How many parameters for our Universe?

Supratik Pal

Indian Statistical Institute Kolkata

December 22, 2022

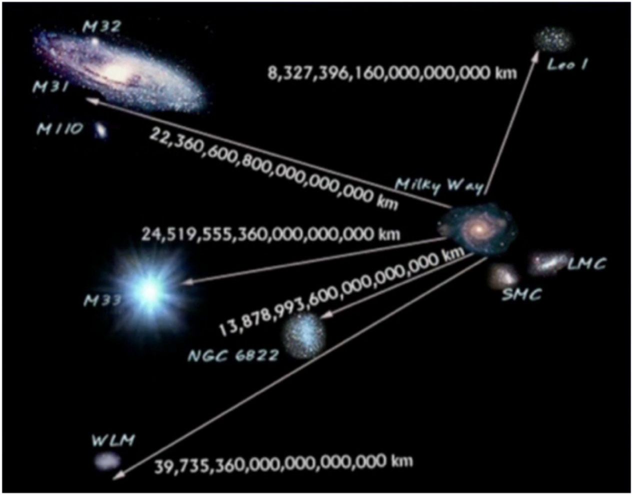
Conclusion

We don't know :)

Some random questions

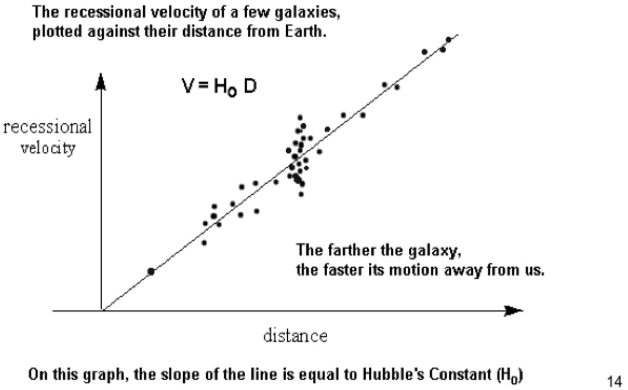
- The accelerating Universe
  - How do we know that?
  - Why is this surprising?
  - Who drives this acceleration?
- What is our current understanding of the Universe?
- How conclusive is our understanding?
- Can future missions help?
- Can other (particle physics) extensions help?

The grand old man of Observational Cosmology: Hubble (1929)

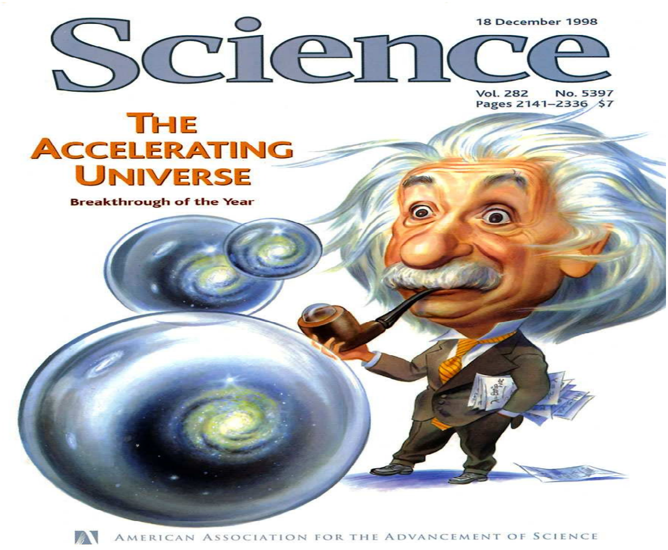


The Universe is expanding

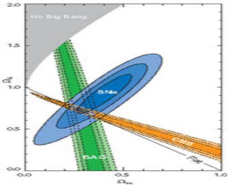
Farther the galaxy, greater is its velocity->
Galaxies are moving away from each other

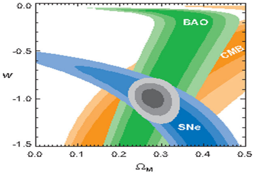


What is the rate of expansion?



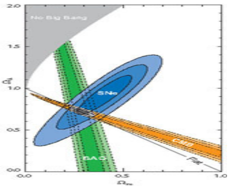
SNIa+CMB+BAO+.... Together they stand

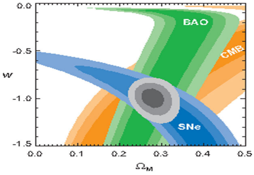




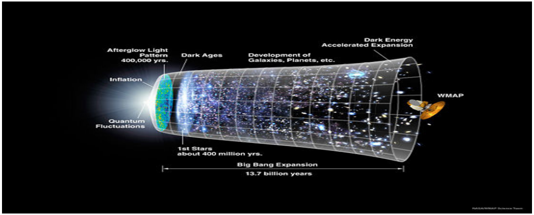
Universe is accelerating at present

SNIa+CMB+BAO+.... Together they stand





Universe is accelerating at present



This is a recent phenomenon

Why is this surprising?

Gravitational force is attractive

$\downarrow$

Evolution of the Universe: Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

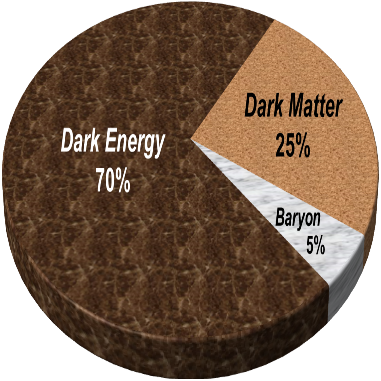
$\downarrow$

Universe should decelerate

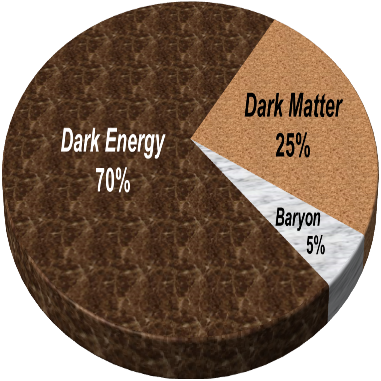
But the Universe is accelerating!

Ordinary matter satisfying Strong Energy Condition $(\rho + 3p) \geq 0$ cannot supply the effective "anti-gravity" required for acceleration!

Who drives this acceleration?



Who drives this acceleration?



If it's not surprising enough, it's not fun!

Dark energy candidates

- Cosmological Constant : Vanilla model 6-parameter ΛCDM
- Dynamical models
  - Quintessence: scalar field minimally coupled to gravity
  - K-essence: Kinetic energy driven
  - Tachyonic: Non-canonical
  - Phantom: Opposite sign in kinetic term
  - Dilatonic
  - (Generalized) Chaplygin gas
  - .....
- Modified gravity
  - Brans-Dicke theory
  - f(R) gravity
  - Braneworld/DGP model
  - ......

Dark energy candidates

- Cosmological Constant : Vanilla model 6-parameter ΛCDM
- Dynamical models
  - Quintessence: scalar field minimally coupled to gravity
  - K-essence: Kinetic energy driven
  - Tachyonic: Non-canonical
  - Phantom: Opposite sign in kinetic term
  - Dilatonic
  - (Generalized) Chaplygin gas
  - .....
- Modified gravity
  - Brans-Dicke theory
  - f(R) gravity
  - Braneworld/DGP model
  - ......

Bottomline: Plethora of models!

From models to parametrizations

Instead of proposing individual models, parametrize dark energy and constrain models by constraining those parameters from observations

From models to parametrizations

Instead of proposing individual models, parametrize dark energy and constrain models by constraining those parameters from observations

Observationally we probe 2 (3) parameters of dark energy:

- Density parameter (Ω_{DE}) (by probing Ω_{M})
- Equation of State (EoS) parameter (w_{DE})
- Sound speed ($C_{\mathrm{s,DE}}$) (if perturbations included)

Parametrization of EoS: Some examples

- CPL Parametrization (6+2)

$$w(a) = w_0 + w_{\rm a}(1-a) = w_0 + w_{\rm a}\frac{z}{1+z}$$

Parametrization of EoS: Some examples

- CPL Parametrization (6+2)

$$w(a) = w_0 + w_{\rm a}(1-a) = w_0 + w_{\rm a}\frac{z}{1+z}$$

- SS Parametrization (6+1)

$$w(a) = (1+w_0)\times$$
$$\left[\sqrt{1+(\Omega_{\rm DE}^{-1}-1)a^{-3}} - (\Omega_{\rm DE}^{-1}-1)a^{-3}\tanh^{-1}\frac{1}{\sqrt{1+(\Omega_{\rm DE}^{-1}-1)a^{-3}}}\right]^2 \times$$
$$\left[\frac{1}{\sqrt{\Omega_{\rm DE}}} - \left(\frac{1}{\Omega_{\rm DE}}-1\right)\tanh^{-1}\sqrt{\Omega_{\rm DE}}\right]^{-2} - 1$$

Both of them represent both phantom and non-phantom DE

Parametrization of EoS: Some examples

- CPL Parametrization (6+2)

$$w(a) = w_0 + w_{\rm a}(1-a) = w_0 + w_{\rm a}\frac{z}{1+z}$$

- SS Parametrization (6+1)

$$w(a) = (1+w_0) \times \left[\sqrt{1+(\Omega_{\rm DE}^{-1}-1)a^{-3}} - (\Omega_{\rm DE}^{-1}-1)a^{-3}\tanh^{-1}\frac{1}{\sqrt{1+(\Omega_{\rm DE}^{-1}-1)a^{-3}}}\right]^2 \times \left[\frac{1}{\sqrt{\Omega_{\rm DE}}} - \left(\frac{1}{\Omega_{\rm DE}}-1\right)\tanh^{-1}\sqrt{\Omega_{\rm DE}}\right]^{-2} - 1$$

Both of them represent both phantom and non-phantom DE

- GCG Parametrization (6+2)

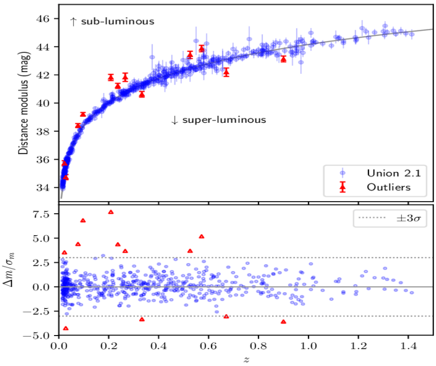
$$w(a) = -\frac{A}{A+(1-A)a^{-3(1+\alpha)}} \; ; \; A = \frac{c}{\rho_{\rm GCG}^{1+\alpha}}$$

Represents non-phantom DE only

Supernova Type Ia Data: Standard Candles

Probe Luminosity distance: $D_L(z) = H_0 d_L(z)$ via distance modulus

$$\mu(z) = 5\log_{10}(D_L(z)) + \mu_0$$



$$\chi^2_{\rm SN}(w_X^0, \Omega_m^0, H_0) = \sum_i \left[\frac{\mu_{\rm obs}(z_i) - \mu(z_i; w_X^0, \Omega_m^0, H_0)}{\sigma_i}\right]^2$$

Marginalizing over the nuisance parameter μ_0,

$$\chi^2_{\rm SN}(w_X^0, \Omega_m^0) = A - B^2/C$$

$$A = \sum_i \left[\frac{\mu_{\rm obs}(z_i) - \mu(z_i; w_X^0, \Omega_m^0, \mu_0 = 0)}{\sigma_i}\right]^2$$

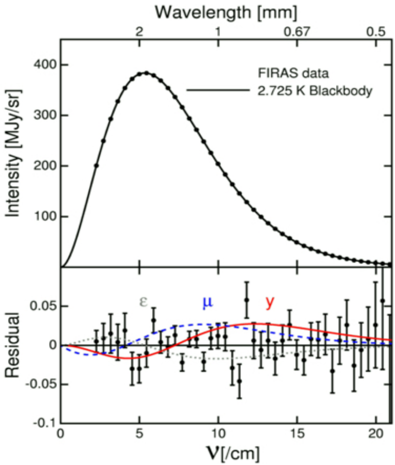
$$B = \sum_i \left[\frac{\mu_{\rm obs}(z_i) - \mu(z_i; w_X^0, \Omega_m^0, \mu_0 = 0)}{\sigma_i}\right] ; C = \sum_i \frac{1}{\sigma_i^2}$$

Datasets

- Union 2.1 compilation of 580 Supernovae
- JLA=SDSS-II+SNLS
- Pantheon
- DES

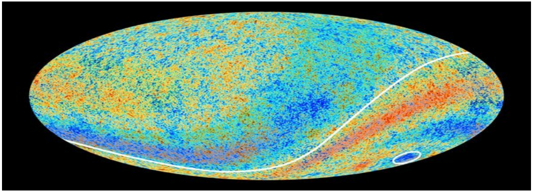
Cosmic Microwave Background (CMB) data

Universe is a Black-body (Penzias & Wilson:1965)

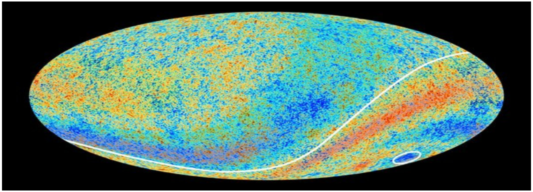


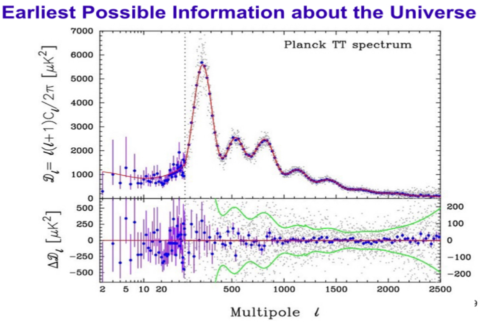
Background : $T_0 = 2.725K \longrightarrow$ Blackbody spectrum

Tiny temperature fluctuations in CMB ($\Delta T \sim 200\mu K$)



Tiny temperature fluctuations in CMB ($\Delta T \sim 200\mu K$)





CMB: Reflection on Dark Energy

- Shift parameter (position of peaks)
- Integrated Sachs-Wolfe effect (low-ℓ)

Shift Parameter

DE $\Leftrightarrow$ Shift in position of peaks by $\sqrt{\Omega_m} D$
D= Angular diameter distance (to LSS) $\Rightarrow$ Shift Parameter

$$R = \sqrt{\frac{\Omega_m h^2}{|\Omega_k| h^2}} \chi(y)$$

$$\chi(y) = \sin y (k < 0) \quad ; \quad = y (k = 0) \quad ; \quad = \sinh y (k > 0)$$

$$y = \sqrt{|\Omega_k|} \int_o^{z_{\rm dec}} \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_X (1+z)^{3(1+\omega_X)}}}$$

$$\chi^2_{\rm CMB}(\omega_X, \Omega_m, H_0) = \left[\frac{R(z_{\rm dec}, \omega_X, \Omega_m, H_0) - R}{\sigma_R}\right]^2$$

Planck 2015 highlights (2018: slightly improved)

Ade et.al., 1502.01589; 1502.01590; 1502.01592; 1502.00612

Primary/fit parameters

Parameters	Planck2015 TT+low P	Planck2015 TT,TE,EE+low P
$\Omega_b h^2$	0.02222 ± 0.00023	0.02225 ± 0.00016
$\Omega_c h^2$	0.1197 ± 0.0022	0.1198 ± 0.0015
$\ln(10^{10} A_S)$	3.089 ± 0.036	3.094 ± 0.034
n_s	0.9655 ± 0.0062	0.9645 ± 0.0049
τ	0.078 ± 0.019	0.079 ± 0.017
H_0	67.31 ± 0.96	67.27 ± 0.66

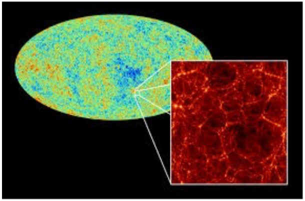
6-parameter ΛCDM

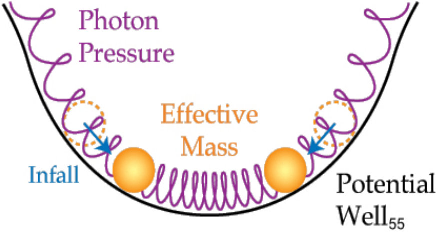
Plus several derived parameters....

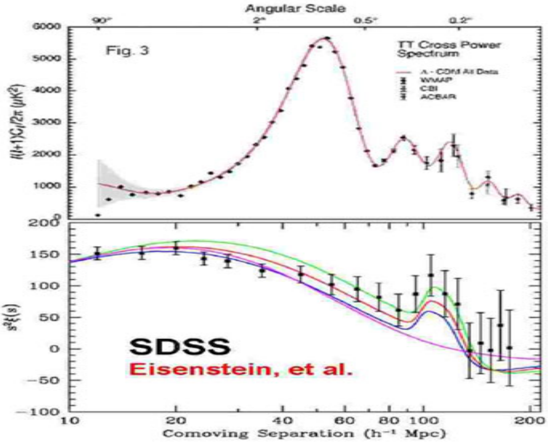
Baryon Acoustic Oscillation (BAO) data

From CMB to Large Scale Structure

BOSS: Baryonic Acoustic Oscillation Spectroscopic Survey







Used to measure $H(z)$ and angular diameter distance $D_A(z)$ via a combination

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)}\right]^{1/3}$$

Confront models via a distance ratio

$$d_z = \frac{r_s(z_{\rm drag})}{D_V(z)}$$

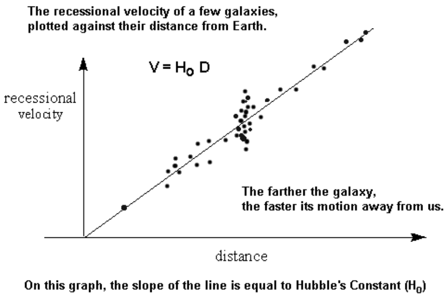
$r_s(z_{\rm drag})$ = comoving sound horizon at a redshift where baryon-drag optical depth is unity

Datasets

- WiggleZ : $z = 0.44, 0.6, 0.73$
- 6dFGS : $z = 0.106$
- SDSS MGS: $z = 0.15$
- BOSS DR12: $z = 0.38, 0.51, 0.61....$

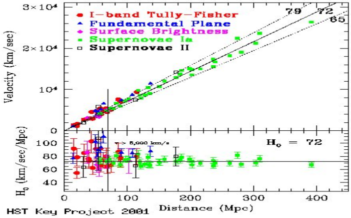
Hence calculate $\chi^2_{\rm BAO}$

Hubble Space Telescope Data (HST)



Old data: Used nearby galaxies as data points.

Now: Constrains the value of H_0 directly by using nearby Type-Ia Supernova data with Cepheid calibrations.



Combine and calculate χ^2 for the analysis of HST data

$$\chi^2_{\mathrm{HST}}(w_X^0, \Omega_m^0, H_0) = \sum_i \left[\frac{H_{\mathrm{obs}}(z_i) - H(z_i; w_X^0, \Omega_m^0, H_0)}{\sigma_i}\right]^2$$

Datasets

- Riess et. al., 2011 (R11)
- Efstathiou, 2014 (E14)
- Riess et. al., 2016 (R16), 2018 (R18), 2020 (R20)...
- Riess+SNe = SH0ES

Analysis: Combine all datasets

Hazra, Majumdar, SP, Panda, Sen: PRD:2015

χ^2 for different models/parametrizations

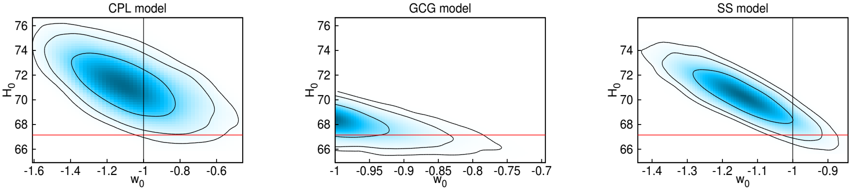
Data	ΛCDM	CPL	SS	GCG
Planck (low-ℓ + high-ℓ)	7789.0	7787.4	7788.1	7789.0
WMAP-9 low-ℓ polarization	2014.4	2014.436	2014.455	2014.383
BAO : SDSS DR7	0.410	0.073	0.265	0.451
BAO : SDSS DR9	0.826	0.793	0.677	0.777
BAO : 6DF	0.058	0.382	0.210	0.052
BAO : WiggleZ	0.020	0.069	0.033	0.019
SN : Union 2.1	545.127	546.1	545.675	545.131
HST	5.090	2.088	2.997	5.189
Total	10355.0	10351.4	10352.4	10355.0

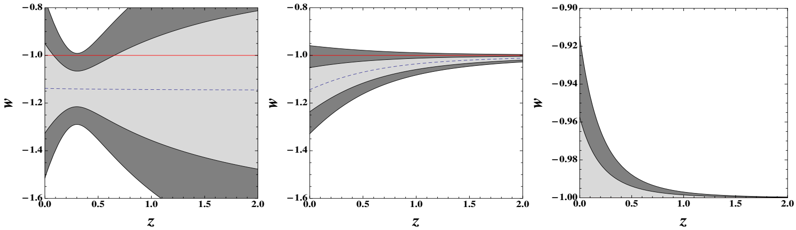
Minimum for CPL but not much improved from ΛCDM

Parameters (mean value with 1σ)

		CPL	SS	GCG
$\Omega_b h^2$	CMB	0.0221 ± 0.00028	0.0221 ± 0.00026	0.022 ± 0.00028
	CMB + non-CMB	0.022 ± 0.00026	$0.0221^{+0.00026}_{-0.00024}$	0.0223 ± 0.00024
	Non-CMB	$0.027^{+0.004}_{-0.005}$	$0.028^{+0.004}_{-0.006}$	0.029 ± 0.005
$\Omega_{CDM} h^2$	CMB	0.1196 ± 0.0027	0.1198 ± 0.0026	$0.1199^{+0.0026}_{-0.0028}$
	CMB + non-CMB	0.1209 ± 0.0023	0.1192 ± 0.0018	0.117 ± 0.0015
	Non-CMB	$0.126^{+0.014}_{-0.017}$	$0.128^{+0.014}_{-0.018}$	$0.127^{+0.015}_{-0.018}$
100θ	CMB	1.041 ± 0.0006	1.041 ± 0.0006	1.041 ± 0.0006
	CMB + non-CMB	1.041 ± 0.0006	1.041 ± 0.00056	1.042 ± 0.00056
	Non-CMB	1.042 ± 0.023	1.048 ± 0.022	$1.05^{+0.019}_{-0.027}$
τ	CMB	$0.09^{+0.012}_{-0.014}$	$0.09^{+0.012}_{-0.015}$	$0.09^{+0.013}_{-0.014}$
	CMB + non-CMB	$0.087^{+0.012}_{-0.014}$	0.091 ± 0.013	0.094 ± 0.014
	Non-CMB	...	...	...
$w_0[-A]$	CMB	$-1.13^{+0.37}_{-0.66}$	$-1.31^{+0.19}_{\text{unbounded}}$	$-0.827^{+0.06}_{\text{non-phantom prior cut}}$
	CMB + non-CMB	$-1.005^{+0.15}_{-0.17}$	$-1.14^{+0.08}_{-0.09}$	$-0.957^{+0.007}_{\text{non-phantom prior cut}}$
	Non-CMB	$-0.995^{+0.23}_{-0.27}$	-1.02 ± 0.12	$-0.92^{+0.018}_{\text{non-phantom prior cut}}$
$w_a[\alpha]$	CMB	$-1.15^{+0.6}_{\text{unbounded}}$	...	$-1.97^{+0.32}_{\text{unbounded}}$
	CMB + non-CMB	$-0.48^{+0.77}_{-0.54}$	...	$-2.0^{+0.29}_{\text{unbounded}}$
	Non-CMB	$-0.5^{+1.64}_{-0.94}$	...	$-1.49^{+0.4}_{\text{unbounded}}$
n_S	CMB	0.9607 ± 0.007	0.9603 ± 0.007	$0.9603 \pm +0.00073$
	CMB + non-CMB	$0.9579^{+0.0063}_{-0.0066}$	$0.9619^{+0.0059}_{-0.0057}$	$0.9669^{+0.00056}_{-0.00059}$
	Non-CMB	...	...	...
$\ln[10^{10} A_S]$	CMB	$3.089^{+0.023}_{-0.027}$	$3.089^{+0.023}_{-0.028}$	3.09 ± 0.025
	CMB + non-CMB	$3.087^{+0.024}_{-0.026}$	3.091 ± 0.025	3.092 ± 0.026
	Non-CMB	...	...	...
Ω_m	CMB	$0.239^{+0.028}_{-0.099}$	$0.27^{+0.04}_{-0.1}$	$0.344^{+0.022}_{-0.032}$
	CMB + non-CMB	$0.291^{+0.011}_{-0.013}$	$0.288^{+0.012}_{-0.013}$	$0.304^{+0.009}_{-0.011}$
	Non-CMB	0.29 ± 0.024	$0.298^{+0.02}_{-0.026}$	$0.3^{+0.021}_{-0.024}$
H_0	CMB	$80^{+17.8}_{-7.8}$	$74.8^{+13.3}_{-9.8}$	$64.6^{+2.61}_{-1.91}$
	CMB + non-CMB	70.26 ± 1.4	70.3 ± 1.4	$67.9^{+0.9}_{-0.7}$
	Non-CMB	72.68 ± 2.2	72.67 ± 2.15	72.4 ± 2.16

Major results





CPL SS GCG

So what is our current understanding of the Universe?

- If phantom is forbidden by theoretical prior, 6-parameter ΛCDM is favoured but if phantom is not forbidden, 2σ tension between early and late time experiments
- 6-parameter ΛCDM is by far the best model but there is apparent tension between high redshift and low redshift data

So what is our current understanding of the Universe?

- If phantom is forbidden by theoretical prior, 6-parameter ΛCDM is favoured but if phantom is not forbidden, 2σ tension between early and late time experiments
- 6-parameter ΛCDM is by far the best model but there is apparent tension between high redshift and low redshift data

So what is our current understanding of the Universe?

- If phantom is forbidden by theoretical prior, 6-parameter ΛCDM is favoured but if phantom is not forbidden, 2σ tension between early and late time experiments
- 6-parameter ΛCDM is by far the best model but there is apparent tension between high redshift and low redshift data
- Can this tension be attributed to unknown systematics?

Unlikely!

So what is our current understanding of the Universe?

- If phantom is forbidden by theoretical prior, 6-parameter ΛCDM is favoured but if phantom is not forbidden, 2σ tension between early and late time experiments
- 6-parameter ΛCDM is by far the best model but there is apparent tension between high redshift and low redshift data
- Can this tension be attributed to unknown systematics?

  Unlikely!
- Can it be due to degeneracy between H_0 and other parameters?

  Most likely yes

So what is our current understanding of the Universe?

- If phantom is forbidden by theoretical prior, 6-parameter ΛCDM is favoured but if phantom is not forbidden, 2σ tension between early and late time experiments
- 6-parameter ΛCDM is by far the best model but there is apparent tension between high redshift and low redshift data
- Can this tension be attributed to unknown systematics?

  Unlikely!
- Can it be due to degeneracy between H_0 and other parameters?

  Most likely yes
- Can it be due to lack of data at intermediate redshifts?

  Most likely yes

So what is our current understanding of the Universe?

- If phantom is forbidden by theoretical prior, 6-parameter ΛCDM is favoured but if phantom is not forbidden, 2σ tension between early and late time experiments
- 6-parameter ΛCDM is by far the best model but there is apparent tension between high redshift and low redshift data
- Can this tension be attributed to unknown systematics?

  Unlikely!
- Can it be due to degeneracy between H_0 and other parameters?

  Most likely yes
- Can it be due to lack of data at intermediate redshifts?

  Most likely yes

So, 6-parameter ΛCDM or 6+N parameter Universe?

Oops! Some more in the queue

- Parallel analysis : Planck13(WMAP9)+ HST+BAO+SNLS3
  ΛCDM is outside 2σ (1σ) confidence regime.
  Durrer et.al., JCAP:2013
- Alternative analysis : Discripancy attributed to the mismatch of value of H_0 due to degeneracy with other parameters ($\Delta N_{\mathrm{eff}}, r_s, Y_{\mathrm{BBN}} \ldots$)
  Riess et.al., JCAP:2016
- Some other datasets in tension with ΛCDM:
  PAN-STARRS1 data $\rightarrow 2.4\sigma$ Rest et.al., ApJ:2013
  Lyα BAO data ($z = 2.34$) $\rightarrow 2.5\sigma$
  Lensing amplitude $\rightarrow A_L = 1.22 \pm 0.10$ (Planck TT + low P)
  H0LiCOW(lensed quasars) $\rightarrow 4.2\sigma$ Suyu et.al., MNRAS:2016
  H0LiCOW XIII +R18 $\rightarrow 5.3\sigma$ Wong et.al., MNRAS:2020

Possible reason in current observations?

- H_0 tension

Bhattacharyya, Alam, Pandey, Das, SP: ApJ:2019

Dataset	Value: km/s/Mpc
WMAP9	69.7 ± 2.1
Riess 2011	72.8 ± 2.4
Planck 2013	67.3 ± 1.2
Efstathiou 2014	70.6 ± 3.3
Planck 2015	67.3 ± 1.0
Riess 2016	73.24 ± 1.74
Riess 2018	74.03 ± 1.42
Riess 2020	$75.8^{+5.2}_{-4.9}$

Possible reason in current observations?

- H_0 tension

Bhattacharyya, Alam, Pandey, Das, SP: ApJ:2019

Dataset	Value: km/s/Mpc
WMAP9	69.7 ± 2.1
Riess 2011	72.8 ± 2.4
Planck 2013	67.3 ± 1.2
Efstathiou 2014	70.6 ± 3.3
Planck 2015	67.3 ± 1.0
Riess 2016	73.24 ± 1.74
Riess 2018	74.03 ± 1.42
Riess 2020	$75.8^{+5.2}_{-4.9}$

- σ_8 ($8h^{-1}$Mpc) tension

Dataset	Value: $S_8 = \sigma_8\sqrt{\Omega_{0m}/0.3}$
Planck 2015	0.851 ± 0.013
Clusters (X-ray)	0.745 ± 0.039
Weak lensing	0.75 ± 0.04
DES Y3 (2021)	0.784 ± 0.013

Generic Parametrization Scheme

- Consider most generic situation: two fluids (DM-DE), interacting, both perturbed
- Write down background and perturbation equations
- Parametrization: $\{w_{DM,eff}\,, w_{DE,eff}\,, C^2_{sDM,eff}\,, C^2_{sDE,eff}\}$
- Boils down to different models for specific choice
  - $w_{DM,eff} = 0, w_{DE,eff} = -1, C^2_{sDM,eff} = 0, C^2_{sDE,eff} = 0$ $\Rightarrow$ ΛCDM
  - $w_{DM,eff} = 0, w_{DE,eff} > -1, C^2_{sDM,eff} = 0, C^2_{sDE,eff} = 1$ $\Rightarrow$ non-phantom, w_zCDM
  - $w_{DM,eff} = 0, w_{DE,eff} < -1, C^2_{sDM,eff} = 0, C^2_{sDE,eff} = 1$ $\Rightarrow$ phantom, w_zCDM
  - $w_{DM,eff} = 0, w_{DE,eff} < -1, C^2_{sDM,eff} = 0, C^2_{sDE,eff} \neq 1$ $\Rightarrow$ modified gravity
  - $w_{DM,eff} \neq 0, w_{DE,eff} = -1, C^2_{sDM,eff} = 0, C^2_{sDE,eff} = 1$ $\Rightarrow$ warm dark matter
  - $w_{DM,eff} \neq 0, w_{DE,eff} < -1 or > -1, C^2_{sDM,eff} \neq 0, C^2_{sDE,eff} \neq 1$ $\Rightarrow$ interacting DM-DE
- Constrain these parameters from data

Parameters (Planck15/ Planck+R16/ Planck+BSH)

Phantom

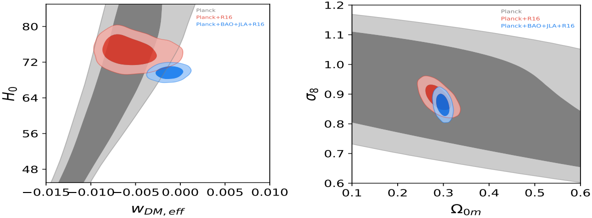
Data	Model	Ω_{0m}	H_0	σ_8	w_0	w_a	$w_{\mathrm{DM,eff}}$	$c^2_{\mathrm{sDE,eff}}$	χ^2_{bf}	$\chi^2_{\Lambda CDM} - \chi^2_{\mathrm{bf}}$
Planck	ΛCDM	$0.30^{+0.02}_{-0.02}$	$68.1^{+1.2}_{-1.2}$	$0.85^{+0.03}_{-0.02}$	-1	0	0	1	781.07	0
	CPLCDM	$0.19^{+0.02}_{-0.04}$	$88.4^{+11.6}_{-3.7}$	$1.02^{+0.08}_{-0.06}$	$-1.5^{+0.3}_{-0.3}$	$-0.13^{+0.27}_{-0.03}$	0	1	779.83	-1.24
	$+w_{\mathrm{DM,eff}}$	$0.62^{+0.32}_{-0.59}$	$66.7^{+32.0}_{-11.1}$	$0.80^{+0.27}_{-0.13}$	$-2.0^{+1.0}_{-1.0}$	$-0.45^{+0.48}_{-1.50}$	$-0.0075^{+0.005}_{-0.004}$	1	778.26	-2.81
	$+c^2_{\mathrm{sDE,eff}}$	$0.68^{+0.32}_{-0.66}$	$64.9^{+31.7}_{-13.5}$	$0.79^{+0.28}_{-0.14}$	$-2.0^{+1.0}_{-1.0}$	$-0.42^{+0.49}_{-1.58}$	$-0.0078^{+0.005}_{-0.004}$	$1.03^{+0.84}_{-0.45}$	778.88	-2.19
Planck +R16	ΛCDM	$0.29^{+0.01}_{-0.01}$	$69.7^{+1.0}_{-1.0}$	$0.86^{+0.02}_{-0.02}$	-1	0	0	1	786.66	0
	CPLCDM	$0.26^{+0.01}_{-0.01}$	$74.0^{+1.7}_{-1.7}$	$0.90^{+0.02}_{-0.03}$	$-1.1^{+0.1}_{-0.1}$	$-0.27^{+0.46}_{-0.26}$	0	1	782.02	-4.64
	$+w_{\mathrm{DM,eff}}$	$0.29^{+0.02}_{-0.02}$	$74.5^{+2.1}_{-2.1}$	$0.88^{+0.03}_{-0.03}$	$-2.0^{+1.0}_{-1.0}$	$-0.96^{+1.10}_{-1.50}$	$-0.005^{+0.001}_{-0.003}$	1	777.65	-9.01
	$+c^2_{\mathrm{sDE,eff}}$	$0.29^{+0.02}_{-0.02}$	$74.5^{+2.1}_{-2.2}$	$0.89^{+0.02}_{-0.02}$	$-2.0^{+1.0}_{-1.0}$	$-0.94^{+1.05}_{-1.65}$	$-0.005^{+0.001}_{-0.002}$	$1.03^{+0.96}_{-0.34}$	780.19	-6.47
Planck +BSH	ΛCDM	$0.30^{+0.01}_{-0.01}$	$68.5^{+0.6}_{-0.6}$	$0.86^{+0.02}_{-0.02}$	-1	0	0	1	1490.66	0
	CPLCDM	$0.29^{+0.01}_{-0.01}$	$69.8^{+1.0}_{-1.0}$	$0.87^{+0.02}_{-0.02}$	$-1.05^{+0.05}_{-0.01}$	$-0.15^{+0.21}_{-1.10}$	0	1	1490.29	-0.37
	$+w_{\mathrm{DM,eff}}$	$0.30^{+0.01}_{-0.01}$	$69.7^{+1.0}_{-1.0}$	$0.86^{+0.02}_{-0.02}$	$-1.06^{+0.06}_{-0.01}$	$-0.33^{+0.39}_{-0.19}$	$-0.0012^{+0.001}_{-0.001}$	1	1488.14	-2.52
	$+c^2_{\mathrm{sDE,eff}}$	$0.30^{+0.01}_{-0.01}$	$69.7^{+1.0}_{-1.0}$	$0.86^{+0.02}_{-0.02}$	$-1.06^{+0.06}_{-0.01}$	$-0.34^{+0.40}_{-0.18}$	$-0.0012^{+0.001}_{-0.001}$	$1.02^{+0.98}_{-1.02}$	1488.83	-1.83

Non-phantom

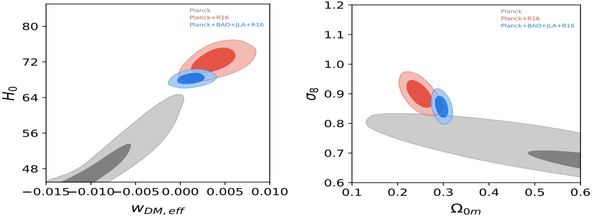
Data	Model	Ω_{0m}	H_0	σ_8	$w_{0,\mathrm{eff}}$	$w_{a,\mathrm{eff}}$	$w_{\mathrm{DM,eff}}$	$c^2_{\mathrm{sDE,eff}}$	χ^2_{bf}	$\chi^2_{\Lambda CDM} - \chi^2_{\mathrm{bf}}$
Planck	ΛCDM	$0.30^{+0.02}_{-0.02}$	$68.1^{+1.2}_{-1.2}$	$0.85^{+0.03}_{-0.02}$	-1	0	0	1	781.07	0
	CPLCDM	$0.37^{+0.03}_{-0.05}$	$62.5^{+4.0}_{-2.7}$	$0.80^{+0.04}_{-0.03}$	$-0.82^{+0.14}_{-0.18}$	$0.03^{+0.22}_{-0.22}$	0	1	782.75	1.68
	$+w_{\mathrm{DM,eff}}$	$1.06^{+0.31}_{-0.47}$	$44.0^{+4.3}_{-7.5}$	$0.60^{+0.05}_{-0.08}$	$-0.68^{+0.35}_{-0.32}$	$0.16^{+0.36}_{-0.40}$	$-0.012^{+0.004}_{-0.006}$	1	782.63	1.56
	$+c^2_{\mathrm{sDE,eff}}$	$1.03^{+0.33}_{-0.43}$	$44.5^{+3.7}_{-8.0}$	$0.60^{+0.04}_{-0.09}$	$-0.68^{+0.05}_{-0.35}$	$0.16^{+0.36}_{-0.40}$	$-0.012^{+0.003}_{-0.006}$	$0.98^{+1.02}_{-0.98}$	780.58	-0.49
Planck +R16	ΛCDM	$0.29^{+0.01}_{-0.01}$	$69.7^{+1.0}_{-1.0}$	$0.86^{+0.02}_{-0.02}$	-1	0	0	1	786.66	0
	CPLCDM	$0.29^{+0.01}_{-0.01}$	$68.6^{+1.3}_{-1.1}$	$0.85^{+0.02}_{-0.02}$	$-0.97^{+0.01}_{-0.03}$	$0.03^{+0.04}_{-0.06}$	0	1	788.97	2.31
	$+w_{\mathrm{DM,eff}}$	$0.25^{+0.02}_{-0.02}$	$72.2^{+1.8}_{-1.8}$	$0.89^{+0.03}_{-0.03}$	$-0.92^{+0.01}_{-0.08}$	$0.05^{+0.48}_{-0.13}$	$0.004^{+0.001}_{-0.001}$	1	785.81	-0.85
	$+c^2_{\mathrm{sDE,eff}}$	$0.25^{+0.02}_{-0.02}$	$72.2^{+1.8}_{-1.8}$	$0.89^{+0.03}_{-0.03}$	$-0.92^{+0.01}_{-0.08}$	$0.05^{+0.10}_{-0.59}$	$0.004^{+0.001}_{-0.002}$	$1.94^{+0.06}_{-1.94}$	785.73	-0.93
Planck +BSH	ΛCDM	$0.30^{+0.01}_{-0.01}$	$68.5^{+0.6}_{-0.6}$	$0.86^{+0.02}_{-0.02}$	-1	0	0	1	1490.66	0
	CPLCDM	$0.30^{+0.01}_{-0.01}$	$67.8^{+0.7}_{-0.7}$	$0.85^{+0.02}_{-0.02}$	$-0.97^{+0.01}_{-0.03}$	$0.04^{+0.04}_{-0.08}$	0	1	1493.36	2.70
	$+w_{\mathrm{DM,eff}}$	$0.30^{+0.01}_{-0.01}$	$68.2^{+0.8}_{-0.8}$	$0.85^{+0.02}_{-0.02}$	$-0.96^{+0.01}_{-0.04}$	$0.10^{+0.07}_{-0.14}$	$0.0012^{+0.001}_{-0.001}$	1	1491.01	0.35
	$+c^2_{\mathrm{sDE,eff}}$	$0.30^{+0.01}_{-0.01}$	$68.2^{+0.8}_{-0.8}$	$0.86^{+0.02}_{-0.02}$	$-0.96^{+0.01}_{-0.04}$	$0.09^{+0.06}_{-0.14}$	$0.0012^{+0.001}_{-0.001}$	$0.98^{+1.02}_{-0.98}$	1491.59	0.93

Major results (Planck+BSH)

Phantom

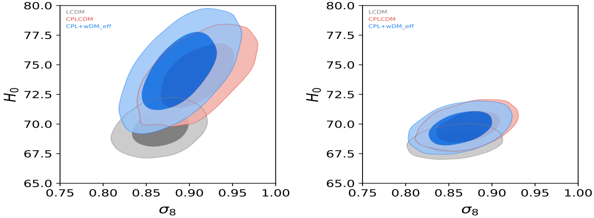


Non-phantom

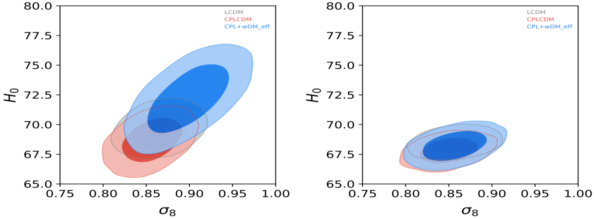


Correlation between H_0 and σ_8

Phantom



Non-phantom



Strong positive correlation between H_0 and σ_8 is generic to present cosmological data

Looking towards future

- CMB missions: LiteBIRD/COrE/CMB-S4/QUaD...
- LSS missions: EUCLID/Gaia...
- Hubble Parameter: JWST
- 21-cm missions: EDGES/SARAS/SKA...
- Thirty Meter Telescope: TMT
- GW missions: LIGO A+/LISA/EINSTEIN...
- New probes....

Future prospects: GW mission eLISA

Baral, Roy, SP: MNRAS:2020

SNIa data: Standard Candles. GW data: Standard Sirens

EOS vs null diagnostics : Sahni et.al.(2008)

$$E(x) = \sqrt{\Omega_{0m} x^{3(1+w_{\rm dm})} + (1-\Omega_{0m}) x^{3(1+w_0+\frac{w_a(x-1)}{x})}}$$

$$Om(x) = \frac{[E(x)]^2 - 1}{x^{3(1+w_{\rm dm})} - 1}$$

$Om(x_1) > Om(x_2) \rightarrow$ phantom
$Om(x_1) < Om(x_2) \rightarrow$ quintessence
$Om(x_1) = Om(x_2) \rightarrow \Lambda$CDM

Fisher matrix forecast analysis

$$F_{ij} = \sum_{n=\{z\}} \frac{1}{\sigma_n^2} \frac{\partial D_L(z_n)}{\partial \theta_i} \frac{\partial D_L(z_n)}{\partial \theta_j}$$

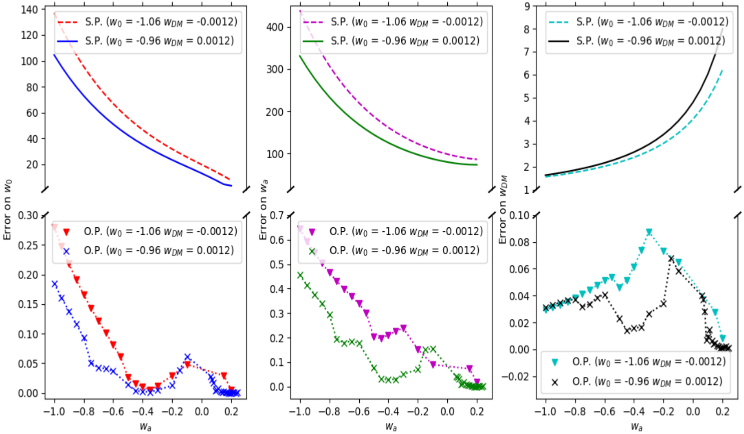
Square root of diagonal elements of covariance matrix $\sqrt{(F^{-1})_{ii}}$ is $1 - \sigma$ error

Fisher matrix forecast analysis

$$F_{ij} = \sum_{n=\{z\}} \frac{1}{\sigma_n^2} \frac{\partial D_L(z_n)}{\partial \theta_i} \frac{\partial D_L(z_n)}{\partial \theta_j}$$

Square root of diagonal elements of covariance matrix $\sqrt{(F^{-1})_{ii}}$ is $1-\sigma$ error

Planck+BSH (separately for phantom and non-phantom)



1-σ error on w_0, w_a and $w_{\rm dm}$ for phantom fiducial values

		Fiducial Values			Error using Stan. Param.			Error using *Om* Param.		
Data	Model	w_0	w_a	$w_{\rm dm}$	Δw_0	Δw_a	$\Delta w_{\rm dm}$	Δw_0	Δw_a	$\Delta w_{\rm dm}$
Planck	w_0CDM	-1	-	-	0.34	-	-	7.5e-4	-	-
+	CPLCDM	-1.1	-0.27	-	11	29	-	3.5e-3	0.016	-
R16	IDE	-2.0	-0.96	-0.005	1800	6800	1.2	6.2	11	0.014
Planck	w_0CDM	-1	-	-	0.35	-	-	4.8e-4	-	-
+	CPLCDM	-1.05	-0.15	-	7.8	22	-	6.2e-4	0.017	-
BSH	IDE	-1.06	-0.33	-0.0012	41	150	2.6	5.4e-3	0.19	0.068

1-σ error on w_0, w_a and $w_{\rm dm}$ for non-phantom fiducial values

		Fiducial Values			Error using Stan. Param.			Error using *Om* Param.		
Data	Model	w_0	w_a	$w_{\rm dm}$	Δw_0	Δw_a	$\Delta w_{\rm dm}$	Δw_0	Δw_a	$\Delta w_{\rm dm}$
Planck	w_0CDM	-1	-	-	0.34	-	-	7.5e-4	-	-
+	CPLCDM	-0.97	0.03	-	4.5	13	-	0.0018	9.1e-4	-
R16	IDE	-0.92	0.05	0.004	7.0	58	5.9	0.013	0.006	0.001
Planck	w_0CDM	-1	-	-	0.35	-	-	4.8e-4	-	-
+	CPLCDM	-0.97	0.04	-	4.5	13	-	8.2e-4	5.1e-4	-
BSH	+IDE	-0.96	0.10	0.0012	7.4	75	6.0	4.6e-3	0.010	9.3e-3

Errors are much improved for null diagnostics in eLISA

Further analysis with realistic catalogues and improved computational tools: Shah, Bhaumik, Mukherjee, SP: in progress

Particle Physics extensions

What if Dark Matter & Neutrinos interact?

Paul, Chatterjee, Ghoshal, SP, JCAP:2021

2 possible scenarios:

1. DM-ν scattering

$$u \equiv \left[\frac{\sigma_{\Psi-\nu}}{\sigma_{\rm Th}}\right]\left[\frac{M_\Psi}{100\ {\rm GeV}}\right]^{-1}$$

2. DM annihilation+Sommerfeld enhancement

$$\frac{\langle \sigma v \rangle}{M_\Psi} = \Gamma a$$

Both DM and Neutrinos taking part in perturbations

Perturbation equations for DM-ν interaction

Evolution of density contrast and velocity divergence of DM and ν perturbations

$$
\begin{aligned}
\dot{\delta}_{\rm DM} &= -\theta_{\rm DM} + 3\dot{\phi} - \frac{\delta\langle\sigma v\rangle}{M_\Psi}\rho_{\rm DM} a - \frac{\langle\sigma v\rangle}{M_\Psi}\rho_{\rm DM}\delta_{\rm DM} a - \frac{\langle\sigma v\rangle}{M_\Psi}\rho_{\rm DM} a\psi \,, \\
\dot{\theta}_{\rm DM} &= k^2\psi - \mathcal{H}\theta_{\rm DM} - S^{-1}\dot{\mu}(\theta_{\rm DM} - \theta_\nu) + 2\frac{\langle\sigma v\rangle}{M_\Psi}\rho_{\rm DM}\theta_{\rm DM} a \,, \\
\dot{\delta}_\nu &= -\frac{4}{3}\theta_{\rm DM} + 4\dot{\phi} + \frac{\delta\langle\sigma v\rangle}{M_\Psi}\frac{\rho^2_{\rm DM}}{\rho_\nu} a + \frac{\langle\sigma v\rangle}{M_\Psi}\frac{\rho^2_{\rm DM}}{\rho_\nu}(2\delta_{\rm DM} - \delta_\nu)\, a + \frac{\langle\sigma v\rangle}{M_\Psi}\frac{\rho^2_{\rm DM}}{\rho_\nu} a\psi, \\
\dot{\theta}_\nu &= k^2\psi + k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right) - \dot{\mu}(\theta_\nu - \theta_{\rm DM}) - a\frac{\langle\sigma v\rangle}{M_\Psi}\frac{\rho^2_{\rm DM}}{\rho_\nu}\left(\frac{3}{4}\theta_{\rm DM} + \theta_\nu\right)
\end{aligned}
$$

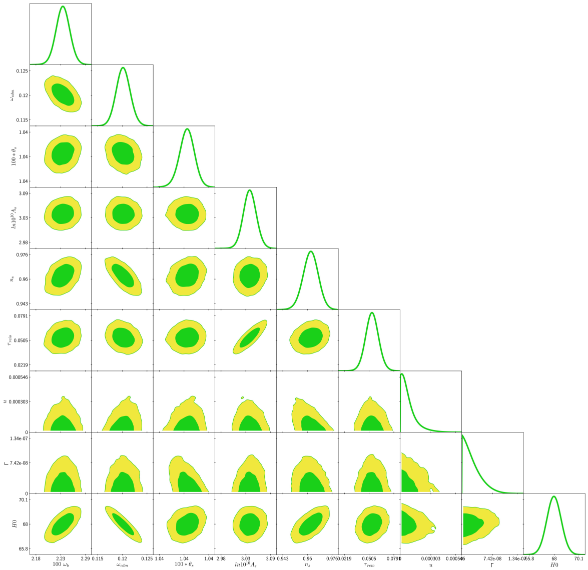
$\dot{\mu} \equiv a\,\sigma_{\Psi-\nu}\, c\, n_{\rm DM}$, $S = \frac{3}{4}(\rho_{\rm DM}/\rho_\nu)$.

6+2 parameter description

$(\{\omega_{\rm b},\ \omega_{\rm cdm},\ \theta_{\rm s},\ A_{\rm s},\ n_{\rm s},\ \tau_{\rm reio},\ u,\ \Gamma\})$

Results for `Planck18`

High-ℓ TT+TE+EE, low-ℓ TT, low-ℓ EE



Constraints from Planck18

High-ℓ TT+TE+EE, low-ℓ TT, low-ℓ EE

Parameter	best-fit	mean$\pm\sigma$	95% lower	95% upper
$100~\omega_b$	2.239	$2.239^{+0.015}_{-0.016}$	2.21	2.269
ω_{cdm}	0.1201	$0.1204^{+0.0014}_{-0.0015}$	0.1176	0.1233
$100 * \theta_s$	1.042	$1.042^{+0.00038}_{-0.00034}$	1.041	1.042
$ln10^{10}A_s$	3.04	$3.045^{+0.016}_{-0.017}$	3.013	3.077
n_s	0.9638	$0.9618^{+0.0055}_{-0.005}$	0.9514	0.9721
τ_{reio}	0.05354	$0.05373^{+0.0074}_{-0.008}$	0.03856	0.06937
u	–	$0.0001003~(1-\sigma~\text{upper})$	–	0.0002373
Γ	–	$3.204\times10^{-8}~(1-\sigma~\text{upper})$	–	6.821×10^{-8}
$H0$	67.96	$67.95^{+0.62}_{-0.66}$	66.7	69.22

Constraints from Planck18

High-ℓ TT+TE+EE, low-ℓ TT, low-ℓ EE

Parameter	best-fit	mean$\pm\sigma$	95% lower	95% upper
$100\ \omega_b$	2.239	$2.239^{+0.015}_{-0.016}$	2.21	2.269
ω_{cdm}	0.1201	$0.1204^{+0.0014}_{-0.0015}$	0.1176	0.1233
$100 * \theta_s$	1.042	$1.042^{+0.00038}_{-0.00034}$	1.041	1.042
$ln10^{10}A_s$	3.04	$3.045^{+0.016}_{-0.017}$	3.013	3.077
n_s	0.9638	$0.9618^{+0.0055}_{-0.005}$	0.9514	0.9721
τ_{reio}	0.05354	$0.05373^{+0.0074}_{-0.008}$	0.03856	0.06937
u	–	0.0001003 $(1-\sigma$ upper)	–	0.0002373
Γ	–	3.204×10^{-8} $(1-\sigma$ upper)	–	6.821×10^{-8}
$H0$	67.96	$67.95^{+0.62}_{-0.66}$	66.7	69.22

Constrains DM-ν interaction but only the upper bound

Future prospects: 21-cm mission SKA

Dey, Paul, SP, arXiv:2207.02451 [astro-ph.CO]

Constraints form Reionization and Non-linear Matter PS

- 50% ionization condition at $z = 8.0$

DM-ν interaction Model (u)	$N_{\rm ion}$	x_{HI}
0.0 (ΛCDM)	23.21	0.5
5.0×10^{-10}	24	0.51
8.8×10^{-8}	300	0.51
6.6×10^{-7}	500	0.51

Table of u against $N_{\rm ion}$ (number of photons entering IGM per baryon in collapsed objects) and x_{HI}(neutral hydrogen fraction).

Future prospects: 21-cm mission SKA

Dey, Paul, SP, arXiv:2207.02451 [astro-ph.CO]

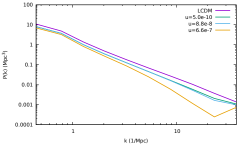
Constraints form Reionization and Non-linear Matter PS

- 50% ionization condition at $z = 8.0$

DM-ν interaction Model (u)	$N_{\rm ion}$	x_{HI}
0.0 (ΛCDM)	23.21	0.5
5.0×10^{-10}	24	0.51
8.8×10^{-8}	300	0.51
6.6×10^{-7}	500	0.51

Table of u against $N_{\rm ion}$ (number of photons entering IGM per baryon in collapsed objects) and x_{HI}(neutral hydrogen fraction).

- Non-linear Matter PS at $z = 8.0$



Increase in u suppresses power (as expected), generates a lesser number of DM halo and less ionizing radiation

N-body simulation and HI maps

- Consider $z = 8.0$ for 50% ionization condition
- Take the linear PS as the initial condition for N-body simulation
- Simulate the HI map and HI PS

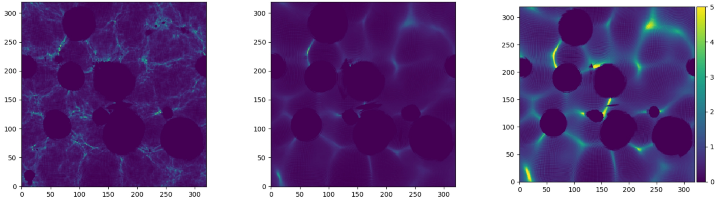
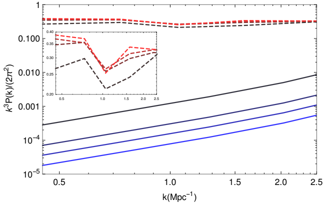


Figure: Two dimensional sections of the simulated HI map at $z = 8.0$. The above figure is for $u = 6.6 \times 10^{-7}$, $u = 8.8 \times 10^{-8}$ and 5.0×10^{-10}.

u is constrained further to $u \leq 6.6 \times 10^{-7}$ by Reionization

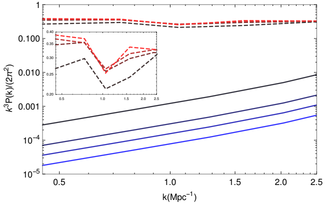
Error Analysis and Prospects of Detection in SKA1-Low



Dashed lines give 21 cm brightness temperature fluctuations PS for ΛCDM and $u = 5 \times 10^{-10},\ 8.8 \times 10^{-8},\ 6.6 \times 10^{-7}$ (darker shade denotes lower u). Solid blue lines denote noise PS generated from SKA1-Low for 128, 512, 1000, 2000 hours of observation (darker shade denotes lower observation time).

Prospects of detection of any DM-ν interaction upto to $u \leq 6.6 \times 10^{-7}$ in SKA1-Low is quite high

Error Analysis and Prospects of Detection in SKA1-Low



Dashed lines give 21 cm brightness temperature fluctuations PS for ΛCDM and $u = 5 \times 10^{-10},\ 8.8 \times 10^{-8},\ 6.6 \times 10^{-7}$ (darker shade denotes lower u). Solid blue lines denote noise PS generated from SKA1-Low for 128, 512, 1000, 2000 hours of observation (darker shade denotes lower observation time).

Prospects of detection of any DM-ν interaction upto to $u \leq 6.6 \times 10^{-7}$ in SKA1-Low is quite high

What if 6 parameter + w_{DE} + DM-ν interaction + $N_{\rm eff}$ + $\Sigma m_\nu + \ldots$? : Dey, Paul, Paul, Dutta Banik, SP: in progress

Take-home message

The point is not to pocket the truth but to chase it