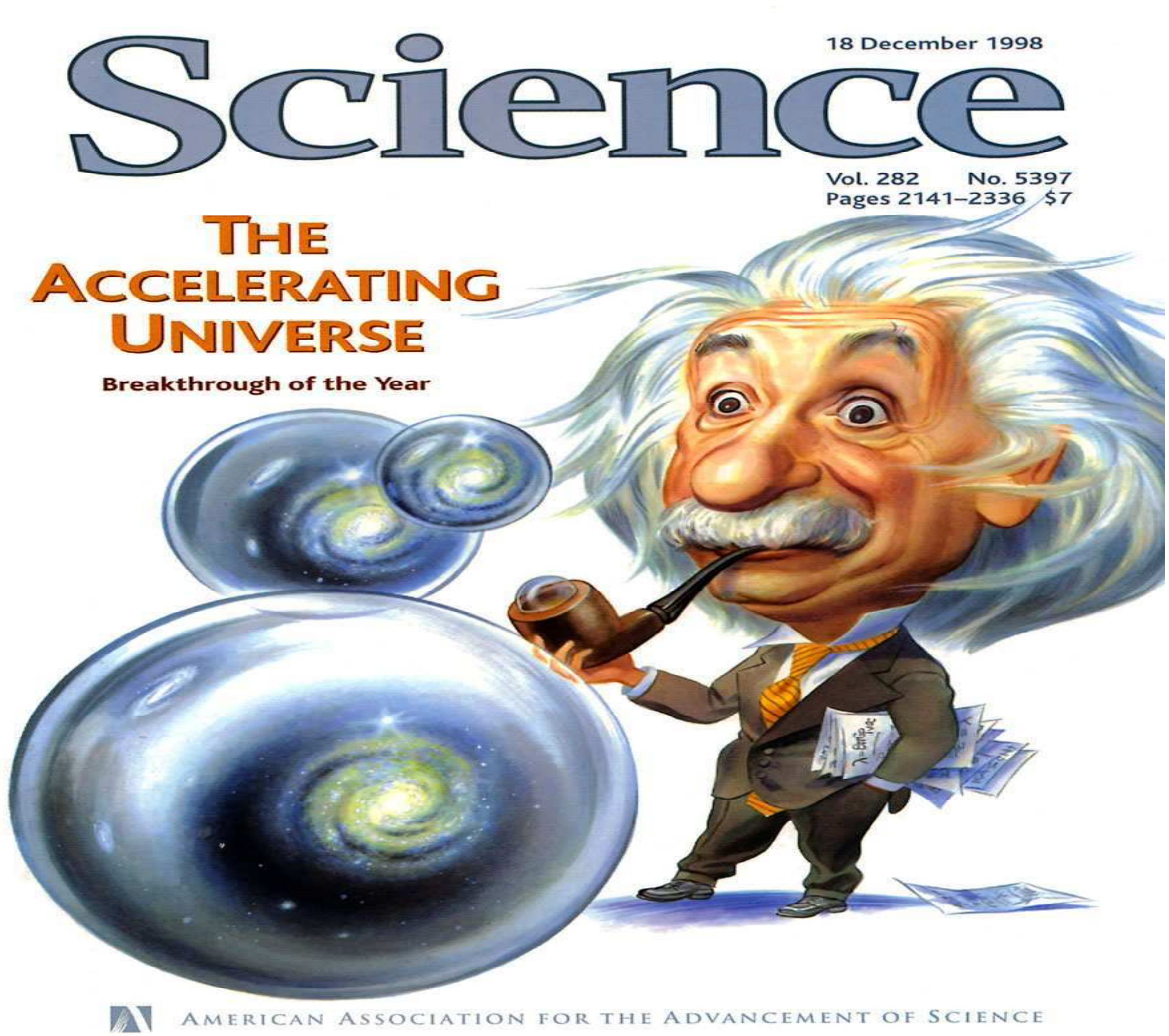
Statefinder parameters: Dynamical models vs Λ

$$r = \frac{\dddot{a}/a}{(\dot{a}/a)^3} = 1 + \left[\frac{H''}{H} + \left(\frac{H'}{H}\right)^2\right](1+z)^2 - 2\frac{H'}{H}(1+z) \; ; \; s = \frac{2}{3}\frac{r-1}{2q-1}$$

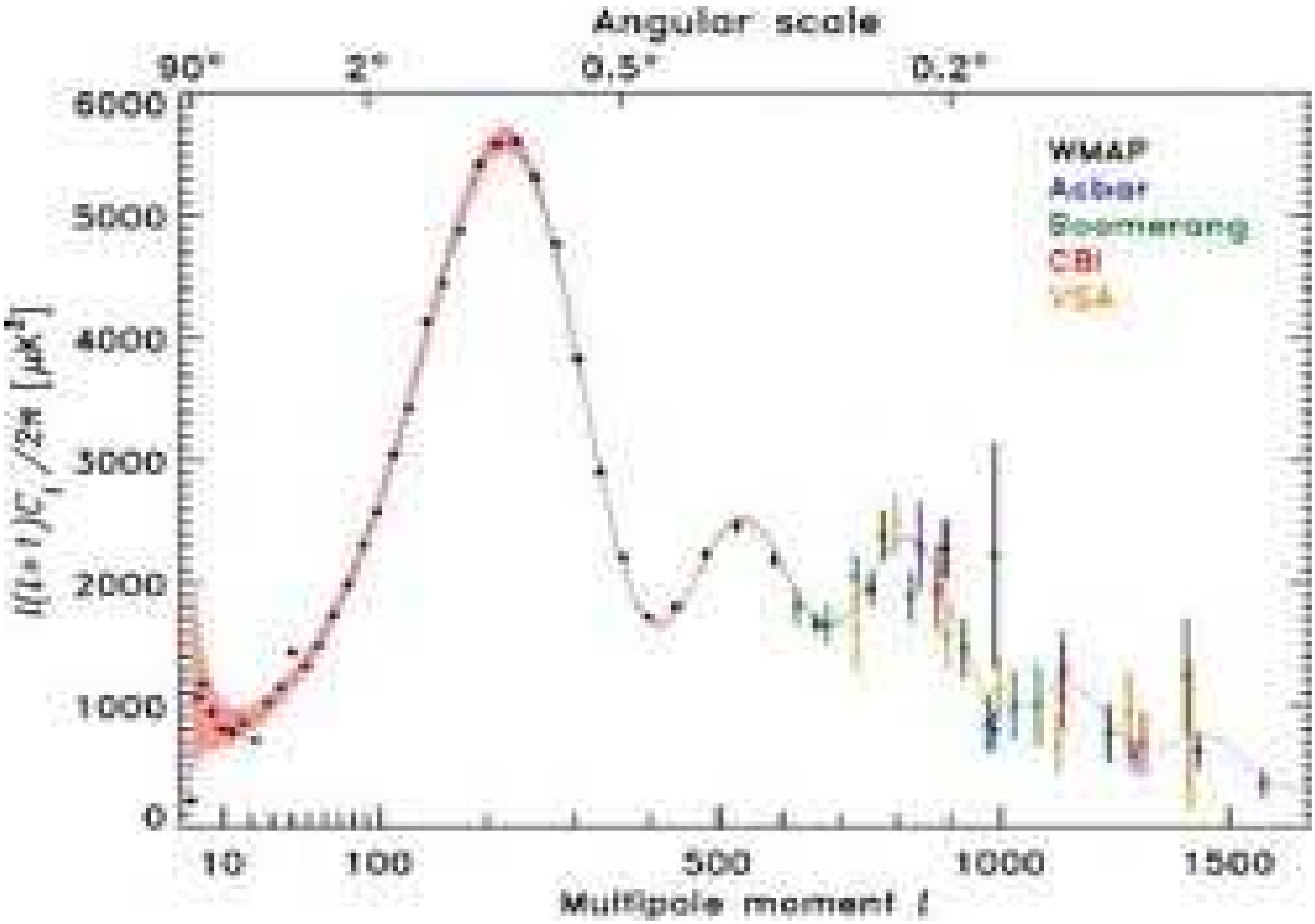
$$r = 1 - \frac{b\Omega_X(1+z)^2}{\Omega_X(1+b(1+z)^2)+\Omega_M(1+z)^3} \; ; \; s = \frac{2}{3}\frac{b(1+z)^2}{[3+b(1+z)^2]}$$

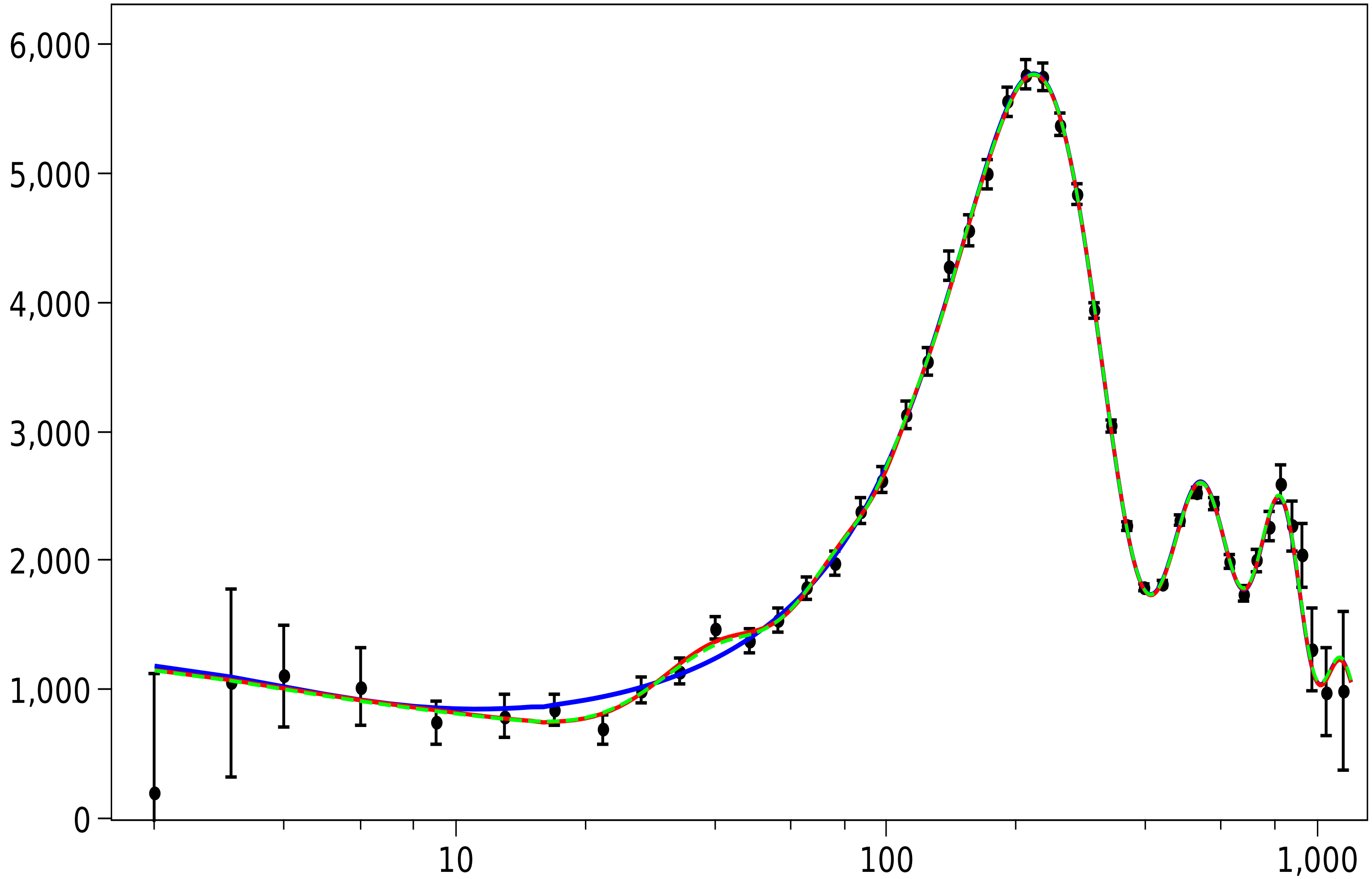


Dynamical model assured. **Matches Sahni et.al.(2003)**


18 December 1998
Science
Vol. 282 No. 5397
Pages 2141–2336 $7
THE ACCELERATING UNIVERSE
Breakthrough of the Year
AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

Cosmic Microwave Background Radiation





6,000
5,000
4,000
3,000
2,000
1,000
0
10
100
1,000

Scalar field model for inflation

EM tensor components $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$; $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

Friedmann Equations

$$H^2 = \frac{1}{3M_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3M_P^2}\left[\dot{\phi}^2 - V(\phi)\right]$$

Energy Conservation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Scalar field model for inflation

EM tensor components $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \;\;;\;\; p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

Friedmann Equations

$$H^2 = \frac{1}{3M_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3M_P^2}\left[\dot{\phi}^2 - V(\phi)\right]$$

Energy Conservation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

For sufficient inflation

$$\dot{\phi}^2 << V(\phi) \quad ; \quad |\ddot{\phi}| << 3H|\dot{\phi}|, V'(\phi)$$

$*$ Slow roll parameters $\epsilon = 2M_P^2\left[\frac{H'}{H}\right]^2 \ll 1$; $\eta = 2M_P^2\left[\frac{H''}{H}\right] \ll 1$

$*$ Number of e-foldings $N = \ln\frac{a_f}{a_i} \qquad \approx 56 - 70$

A typical model Pal, **SP**, Basu, JCAP(2010)

$$V(\phi) = V_0\,(1 - \mathrm{sech}[\alpha\phi])$$

Friedmann Equations with Slow-Roll

$$H^2 = \frac{V_0}{3M_P^2}[1 - \mathrm{sech}(\alpha\phi)]$$

A typical model Pal, **SP**, Basu, JCAP(2010)

$$V(\phi) = V_0\,(1 - \mathrm{sech}[\alpha\phi])$$

Friedmann Equations with Slow-Roll

$$H^2 = \frac{V_0}{3M_P^2}[1 - \mathrm{sech}(\alpha\phi)]$$

- Scalar field

$$\phi = \alpha^{-1}\sinh^{-1}\left[\alpha^2\sqrt{\frac{V_0}{3}}M_P(d-t)\right]$$

- Scale factor

$$a(t) = a_1\ \exp\left[-(\alpha M_p)^{-2}\sqrt{1+\alpha^4\frac{V_0}{3}M_p^2(d-t)^2}\right]$$

- e-foldings

$$N = (\alpha^2 M_p)^{-1}[\cosh(\alpha\phi) - \ln\cosh^2(\frac{\alpha\phi)}{2})]_{\phi_{end}}^{\phi_{in}}$$

α M_P^{-1}	$\epsilon_H < 1$ $\phi \geq M_P$	$\|\eta_H\| < 1$ $\phi \geq M_P$	ϕ_{end} M_P	ϕ_{in} M_P	N
				2.44625	70
2.9	0.59886	1.02192	1.02192	2.39431	60
				2.37111	56
				2.38713	70
3.0	0.58759	1.00796	1.00796	2.33689	60
				2.31446	56
				2.33112	70
3.1	0.57681	0.99435	0.99435	2.28248	60
				2.26076	56

How are the cosmic structures formed? $\Rightarrow$ **Perturbations**

Quantum fluctuations of inflaton are transformed to macroscopic cosmological perturbations

Perturbations in the metric

$\Downarrow$

$$\boxed{\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}}$$

$\swarrow$ $\downarrow$ $\searrow$

Scalar perturbation	Vector perturbation	Tensor perturbation
$\Downarrow$		$\Downarrow$
Matter		**Gravitational waves**

Scalar perturbation

In terms of conformal time $\eta = \int \frac{dt}{a}$ define a new scalar $v = a(\eta)\phi$

In Fourier space

$$\hat{v}(\eta, \overrightarrow{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[\hat{a}_k v_k(\eta) e^{i\overrightarrow{k}.\overrightarrow{x}} + \hat{a}_k^\dagger v_k^*(\eta) e^{-i\overrightarrow{k}.\overrightarrow{x}} \right]$$

Eqn of motion for k-th Fourier mode

$$v_k'' + \left(k^2 - \frac{z''}{z} \right) v_k = 0$$

Apply Slow roll, normalization, boundary conditions

Scalar perturbation

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Eqn of motion for k-th Fourier mode

$$v_k'' + \left(k^2 - \frac{z''}{z} \right) v_k = 0$$

Apply Slow roll, normalization, boundary conditions

Solution

$$v_k = \sqrt{\frac{1}{2k}} \exp(-ik\eta) \left(1 - \frac{i}{k\eta} \right)$$

Scalar perturbation

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In Fourier space

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Apply Slow roll, normalization, boundary conditions

Solution

$$v_k = \sqrt{\frac{1}{2k}} \exp(-ik\eta) \left(1 - \frac{i}{k\eta}\right)$$

Power spectrum for comoving curvature perturbation

$$P_R(k) = \frac{k^3}{2\pi^2} \frac{|v_k|^2}{z^2} = \frac{\alpha^2 V_0}{12\pi^2 M_P^2} (1 + k^2\eta^2) \left[\ln\left(a_1 M_P^{-1} \sqrt{\frac{V_0}{3}} |\eta|\right)\right]^2$$

Tensor perturbation

Fluctuation equation for the tensor amplitudes

$$h_k'' + 2\mathcal{H}h_k' + k^2 h_k = 0$$

Define a new variable $h_k = \frac{\sqrt{2}}{M_P}\frac{u_k}{a}$

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)u_k = 0$$

Apply the same conditions

Tensor perturbation

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Solution

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$$u_k'' + \left(k^2 - \frac{a''}{a}\right)u_k = 0$$

Apply the same conditions

Solution

$$u_k = \sqrt{\frac{1}{2k}}\exp(-ik\eta)\left(1 - \frac{i}{k\eta}\right)$$

Dimensionless power spectrum for tensor fluctuation

$$P_T = 2P_{h_k} = \frac{V_0}{3\pi^2 M_P^4}(1 + k^2\eta^2)$$

Observable quantities

Observable quantities

- Power spectrum for comoving curvature perturbation

$$P_R(k) = \frac{k^3}{2\pi^2}|R_k|^2 = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2} \quad \Rightarrow P_R^{1/2}|_{k=aH} \sim 5 \times 10^{-5}$$

Observable quantities

- Power spectrum for comoving curvature perturbation

$$P_R(k) = \frac{k^3}{2\pi^2}|R_k|^2 = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2} \quad \Rightarrow P_R^{1/2}|_{k=aH} \sim 5 \times 10^{-5}$$

- Scalar spectral index

$$n_s = 1 + \frac{d \ln P_R(k)}{d \ln k}|_{k=aH} \qquad \Rightarrow 0.948 < n_s < 1$$

Observable quantities

- Power spectrum for comoving curvature perturbation

$$P_R(k) = \frac{k^3}{2\pi^2}|R_k|^2 = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2} \qquad \Rightarrow P_R^{1/2}|_{k=aH} \sim 5 \times 10^{-5}$$

- Scalar spectral index

$$n_s = 1 + \frac{d\ln P_R(k)}{d\ln k}|_{k=aH} \qquad \Rightarrow 0.948 < n_s < 1$$

- Running of spectral index

$$\frac{dn_s}{d\ln k}|_{k=aH} \qquad \neq 0$$

Crucial constraint from WMAP3

Observable quantities

- Power spectrum for comoving curvature perturbation

$$P_R(k) = \frac{k^3}{2\pi^2}|R_k|^2 = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2} \quad \Rightarrow P_R^{1/2}|_{k=aH} \sim 5\times 10^{-5}$$

- Scalar spectral index

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$$\frac{dn_s}{d\ln k}|_{k=aH} \quad \neq 0$$

Crucial constraint from WMAP3

- Ratio of tensor to scalar amplitudes

$$r = \frac{P_T|_{k=aH}}{P_R|_{k=aH}} \quad < 0.002$$

Observable quantities from the model

- Power spectrum for comoving curvature perturbation

$$P_R|_{k=aH} = \frac{\alpha^2 V_0}{6\pi^2 M_P^2}\left[\ln\left(a_1 M_P^{-1}\sqrt{\frac{V_0}{3}}|\eta|\right)\right]^2$$

- Scalar spectral index

$$n_s = 1 - 2\left[\ln\left(a_1 M_P^{-1}\sqrt{\frac{V_0}{3}}|\eta|\right)\right]^{-1}$$

- Running of spectral index

$$\frac{dn_s}{d\ln k}|_{k=aH} = -2\left[\ln\left(a_1 M_P^{-1}\sqrt{\frac{V_0}{3}}|\eta|\right)\right]^{-2}$$

- Ratio of tensor to scalar amplitudes

$$r = \frac{4}{\alpha^2 M_P^2}\left[\ln\left(a_1 M_P^{-1}\sqrt{\frac{V_0}{3}}|\eta|\right)\right]^{-2}$$

α M_P^{-1}	t_{hc} M_P^{-1}	$P_R^{1/2}$	n_s	r
		$\sim 5 \times 10^{-5}$	$0.948 < n_s < 1$	< 0.002
2.9	3.7×10^{10}	3.7784×10^{-5}	0.9609	1.82×10^{-4}
3.0	3.7×10^{10}	3.9080×10^{-5}	0.9609	1.70×10^{-4}
3.1	3.7×10^{10}	4.0377×10^{-5}	0.9609	1.60×10^{-4}

Running of spectral index $\frac{dn_s}{d\ln k}|_{k=aH} \simeq -8 \times 10^{-4} \neq 0$

$\Longrightarrow$ **Matches WMAP**

Post-inflationary perturbations and CMB spectrum

Perturbed mertic

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

Diagonal Energy-momentum tensor $\Longrightarrow \Phi = \Psi$

At superhorizon scale $k \ll aH$, curvature perturbation

$$\mathcal{R}_k = \zeta_k \equiv -\left(\frac{2}{3}\frac{\mathcal{H}^{-1}\Phi_k' + \Phi_k}{1+\omega} + \Phi_k\right)$$

Hence calculate Φ_k from $\mathcal{R}_k$ by

$$\boxed{\Phi_k = -\frac{3+3\omega}{5+3\omega}\mathcal{R}_k}$$

Φ_k carries direct imprints of a model of inflation on CMB

Large scale ($0 \leq l \leq 30$) : Sachs-Wolfe effect

Red(blue)shift due to fluctuation in gravitational potential at LSS

Temperature fluctuation in CMB $\Theta(\eta, x, \hat{n}) \equiv \frac{\Delta T(\eta, x, \hat{n})}{T(\eta)}$

Total CMB anisotropy (sudden decoupling approximation)

$$\Theta(\hat{e}) = \left[\left(\frac{1}{4}\delta_\gamma + \Psi\right) + e v_\gamma\right]_{LS} + \int_{\eta_{LS}}^{\eta_0} \frac{\partial}{\partial \eta}(\Psi + \Phi) d\eta$$

Contribution from Sachs-Wolfe effect

$$\Theta(\hat{e}) = \left(\frac{1}{4}\delta_\gamma + \Psi\right)_{LS} = \frac{1}{3}\Phi_{LS} = \left(-\frac{1}{5}\mathcal{R}_k\right)_{LS}$$

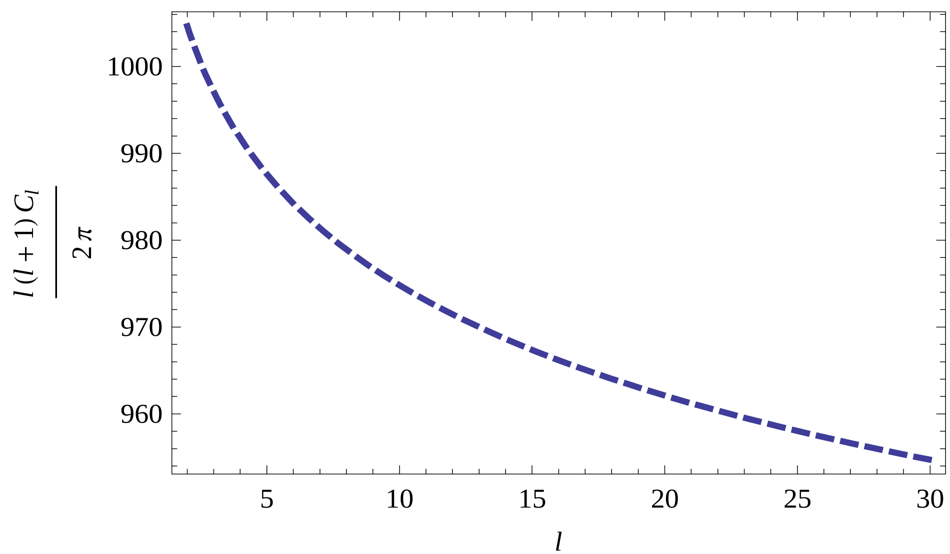
CMB Spectrum for Sachs-Wolfe effect

$$\boxed{C_l^{\mathrm{SW}} = \frac{4\pi}{9} \int_0^\infty \frac{dk}{k} j_l^2(k\eta_0) P_\Phi(k)}$$

$j_l^2(k\eta_0)$ is sharply peaked at ($k\eta_0 = l$) $\Rightarrow$ take $P_\Phi(k)$ outside integral

For our typical model

$$C_l^{\mathrm{SW}} = \frac{\alpha^2 V_0}{75\pi M_P^2} \left(\ln \left[a_1 M_P^{-1} \sqrt{\frac{V_0}{3}} \frac{2\eta_0}{l} \right] \right)^2 \frac{1}{2l(l+1)}$$



Sachs-Wolfe pleatue is nearly flat $\Longrightarrow$ Matches CMB

Small scale ($30 \leq l \leq 1500$) : Baryon Acoustic Oscialltions

Perturbation equation for the photon-baryon fluid

$$\frac{1}{4}\delta''_{\gamma k} + \frac{1}{4}\frac{R'}{1+R}\delta'_{\gamma k} + \frac{1}{4}k^2 c_s^2 \delta_{\gamma k} = -\frac{k^2}{3}\Phi_k(\eta) + \frac{R'}{1+R}\Phi'_k(\eta) + \Phi''_k(\eta)$$

$\Longrightarrow$ An equation for forced oscillation

Use transfer function for small scale $\Rightarrow \Phi_k \equiv -\frac{3}{5}T_k\mathcal{R}_k$

Solution of the BAO equation

$$\frac{1}{4}\delta_{\gamma k} = -(1+R)\Phi_k + A_k\cos(kr_s) + B_k\sin(kr_s)$$

Intial condition

Adiabatic $\Longrightarrow B_k = 0$; $\eta \to 0$ limit $\Longrightarrow A_k = \frac{1}{2}T_k^0\Phi_k^0$

$$\boxed{\frac{1}{4}\delta_{\gamma k} = -(1+R)\Phi_k + \frac{1}{2}T_k^0\Phi_k^0\cos(kr_s)}$$

Introduce Silk damping for large l

Diffused photons carry some baryons $\Longrightarrow$ less inhomogenieties

$$\frac{1}{4}\delta_\gamma(\eta_{LS},k) = -(1+R)\Phi(\eta_{LS},k)+\frac{1}{2}T_k^0\Phi_k^0\left[\exp(-\frac{k^2}{k_D^2})\cos(kr_s)\right]_{\eta=\eta_{LS}}$$

CMB multipoles (with finite thickness) $\Theta_l =$

$$\left[\left[\frac{1}{4}\delta_\gamma(\eta_{LS},k)+\Phi(\eta_{LS},k)\right]j_l(k\eta_0)+V_\gamma(\eta_{LS},k)\frac{dj_l(k\eta_0)}{d(k\eta_0)}\right]e^{-(\sigma k\eta_{LS})^2}$$

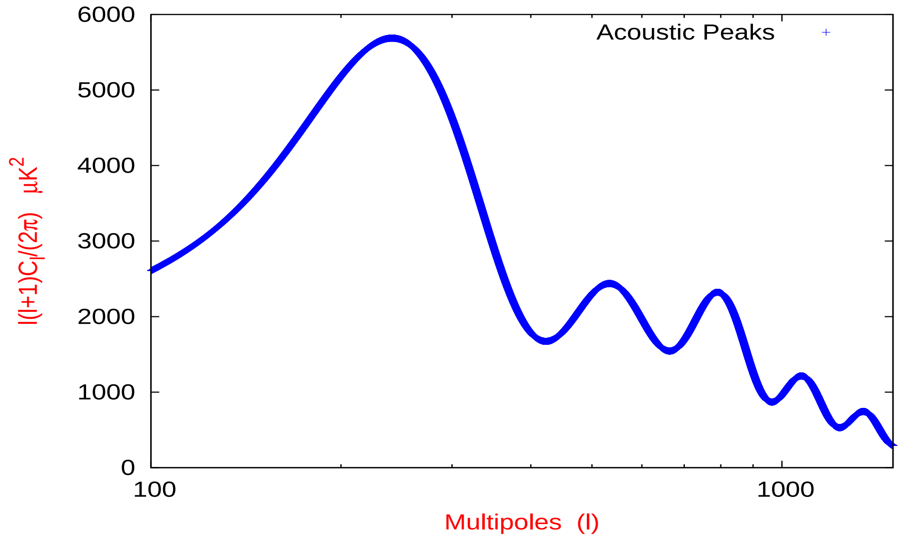
CMB spectrum for BAO $C_l =$

$$\boxed{4\pi P_{\Phi^0}\int_0^\infty\left[C-D\cos(\rho l x)+E\cos^2(\rho l x)+F\left(1-\frac{l(l+1)}{l^2x^2}\right)\sin^2(\rho l x)\right]j_l^2(xl)\frac{dx}{x}}$$

$x=\frac{k\eta_0}{l}$; C,D,E,F contain transfer functions, Silk damping scale...

$\Longrightarrow$ **Highly complicated numerical integration**

For our typical model

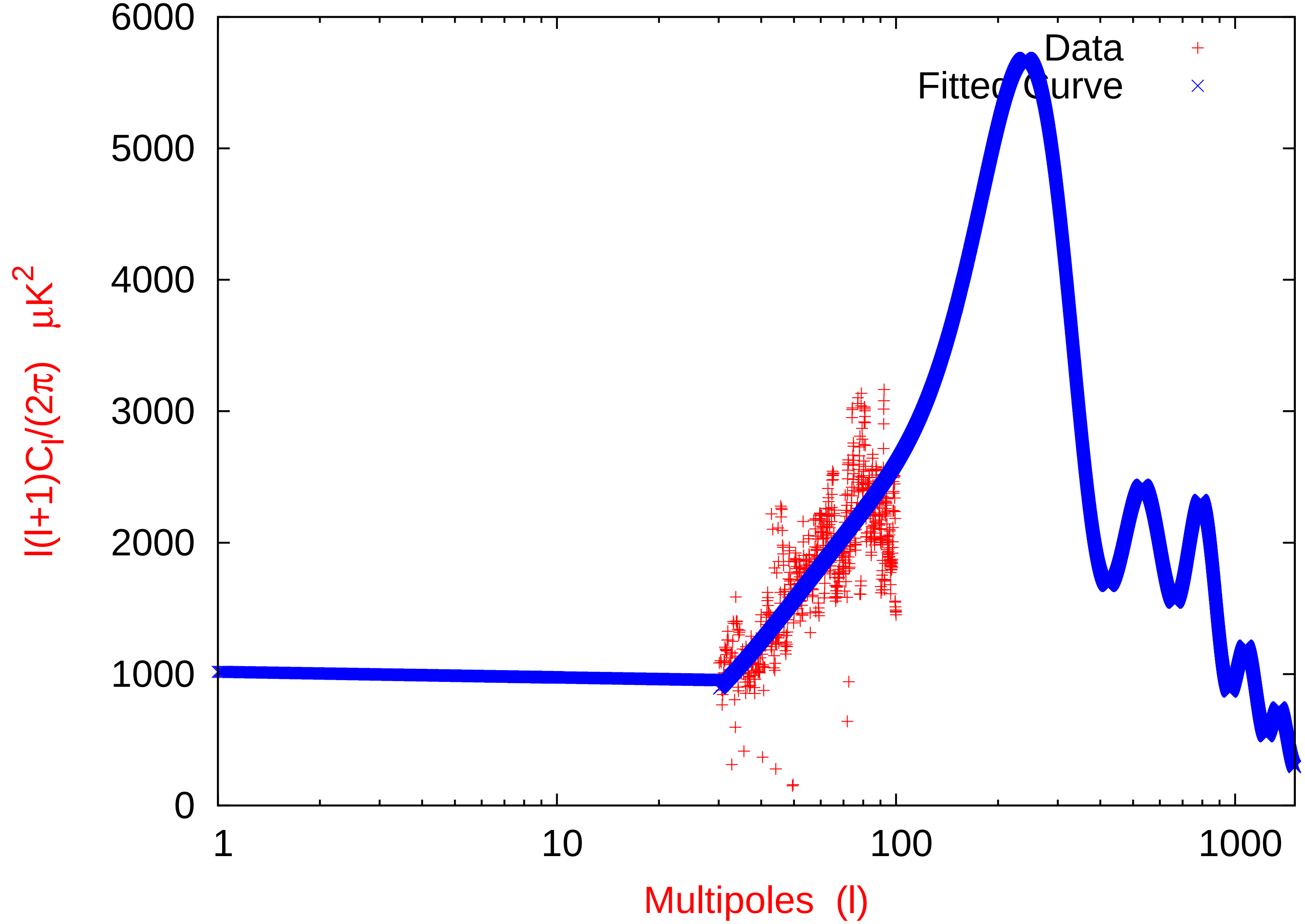


1st peak at $l \approx 241$ **confirms no spatial curvature**

2nd and 3rd peaks at $l \approx 533,\ 791$ **confirm adiabatic perturbation**

Peak positions and heights confirm $\Omega_b \approx 0.04, \Omega_M \approx 0.3, \Omega_{\rm DE} \approx 0.7$

Total CMB anisotropy spectrum



And miles to go ....