

The Haves and Have-nots of Brane Cosmology

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and

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The Haves

Dvali, Arkani-Hamed, Antoniadis, Randall, Sundrum, Gabadedge, Brandenberger, Henry-Tye, Poratti, Hawking, Maartens, Barrow, Wands, Bassett, Rubakov, Clarkson, Seahra, Nilles, Starobinski, Sahni, Shtanov, Sasaki, Koyama, Gregory, Chen, Gergely, Brux, Trodden, Tsujikawa, Cooray, Copeland, Sami, de Rahm, Burgess, Himemoto, Kobayashi, Maeda, Binetruy, Ellwanger, Deffayet, Langlois, Kar, Papantonopoulos, Panotopoulos, Seery, Sakharov, Park, Gogberashvili, Shiromizu, Garriga, Tanaka, Dimopoulos, Csaki, Stojkovic, Perivolaropoulos, van de Bruck, Germani ...

The Haves

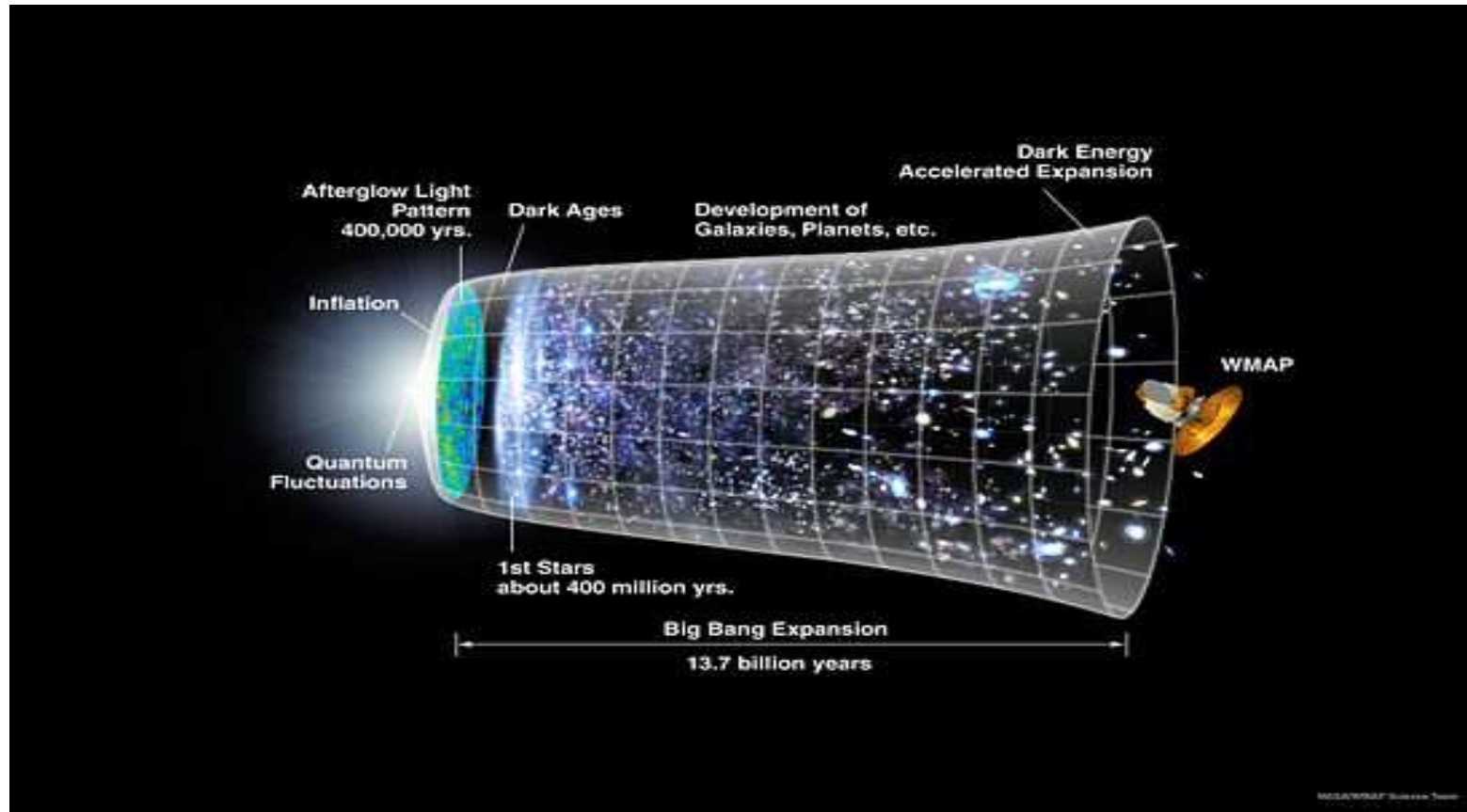
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The Have-nots

Supratik Pal :) :) :)

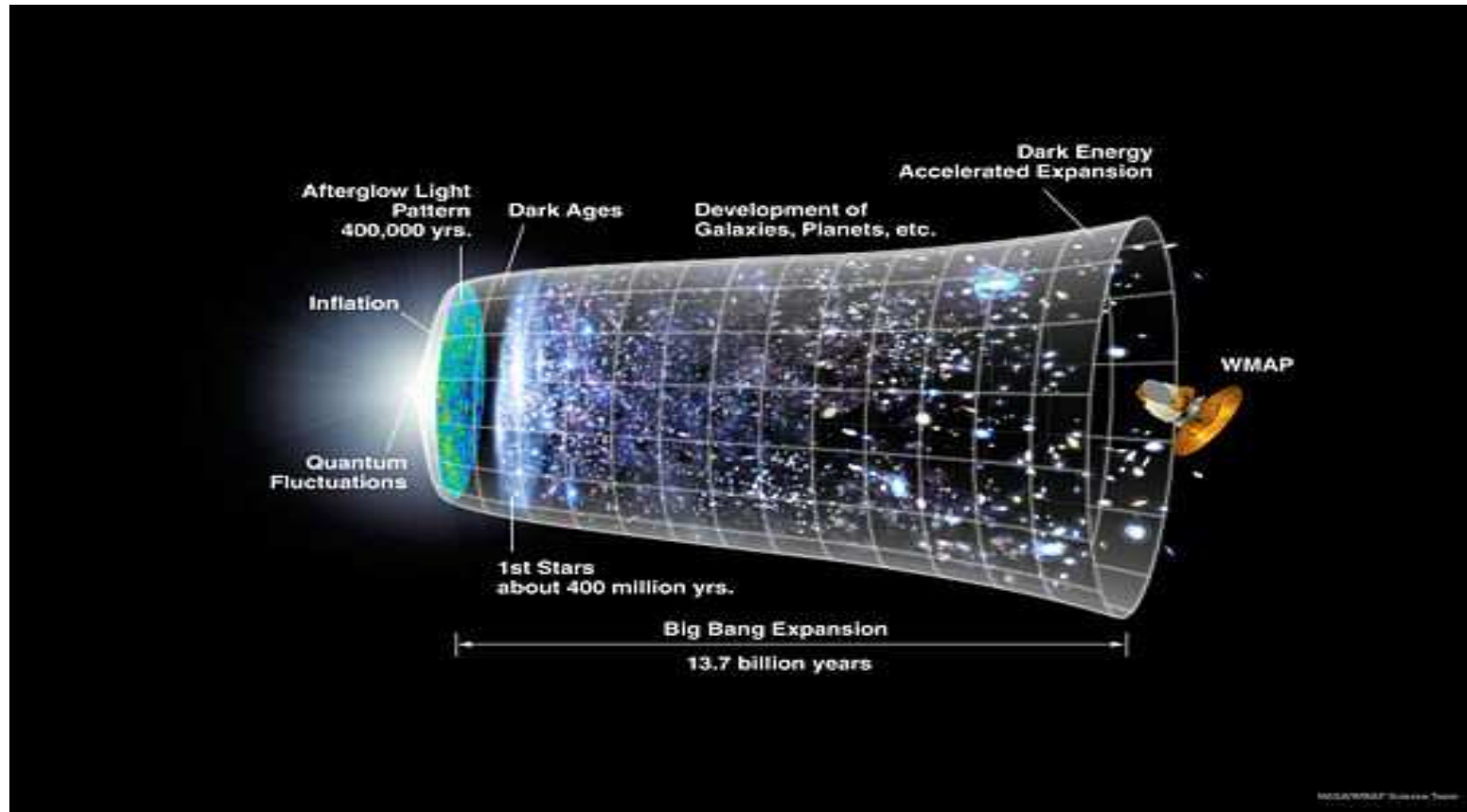
A cosmologist's wishlist

To explain...



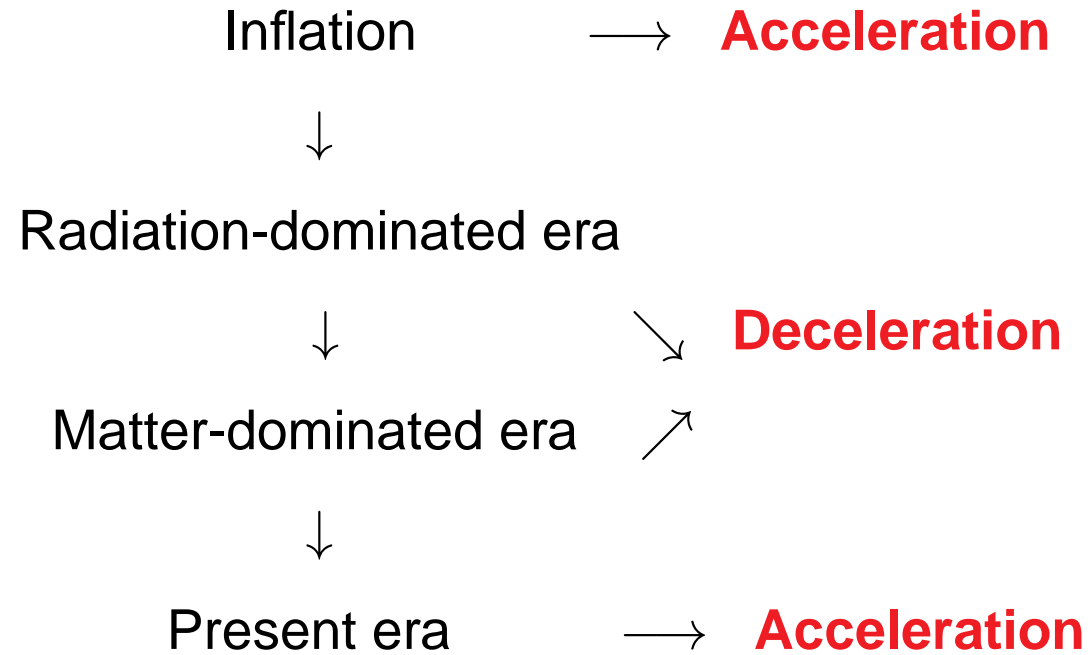
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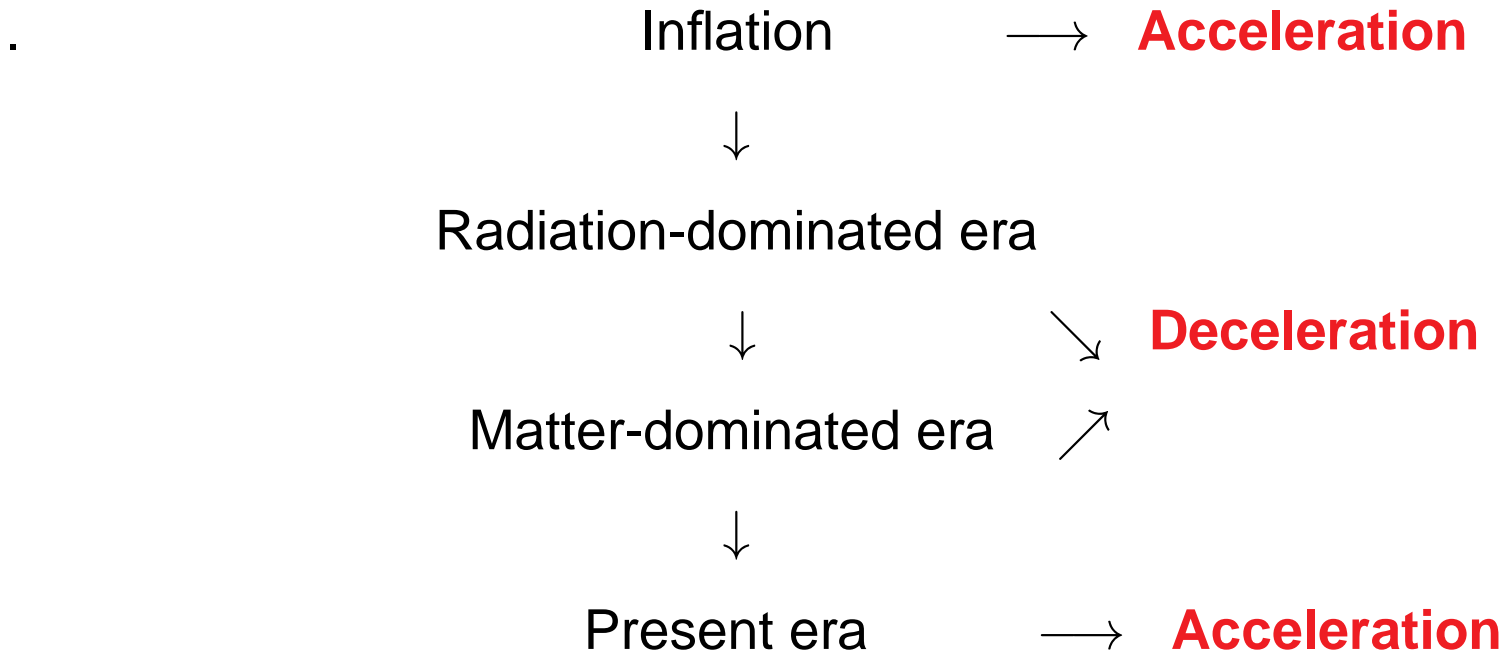


...in an unambiguous way !

A brief history of “adventure”



A brief history of “adventure”



Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Ordinary matter satisfy Strong Energy Condition $(\rho + 3p) \geq 0$

Who gives this “anti-gravity” ?

Cosmological Constant

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

$$M = \frac{4}{3}\pi a^3(\rho + 3p) \implies \boxed{\ddot{a} = -\frac{GM}{a^2} + \frac{\Lambda}{3}a}$$

↓

Effective mass

↙

Attraction

↘

Repulsion

$\Lambda > 0$ guarantees repulsion, hence accelerated expansion

Inflationary solution

⇓

$$a(t) \propto \text{Exp}\left[\sqrt{\frac{\Lambda_i}{3}}t\right]$$

Late time solution

⇓

$$a(t) \propto \left[\sinh\left(\frac{3}{2}\sqrt{\frac{\Lambda_0}{3}}t\right)\right]^{2/3}$$

-
- **Extreme fine-tuning** to match present value $\rho_{\Lambda_0} \sim 10^{-47} GeV^4$.
How to generate this from a large inflationary value?
 - Gives **ever-accelerating universe!!!** How to produce intermediate decelerating phases (RDE and MDE)?
 - How to explain formation of matter *aka* **(p)reheating**?
 - How to get correct power spectra and several other aspects of **CMB observations**?
 - Signatures of **dynamical dark energy**?
 - Is **cosmic coincidence** a solution or a problem?

Mukhanov; Dodelson; Weinberg; Carroll; Turner; Smoot...

Several fundamental puzzles confronting modern cosmology !

Dynamical models

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i)$$

$p_i < -\rho_i/3 \Rightarrow w_i < -1/3$ leads to acceleration \iff violates SEC

Guth; Linde; Starobinsky; Liddle; Lyth; Sahni...

Too many candidates with wide range of initial conditions

Dynamical models

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Scalar field models

Lagrangian density $\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

EM tensor components $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$; $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

Choose the potential to be sufficiently steep so that $V'' V/V'^2 \geq 1$

Scalar field rolls down the potential : “Tracker potential”

Includes Quintessence, Kessence, Chaplygin gas...

Guth; Linde; Starobinsky; Liddle; Lyth; Sahni...

Too many candidates with wide range of initial conditions

Modified gravity models

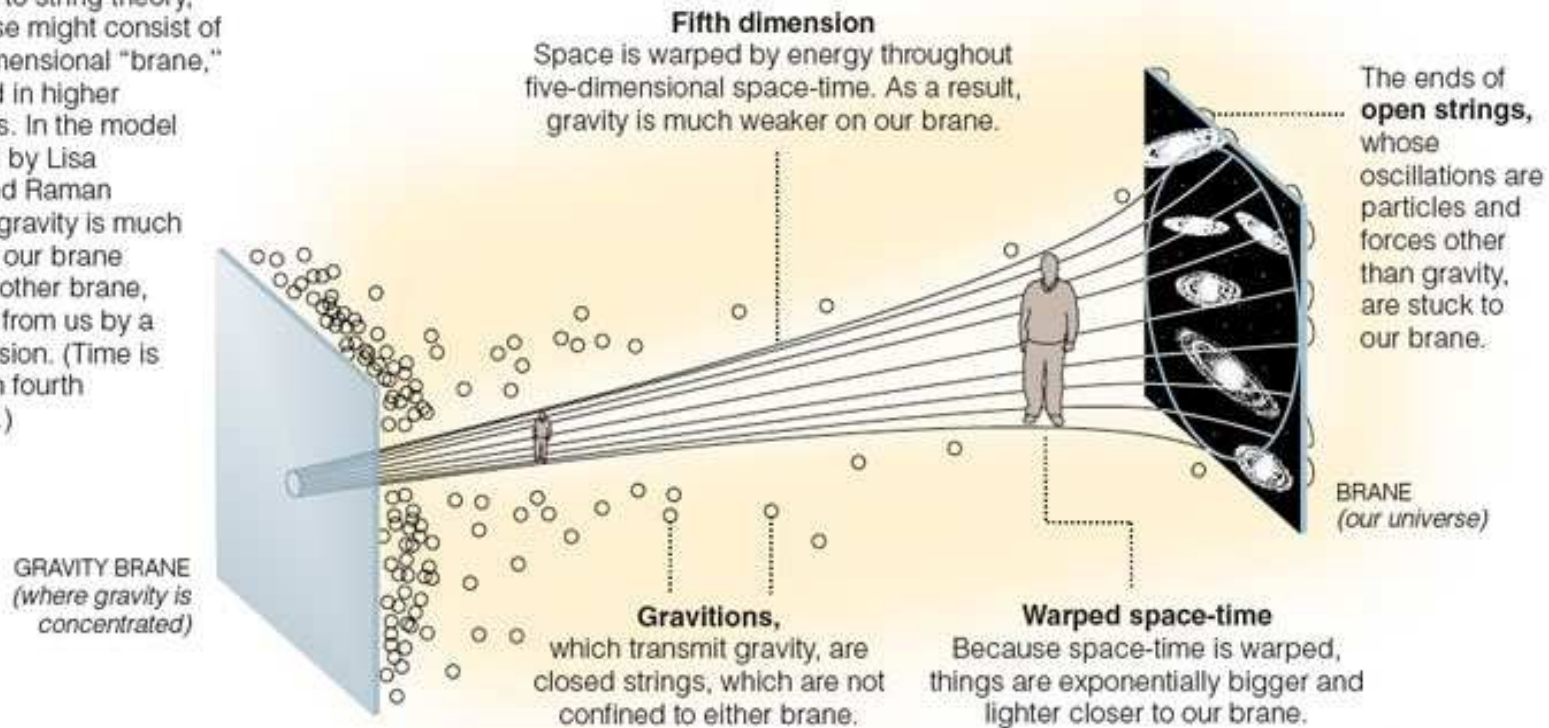
- * Einstein's theory is not directly tested in cosmological scales.
- * Modify the gravity sector rather than the matter sector.
 - **Brans-Dicke theory:** G as a VEV of a geometric field, need vastly different values for solar system and cosmic scales
 - **Gauss-Bonnet theory:** Higher derivative corrections to Einstein's gravity
 - **f(R) gravity:** R in the action replaced by $f(R)$, [e.g. $R - \frac{\mu}{R}$],
 - **Phenomenological models:** MOND, MOG, Scalar-Tensor...
Brans; Lovelock; Odintsov; Polarski; Tsujikawa; Bekenstein...

Tools to discriminate \implies Observational constraints

Brane cosmology

Island Universes in Warped Space-Time

According to string theory, our universe might consist of a three-dimensional "brane," embedded in higher dimensions. In the model developed by Lisa Randall and Raman Sundrum, gravity is much weaker on our brane than on another brane, separated from us by a fifth dimension. (Time is the unseen fourth dimension.)



ADD, RS, DGP... depending on bulk, embedding, hierarchy issue...

No unique candidate to address all the points

Have-Not No.1

Brane cosmology: How to visualize?

FRW \Rightarrow **Brane** ; 5D Static, spherically symmetric metric \Rightarrow **Bulk**

$$ds_5^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_3^2$$

Embedding mechanism \Rightarrow Induced metric on the brane

$$ds_4^2 = -d\tau^2 + r^2(\tau)d\Omega_3^2$$

Identify $r(\tau)$ with the scale factor $a(\tau) \implies$ **FRW !**

Visser, PLB(2000); Sahni, JCAP(2003); Maartens, LRR(2004)

SP, PRD(2006),(2006),(2008); Mukherji, **SP**, MPLA(2010)

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Expanding 4D universe \equiv Moving brane in the bulk



Brane-based observer



Bulk-based observer

Accelerated universe is the manifestation of geodesic motion

Visser, PLB(2000); Sahni, JCAP(2003); Maartens, LRR(2004)

SP, PRD(2006),(2006),(2008); Mukherji, **SP**, MPLA(2010)

RS-type brane: effective field equations

$$G_{\mu\nu} = \underbrace{-\Lambda g_{\mu\nu} + \kappa_4^2 T_{\mu\nu}}_{\substack{\Downarrow \\ \text{4D GR}}} + \underbrace{\kappa_5^4 S_{\mu\nu}}_{\substack{\Downarrow \\ \text{Quadratic} \\ T_{\mu\nu}}} - \underbrace{\mathcal{E}_{\mu\nu}}_{\substack{\Downarrow \\ \text{Weyl} \\ \text{term}}} + \underbrace{\mathcal{F}_{\mu\nu}}_{\substack{\Downarrow \\ \text{Bulk} \\ \text{matter}}}$$

Langlois, PRL(2002); Maartens, LRR(2004)

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Friedmann equations

$$H^2 = \frac{\kappa_4^2}{3} \left[\rho + \frac{\rho^2}{2\lambda_b} + \rho^* \right] + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\dot{H} = -\frac{\kappa_4^2}{2} \left[(\rho + p) \left(1 + \frac{\rho}{\lambda_b} \right) + \frac{4}{3} \rho^* \right] + \frac{k}{a^2}$$

Langlois, PRL(2002); Maartens, LRR(2004)

Quadratic term

Early time: $\rho^2 \gg \lambda_b > (100\text{GeV})^4$

results in non-standard evolution during inflation

Maartens, PRD(2001); PRL(2001); Sasaki, PRD(2001)

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Maartens, PRD(2001); PRL(2001); Sasaki, PRD(2001)

Weyl term

For empty bulk: $\rho^* = \frac{C}{a^4} \implies$ dark radiation

$\leq .03\%$ of radiation density (Nucleosynthesis data)

For radiative bulk: $\rho^* = \frac{C(\tau)}{a^4}$

may be significant at late time as well

Barrow, PLB(2002); Langlois, PRL(2002); **SP**, PRD(2006),(2006),(2008)

Brane inflation: governing equations

$$H^2 = \frac{8\pi}{3M_P^2} V \left[1 + \frac{V}{2\lambda} \right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Slow roll parameters

$$\epsilon_V = \frac{M_P^2}{16\pi} \left(\frac{V'}{V} \right)^2 \frac{1 + \frac{V}{\lambda}}{\left(1 + \frac{V}{2\lambda}\right)^2} \ll 1 \quad ; \quad \eta_V = \frac{M_P^2}{8\pi} \left(\frac{V''}{V} \right) \frac{1}{\left(1 + \frac{V}{2\lambda}\right)} \ll 1$$

$$\xi_V = \frac{M_P^4}{(8\pi)^2} \left(\frac{V' V'''}{V^2} \right) \frac{1}{\left(1 + \frac{V}{2\lambda}\right)^2} \quad ; \quad \sigma_V = \frac{M_P^6}{(8\pi)^3} \frac{(V')^2 V''''}{V^3} \frac{1}{\left(1 + \frac{V}{2\lambda}\right)^3}$$

Number of e-foldings

$$N = \frac{8\pi}{M_P^2} \int_{\phi_f}^{\phi_i} \left(\frac{V}{V'} \right) \left(1 + \frac{V}{2\lambda} \right) d\phi \approx 56 - 70$$

Chaotic inflation on the brane

$$V = \frac{1}{2}m^2\phi^2$$

$$N \simeq \frac{2\pi}{M_4^2} (\phi_i^2 - \phi_f^2) + \frac{\pi^2 m^2}{3M_5^6} (\phi_i^4 - \phi_f^4)$$

$$\rho \gg \lambda \ll 10^{16} \text{ GeV} \Rightarrow M_5 < 10^{17} \text{ GeV} \Rightarrow$$

$$\phi_{\text{end}}^4 \simeq \frac{5}{4\pi^2} \left(\frac{M_5}{m}\right)^2 M_5^4$$

$$N_{\text{COBE}} \approx 55 \Rightarrow$$

$$m \approx 5 \times 10^{-5} M_5, \quad \phi_{\text{cobe}} \approx 3 \times 10^2 M_5$$

$\phi_{\text{cobe}} < M_4 \Rightarrow \eta$ -problem softened by quadratic term *Have No.1*

Maartens(2000), Sasaki(2001), (2002), Bertolami(2003)

Likewise calculate observable quantities *will come to this point*

But...

The 4-D field equations are now “derived field equations”.

Effective inflaton potential has to be generated from bulk...

Have-not No.2

Parameter estimation....

Have-not No.3

Effects on reheating phenomenology, leptogenesis....

Have-not No.4

Brane inflation from bulk SUGRA: Schematics

Choudhury, **SP**, PRD(2012), NPB(2012)

$$S = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{g_5} \left[M_5^3 (R_5 - 2\Lambda_5) + L_{SUGRA}^5 + \sum_i \delta(y - y_i) L_{4i} \right]$$

* **$N = 2, D = 5$ SUGRA** $\frac{L_{SUGRA}^5}{e_5} =$
 $-\frac{R^5}{2} + \frac{i}{2} \bar{\Psi}_{i\tilde{m}} \Gamma^{\tilde{m}\tilde{n}\tilde{q}} \nabla_{\tilde{n}} \Psi_{\tilde{q}}^i - S_{IJ} F_{\tilde{m}\tilde{n}}^I F^{I\tilde{m}\tilde{n}} - \frac{1}{2} g_{\alpha\beta} D_{\tilde{m}} \phi^\mu D^{\tilde{m}} \phi^\nu + F + CS$

* **Radion fields:** $\chi = -\psi_5^2$; $T = \frac{1}{\sqrt{2}} \left(e_{\tilde{5}}^{\tilde{5}} - i\sqrt{\frac{2}{3}} A_5^0 \right)$

* **Kähler function:** $H(G) =$
 $\exp\left(\frac{G}{M^2}\right) \left[\left(\frac{\partial W}{\partial \phi_m} + \frac{\partial G}{\partial \phi_m} \frac{W}{M^2} \right)^\dagger (G_m^n)^{-1} \left(\frac{\partial W}{\partial \phi^n} + \frac{\partial G}{\partial \phi^n} \frac{W}{M^2} \right) - 3 \frac{|W|^2}{M^2} \right]$

* Z_2 symmetry

* Compactification around a circle S^1

* Dimensional reduction $ds_5^2 = e^{2A(y)} (ds_4^2 + R^2 \beta^2 dy^2)$

* $V_D = 0 \Leftrightarrow U(1)$ gauge interaction is absent

$$V = V_F = \exp \left[\frac{1}{M^2} \sum_{\alpha} \phi_{\alpha}^{\dagger} \phi^{\alpha} \right] \left[\sum_{\beta} \left| \frac{\partial W}{\partial \phi_{\beta}} \right|^2 - 3 \frac{|W|^2}{M^2} \right]$$

$N = 2, D = 5$ bulk SUGRA $\Rightarrow N = 1, D = 4$ brane SUGRA

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* Include one-loop corrections

* Choose $M_5 \leq 10^{17} \text{ GeV}$ (softening η -problem \Rightarrow fine-tuning!!)

$$V(\phi) = V_0 \left[1 + \left(D_4 + K_4 \ln \left(\frac{\phi}{M} \right) \right) \left(\frac{\phi}{M} \right)^4 \right]$$

Coleman Weinberg potential

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Coleman Weinberg potential

$$N = \frac{M^2}{U} \left[\frac{1}{2} \left(1 + \frac{\alpha}{2} \right) \left(\frac{1}{\phi_f^2} - \frac{1}{\phi_i^2} \right) + \frac{D_4}{2M^4} (1 + \alpha) (\phi_i^2 - \phi_f^2) + \frac{\alpha D_4^2}{12M^8} (\phi_i^6 - \phi_f^6) \right]$$

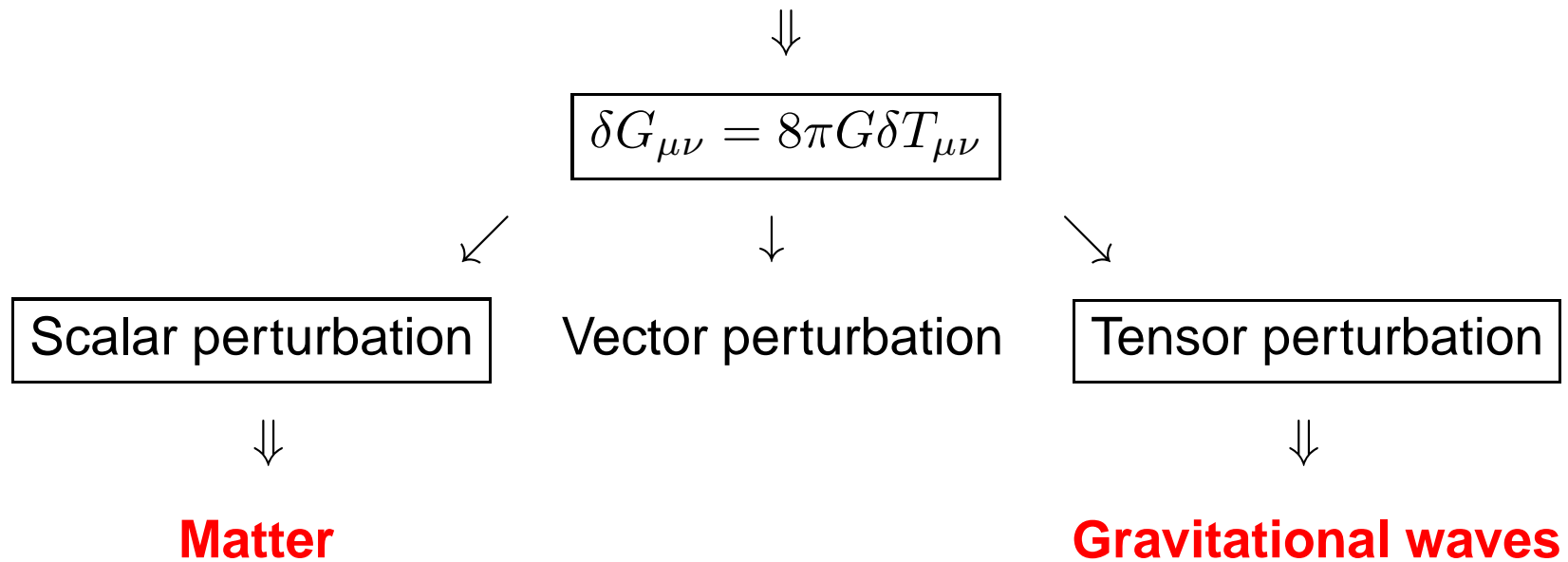
$$V_0^{1/4} \simeq 2 \times 10^{15} \text{ GeV}$$

Have No.2

Cosmological perturbations

Quantum fluctuations of inflaton are transformed to macroscopic cosmological perturbations

Perturbations in the metric



Observable quantities

Observable quantities

- Amplitude of scalar perturbation

$$\Delta_s^2 \simeq \frac{512\pi}{75M_P^6} \left[\frac{V^3}{(V')^2} \left[1 + \frac{V}{2\lambda} \right]^3 \right]_{k=aH} \Rightarrow \Delta_s^2|_{k=aH} \sim 2 \times 10^{-9}$$

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- Amplitude of tensor perturbation

$$\Delta_t^2 = \frac{32}{75M_P^4} \left[\frac{V \left[1 + \frac{V}{2\lambda} \right]}{\sqrt{1 + \frac{2V}{\lambda} \left(1 + \frac{V}{2\lambda} \right)} - \frac{2V}{\lambda} \left(1 + \frac{V}{2\lambda} \right) \sinh^{-1} \frac{1}{\sqrt{\frac{2V}{\lambda} \left(1 + \frac{V}{2\lambda} \right)}}} \right]_{k=aH}$$

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- Ratio of tensor to scalar amplitudes

$$r = 16 \frac{\Delta_t^2}{\Delta_s^2} < 0.36 : \text{WMAP7}$$

$\sim 10^{-2}$: what PLANCK can probe

- Scalar spectral index

$$n_s - 1 = \frac{d(\ln(\Delta_s^2))}{d(\ln(k))} \simeq 2\eta_V^* - 6\epsilon_V^* \quad \Rightarrow 0.948 < n_s < 1$$

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- Running of scalar spectral index

$$\alpha_s = \left. \frac{dn_s}{d \ln k} \right|_{k=aH} = 16\eta\epsilon - 18\epsilon^2 - 2\xi \quad \sim -10^{-3}$$

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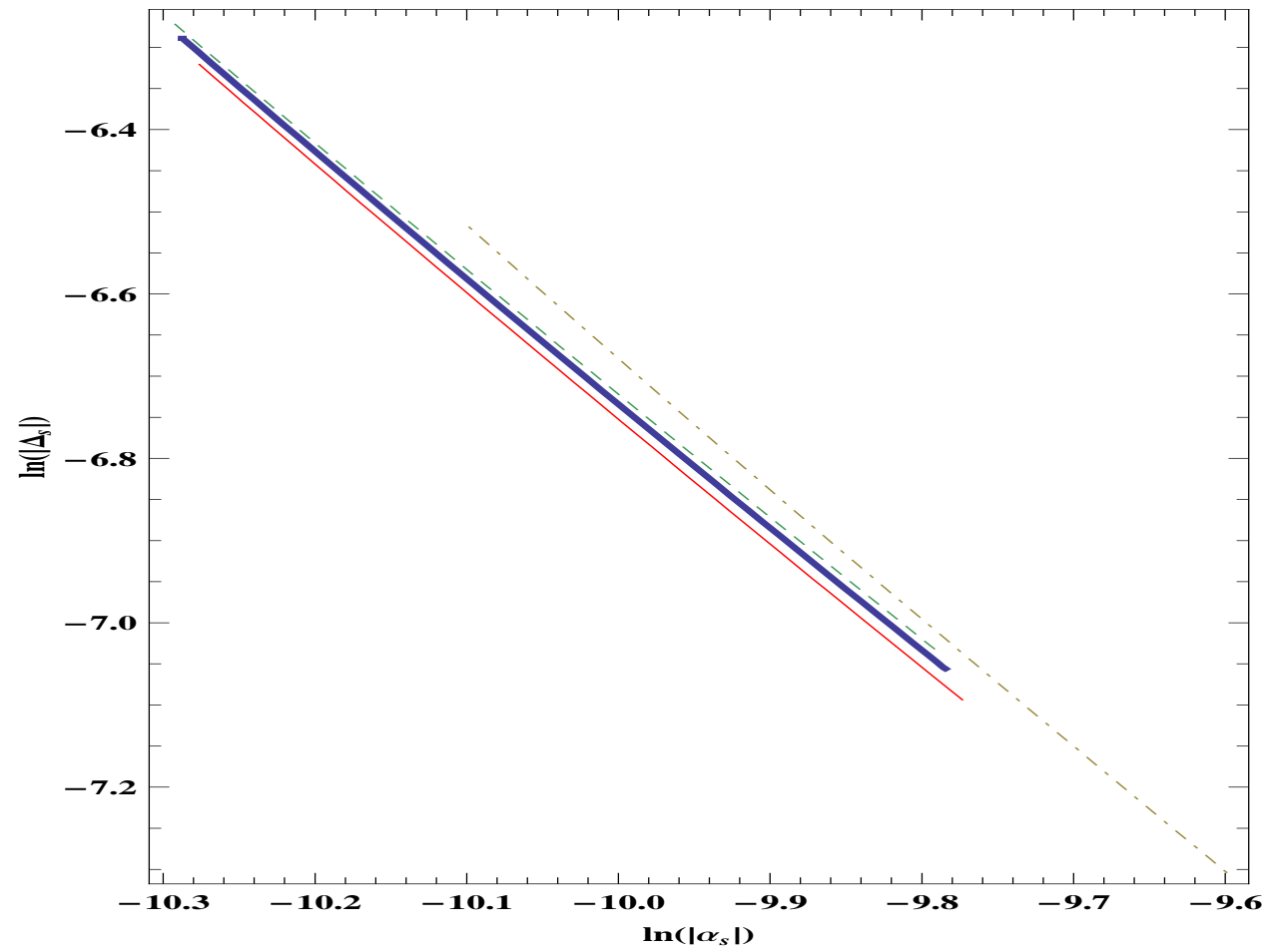
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- Running of tensor spectral index

$$\alpha_t = \left. \frac{dn_t}{d \ln k} \right|_{k=aH} = 6\epsilon\eta - 9\epsilon^2$$

$\ln(\Delta_s)$ versus $\ln(|\alpha_s|)$



C_4 $\simeq D_4$	Δ_s^2 $\times 10^{-9}$	Δ_t^2 $\times 10^{-14}$	n_s	n_t $\times 10^{-5}$	r $\times 10^{-5}$	α_s $\times 10^{-3}$	α_t $\times 10^{-6}$
-0.70	3.126	6.803	0.951	-4.352	2.176	-0.798	-2.125
	1.835		0.941	-7.412	3.706	-1.142	-4.323
	1.440		0.936	-9.447	4.723	-1.345	-5.975
-0.65	2.902	6.317	0.951	-4.352	2.176	-0.798	-2.125
	1.704		0.941	-7.412	3.706	-1.142	-4.323
	1.327		0.936	-9.447	4.723	-1.345	-5.975
-0.60	2.679	5.831	0.951	-4.352	2.176	-0.798	-2.125
	1.573		0.941	-7.412	3.706	-1.142	-4.323
	1.234		0.936	-9.447	4.723	-1.345	-5.975

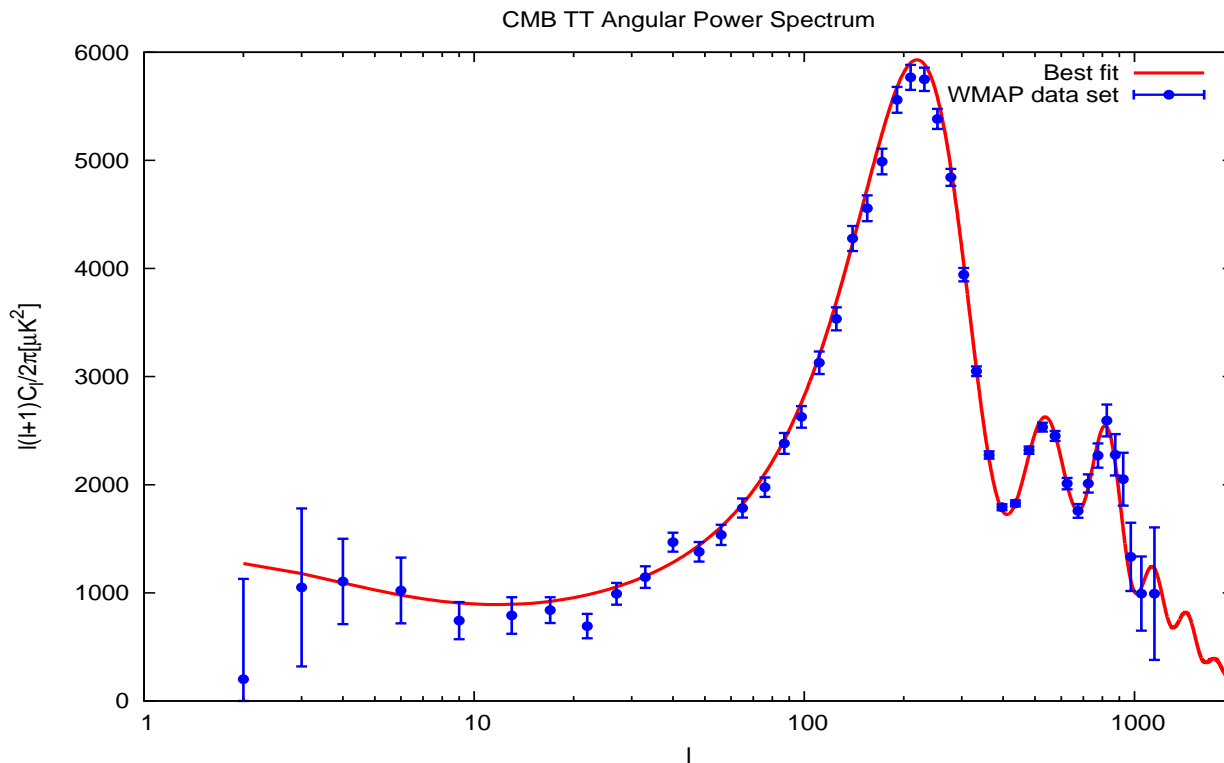
Parameter estimation with CAMB

Input parameters in CAMB

H_0 km/sec/Mpc	τ_{Reion}	$\Omega_b h^2$	$\Omega_c h^2$	T_{CMB} K
71.0	0.09	0.0226	0.1119	2.725

Output parameters from CAMB

t_0 Gyr	z_{Reion}	Ω_m	Ω_Λ	Ω_k	η_{Rec} Mpc	η_0 Mpc
13.707	10.704	0.2670	0.7329	0.0	285.10	14345.1

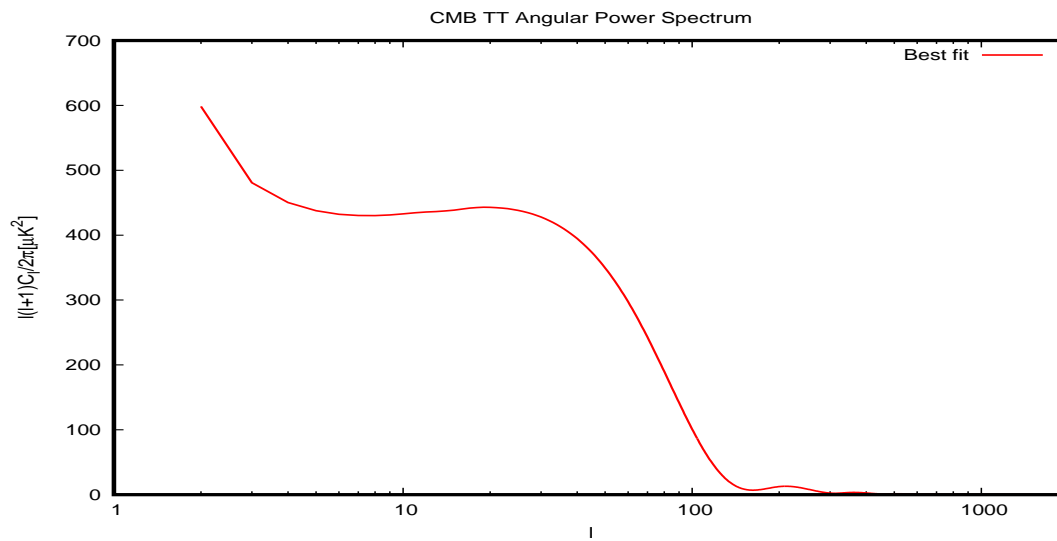
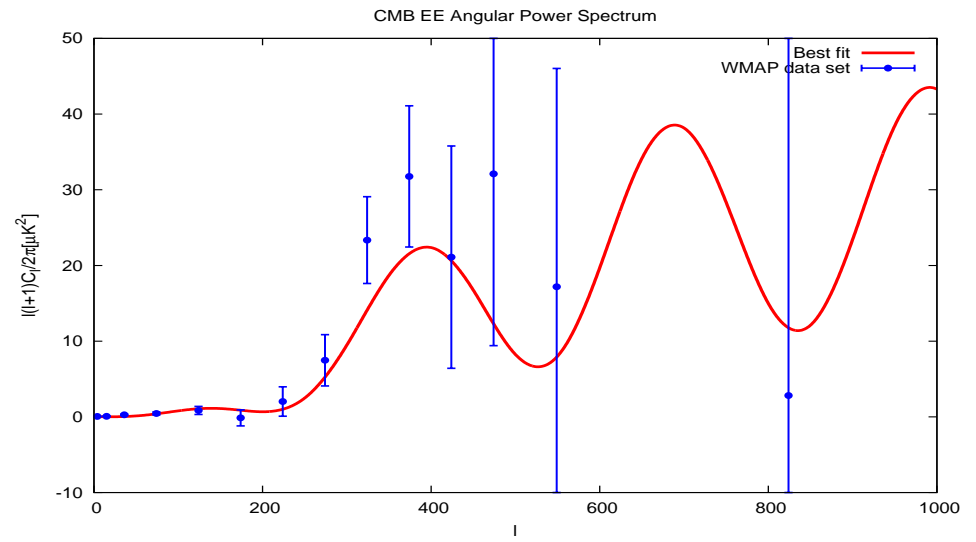
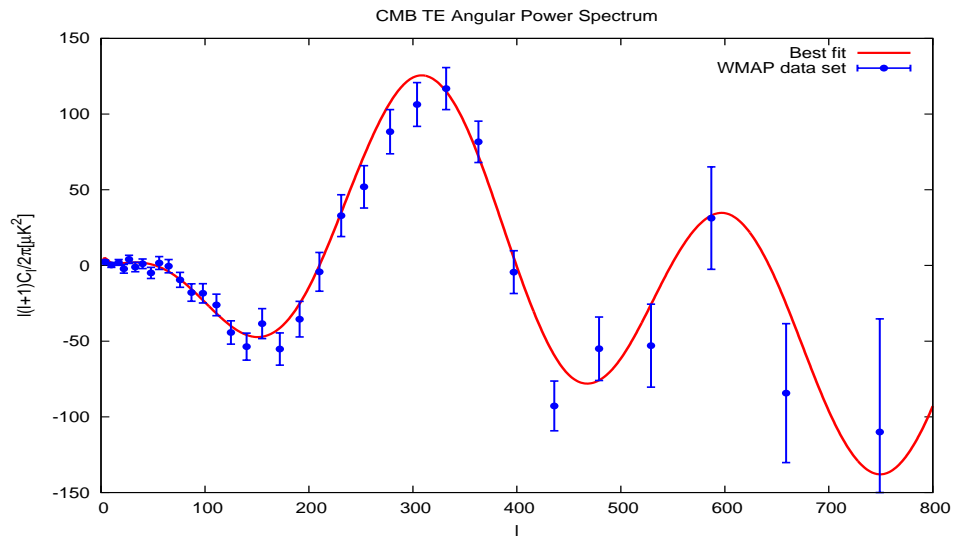


1st peak at $l \approx 241$ confirms $\Omega_k \approx 0$

2nd and 3rd peaks at $l \approx 533, 791$ confirm adiabatic perturbation

Peak positions and heights confirm $\Omega_b \approx 0.04, \Omega_M \approx 0.3, \Omega_{DE} \approx 0.7$

Have No.3



Late time evolutions

Quadratic term

Late time: $\rho^2 \ll \lambda_b > (100\text{GeV})^4 \Rightarrow$ negligible

Weyl term

For empty bulk: $\rho^* = \frac{C}{a^4} \implies$ dark radiation

$\leq .03\%$ of radiation density (Nucleosynthesis data)

\Rightarrow negligible

Results in standard evolution at late time

How to get dark energy?

Have-not No.5

How to get dark energy?



Choose different bulk and/or embedding



RS, radiative bulk

Generalized dynamics

DGP braneworlds



$$\rho^* = \frac{C(\tau)}{a^4}$$

(SP, Maartens, Langlois...)



Modifications at large scale

(Maartens, Koyama, Chen...)



New Avatar **Galileon**

(Burgess, Brux, de Rahm, Tsujikawa, Trodden, Copeland...)

Dark energy from generalized bulk dynamics

Das, Ghosh, van Holten, **SP**, JHEP(2009)

The general bulk action

$$S = m \int d\tau \left[\frac{1}{2e} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \frac{e}{2} - \lambda g_{\mu\nu} \xi^\mu \dot{x}^\nu + \frac{e\lambda^2}{2} g_{\mu\nu} \xi^\mu \xi^\nu + \frac{e\beta\lambda^2}{2} \right]$$

The action has been derived by Kaluza-Klein decomposition

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τ = worldline evolution parameter

ξ^μ = Killing vectors associated with symmetry of spacetime

$e(\tau)$ = worldline einbein to maintain reparametrization-invariance

$\lambda(\tau)$ = auxiliary worldline scalar variable

β = a nonzero parameter of the theory

The action has been derived by Kaluza-Klein decomposition

Study geodesics in the background

$$ds_5^2 = - \left(k - \frac{2M}{r^2} + \Lambda_5 r^2 \right) dt^2 + \frac{dr^2}{k - \frac{2M}{r^2} + \Lambda_5 r^2} + r^2 d\Omega_3^2$$

Using the Killing vectors, radial geodesics look

$$\dot{r}^2 + V_{\text{eff}}(r) = \varepsilon^2$$

Effective potential : $V_{\text{eff}} = \left(k - \frac{2M}{r^2} + \Lambda_5 r^2 \right) \left(1 + \frac{l^2}{r^2} + \frac{l^2}{\beta} \right)$

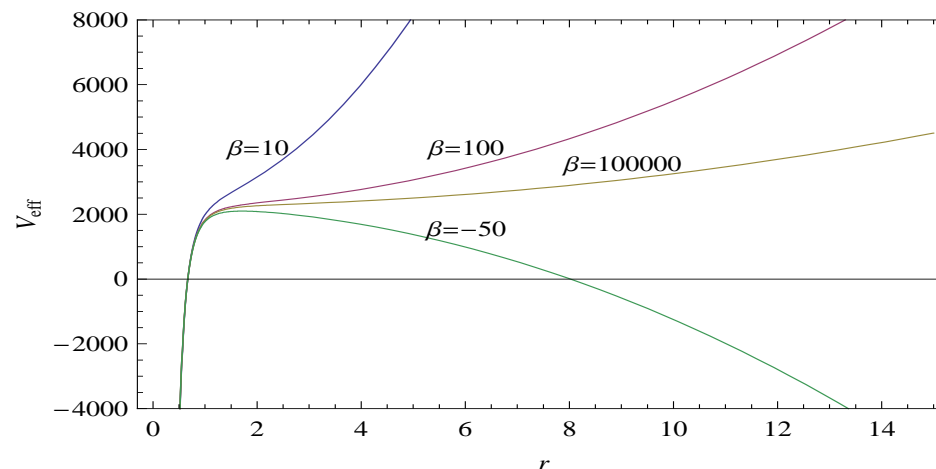
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Repulsive force is generated for $\beta < 0 \Rightarrow$ Dark Energy?

Brane Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_4}{3}\rho + \frac{2M}{a^4} + \left[\alpha - \Lambda_5 + \frac{l^2(2M/a^2 - \Lambda_5 a^2)}{\beta(a^2 + l^2)}\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_4}{3}(\rho + 3p) - \frac{2M}{a^4} + \left[\alpha - \Lambda_5 - \frac{l^2(\Lambda_5 a^4 + 2M + 2\Lambda_5 l^2 a^2)}{\beta(a^2 + l^2)^2}\right]$$

Research Highlight in Nature(2009)

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Approximate solution

$$a(t) \approx M + \frac{1}{2} \left[e^{t/\sqrt{-\beta}} + M^2 e^{-t/\sqrt{-\beta}} \right]$$

\implies **Accelerating solution !**

Behaves pretty close to Λ CDM, with $\beta^{-1} \approx \Lambda$

Research Highlight in Nature(2009)

Observable quantities

Stringent constraints on dark energy models

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- Luminosity distance $d_L(z) = c(1+z) \int_0^z \frac{dz'}{H(z')} \Rightarrow \Omega_{DE} \approx 0.7$

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$$r = \frac{\ddot{\ddot{a}}/a}{(\dot{a}/a)^3} = 1 + \left[\frac{H''}{H} + \left(\frac{H'}{H} \right)^2 \right] (1+z)^2 - 2 \frac{H'}{H} (1+z)$$

$$s = \frac{r-1}{3(q-1/2)}$$

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- Averaging over entire redshift $Om(z)$ and $\bar{q}(z) \Rightarrow \text{test for } \Lambda$

Stringent constraints on dark energy models

Observational aspects of generalized dynamics

Das, Ghosh, van Holten, **SP**, IJMPD(2011)

Express Friedmann equations in terms of redshift s.t. $a \propto \frac{1}{1+z}$

Neglect all terms $\geq (1+z)^4$

Hubble parameter boils down to

$$H^2 = H_0^2 \left[\underbrace{\Omega_X}_{\text{Dark Energy}} (1 + b(1+z)^2) + \underbrace{\Omega_M}_{\text{Matter Sector}} (1+z)^3 \right]$$

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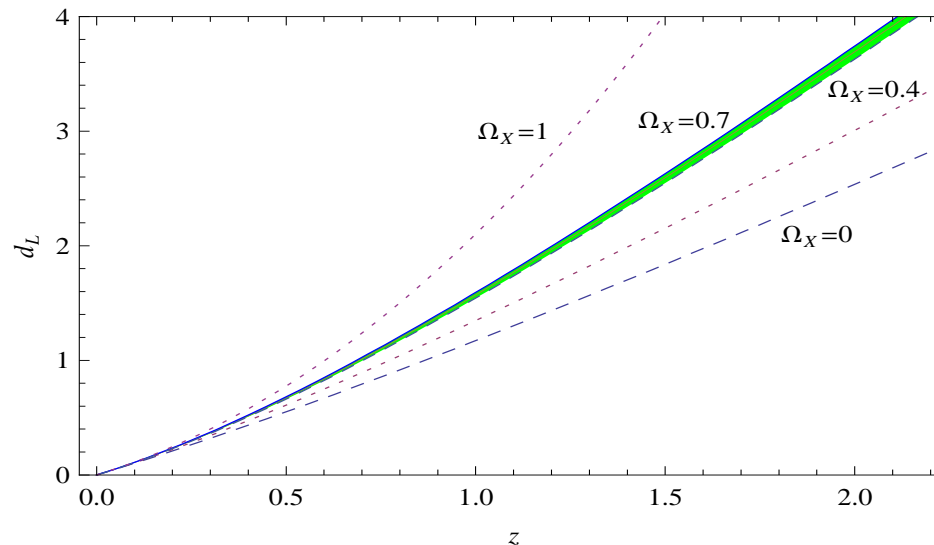
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- $H_0 = 74.2 \pm 3.6$ km/s/Mpc : WMAP5/SHOES
- $\Omega_M = 8\pi G_4 \rho / 3H_0^2 = 0.28 \pm 0.08, 95\% \text{ CL ?}$: CMB/LSST
- $\Omega_X = (\alpha - \Lambda_5 \mu^2) / H_0^2 = 0.726 \pm 0.015, 95\% \text{ CL ?}$: WMAP5/SNIa
- $b = \Lambda_5 \beta (\mu^2 - 1)^2 / (\alpha - \Lambda_5 \mu^2) = ?$

Luminosity distance: SNIa: $\Omega_X \approx 0.7$; CMB+LSST: $\Omega_M = 0.3$

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}$$
$$= \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_X(1+b(1+z')^2) + \Omega_M(1+z')^3}}$$



Observational data from all of the SNIa fall on **green** line.

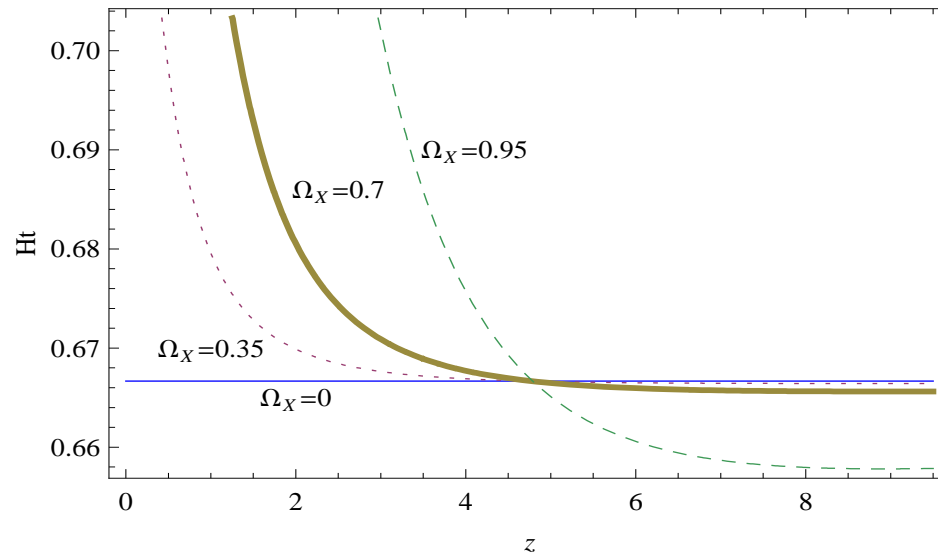
$$\boxed{\Omega_M = 0.3 \ ; \ \Omega_X = 0.7 \ ; \ -0.07 \leq b < 0}$$

Matches observations for Dark Energy and Matter density

Age of the universe: Latest accepted value 13.7 ± 0.02 Gyr

Correct Dark Energy density results in correct calculation of age

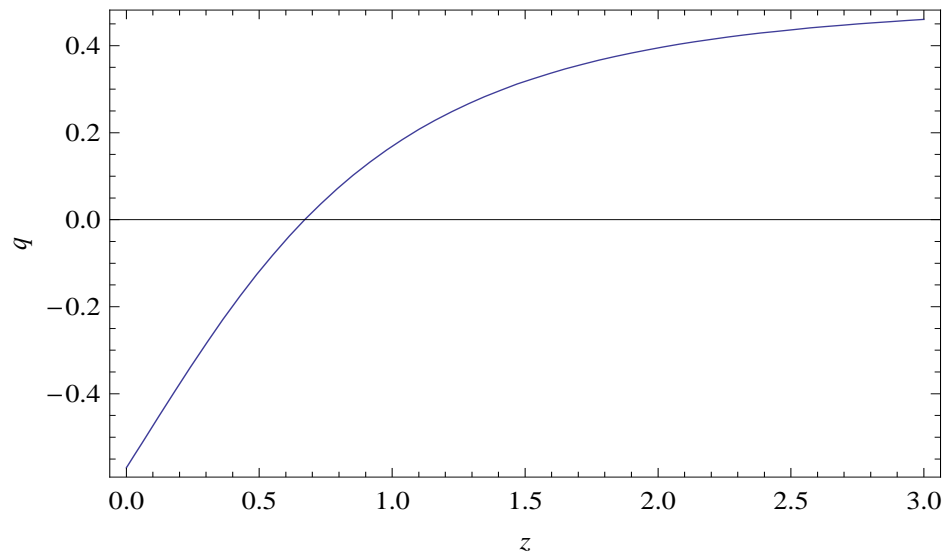
$$t(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$
$$= \frac{1}{H_0} \int_z^\infty \frac{dz'}{(1+z')\sqrt{\Omega_X(1+b(1+z')^2)+\Omega_M(1+z')^3}}$$



$\Omega_X = 0.7$ matches the plot obtained from observational data

Deceleration parameter: $q < 0$, onset of acceleration $z = 0.6$

$$q(z) = \frac{-\ddot{a}/a}{\dot{a}^2/a^2} = \frac{H'(z)}{H(z)}(1+z) - 1$$
$$= \frac{\Omega_M(1+z)^3 - 2\Omega_X}{2[\Omega_X(1+b(1+z)^2) + \Omega_M(1+z)^3]}$$



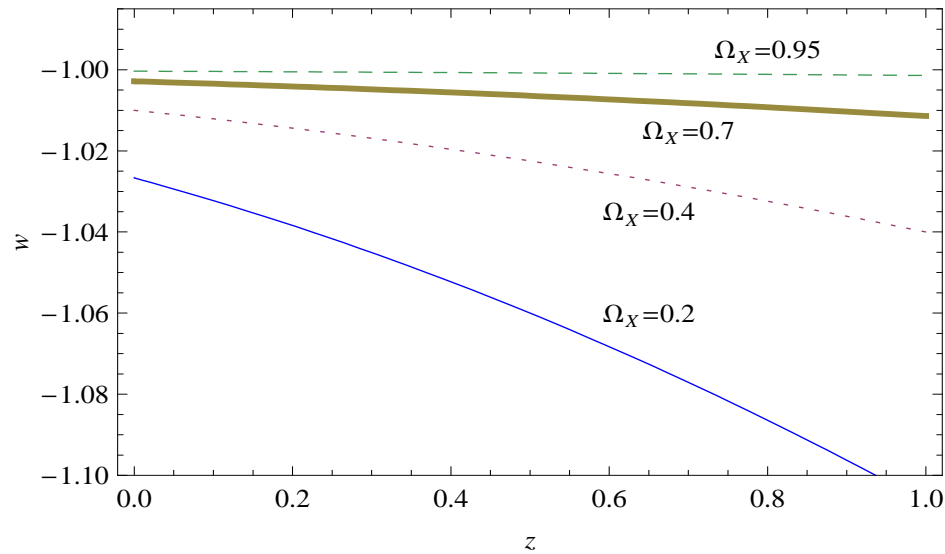
Plot for $\Omega_M = 0.3$; $\Omega_X = 0.7$; $b = -0.05$

* $q < 0$ at present

* **Onset of recent acceleration $z \approx 0.6$ confirmed**

Equation of state: SNIa Gold dataset: $-1.11 < w_X < -1$ at 2σ

$$w_X(z) = \frac{2q(z)-1}{3[1-\Omega_M(z)]} \approx -1 + \frac{2b(1+z)^2}{3}$$



* **Shows phantom behavior without any phantom field**

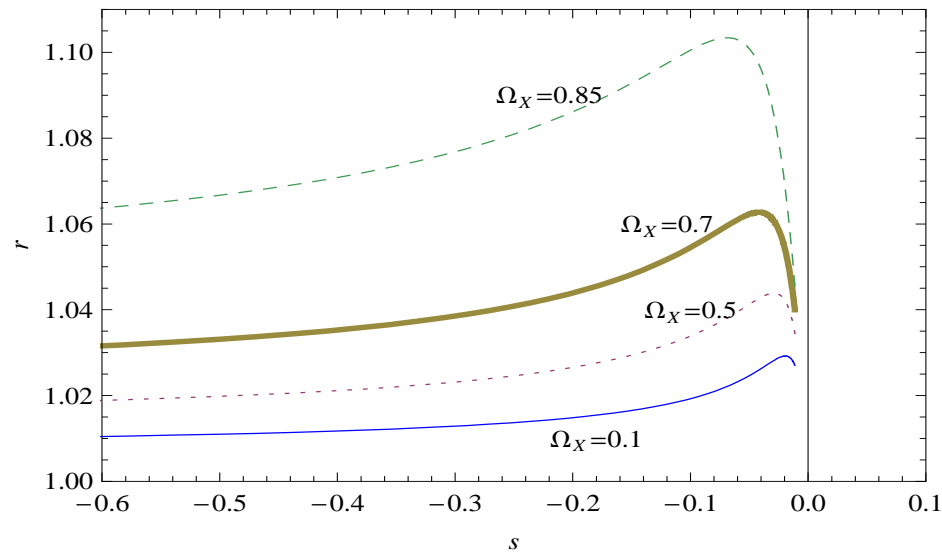
$$\boxed{-1.11 < w_X < -1 \Rightarrow -0.15 < b < 0 ; d_L \Rightarrow -0.07 \leq b < 0}$$

* **Will fit well with more precise observational data too**

Statefinder parameters: Dynamical models vs Λ

$$r = \frac{\ddot{a}/a}{(\dot{a}/a)^3} = 1 + \left[\frac{H''}{H} + \left(\frac{H'}{H} \right)^2 \right] (1+z)^2 - 2 \frac{H'}{H} (1+z) ; s = \frac{2}{3} \frac{r-1}{2q-1}$$

$$r = 1 - \frac{b\Omega_X(1+z)^2}{\Omega_X(1+b(1+z)^2) + \Omega_M(1+z)^3} ; s = \frac{2}{3} \frac{b(1+z)^2}{[3+b(1+z)^2]}$$



Dynamical model assured

Have No.4

A cosmologist's checklist

- Inflation? Yes
- SUGRA origin? Yes
- String \mapsto Brane cosmology? No
- Reheating and leptogenesis? Yes (?)
- Parameter estimation with CAMB? Yes
- Detailed CMB physics with Monte-Carlo? No
- Dark energy from RS? Not possible
- Dark energy from modified bulk? Yes
- Maximum likelihood analysis? No
- Dark matter? Not possible (?)
- Effects on reionization? No

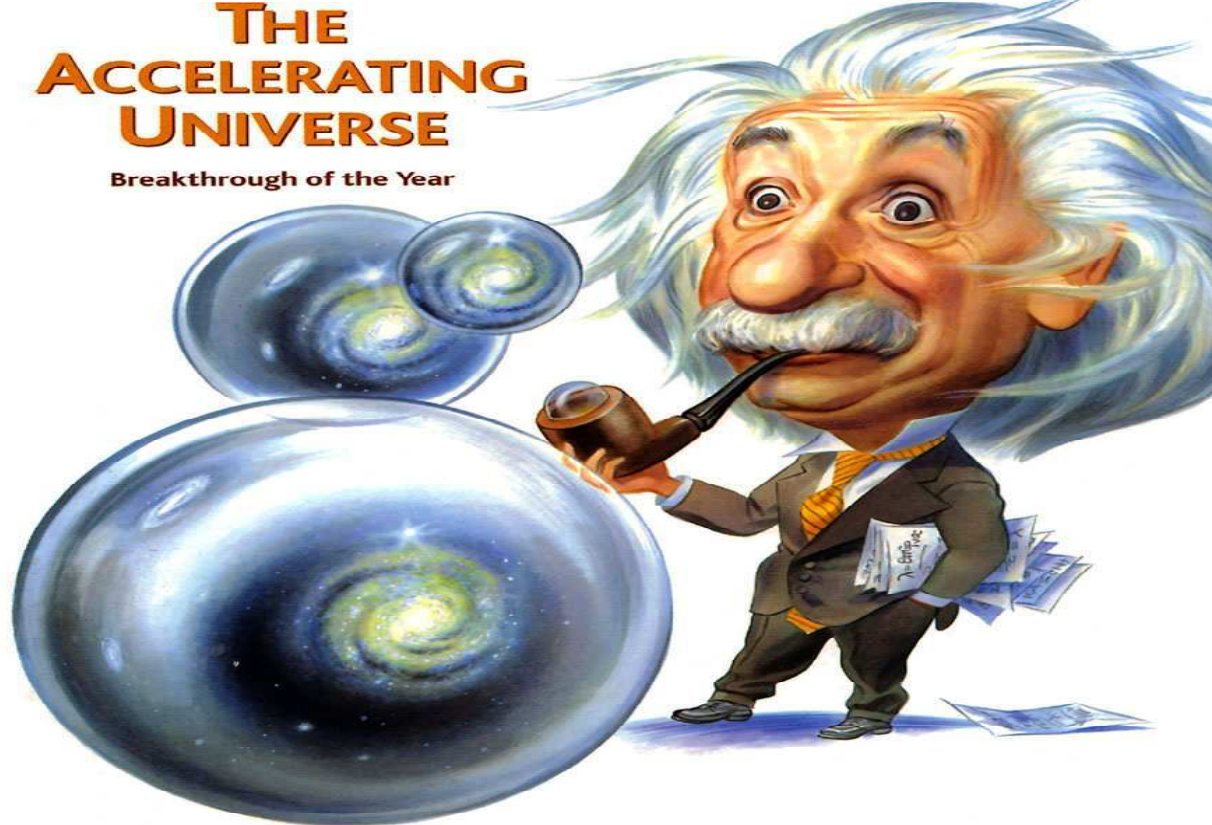
Science

18 December 1998

Vol. 282 No. 5397
Pages 2141-2336 \$7

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