

The Impact of Public Ownership in the Lending Sector

by

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Abstract

We examine the effects of increased government ownership of suppliers in the lending sector, which induces increased concern with total welfare and reduced concern with profit. Such increased ownership of a lender can have unanticipated effects. For instance, it can increase lender profit. Furthermore, borrower welfare often declines as government ownership increases in a lender with a relatively limited ability to discern the true quality of borrowers' projects. In addition, there are settings in which increased government ownership of a lender has no impact on either lender profit or borrower welfare.

Keywords: Public ownership, lending policy.

JEL Classifications: G24, H10, L32.

June 2011

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1 Introduction.

In their review of the extent of government ownership of banks, La Porta et al. (2002, p. 290) conclude that “government ownership of banks is large and pervasive around the world.” The authors report that, on average, 42% of the equity in the largest banks in a country is owned by the government.¹ Furthermore, the recent financial crisis has raised the specter of expanded government participation in the financial sector. Despite the pronounced scope of prevailing and likely future government activity in the lending sector, our knowledge of the impact of government ownership of lenders is limited. Although the literature provides many studies of competition between public and private suppliers of generic retail products, the literature offers few explicit analyses of the interaction between public and private lenders. The purpose of this research is to help fill this void.

We analyze the impact of government (public) ownership in a canonical setting where two lenders compete to finance the projects of borrowers who have privileged knowledge of the quality of their projects. The lenders announce the terms on which they will finance approved projects and screen the projects of potential borrowers. If only one lender approves a borrower’s project, the borrower accepts the terms offered by the lender, provided they generate an expected payoff in excess of the borrower’s transactions costs. If a borrower’s project is approved by both lenders, the borrower accepts the terms that provide the highest net return.²

Increased public ownership of a lender increases the lender’s concern with total industry welfare and reduces its concern with its own profit. Consequently, one might suspect that increased public ownership would induce a lender to offer more generous financing terms to

¹La Porta et al. (2002, p. 267). This average reflects considerable variation across countries. The authors report that in 1995 government ownership of the ten largest commercial and development banks in the country ranged from 0% in Canada, Japan, the UK, and the US to 12% in Australia, 17% in France, 65% in Israel, 84% in Poland, 85% in India, 89% in Egypt, 99% in China, and 100% in Iran and Syria (La Porta et al., 2002, pp. 272-273). Zhou et al. (2009) describe the extensive government participation that prevails in the Chinese banking sector.

²A borrower cannot pursue his project if it is not approved by a lender. A borrower will choose not to pursue his approved project if the highest expected payoff he anticipates from the project is less than the relevant transactions costs.

borrowers at the expense of its own profit. This will be the case when the lender's ability to discern the true quality of borrowers' projects is relatively pronounced. However, increased public ownership of a lender with relatively limited screening ability will induce the lender to set less generous financing terms in order to render these terms more similar to those of the rival lender. The more congruent terms encourage borrowers to secure financing from the closest lender, thereby reducing borrower transactions costs (i.e., Hotelling transportation costs) and increasing total welfare. The less generous financing terms that result from increased public ownership serve to increase the equilibrium profit of lenders.

Increased public ownership of a lender and the associated increased concern with total welfare typically generates higher equilibrium welfare in our model. However, there are instances in which increased public ownership has no impact on equilibrium financing terms, lender profit, or borrower welfare. This is the case when: (i) the two lenders are equally adept at screening the projects of potential borrowers; or (ii) one of the lenders is fully publicly owned. In case (i), the equally situated lenders offer the same financing terms to approved borrowers. Consequently, all borrowers obtain financing from the closest lender, which ensures that borrower transactions costs are minimized and industry welfare is maximized. When welfare is maximized, increased public ownership does not alter a lender's behavior, and so equilibrium outcomes do not change.

In case (ii), the lender that is fully publicly owned will set its financing terms to match those of the rival lender in order to minimize borrower transactions costs and maximize industry welfare. Again, then, increased public ownership in the rival lender will not change its behavior because the prevailing behavior already maximizes industry welfare. Therefore, the increased public ownership does not alter lender profit, borrower welfare, or total welfare in this case either.

We develop these conclusions and others as follows. Section 2 describes the key elements of our model. Section 3 presents our main findings. Section 4 illustrates the magnitudes of the key qualitative effects in our model in selected settings. Section 5 provides conclud-

ing observations and discusses some extensions of the analysis. The proofs of all formal conclusions are presented in the Appendix.

Before proceeding, we explain briefly how our research contributes to the literature. As noted at the outset, many studies examine the outcome of competition between publicly-owned and privately-owned suppliers of generic retail products. These studies analyze, for example, different forms of industry competition (e.g., De Fraja and Delbono, 1989; Crémer et al., 1991; Pal, 1998), the optimal extent of public ownership (e.g., Crémer et al., 1989; Matsumura, 1998), the effect of such “mixed oligopoly” on industry innovation (e.g., Delbono and Denicolo, 1993; Ishibashi and Matsumura, 2006), and how government ownership of domestic firms affects the terms of international trade (e.g., Fjell and Pal, 1996; Pal and White, 1998).³ Our investigation differs from these studies in part because we focus on settings in which lenders compete to finance projects in the presence of asymmetric information. The screening abilities of lenders play a central role in determining the effects of increased public ownership in our model. Our study also differs from many of its predecessors by allowing partial government ownership of some or all industry producers.⁴

Our focus on screening in the presence of asymmetric information also distinguishes our work from most studies of government ownership in the banking sector. To illustrate, Zhou et al. (2009) consider duopoly competition for deposits between a private bank and a public bank, abstracting from the adverse selection considerations that are central in our model. The theoretical model that underlies Barros and Modesto (1999)’s empirical analysis of competition in the Portuguese banking sector similarly abstracts from these adverse selection problems and from lender screening of borrowers’ projects. Andrianova et al. (2008) examine how the relatively pronounced propensity of profit-maximizing enterprises to abscond with the deposits they receive affects their competition with government enterprises.

Our analysis may be most similar to De Fraja (2009)’s investigation of duopoly com-

³See De Fraja and Delbono (1990) for a useful review of the literature.

⁴Fershtman (1990), Matsumura (1998), and Kumar and Saha (2008) also admit partial public ownership.

petition between a public and a private lender. Our study differs from his in at least four important respects. First, we analyze competition for a continuum of borrowers rather than for a single borrower. Second, we allow for partial government ownership of one or both lenders and analyze the effects of varying degrees of ownership. Third, the lenders in our model can have a range of screening abilities, whereas De Fraja assumes that a lender either knows the quality of a lender’s project or has no private knowledge of this quality. Fourth, our model generates a unique equilibrium, whereas multiple equilibria can arise in De Fraja’s model where the borrower approaches lenders sequentially and the second lender can observe the financing decision of the first lender. In practice, it can be difficult for a lender to determine whether rival lenders have declined to finance a borrower’s project.

2 Elements of the Model.

We analyze competition between two lenders to finance the projects of potential borrowers. These risk-neutral borrowers have no wealth and so must secure funding from a lender in order to pursue their projects. Each project requires an up-front investment of $I > 0$. The project subsequently either succeeds or fails. A borrower has either a high quality project or a low quality project. A high quality project has a higher probability of success ($p_H \in (0, 1)$) than does a low quality project ($p_L \in (0, p_H)$). A project generates payoff $V > 0$ when it succeeds and 0 when it fails. A high quality project generates positive net surplus, whereas a low quality project generates negative net surplus, i.e., $p_L V - I < 0 < p_H V - I$.

Each borrower knows the quality of his project. The lenders do not share this knowledge. Initially, the lenders know only that the fraction $\phi_H \in (0, 1)$ of borrowers have high quality projects and the complementary fraction $\phi_L = 1 - \phi_H$ have low quality projects. Each lender subsequently observes a private signal ($s \in \{p_L, p_H\}$) about the quality (i.e., the success probability) of the project of each borrower that applies for funding. The signal of lender $i \in \{1, 2\}$ reveals the true project quality with probability $q_i \in (\frac{1}{2}, 1]$ and reports the incorrect project quality with probability $1 - q_i$. We will refer to q_i as lender i ’s screening accuracy. For simplicity, each lender’s screening accuracy is taken to be exogenous and

independent of its ownership structure.⁵ Each lender funds a borrower’s project if and only if the project produces a favorable signal ($s = p_H$).⁶

Borrowers potentially face initial application costs and ongoing transactions costs in securing financing for their projects. For expositional simplicity, we abstract from lender-specific initial application costs. In practice, the cost of preparing additional applications often is small once a borrower has assembled the information that is commonly requested on financing applications. Furthermore, the cost of this information gathering typically is small relative to the potential benefit of securing financing.

Ongoing transactions costs of financing can be more pronounced and can vary considerably across borrowers. These transactions costs also can vary with a lender’s monitoring policies and reporting requirements. Even in settings where the lender has no direct involvement in the borrower’s project, the costs of preparing and delivering progress reports (to a potentially distant lender) can be substantial and idiosyncratic. In settings where the lender has a more direct oversight or collaborative role in the project, the borrower’s transactions costs can include the costs of working directly with a lender whose preferred methods of operation may be similar to or quite distinct from those of the borrower.

The non-trivial and idiosyncratic transactions costs that borrowers experience when they interact with lenders are captured in standard Hotelling fashion. Borrowers with low quality projects (“ L borrowers”) and borrowers with high quality projects (“ H borrowers”) are both distributed uniformly on the $[0, 1]$ interval. Lender 1 is located at 0 and lender 2 is located at 1. The total number of borrowers is normalized to 1. A borrower located at point $x \in [0, 1]$ incurs transactions cost tx when he secures funding from lender 1. This borrower incurs transactions cost $t[1 - x]$ when he secures funding from lender 2.

When deciding whether to accept the financing offered by a lender, a borrower considers

⁵The concluding section offers some thoughts on settings with endogenous screening accuracies.

⁶This policy is optimal for a lender as long as its screening accuracy is not too limited (i.e., as long as q_i is not too close to $\frac{1}{2}$) and as long as the fraction of high quality projects in the population is not too pronounced (i.e., as long as ϕ_H is not too close to 1).

both the associated transactions costs and the financing terms specified by the lenders. Because the borrowers have no wealth, a lender can only secure a payment from the borrower when the borrower’s project succeeds. $[1 - \gamma_i]V$ will denote the payment that lender i requires from the borrower when his project succeeds. $\gamma_i \in (0, 1)$, the fraction of the payoff from a successful project that lender i offers to the borrower, will be referred to as lender i ’s *sharing rate*. A lender optimally delivers no payment to the borrower when his project fails.⁷

The lenders set their sharing rates simultaneously and independently at the outset of their interaction with the borrowers. The borrowers then apply for financing. After observing its private signal about a borrower’s project, a lender informs the borrower whether it is willing to finance the borrower’s project. *Contested borrowers* – those who receive financing offers from both lenders – then decide which offer, if either, to accept. *Captive borrowers* – those who receive only one financing offer – decide whether to accept the offer.

A borrower will only pursue his project if his expected net return from doing so exceeds his reservation profit. We normalize the reservation profit of L borrowers to 0 and allow a potentially higher reservation profit for H borrowers ($R \geq 0$). This higher reservation profit might arise, for example, if the quality of a borrower’s project reflects in part the borrower’s innate skills and thus his expected financial return from self-employment.

We focus on the setting of primary interest in which all captive and contested borrowers undertake their projects and both lenders finance the projects of some contested borrowers in equilibrium. This outcome in which the lenders compete to serve contested borrowers rather than effectively acting as local monopoly lenders will arise when R and t are moderately large and q_1 and q_2 are sufficiently large.⁸ If R or t were very large, welfare and/or profit could be maximized when the projects of some H borrowers are not financed. If R and/or

⁷Among all feasible payment structures, this payment structure creates the strongest incentive for H borrowers to accept financing relative to the corresponding incentive for L borrowers. Consequently, this payment structure is optimal for each lender.

⁸Appendix B provides a formal statement of the relevant conditions. The conditions effectively are the counterpart to the conditions that ensure full market coverage in standard models of Hotelling competition.

t were close to 0, intense competition could make it unprofitable to serve distant contested borrowers. Relatively small values of q_1 and q_2 could generate relatively few contested H borrowers and an associated lender focus on serving captive borrowers.

For expositional simplicity, we also presume that Assumption 1 holds throughout the ensuing analysis.

Assumption 1. $\phi_H p_H - \phi_L p_L > 0$.

Assumption 1 ensures that the proportion of L borrowers in the population is not too pronounced. The concluding section reviews the key changes that arise in alternative settings.

The sharing rates that the lenders offer influence the number of projects that they ultimately finance. Lemma 1 specifies x_L and x_H , which are the locations of the contested L borrower and the contested H borrower, respectively, who is indifferent between accepting the financing offers of the two lenders when lender 1 sets sharing rate γ_1 and lender 2 sets sharing rate γ_2 .⁹

Lemma 1.

$$x_L = \frac{1}{2} + \frac{[\gamma_1 - \gamma_2] p_L V}{2t} \quad \text{and} \quad x_H = \frac{1}{2} + \frac{[\gamma_1 - \gamma_2] p_H V}{2t}. \quad (1)$$

Lemma 1 reflects the fact that a borrower at location $x \in [0, 1]$ whose project succeeds with probability p_j anticipates net payoff $p_j \gamma_1 V - t x$ when he secures financing from lender 1 and net payoff $p_j \gamma_2 V - t [1 - x]$ when he obtains funding from lender 2. The expressions for x_L and x_H in Lemma 1 reflect the value of x at which these two net payoffs are the same when $p_j = p_L$ and when $p_j = p_H$, respectively. Lemma 1 implies that each lender will serve one half of the contested borrowers if the two lenders set the same sharing rate. A lender will serve more than half of the contested borrowers if it sets a higher sharing rate than its rival.

Lender 1's expected profit when it sets sharing rate γ_1 and lender 2 sets sharing rate γ_2

⁹For expositional ease, we suppress the dependence of x_L and x_H on the sharing rates γ_1 and γ_2 in Lemma 1 and throughout the ensuing discussion.

is:

$$\begin{aligned} \pi_1(\gamma_1, \gamma_2) &= \phi_L q_2 [1 - q_1] [p_L (1 - \gamma_1) V - I] + \phi_H q_1 [1 - q_2] [p_H (1 - \gamma_1) V - I] \\ &+ \phi_L [1 - q_1] [1 - q_2] x_L [p_L (1 - \gamma_1) V - I] + \phi_H q_1 q_2 x_H [p_H (1 - \gamma_1) V - I]. \end{aligned} \quad (2)$$

The first of the four terms in equation (2) reflects lender 1's expected profit from captive L borrowers. Lender 1 secures a captive L borrower when it is the only lender to observe a favorable signal about the borrower's low quality project. This event occurs with probability $q_2 [1 - q_1]$. The second of the four terms in equation (2) reflects lender 1's corresponding expected profit from captive H borrowers. The last two terms in equation (2) reflect lender 1's expected profit from contested L borrowers and H borrowers, respectively. A borrower will be contested when both lenders observe a favorable signal about the borrower's project. This outcome occurs with probability $[1 - q_1] [1 - q_2]$ for a low quality project and with probability $q_1 q_2$ for a high quality project.

Lender 2's expected profit when it sets sharing rate γ_2 and lender 1 sets sharing rate γ_1 is defined in parallel fashion:

$$\begin{aligned} \pi_2(\gamma_1, \gamma_2) &= \phi_L q_1 [1 - q_2] [p_L (1 - \gamma_2) V - I] + \phi_H q_2 [1 - q_1] [p_H (1 - \gamma_2) V - I] \\ &+ \phi_L [1 - q_1] [1 - q_2] [1 - x_L] [p_L (1 - \gamma_2) V - I] + \phi_H q_1 q_2 [1 - x_H] [p_H (1 - \gamma_2) V - I]. \end{aligned} \quad (3)$$

The (expected) welfare of L borrowers when lender i offers sharing rate γ_i is:

$$\begin{aligned} W_L(\gamma_1, \gamma_2) &= \phi_L \left\{ q_2 [1 - q_1] \left[p_L \gamma_1 V - \frac{t}{2} \right] + q_1 [1 - q_2] \left[p_L \gamma_2 V - \frac{t}{2} \right] \right. \\ &\left. + [1 - q_1] [1 - q_2] \left[[x_L \gamma_1 + (1 - x_L) \gamma_2] p_L V - \frac{t(x_L)^2}{2} - \frac{t(1 - x_L)^2}{2} \right] \right\}. \end{aligned} \quad (4)$$

The first of the three terms in equation (4) reflects the expected welfare of captive L borrowers who secure financing from lender 1. This welfare is the difference between the expected monetary payoff for these borrowers ($p_L \gamma_1 V$) and their average transactions costs ($\frac{t}{2}$), weighted by the probability ($q_2 [1 - q_1]$) that only lender 1 offers to finance the project of an L borrower. The second term in equation (4) reflects the corresponding expected wel-

fare of captive L borrowers who secure financing from lender 2. The last term in equation (4) captures the expected welfare of contested L borrowers. This term reflects the expected payoff of the x_L contested L borrowers that secure funding from lender 1, the corresponding payoff of the $1 - x_L$ contested L borrowers that secure funding from lender 2, and the average transactions costs of these borrowers. As noted above, L borrowers receive financing offers from both lenders with probability $[1 - q_1][1 - q_2]$.

The corresponding (expected) welfare of H borrowers is:

$$W_H(\gamma_1, \gamma_2) = \phi_H \left\{ q_1 [1 - q_2] \left[p_H \gamma_1 V - \frac{t}{2} \right] + q_2 [1 - q_1] \left[p_H \gamma_2 V - \frac{t}{2} \right] + q_1 q_2 \left[[x_H \gamma_1 + (1 - x_H) \gamma_2] p_H V - \frac{t(x_H)^2}{2} - \frac{t(1 - x_H)^2}{2} \right] + [1 - q_1][1 - q_2] R \right\}. \quad (5)$$

The last term in equation (5) reflects the welfare of H borrowers whose projects are not financed.

A lender that is completely owned by private shareholders is assumed to maximize its expected profit. In contrast, a lender that is fully owned by the government is assumed to maximize total welfare, which is the sum of industry profit and borrower welfare.¹⁰ Formally, when lender 1 sets sharing rate γ_1 and lender 2 sets sharing rate γ_2 , total welfare is:

$$W(\gamma_1, \gamma_2) = \pi_1(\gamma_1, \gamma_2) + \pi_2(\gamma_1, \gamma_2) + W_L(\gamma_1, \gamma_2) + W_H(\gamma_1, \gamma_2). \quad (6)$$

We further assume that when the government owns the fraction $\alpha_i \in [0, 1]$ of lender i 's shares and private investors own the remaining shares, lender i will set γ_i to maximize:

$$\widetilde{W}_i(\gamma_1, \gamma_2) = \alpha_i W(\gamma_1, \gamma_2) + [1 - \alpha_i] \pi_i(\gamma_1, \gamma_2), \quad (7)$$

taking as given the sharing rate (γ_{-i}) set by the rival lender. Thus, we assume that lenders pursue the objectives of their shareholders in proportion to their ownership shares. The ensuing analysis will consider how industry outcomes change as α_1 and α_2 – hereafter referred

¹⁰The assumption that a government-owned firm seeks to maximize total welfare is standard in the literature. See, for example, Crémer et al (1989, 1991), Delbono and Denicolo (1993), Barros (1994), Anderson et al. (1997), Matsushima and Matsumura (2003), Ishibashi and Matsumura (2006), and De Fraja (2008). The concluding section considers the implications of alternative objectives for a government-owned firm.

to as the (extent of) public ownership of lenders 1 and 2, respectively – change.

3 Findings.

To characterize the equilibrium outcomes in this setting, we first identify the reaction functions of the two lenders. Lender i 's reaction function specifies the sharing rate (γ_i) that maximizes its objective (\widetilde{W}_i) given the sharing rate set by the rival lender (γ_{-i}).

Lemma 2. *The reaction function of lender i is:*

$$\gamma_i = \left[\frac{1}{2 - \alpha_i} \right] \gamma_{-i} + \left[\frac{1 - \alpha_i}{2 - \alpha_i} \right] D_i \quad \text{for } -i \neq i, \quad i, -i \in \{1, 2\} \quad (8)$$

where:

$$D_i = 1 - 2t \left[\frac{B_i}{A} \right] - [I + t] \frac{C}{A}; \quad (9)$$

$$A = [1 - q_1][1 - q_2] \phi_L (p_L V)^2 + q_1 q_2 \phi_H (p_H V)^2; \quad (10)$$

$$B_i = [1 - q_i] q_{-i} \phi_L p_L V + q_i [1 - q_{-i}] \phi_H p_H V; \quad \text{and} \quad (11)$$

$$C = [1 - q_1][1 - q_2] \phi_L p_L V + q_1 q_2 \phi_H p_H V. \quad (12)$$

Lemma 2 indicates that each lender increases its sharing rate as the rival lender increases its sharing rate. Furthermore, the rate ($\frac{1}{2 - \alpha_i}$) at which lender i increases γ_i as the sharing rate of its rival (γ_{-i}) increases is higher the more pronounced is α_i , the extent of public ownership of lender i . Increased public ownership renders a lender more intent on limiting aggregate borrower transactions costs in order to increase total welfare. The aggregate transactions costs of contested borrowers decline as more of them secure financing from the closest lender, which is facilitated when the two lenders set more similar sharing rates. (Recall Lemma 1.)

The equilibrium sharing rates for the lenders are identified by the intersection of the reaction functions specified in Lemma 2. These rates are identified in Lemma 3.

Lemma 3. *When $(\alpha_1, \alpha_2) \neq (1, 1)$, the equilibrium sharing rates are:*

$$\gamma_1^* = \frac{[1 - \alpha_2] D_2 + [1 - \alpha_1] [2 - \alpha_2] D_1}{[2 - \alpha_1] [2 - \alpha_2] - 1}; \quad \text{and} \quad (13)$$

$$\gamma_2^* = \frac{[1 - \alpha_1] D_1 + [1 - \alpha_2] [2 - \alpha_1] D_2}{[2 - \alpha_1] [2 - \alpha_2] - 1}. \quad (14)$$

If both lenders are fully publicly owned (so $\alpha_1 = \alpha_2 = 1$), the reaction function of both lenders is the 45° line in (γ_1, γ_2) space. Therefore, there are a continuum of equilibrium sharing rates. In each equilibrium, the two lenders set the same sharing rate. One immediate implication of Lemma 3 is presented in Corollary 1.

Corollary 1. $\gamma_i^* - \gamma_{-i}^* = \left[\frac{(1-\alpha_2)(1-\alpha_1)}{(2-\alpha_1)(2-\alpha_2)-1} \right] \frac{2t}{A} [q_{-i} - q_i] [\phi_H p_H - \phi_L p_L] V$ for $i, -i \in \{1, 2\}$ where $(\alpha_1, \alpha_2) \neq (1, 1)$.

Corollary 1 and Assumption 2 underlie Proposition 1, which explains how equilibrium sharing rates vary with the lenders' screening accuracies.

Proposition 1. $\gamma_i^* \geq \gamma_{-i}^*$ as $q_i \leq q_{-i}$ for all $\alpha_1, \alpha_2 \in [0, 1)$. Furthermore, $\gamma_1^* = \gamma_2^*$ when $\alpha_1 = 1$ and/or $\alpha_2 = 1$.

Proposition 1 reports that in the absence of full public ownership, the lender with the lowest screening accuracy sets the highest equilibrium sharing rate. This conclusion reflects the following considerations. Although a higher sharing rate can reduce a lender's profit by delivering a larger payment to borrowers when their projects succeed, it may increase the lender's profit by helping it to attract more contested borrowers. A lender with a relatively low screening accuracy will be particularly intent on attracting contested borrowers because a low screening accuracy implies that the lender's pool of captive borrowers will contain relatively many unprofitable L borrowers. Consequently, the lender with the lower screening accuracy finds it profitable to compete relatively aggressively for contested borrowers by setting a relatively high sharing rate.

Proposition 1 also reports that the two lenders will set the same sharing rate when one or both lenders are fully publicly owned. This conclusion reflects the fact that when a lender is fully publicly owned, it seeks to maximize total surplus. The lender achieves this objective

by setting the same sharing rate as its rival, which serves to minimize the transactions costs that borrowers incur in equilibrium.

Now consider how equilibrium sharing rates change as the extent of public ownership changes. Proposition 2 reports that equilibrium sharing rates decline as public ownership of the lender with the lowest screening accuracy increases. In contrast, equilibrium sharing rates increase as public ownership of the lender with the highest screening accuracy increases.

Proposition 2. $\frac{d\gamma_i^*}{d\alpha_i} \leq 0$ and $\frac{d\gamma_{-i}^*}{d\alpha_i} \geq 0$ as $q_i \leq q_{-i}$ for all $\alpha_1, \alpha_2 \in [0, 1)$, for $i = 1, 2$.

Proposition 2 reflects the fact that as public ownership of a lender increases, the lender moves its sharing rate closer to the rate set by the rival lender in order to limit equilibrium borrower transactions costs. Consequently, the lender with the relatively low screening accuracy that sets a relatively high sharing rate will reduce this rate as the extent of its public ownership increases. Recall from Lemma 2 that the sharing rates of the two lenders are strategic complements (Bulow et al., 1985). Therefore, as one lender reduces its sharing rate the rival lender also reduces its sharing rate, causing the equilibrium sharing rates of both lenders to decline as the extent of public ownership in the lender with the lower screening accuracy increases.

In contrast, the equilibrium sharing rates of both lenders increase as the extent of public ownership in the lender with the higher screening accuracy increases. Recall from Proposition 1 that the lender with the higher screening accuracy sets the lower sharing rate in order to avoid ceding excessive rent to borrowers. As this lender becomes more concerned with total welfare, it increases its sharing rate in order to reduce aggregate borrower transactions costs. The rival lender increases its sharing rate in response (recall Lemma 2) and so both lenders set higher sharing rates in equilibrium.

When the two lenders have the same screening accuracy, they set the same sharing rate in equilibrium. (Recall Proposition 1.) Aggregate borrower transactions costs are minimized in this event, and so total welfare is maximized. Consequently, a change in the extent of

public ownership of a lender will not alter equilibrium sharing rates when the two lenders have the same screening accuracy.

The welfare of L borrowers and H borrowers both increase as both lenders increase their sharing rates. In contrast, borrower welfare declines as both lenders reduce their sharing rates. Consequently, from Proposition 2, the welfare of L borrowers and H borrowers both increase as public ownership of the lender with the higher screening accuracy increases. In contrast, the welfare of L borrowers and H borrowers both decrease as public ownership of the lender with the lower screening accuracy increases. Furthermore, because equilibrium sharing rates do not change as the extent of public ownership changes when both lenders have the same screening accuracy, borrower welfare does not change as the extent of public ownership changes in this case. These conclusions are recorded formally as Proposition 3.

Proposition 3. $\frac{dW_j^*}{d\alpha_i} \gtrless 0$ as $q_i \gtrless q_{-i}$ for $j = L, H$.

Recall that when a lender is fully publicly owned, it sets the same sharing rate that its rival sets in order to minimize aggregate borrower transactions costs. When transactions costs are minimized, a change in the public ownership of the rival lender will not alter its preferred sharing rate.¹¹ Therefore, a change in the rival's public ownership will not affect equilibrium sharing rates, and so lender profit, borrower welfare, and total welfare will not change. This conclusion is recorded formally as Proposition 4.

Proposition 4. *Suppose $\alpha_i = 1$ for $i = 1$ or 2 . Then the equilibrium sharing rates, market shares, profits, borrower welfare, and total welfare do not change as α_{-i} changes, for $-i \neq i$.*

Together, Propositions 2, 3, and 4 identify two settings in which changes in public ownership do not alter equilibrium outcomes. For emphasis, these settings are summarized in

¹¹Formally, when $\alpha_i = 1$, an increase in α_{-i} causes lender $-i$'s reaction function to rotate around the same point of intersection with lender i 's reaction function. Consequently, equilibrium sharing rules do not change as α_{-i} changes.

Corollary 2.

Corollary 2. *A change in the public ownership of a lender has no effect on equilibrium sharing rates, lender profit, borrower welfare, or total welfare if the other lender is fully publicly owned or if both lenders have the same screening accuracy.*

When neither lender is fully publicly owned (so $\alpha_1 < 1$ and $\alpha_2 < 1$) and the lenders have distinct screening accuracies, a change in the public ownership of a lender will alter equilibrium sharing rates and thereby affect total welfare. The key impact of increased public ownership is to reduce the difference between the lenders' equilibrium sharing rates, as Lemma 4 reports.

Lemma 4. $\frac{d|\gamma_i^* - \gamma_{-i}^*|}{d\alpha_i} < 0$ for $i = 1, 2$, for all $\alpha_1, \alpha_2 \in [0, 1)$ when $q_1 \neq q_2$.

Lemma 4 reflects the fact that increased public ownership induces the lender with the lower (respectively, the higher) screening accuracy to increase (respectively, reduce) its screening rate. (Recall Propositions 1 and 2.) The rival lender follows suit (as indicated in Lemma 2), but to a more limited extent. (Observe from Lemma 2 that $\left. \frac{d\gamma_i^*}{d\gamma_{-i}^*} \right|_{d\alpha_1 = d\alpha_2 = 0} = \frac{1}{2 - \alpha_i} < 1$.) Therefore, the difference between the lenders' screening rates declines.

Less disparate sharing rates encourage contested borrowers to secure financing from the closest lender. The associated reduction in borrower transactions costs causes total welfare to increase, as Proposition 5 reports.

Proposition 5. $\frac{dW}{d\alpha_i} > 0$ for $i = 1, 2$ whenever $\alpha_1 < 1$, $\alpha_2 < 1$, and $q_1 \neq q_2$.

It is not surprising that increased public ownership of a lender and the lender's resulting increased concern with total welfare serves to increase total welfare. It may be more surprising that total welfare is somewhat insensitive to the particular lender in which the public ownership is secured, as Lemma 5 suggests.

Lemma 5. $W^* = G - \frac{A}{4t} [\gamma_2^* - \gamma_1^*]^2$, where G is independent of α_1 , α_2 , γ_1 , and γ_2 .

Lemma 5 reports that equilibrium total welfare varies with γ_1^* and γ_2^* to the extent that $\gamma_2^* - \gamma_1^*$ differ. This is the case because aggregate transactions costs are determined by the difference in equilibrium sharing rates. Due to the largely symmetric nature of the setting under consideration, the difference in equilibrium sharing rates does not change if the extent of public ownership of the two lenders is interchanged. (Recall Corollary 1.) Consequently, such an interchange does not affect total welfare, as Proposition 6 reports. The proposition refers to $W^*(\hat{\alpha}_1, \hat{\alpha}_2)$, which is the equilibrium level of total welfare when $\hat{\alpha}_i$ is the extent of public ownership of lender i , for $i = 1, 2$.

Proposition 6. $W^*(\hat{\alpha}_1, \hat{\alpha}_2) = W^*(\hat{\alpha}_2, \hat{\alpha}_1)$.

Proposition 6 identifies the sense in which total welfare is somewhat insensitive to the specific lender in which public ownership is secured. In particular, suppose a government decides to acquire a fixed ownership interest (e.g., $z\%$) in one of two private, profit-maximizing duopoly lenders. Also continue to assume that a $z\%$ public ownership share in lender i implies that lender i will seek to maximize $zW + [1 - z]\pi_i$. Then Proposition 6 implies that total welfare will be the same in the setting under consideration whether the government secures the $z\%$ ownership interest in lender 1 or in lender 2, regardless of the lenders' relative screening accuracies.

Although the exact locus of government ownership may not alter total welfare, it can affect the individual components of welfare substantially. As explained above, increased public ownership in the lender with the higher screening accuracy typically increases borrower welfare whereas an identical increase in ownership in the lender with the lower screening accuracy typically reduces borrower welfare. As the discussion in Section 4 reveals, these distinct qualitative effects can be of substantial magnitude.

Before illustrating the magnitudes of the effects of changes in public ownership on bor-

lower welfare, we consider the qualitative effects of such changes on lender profit. Recall that an increase in public ownership induces a lender to become relatively more concerned with total welfare and relatively less concerned with its own profit. Therefore, it would seem natural to expect that a lender's profit would decline as the extent of its public ownership increases. Proposition 7 reports that this is the case when the lender has the highest screening accuracy, whereas this may not be the case more generally.

Proposition 7. $\frac{d\pi_i^*}{d\alpha_i} < 0$ if $q_i > q_{-i}$. In contrast, $\frac{d\pi_i^*}{d\alpha_i} > 0$ if $q_i < q_{-i}$ and: (i) α_i is sufficiently small; and/or (ii) $|q_{-i} - q_i|$ is sufficiently small.¹²

Recall from Proposition 1 that when a lender (e.g., lender 1) has the highest screening accuracy, it will set the lowest sharing rate ($\gamma_1^* < \gamma_2^*$). Lender 1 will increase γ_1 toward γ_2 as its relative concern with total welfare increases. A direct effect of the higher sharing rate is to reduce the profit that lender 1 secures from captive borrowers. A potential countervailing effect is the reduction in $\gamma_2^* - \gamma_1^*$ that arises (recall Lemma 4), which allows lender 1 to capture more contested borrowers. However, in part because lender 1 serves a relatively large number of captive borrowers when it has a relatively high screening accuracy, the direct effect of the increase in γ_1 outweighs the potential countervailing effect, resulting in reduced profit for lender 1.

In contrast, when a lender (e.g., lender 1) has a relatively low screening accuracy, it will set a relatively high sharing rate ($\gamma_1^* > \gamma_2^*$). Lender 1 will reduce γ_1 toward γ_2 as it becomes more concerned with aggregate welfare. The reduced sharing rate increases the profit that lender 1 secures from captive borrowers. A potential countervailing effect is the reduction in $\gamma_1^* - \gamma_2^*$ that arises in equilibrium, which reduces the number of contested borrowers that lender 1 serves. Because lender 1 serves a relatively large number of contested borrowers when it sets a relatively high sharing rate, this countervailing effect can outweigh the direct effect. However, it will not do so when $q_2 - q_1$ or α_1 are sufficiently small. This is the case

¹²Formally, the relevant sufficient conditions are: (i) $\alpha_i \leq \frac{1}{2-\alpha_i}$; and (ii) $|q_{-i} - q_i| < \frac{[2-\alpha_i][2-\alpha_{-i}]-1}{4[1-\alpha_{-i}][2\alpha_i-1-\alpha_i\alpha_{-i}]}$.

because $\gamma_1^* - \gamma_2^*$ is small when $q_2 - q_1$ is small (recall Corollary 1), and so the reduction in $\gamma_1^* - \gamma_2^*$ is relatively small. Furthermore, the rate at which $\gamma_1^* - \gamma_2^*$ declines as α_1 increases is relatively small when α_1 is small (since $\frac{\partial}{\partial \alpha_1} \left| \frac{\partial |\gamma_1^* - \gamma_2^*|}{\partial \alpha_1} \right| > 0$).¹³

The impact of increased public ownership in one lender on the profit of the rival lender is less ambiguous. Recall from Proposition 1 that when a lender (say, lender 1) has a relatively low screening accuracy, it sets a relatively high sharing rate ($\gamma_1 > \gamma_2$). Lender 1 will reduce γ_1 toward γ_2 as it becomes more concerned with total welfare. Contested borrowers become more inclined to borrow from lender 2 when γ_1 declines, and so lender 2's equilibrium profit increases.

In contrast, when lender 1 has a relatively high screening accuracy, it will set a relatively low sharing rate ($\gamma_1 < \gamma_2$). Lender 1 will increase γ_1 toward γ_2 as it becomes more concerned with aggregate welfare. Contested borrowers become less inclined to borrow from lender 2 when γ_1 increases, and so lender 2's equilibrium profit declines.

Finally, recall from Proposition 1 that when the two lenders have the same screening accuracy, they will set the same sharing rate. Aggregate transportation costs are minimized, and thus total welfare is maximized, when the two lenders set the same sharing rate. Consequently, increased public ownership does not alter equilibrium sharing rates, and so equilibrium profit levels do not change. These conclusions are stated formally in Proposition 8.

Proposition 8. $\frac{d\pi_i^*}{d\alpha_{-i}} \geq 0$ as $q_i \geq q_{-i}$ for all $\alpha_1, \alpha_2 \in [0, 1)$, for $i \neq -i$, $i, -i \in \{1, 2\}$.

Having analyzed the key qualitative impacts of increased public ownership of lenders, we now illustrate the magnitudes of these effects in Section 4.

¹³Fershtam (1990) also shows that a partially nationalized firm may secure higher profit than a privately owned competitor.

4 Examples.

Before concluding, we demonstrate that some of the qualitative effects identified in Section 3 can be of considerable magnitude under plausible conditions. To do so, consider the setting where the payoff from a successful project is $V = 60$, the required up-front investment is $I = 20$, sixty percent of borrowers have high-quality projects (so $\phi_H = .60$ and $\phi_L = .40$), a high-quality project succeeds with probability $p_H = .75$, and a low-quality project succeeds with probability $p_L = .25$. Also suppose the borrowers' unit transactions cost is $t = 5$, the reservation profit of H borrowers is $R = 6$, lender 1 identifies the true quality of a borrower's project with probability $q_1 = .95$, and lender 2 identifies the true quality of a borrower's project with probability $q_2 = .75$.¹⁴

Consider the case where each lender initially is fully owned by private shareholders and so acts to maximize its profit. The first column of data (labeled " $\alpha_1 = 0$ ") in Table 1 presents equilibrium outcomes – sharing rates (γ_1^* and γ_2^*), lender profit (π_1^* and π_2^*), borrower welfare (W_L^* and W_H^*), and total welfare (W^*) – in this setting. The second column of data (labeled " $\alpha_1 = 0.5$ ") provides the corresponding outcomes when the government owns one-half of the shares of lender 1. The last column in Table 1 pertains to the case where the government secures full ownership of lender 1 and so the lender seeks to maximize total welfare.¹⁵

¹⁴It can be verified that for all values of $\alpha_1, \alpha_2 \in [0, 1]$, the equilibrium in this setting is the one in which all captive and contested borrowers undertake their projects and both lenders finance the projects of some contested borrowers.

¹⁵Lender 2 is fully owned by private shareholders (so $\alpha_2 = 0$) in all cases considered in Table 1.

	$\alpha_1 = 0$	$\alpha_1 = 0.5$	$\alpha_1 = 1$
γ_1^*	0.38258	0.39066	0.41488
γ_2^*	0.39873	0.40277	0.41488
π_1^*	2.34454	2.28157	2.05876
π_2^*	0.81454	0.70698	0.40127
W_L^*	0.40234	0.41036	0.43442
W_H^*	9.43666	9.60411	10.1149
W^*	13.0731	13.0780	13.0844

Table 1. The Effects of Public Ownership of Lender 1.

Table 1 reveals that increased public ownership of the lender with the highest screening accuracy induces the lender to increase its sharing rate toward the rate set by its rival. The higher equilibrium sharing rates increase borrower welfare and reduce lender profit. Relative to the case where both lenders are fully owned by private investors, full government ownership of lender 1 causes its profit to decline by approximately 12% (from 2.34454 to 2.05876). Government ownership of lender 1 has a more pronounced impact on the rival's profit: lender 2's profit declines by more than 50% (from 0.81454 to 0.40127). Borrowers benefit from the higher sharing rates induced by public ownership of lender 1. The welfare of L borrowers increases by nearly 8% while the welfare of H borrowers increases by approximately 7%. The increases in borrower welfare largely offset the reductions in lender profit, so total welfare is little changed by the increased public ownership of lender 1.

Table 2 summarizes the corresponding effects of increased public ownership of lender 2. The lower sharing rates induced by increased public ownership of the lender with the lower screening accuracy cause lender profit to increase and borrower welfare to decline. As Table 2 reveals, when full government ownership of lender 2 causes it to maximize total welfare rather than its own profit, the profit of both lenders increases by approximately 21%. The lower equilibrium sharing rates caused by increased public ownership of lender 2 reduce the

welfare of H borrowers by approximately 6.5% and they reduce the welfare of L borrowers by approximately twice that amount.

	$\alpha_1 = 0$	$\alpha_1 = 0.5$	$\alpha_1 = 1$
γ_1^*	0.38258	0.37854	0.36643
γ_2^*	0.39873	0.39066	0.36643
π_1^*	2.34454	2.46623	2.84825
π_2^*	0.81454	0.86620	0.98725
W_L^*	0.40234	0.38946	0.35084
W_H^*	9.43666	9.28113	8.82303
W^*	13.0731	13.0780	13.0844

Table 2. The Effects of Public Ownership of Lender 2.

In summary, Tables 1 and 2 illustrate the more general conclusion that public ownership of lenders can have a substantial impact on lender profit and borrower welfare. In particular, increased public ownership of a lender with a relatively high screening accuracy can reduce the lender's profit substantially and can reduce the profit of the rival profit-maximizing lender even more dramatically. Also, increased public ownership of a lender with a relatively low screening accuracy can cause substantial reductions in the welfare of all borrowers, those with high-quality projects and those with low-quality projects alike.

5 Extensions and Conclusions.

We have analyzed a streamlined model of the lending sector in order to examine the effects of increased government ownership of lenders. We found that the impact of such increased ownership is sensitive to the lenders' abilities to discern the quality of borrowers' projects. When borrowers with high quality projects are relatively common (so Assumption 1 holds), increased public ownership of a lender with a relatively pronounced screening accuracy typically induces the lender to offer more generous financing terms to borrowers,

which increases borrower welfare. In contrast, increased public ownership of a lender with a relatively limited screening accuracy typically reduces borrower welfare by inducing the lender to offer less generous terms to potential borrowers. These less generous terms for borrowers enhance the lenders' equilibrium profit. Consequently, expanded public ownership and its associated increased focus on total welfare and reduced focus on profit can generate higher lender profit and reduced borrower welfare.

Increased public ownership of a lender also can increase lender profit and reduce borrower welfare when a relatively large proportion of borrowers have low quality projects (so $\phi_L p_L > \phi_H p_H$). In this case, a lender with a relatively low screening accuracy will set a relatively low sharing rate in order to avoid attracting too many unprofitable L borrowers (see Corollary 1). Increased public ownership of the lender with the higher screening accuracy will cause the lender to reduce its sharing rate toward the rate of the rival lender in order to reduce borrower transactions costs. The reduced sharing rates that arise in equilibrium increase lender profit and reduce borrower welfare.

We also identified settings in which increased public ownership of a lender has no impact on lender profit or borrower welfare. This is the case when one lender is fully publicly owned or when the two lenders are equally adept at discerning the true quality of borrowers' projects.¹⁶

We conjecture that many of the key qualitative conclusions drawn above are robust to generalizations of our streamlined model. These generalizations include richer project payoff structures, more than two lenders, additional borrower types, and alternative screening technologies. However, distinct formulations can introduce new conclusions. For instance, suppose increased public ownership of a lender induces the lender to value borrower welfare more highly than it values its own profit (as in Matsumura (1998), for example). In this case, increased public ownership of a lender can reduce total welfare. A lender's pronounced concern with borrower welfare can induce it to offer particularly generous financing

¹⁶This conclusion holds regardless of whether Assumption 1 holds.

terms. Borrowers may incur substantial transactions costs in order to take advantage of these generous terms. The increased transactions costs can reduce total welfare.

Future research should consider extensions of our model in which lenders' locations and screening accuracies are endogenous. Increased concern with total welfare likely will increase a lender's preference for a more central location.¹⁷ Increased concern with borrower welfare and reduced concern with profit may reduce a lender's efforts to screen out unprofitable projects.¹⁸

Future research also might admit borrower moral hazard. In this case, sharing rates will affect both a borrower's choice among lenders and the borrower's effort to ensure the success of his project. A more complete model of the banking sector also would allow lenders to compete for both borrowers and depositors.¹⁹

¹⁷Crémer et al. (1991) and Matsushima and Matsumura (2003), among others, analyze competition between public and private suppliers with endogenous locations.

¹⁸Manove et al. (2001), Hauswald and Marquez (2003), Dell'Ariccia and Marquez (2006), and Bose et al. (2011), among others, analyze optimal screening activity by profit-maximizing lenders.

¹⁹See Saha and Sensarma (2009), for example.

Appendix A

Proof of Lemma 1.

An L borrower at location x is indifferent between securing financing from the two lenders when lender i offers sharing rate γ_i if:

$$\begin{aligned} p_L \gamma_1 V - t x &= p_L \gamma_2 V - t[1 - x] \Leftrightarrow t[1 - 2x] = p_L V [\gamma_2 - \gamma_1] \\ \Leftrightarrow 2 t x &= t - p_L V [\gamma_2 - \gamma_1] \Leftrightarrow x = \frac{1}{2} + \frac{p_L V [\gamma_1 - \gamma_2]}{2 t}. \end{aligned}$$

The corresponding derivation of x_H is analogous and so is omitted. ■

Proof of Lemma 2.

The welfare of the $\phi_L q_2 [1 - q_1]$ L borrowers approved only by lender 1 is:

$$\phi_L q_2 [1 - q_1] \left[p_L \gamma_1 V - \int_0^1 t \xi d\xi \right] = \phi_L q_2 [1 - q_1] \left[p_L \gamma_1 V - \frac{t}{2} \right]. \quad (15)$$

The welfare of the $\phi_L q_1 [1 - q_2]$ L borrowers approved only by lender 2 is:

$$\phi_L q_1 [1 - q_2] \left[p_L \gamma_2 V - \int_0^1 t(1 - \xi) d\xi \right] = \phi_L q_1 [1 - q_2] \left[p_L \gamma_2 V - \frac{t}{2} \right]. \quad (16)$$

$\phi_L [1 - q_2][1 - q_1]$ L borrowers are approved by both lenders. Of these contested L borrowers, those on $[0, x_L]$ secure welfare:

$$\begin{aligned} \phi_L [1 - q_2][1 - q_1] \left[x_L p_L \gamma_1 V - \int_0^{x_L} t \xi d\xi \right] \\ = \phi_L [1 - q_2][1 - q_1] \left[x_L p_L \gamma_1 V - \frac{t(x_L)^2}{2} \right]. \end{aligned} \quad (17)$$

The corresponding welfare of the contested L type borrowers on $[x_L, 1]$ is:

$$\begin{aligned} \phi_L [1 - q_2][1 - q_1] \left[(1 - x_L) p_L \gamma_2 V - \int_{x_L}^1 t(1 - \xi) d\xi \right] \\ = \phi_L [1 - q_2][1 - q_1] \left\{ [1 - x_L] p_L \gamma_2 V - t \left[\frac{2(1 - x_L) - 1 + (x_L)^2}{2} \right] \right\} \\ = \phi_L [1 - q_2][1 - q_1] \left[[1 - x_L] p_L \gamma_2 V - \frac{t(1 - x_L)^2}{2} \right]. \end{aligned} \quad (18)$$

(17) and (18) imply that the welfare of all contested L borrowers is:

$$\phi_L [1 - q_2][1 - q_1] \left[[x_L p_L \gamma_1 + (1 - x_L) p_L \gamma_2] V - \frac{t(x_L)^2}{2} - \frac{t(1 - x_L)^2}{2} \right]. \quad (19)$$

(15), (16), and (19) imply that the welfare of L borrowers is as specified in (4). Analogous calculations reveal that the welfare of H borrowers is as specified in (5). (2), (3), (4), and (5) imply that total welfare is:

$$\begin{aligned}
W(\gamma_1, \gamma_2) &= \phi_L \left\{ q_2 [1 - q_1] \left[p_L \gamma_1 V - \frac{t}{2} \right] + q_1 [1 - q_2] \left[p_L \gamma_2 V - \frac{t}{2} \right] \right. \\
&\quad \left. + [1 - q_2] [1 - q_1] \left[[x_L \gamma_1 + (1 - x_L) \gamma_2] p_L V - \frac{t(x_L)^2}{2} - \frac{t(1 - x_L)^2}{2} \right] \right\} \\
&\quad + \phi_H \left\{ q_1 [1 - q_2] \left[p_H \gamma_1 V - \frac{t}{2} \right] + q_2 [1 - q_1] \left[p_H \gamma_2 V - \frac{t}{2} \right] \right. \\
&\quad \left. + q_2 q_1 \left[[x_H \gamma_1 + (1 - x_H) \gamma_2] p_H V - \frac{t(x_H)^2}{2} - \frac{t(1 - x_H)^2}{2} \right] + [1 - q_2] [1 - q_1] R \right\} \\
&\quad + \phi_L q_2 [1 - q_1] [p_L (1 - \gamma_1) V - I] + \phi_H q_1 [1 - q_2] [p_H (1 - \gamma_1) V - I] \\
&\quad + \phi_L [1 - q_2] [1 - q_1] x_L [p_L (1 - \gamma_1) V - I] + \phi_H q_2 q_1 x_H [p_H (1 - \gamma_1) V - I] \\
&\quad + \phi_L q_1 [1 - q_2] [p_L (1 - \gamma_2) V - I] + \phi_H q_2 [1 - q_1] [p_H (1 - \gamma_2) V - I] \\
&\quad + \phi_L [1 - q_2] [1 - q_1] [1 - x_L] [p_L (1 - \gamma_2) V - I] + \phi_H q_2 q_1 [1 - x_H] [p_H (1 - \gamma_2) V - I] \\
&= \phi_L \left\{ q_2 [1 - q_1] \left[-\frac{t}{2} \right] + q_1 [1 - q_2] \left[-\frac{t}{2} \right] + [1 - q_2] [1 - q_1] \left[-\frac{t(x_L)^2}{2} - \frac{t(1 - x_L)^2}{2} \right] \right\} \\
&\quad + \phi_H \left\{ q_1 [1 - q_2] \left[-\frac{t}{2} \right] + q_2 [1 - q_1] \left[-\frac{t}{2} \right] + q_2 q_1 \left[-\frac{t(x_H)^2}{2} - \frac{t(1 - x_H)^2}{2} \right] \right\} \\
&\quad + \phi_L q_2 [1 - q_1] [p_L V - I] + \phi_H q_1 [1 - q_2] [p_H V - I] + \phi_H [1 - q_2] [1 - q_1] R \\
&\quad + \phi_L [1 - q_2] [1 - q_1] x_L [p_L V - I] + \phi_H q_2 q_1 x_H [p_H V - I] \\
&\quad + \phi_L q_1 [1 - q_2] [p_L V - I] + \phi_H q_2 [1 - q_1] [p_H V - I] \\
&\quad + \phi_L [1 - q_2] [1 - q_1] [1 - x_L] [p_L V - I] + \phi_H q_2 q_1 [1 - x_H] [p_H V - I] \\
&= \phi_L \left\{ q_2 [1 - q_1] \left[-\frac{t}{2} \right] + q_1 [1 - q_2] \left[-\frac{t}{2} \right] + [1 - q_2] [1 - q_1] \left[-\frac{t(x_L)^2}{2} - \frac{t(1 - x_L)^2}{2} \right] \right\} \\
&\quad + \phi_H \left\{ q_1 [1 - q_2] \left[-\frac{t}{2} \right] + q_2 [1 - q_1] \left[-\frac{t}{2} \right] + q_2 q_1 \left[-\frac{t(x_H)^2}{2} - \frac{t(1 - x_H)^2}{2} \right] \right\} \\
&\quad + \phi_L q_2 [1 - q_1] [p_L V - I] + \phi_H q_1 [1 - q_2] [p_H V - I] + \phi_H [1 - q_2] [1 - q_1] R \\
&\quad + \phi_L [1 - q_2] [1 - q_1] [p_L V - I] + \phi_H q_2 q_1 [p_H V - I] \\
&\quad + \phi_L q_1 [1 - q_2] [p_L V - I] + \phi_H q_2 [1 - q_1] [p_H V - I]
\end{aligned}$$

$$\begin{aligned}
&= q_2 [1 - q_1] \left[-\frac{t}{2} \right] + q_1 [1 - q_2] \left[-\frac{t}{2} \right] - \phi_L [1 - q_2] [1 - q_1] \left[\frac{t(x_L)^2}{2} + \frac{t(1 - x_L)^2}{2} \right] \\
&\quad - \phi_H q_2 q_1 \left[\frac{t(x_H)^2}{2} + \frac{t(1 - x_H)^2}{2} \right] + \phi_H [p_H V - I] \{q_1 [1 - q_2] + q_2 q_1 + q_2 [1 - q_1]\} \\
&\quad + \phi_L [p_L V - I] \{q_2 [1 - q_1] + [1 - q_2] [1 - q_1] + q_1 [1 - q_2]\} + \phi_H [1 - q_2] [1 - q_1] R \\
&= -\frac{t}{2} [q_2 (1 - q_1) + q_1 (1 - q_2)] - \frac{t}{2} \phi_L [1 - q_2] [1 - q_1] [(x_L)^2 + (1 - x_L)^2] \\
&\quad - \frac{t}{2} \phi_H q_2 q_1 [(x_H)^2 + (1 - x_H)^2] + \phi_L [p_L V - I] [1 - q_1 q_2] \\
&\quad + \phi_H [p_H V - I] [q_1 + q_2 - q_1 q_2] + \phi_H [1 - q_2] [1 - q_1] R. \tag{20}
\end{aligned}$$

To characterize the equilibrium sharing rates, first observe from (1) and (2) that:

$$\begin{aligned}
\frac{\partial \pi_1(\gamma_1, \gamma_2)}{\partial \gamma_1} &= -[1 - q_1] q_2 \phi_L p_L V - q_1 [1 - q_2] \phi_H p_H V \\
&\quad - [1 - q_1] [1 - q_2] \phi_L p_L V x_L - q_1 q_2 \phi_H p_H V x_H \\
&\quad + [1 - q_1] [1 - q_2] \phi_L [p_L V (1 - \gamma_1) - I] \frac{\partial x_L}{\partial \gamma_1} + q_1 q_2 \phi_H [p_H V (1 - \gamma_1) - I] \frac{\partial x_H}{\partial \gamma_1} \\
&= -[1 - q_1] q_2 \phi_L p_L V - q_1 [1 - q_2] \phi_H p_H V \\
&\quad - [1 - q_1] [1 - q_2] \phi_L p_L V \left[\frac{1}{2} - \frac{(\gamma_2 - \gamma_1) p_L V}{2t} \right] - q_1 q_2 \phi_H p_H V \left[\frac{1}{2} - \frac{(\gamma_2 - \gamma_1) p_H V}{2t} \right] \\
&\quad + [1 - q_1] [1 - q_2] \phi_L [p_L V (1 - \gamma_1) - I] \frac{p_L V}{2t} + q_1 q_2 \phi_H [p_H V (1 - \gamma_1) - I] \frac{p_H V}{2t} \\
&= -[(1 - q_1) q_2 \phi_L p_L V + q_1 (1 - q_2) \phi_H p_H V] - \frac{1}{2} [(1 - q_1) (1 - q_2) \phi_L p_L V + q_1 q_2 \phi_H p_H V] \\
&\quad + \frac{1}{2t} [(1 - q_1) (1 - q_2) \phi_L (p_L V)^2 + q_1 q_2 \phi_H (p_H V)^2] [\gamma_2 - \gamma_1] \\
&\quad + \frac{1}{2t} [(1 - q_1) (1 - q_2) \phi_L (p_L V)^2 + q_1 q_2 \phi_H (p_H V)^2] [1 - \gamma_1] \\
&\quad - \frac{1}{2t} [(1 - q_1) (1 - q_2) \phi_L p_L V + q_1 q_2 \phi_H p_H V] I \\
&= -B_1 - \frac{C}{2} + \frac{A}{2t} [1 + \gamma_2 - 2\gamma_1] - \frac{CI}{2t}, \tag{21}
\end{aligned}$$

where A , B_1 , and C are defined in the statement of the lemma. (21) and symmetry provide:

$$\frac{\partial \pi_2}{\partial \gamma_2} = -B_2 - \frac{C}{2} + \frac{A}{2t} [1 + \gamma_1 - 2\gamma_2] - \frac{CI}{2t}, \tag{22}$$

where B_2 is defined in (11). For future reference, notice from (11) that:

$$\begin{aligned} B_2 - B_1 &= [q_1(1 - q_2) - q_2(1 - q_1)] \phi_L p_L V + [q_2(1 - q_1) - q_1(1 - q_2)] \phi_H p_H V \\ &= [q_2 - q_1] [\phi_H p_H - \phi_L p_L] V. \end{aligned} \quad (23)$$

To characterize lender 2's preferred sharing rate, notice from (7) that:

$$\frac{\partial \widetilde{W}_2(\gamma_1, \gamma_2)}{\partial \gamma_2} = [1 - \alpha_2] \frac{\partial \pi_2}{\partial \gamma_2} + \alpha_2 \left[\frac{\partial W}{\partial \gamma_2} \right]. \quad (24)$$

From (20):

$$\begin{aligned} \frac{\partial W(\gamma_1, \gamma_2)}{\partial \gamma_2} &= -\frac{t}{2} \phi_L [1 - q_2] [1 - q_1] \left[2x_L \left(\frac{\partial x_L}{\partial \gamma_2} \right) - 2(1 - x_L) \left(\frac{\partial x_L}{\partial \gamma_2} \right) \right] \\ &\quad - \frac{t}{2} \phi_H q_2 q_1 \left[2x_H \left(\frac{\partial x_H}{\partial \gamma_2} \right) - 2(1 - x_H) \left(\frac{\partial x_H}{\partial \gamma_2} \right) \right] \\ &= -t \phi_L [1 - q_2] [1 - q_1] [2x_L - 1] \frac{\partial x_L}{\partial \gamma_2} - t \phi_H q_2 q_1 [2x_H - 1] \frac{\partial x_H}{\partial \gamma_2} \\ &= t \phi_L [1 - q_2] [1 - q_1] \frac{p_L V}{2t} [2x_L - 1] + t \phi_H q_2 q_1 \frac{p_H V}{2t} [2x_H - 1] \\ &= t \phi_L [1 - q_2] [1 - q_1] \frac{p_L V}{2t} \left[1 - \frac{(\gamma_2 - \gamma_1) p_L V}{t} - 1 \right] \\ &\quad + t \phi_H q_2 q_1 \frac{p_H V}{2t} \left[1 - \frac{(\gamma_2 - \gamma_1) p_H V}{t} - 1 \right] \\ &= -\frac{1}{2t} [\phi_L (1 - q_2) (1 - q_1) (p_L V)^2 + \phi_H q_2 q_1 (p_H V)^2] [\gamma_2 - \gamma_1] = -\frac{A}{2t} [\gamma_2 - \gamma_1]. \end{aligned} \quad (25)$$

The equality in (25) follows from (1)

(22), (24), and (26) provide:

$$\begin{aligned} \frac{\partial \widetilde{W}_2(\gamma_1, \gamma_2)}{\partial \gamma_2} &= [1 - \alpha_2] \left[-B_2 - \frac{C}{2} + \frac{A}{2t} [1 + \gamma_1 - 2\gamma_2] - \frac{C I}{2t} \right] - \alpha_2 \frac{A}{2t} [\gamma_2 - \gamma_1] = 0 \\ \Leftrightarrow & [1 - \alpha_2] \{-B_2 2t + A [1 + \gamma_1 - 2\gamma_2] - C [I + t]\} - \alpha_2 A [\gamma_2 - \gamma_1] = 0 \\ \Leftrightarrow & A [\gamma_1 - 2\gamma_2] [1 - \alpha_2] - \alpha_2 A [\gamma_2 - \gamma_1] + [1 - \alpha_2] [A - 2t B_2 - C (I + t)] = 0 \\ \Leftrightarrow & A \gamma_1 [1 - \alpha_2 + \alpha_2] - A \gamma_2 [2(1 - \alpha_2) + \alpha_2] + [1 - \alpha_2] [A - 2t B_2 - C (I + t)] = 0 \\ \Leftrightarrow & A \gamma_2 [2 - \alpha_2] = A \gamma_1 + [1 - \alpha_2] [A - 2t B_2 - C (I + t)] \\ \Leftrightarrow & \gamma_2 = \left[\frac{1}{2 - \alpha_2} \right] \gamma_1 + \left[\frac{1 - \alpha_2}{2 - \alpha_2} \right] \left[1 - 2t \left(\frac{B_2}{A} \right) - [I + t] \frac{C}{A} \right]. \end{aligned} \quad (27)$$

By symmetry, (27) implies that the lenders' reaction functions can be rewritten as:

$$\gamma_i = \left[\frac{1}{2 - \alpha_i} \right] \gamma_{-i} + \left[\frac{1 - \alpha_i}{2 - \alpha_i} \right] D_i,$$

as specified in the lemma. ■

Proof of Lemma 3.

(8) and (9) imply that in equilibrium, for $-i \neq i$ and for $(\alpha_1, \alpha_2) \neq (1, 1)$:

$$\begin{aligned} \gamma_i &= \left[\frac{1}{2 - \alpha_i} \right] \left\{ \left[\frac{1}{2 - \alpha_{-i}} \right] \gamma_i + \left[\frac{1 - \alpha_{-i}}{2 - \alpha_{-i}} \right] D_{-i} \right\} + \left[\frac{1 - \alpha_i}{2 - \alpha_i} \right] D_i \\ \Rightarrow \gamma_i \left[1 - \frac{1}{(2 - \alpha_i)(2 - \alpha_{-i})} \right] &= \left[\frac{1}{2 - \alpha_i} \right] \left[\frac{1 - \alpha_{-i}}{2 - \alpha_{-i}} \right] D_{-i} + \left[\frac{(1 - \alpha_i)(2 - \alpha_{-i})}{(2 - \alpha_i)(2 - \alpha_{-i})} \right] D_i \\ \Rightarrow \gamma_i [(2 - \alpha_i)(2 - \alpha_{-i}) - 1] &= [1 - \alpha_{-i}] D_{-i} + [1 - \alpha_i] [2 - \alpha_{-i}] D_i \\ \Rightarrow \gamma_i^* &= \frac{[1 - \alpha_{-i}] D_{-i} + [1 - \alpha_i] [2 - \alpha_{-i}] D_i}{[2 - \alpha_1] [2 - \alpha_2] - 1}. \quad \blacksquare \end{aligned} \quad (28)$$

Proof of Corollary 1.

From (11) and (23):

$$D_i - D_{-i} = \frac{2t}{A} [B_{-i} - B_i] = \frac{2t}{A} [q_{-i} - q_i] [\phi_H p_H - \phi_L p_L] V. \quad (29)$$

(28) and (29) imply:

$$\begin{aligned} \gamma_i^* - \gamma_{-i}^* &= \frac{[1 - \alpha_i] [1 - \alpha_{-i}] [D_i - D_{-i}]}{[2 - \alpha_1] [2 - \alpha_2] - 1} \\ &= \left[\frac{(1 - \alpha_i)(1 - \alpha_{-i})}{(2 - \alpha_1)(2 - \alpha_2) - 1} \right] \frac{2t}{A} [q_{-i} - q_i] [\phi_H p_H - \phi_L p_L] V. \quad \blacksquare \end{aligned} \quad (30)$$

Proof of Proposition 1

The proof follows immediately from (30) and Assumption 1. ■

Proof of Proposition 2.

From (13):

$$\gamma_1^* = \frac{[1 - \alpha_2] D_2 + [1 - \alpha_1] [2 - \alpha_2] D_1}{[2 - \alpha_1] [2 - \alpha_2] - 1} \quad (31)$$

$$\Rightarrow \frac{\partial \gamma_1^*}{\partial \alpha_1} \stackrel{s}{=} - \{ [2 - \alpha_1] [2 - \alpha_2] - 1 \} [2 - \alpha_2] D_1$$

$$\begin{aligned}
& + \{[1 - \alpha_2] D_2 + [1 - \alpha_1] [2 - \alpha_2] D_1\} [2 - \alpha_2] \\
& = D_1 [2 - \alpha_2] \{[1 - \alpha_1] [2 - \alpha_2] - [2 - \alpha_1] [2 - \alpha_2] + 1\} + D_2 [1 - \alpha_2] [2 - \alpha_2] \\
& = [1 - \alpha_2] [2 - \alpha_2] [D_2 - D_1] \\
& = [1 - \alpha_2] [2 - \alpha_2] \frac{2t}{A} [q_1 - q_2] [\phi_H p_H - \phi_L p_L] V \geq 0 \text{ as } q_1 \geq q_2. \tag{32}
\end{aligned}$$

The equality in (32) follows from (29). The inequalities in (32) reflect Assumption 1.

From (14):

$$\begin{aligned}
\frac{\partial \gamma_2^*}{\partial \alpha_1} &= \frac{\partial}{\partial \alpha_1} \left\{ \left[\frac{1 - \alpha_1}{(2 - \alpha_1)(2 - \alpha_2) - 1} \right] D_1 + \left[\frac{(1 - \alpha_2)(2 - \alpha_1)}{(2 - \alpha_1)(2 - \alpha_2) - 1} \right] D_2 \right\} \\
&= - \left\{ \frac{-[(2 - \alpha_1)(2 - \alpha_2) - 1] + [1 - \alpha_1][2 - \alpha_2]}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right\} D_1 \\
&\quad + \left\{ \frac{-[(2 - \alpha_1)(2 - \alpha_2) - 1][1 - \alpha_2] + [1 - \alpha_2][2 - \alpha_1][2 - \alpha_2]}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right\} D_2 \\
&= - \left[\frac{1 - \alpha_2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right] D_1 + \left[\frac{1 - \alpha_2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right] D_2 \\
&= \left[\frac{1 - \alpha_2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right] [D_2 - D_1] \\
&= \left[\frac{1 - \alpha_2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right] \frac{2t}{A} [q_1 - q_2] [\phi_H p_H - \phi_L p_L] V \geq 0 \text{ as } q_1 \geq q_2. \tag{33}
\end{aligned}$$

The equality in (33) follows from (29). The inequalities in (33) reflect Assumption 1. ■

Proof of Proposition 3.

The proof follows from Proposition 2 as long as the welfare of each borrower declines as γ_1 and γ_2 decline. Consider a reduction in the sharing rate of lender i from $\hat{\gamma}_i$ to $\tilde{\gamma}_i$, for $i = 1, 2$. Any borrower that continues to secure financing from the same lender after sharing rates decline experiences a reduction in welfare because his transactions cost is unchanged and his payoff from a successful project is smaller. Now consider the welfare of a $j \in \{L, H\}$ borrower who originally secures financing from lender 1 but secures financing from lender 2 after the sharing rates decline. In this case:

$$p_j V \hat{\gamma}_1 - t x \geq p_j V \hat{\gamma}_2 - t [1 - x] > p_j V \tilde{\gamma}_2 - t [1 - x]. \tag{34}$$

The weak inequality in (34) holds because the borrower initially prefers to secure financing from lender 1. The strict inequality in (34) holds because $\tilde{\gamma}_2 < \hat{\gamma}_2$. (34) implies that the borrower's welfare declines when the sharing rates of both lenders decline. The analysis for the other relevant cases is analogous. ■

Proof of Proposition 4.

From (28):

$$\gamma_i^* = \frac{[1 - \alpha_{-i}] D_{-i}}{[2 - \alpha_{-i}] - 1} = D_{-i} \text{ when } \alpha_i = 1. \quad (35)$$

The proposition follows from (35), since D_1 and D_2 are independent of α_1 and α_2 , from (9) – (12). ■

Proof of Corollary 2.

The proof follows immediately from Propositions 2, 3, and 4. ■

Proof of Lemma 4.

$$\begin{aligned} \frac{\partial}{\partial \alpha_1} \left\{ \frac{[1 - \alpha_2][1 - \alpha_1]}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} &= \frac{-[(2 - \alpha_1)(2 - \alpha_2) - 1][1 - \alpha_2] + [1 - \alpha_2][1 - \alpha_1][2 - \alpha_2]}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \\ &= \frac{[1 - \alpha_2] \{1 - (2 - \alpha_1)(2 - \alpha_2) + [2 - \alpha_2][1 - \alpha_1]\}}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \\ &= - \frac{[1 - \alpha_2]^2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} < 0. \end{aligned} \quad (36)$$

Analogously:

$$\frac{\partial}{\partial \alpha_2} \left\{ \frac{[1 - \alpha_2][1 - \alpha_1]}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} = - \frac{[1 - \alpha_1]^2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} < 0. \quad (37)$$

The lemma follows from (30), (36), and (37). ■

Proof of Lemma 5.

From (20):

$$W = G_1 - \frac{t}{2} \phi_L [1 - q_1][1 - q_2] [(x_L)^2 + (1 - x_L)^2] - \frac{t}{2} \phi_H q_1 q_2 [(x_H)^2 + (1 - x_H)^2], \quad (38)$$

where:

$$\begin{aligned} G_1 = & - \frac{t}{2} [q_2(1 - q_1) + q_1(1 - q_2)] + \phi_L [p_L V - I] [1 - q_1 q_2] \\ & + \phi_H [p_H V - I] [q_1 + q_2 - q_1 q_2] + \phi_H [1 - q_2] [1 - q_1] R. \end{aligned}$$

Notice that G_1 is independent of α_1 , α_2 , γ_1 , and γ_2 . From (1):

$$(x_L)^2 + (1 - x_L)^2 = \left[\frac{1}{2} - \frac{(\gamma_2 - \gamma_1) p_L V}{2t} \right]^2 + \left[\frac{1}{2} + \frac{(\gamma_2 - \gamma_1) p_L V}{2t} \right]^2$$

$$= \frac{1}{2} + \frac{(\gamma_2 - \gamma_1)^2 (p_L V)^2}{2t^2}, \text{ and} \quad (39)$$

$$\begin{aligned} (x_H)^2 + (1 - x_H)^2 &= \left[\frac{1}{2} - \frac{(\gamma_2 - \gamma_1) p_H V}{2t} \right]^2 + \left[\frac{1}{2} + \frac{(\gamma_2 - \gamma_1) p_H V}{2t} \right]^2 \\ &= \frac{1}{2} + \frac{(\gamma_2 - \gamma_1)^2 (p_H V)^2}{2t^2}. \end{aligned} \quad (40)$$

(10), (38), (39), and (40) provide:

$$\begin{aligned} W &= G_1 - \frac{t}{2} \phi_L [1 - q_1] [1 - q_2] \left[\frac{1}{2} + \frac{(\gamma_2 - \gamma_1)^2 (p_L V)^2}{2t^2} \right] - \frac{t}{2} \phi_H q_1 q_2 \left[\frac{1}{2} + \frac{(\gamma_2 - \gamma_1)^2 (p_H V)^2}{2t^2} \right] \\ &= G - \frac{1}{4t} \phi_L [1 - q_1] [1 - q_2] [\gamma_2 - \gamma_1]^2 (p_L V)^2 - \frac{1}{4t} \phi_H q_1 q_2 [\gamma_2 - \gamma_1]^2 (p_H V)^2 \\ &= G - \frac{A}{4t} [\gamma_2 - \gamma_1]^2. \end{aligned} \quad (41)$$

where:

$$G = G_1 - \frac{t}{4} \phi_L [1 - q_1] [1 - q_2] - \frac{t}{4} \phi_H q_1 q_2,$$

so G is independent of α_1 , α_2 , γ_1 , and γ_2 . ■

Proof of Proposition 5.

From (30) and (41), W can be written as:

$$W = G - \frac{A}{4t} \left\{ \left[\frac{[1 - \alpha_2] [1 - \alpha_1]}{[2 - \alpha_1] [2 - \alpha_2] - 1} \right] G_3 \right\}^2, \quad (42)$$

where:

$$G_3 = \frac{2t}{A} [q_1 - q_2] [\phi_H p_H - \phi_L p_L] V. \quad (43)$$

Notice that G_3 is independent of α_1 , α_2 , γ_1 , and γ_2 . Therefore, the proposition follows from (36), (37), and (42). ■

Proof of Proposition 6.

From (41), α_1 and α_2 affect W only through $(\gamma_2 - \gamma_1)$. The proposition then holds because $\gamma_2^* - \gamma_1^*$ is symmetric in α_1 and α_2 , from (30). ■

Proof of Proposition 7.

From (1) and (2):

$$\begin{aligned}
\frac{\partial \pi_1}{\partial \alpha_1} &= -\phi_L q_2 [1 - q_1] p_L V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] - \phi_H q_1 [1 - q_2] p_H V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] \\
&\quad - \phi_L [1 - q_2] [1 - q_1] x_L p_L V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] + \phi_L [1 - q_2] [1 - q_1] [p_L (1 - \gamma_1) V - I] \frac{\partial x_L}{\partial \alpha_1} \\
&\quad - \phi_H q_2 q_1 x_H p_H V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] + \phi_H q_2 q_1 [p_H (1 - \gamma_1) V - I] \frac{\partial x_H}{\partial \alpha_1} \\
&= -[\phi_L q_2 (1 - q_1) p_L V + \phi_H q_1 (1 - q_2) p_H V] \frac{\partial \gamma_1}{\partial \alpha_1} \\
&\quad - \phi_L [1 - q_2] [1 - q_1] x_L p_L V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] - \phi_H q_2 q_1 x_H p_H V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] \\
&\quad + \phi_L [1 - q_2] [1 - q_1] p_L V [1 - \gamma_1] \frac{\partial x_L}{\partial \alpha_1} - \phi_L [1 - q_2] [1 - q_1] I \frac{\partial x_L}{\partial \alpha_1} \\
&\quad + \phi_H q_2 q_1 p_H V [1 - \gamma_1] \frac{\partial x_H}{\partial \alpha_1} - \phi_H q_2 q_1 I \frac{\partial x_H}{\partial \alpha_1} \\
&= -[\phi_L q_2 (1 - q_1) p_L V + \phi_H q_1 (1 - q_2) p_H V] \frac{\partial \gamma_1}{\partial \alpha_1} \\
&\quad - \phi_L [1 - q_2] [1 - q_1] \left[\frac{1}{2} - \frac{(\gamma_2 - \gamma_1) p_L V}{2t} \right] p_L V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] \\
&\quad - \phi_H q_2 q_1 \left[\frac{1}{2} - \frac{(\gamma_2 - \gamma_1) p_H V}{2t} \right] p_H V \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] \\
&\quad - \frac{1}{2t} \phi_L [1 - q_2] [1 - q_1] (p_L V)^2 [1 - \gamma_1] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} \\
&\quad + \frac{1}{2t} \phi_L [1 - q_2] [1 - q_1] p_L V I \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} \\
&\quad - \frac{1}{2t} \phi_H q_2 q_1 (p_H V)^2 [1 - \gamma_1] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} + \frac{1}{2t} \phi_H q_2 q_1 p_H V I \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} \\
&= -[\phi_L q_2 (1 - q_1) p_L V + \phi_H q_1 (1 - q_2) p_H V] \frac{\partial \gamma_1}{\partial \alpha_1} \\
&\quad - \frac{1}{2} [\phi_L (1 - q_2) (1 - q_1) p_L V + \phi_H q_2 q_1 p_H V] \frac{\partial \gamma_1}{\partial \alpha_1} \\
&\quad + [\phi_L (1 - q_2) (1 - q_1) (p_L V)^2 + \phi_H q_2 q_1 (p_H V)^2] \left[\frac{\gamma_2 - \gamma_1}{2t} \right] \frac{\partial \gamma_1}{\partial \alpha_1} \\
&\quad - \frac{1}{2t} [\phi_L (1 - q_2) (1 - q_1) (p_L V)^2 + \phi_H q_2 q_1 (p_H V)^2] [1 - \gamma_1] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} \\
&\quad + \frac{1}{2t} [\phi_L (1 - q_2) (1 - q_1) p_L V + \phi_H q_2 q_1 p_H V] I \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1}
\end{aligned}$$

$$\begin{aligned}
&= -B_1 \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] - \frac{C}{2} \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] + \frac{A}{2t} [\gamma_2 - \gamma_1] \frac{\partial \gamma_1}{\partial \alpha_1} \\
&\quad - \frac{A}{2t} [1 - \gamma_1] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} + \frac{C}{2t} I \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1}.
\end{aligned} \tag{44}$$

The last equality in (44) reflects (10), (11), and (12). From (8):

$$\begin{aligned}
\gamma_1 &= \left[\frac{1}{2 - \alpha_1} \right] \gamma_2 + \left[\frac{1 - \alpha_1}{2 - \alpha_1} \right] D_1 \Rightarrow [2 - \alpha_1] \gamma_1 = \gamma_2 + [1 - \alpha_1] D_1 \\
\Rightarrow [1 - \alpha_1] \gamma_1 &= \gamma_2 - \gamma_1 + [1 - \alpha_1] D_1 \Rightarrow \gamma_1 = \left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right] + D_1 \\
\Rightarrow 1 - \gamma_1 &= 1 - \left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} + D_1 \right] = 1 - D_1 - \left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right].
\end{aligned} \tag{45}$$

From (9):

$$\frac{A}{2t} [1 - D_1] = \frac{A}{2t} \left[\left(\frac{2t}{A} \right) B_1 + [I + t] \left(\frac{C}{A} \right) \right] = B_1 + I \frac{C}{2t} + \frac{C}{2}. \tag{46}$$

(45) and (46) provide:

$$\begin{aligned}
\frac{A}{2t} [1 - \gamma_1] &= -\frac{A}{2t} \left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right] + \frac{A}{2t} [1 - D_1] \\
&= -\frac{A}{2t} \left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right] + B_1 + I \frac{C}{2t} + \frac{C}{2}.
\end{aligned} \tag{47}$$

From (44) and (47):

$$\begin{aligned}
\frac{\partial \pi_1}{\partial \alpha_1} &= -B_1 \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] - \frac{C}{2} \left[\frac{\partial \gamma_1}{\partial \alpha_1} \right] + \frac{A}{2t} [\gamma_2 - \gamma_1] \frac{\partial \gamma_1}{\partial \alpha_1} \\
&\quad - \left[-\frac{A}{2t} \left(\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right) + B_1 + I \frac{C}{2t} + \frac{C}{2} \right] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} + \frac{C}{2t} I \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} \\
&= \frac{A}{2t} [\gamma_2 - \gamma_1] \frac{\partial \gamma_1}{\partial \alpha_1} + \frac{A}{2t} \left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} - \left[B_1 + \frac{C}{2} \right] \frac{\partial \gamma_2}{\partial \alpha_1}.
\end{aligned} \tag{48}$$

From (30):

$$\begin{aligned}
\frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} &= \left\{ \frac{1 - \alpha_2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} - \frac{(1 - \alpha_2)(2 - \alpha_2)}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right\} [D_2 - D_1] \\
&= - \left[\frac{(1 - \alpha_2)^2}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^2} \right] [D_2 - D_1].
\end{aligned} \tag{49}$$

From (30) and (32):

$$[\gamma_2 - \gamma_1] \frac{\partial \gamma_1}{\partial \alpha_1} = \left[\frac{(1 - \alpha_2)^2 (2 - \alpha_2) (1 - \alpha_1)}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} \right] [D_2 - D_1]^2. \quad (50)$$

(30) and (49) provide:

$$\left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} = - \left[\frac{(1 - \alpha_2)^3}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} \right] [D_2 - D_1]^2. \quad (51)$$

(50) and (51) provide:

$$\begin{aligned} & \frac{A}{2t} [\gamma_2 - \gamma_1] \frac{\partial \gamma_1}{\partial \alpha_1} + \frac{A}{2t} \left[\frac{\gamma_2 - \gamma_1}{1 - \alpha_1} \right] \frac{\partial (\gamma_2 - \gamma_1)}{\partial \alpha_1} \\ &= \frac{A}{2t} \left\{ \frac{(1 - \alpha_2)^2 (2 - \alpha_2) (1 - \alpha_1)}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} - \frac{(1 - \alpha_2)^3}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} \right\} [D_2 - D_1]^2 \\ &= \frac{A}{2t} \left[\frac{(1 - \alpha_2)^2 [(2 - \alpha_2) (1 - \alpha_1) - (1 - \alpha_2)]}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} \right] [D_2 - D_1]^2 \\ &= \left[\frac{(1 - \alpha_2)^2 [1 - 2\alpha_1 + \alpha_1 \alpha_2]}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} \right] \frac{2t}{A} [q_2 - q_1]^2 [\phi_H p_H - \phi_L p_L]^2 V^2. \end{aligned} \quad (52)$$

The last equality in (52) reflects (10) and (29). Also, from (33):

$$\left[B_1 + \frac{C}{2} \right] \frac{\partial \gamma_2}{\partial \alpha_1} = - \left[B_1 + \frac{C}{2} \right] \left[\frac{1 - \alpha_2}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^2} \right] \frac{2t}{A} [q_2 - q_1] [\phi_H p_H - \phi_L p_L] V. \quad (53)$$

(48), (52), and (53) provide (using (10), (11), and (12)):

$$\begin{aligned} \frac{\partial \pi_1}{\partial \alpha_1} &= \left[\frac{(1 - \alpha_2) (q_2 - q_1) [\phi_H p_H - \phi_L p_L] V}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} \right] \frac{2t}{A} \\ &\quad \cdot \{ (1 - \alpha_2) [1 - 2\alpha_1 + \alpha_1 \alpha_2] [q_2 - q_1] [\phi_H p_H - \phi_L p_L] V \\ &\quad \quad + [(2 - \alpha_1) (2 - \alpha_2) - 1] \left[B_1 + \frac{C}{2} \right] \} \\ &= \left[\frac{(1 - \alpha_2) (q_2 - q_1) [\phi_H p_H - \phi_L p_L] V}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3} \right] \frac{t}{A} \\ &\quad \cdot \{ 2 [1 - \alpha_2] [1 - 2\alpha_1 + \alpha_1 \alpha_2] [q_2 - q_1] [\phi_H p_H - \phi_L p_L] V \\ &\quad \quad + [(2 - \alpha_1) (2 - \alpha_2) - 1] [2B_1 + C] \} \\ &= \frac{[1 - \alpha_2] [q_2 - q_1] [\phi_H p_H - \phi_L p_L] t V}{[(2 - \alpha_1) (2 - \alpha_2) - 1]^3 [\phi_L [1 - q_2] [1 - q_1] (p_L)^2 + \phi_H q_2 q_1 (p_H)^2] V^2} \\ &\quad \cdot \{ 2 [1 - \alpha_2] [1 - 2\alpha_1 + \alpha_1 \alpha_2] [q_2 - q_1] [\phi_H p_H - \phi_L p_L] V \end{aligned}$$

$$\begin{aligned}
& + 2[(2 - \alpha_1)(2 - \alpha_2) - 1][\phi_L p_L q_2(1 - q_1) + \phi_H p_H q_1(1 - q_2)]V \\
& + [(2 - \alpha_1)(2 - \alpha_2) - 1][\phi_L p_L(1 - q_2)(1 - q_1) + \phi_H p_H q_2 q_1]V \\
= & \frac{[1 - \alpha_2][q_2 - q_1][\phi_H p_H - \phi_L p_L]t[\phi_H p_H M_H + \phi_L p_L M_L]}{[(2 - \alpha_1)(2 - \alpha_2) - 1]^3 [\phi_L(1 - q_2)(1 - q_1)(p_L)^2 + \phi_H q_2 q_1 (p_H)^2]}, \tag{54}
\end{aligned}$$

where:

$$\begin{aligned}
M_H = & 2[1 - \alpha_2][1 - 2\alpha_1 + \alpha_1\alpha_2][q_2 - q_1] + 2[(2 - \alpha_1)(2 - \alpha_2) - 1]q_1[1 - q_2] \\
& + [(2 - \alpha_1)(2 - \alpha_2) - 1]q_2 q_1, \quad \text{and} \tag{55}
\end{aligned}$$

$$\begin{aligned}
M_L = & -2[1 - \alpha_2][1 - 2\alpha_1 + \alpha_1\alpha_2][q_2 - q_1] + 2[(2 - \alpha_1)(2 - \alpha_2) - 1]q_2[1 - q_1] \\
& + [(2 - \alpha_1)(2 - \alpha_2) - 1][1 - q_2][1 - q_1]. \tag{56}
\end{aligned}$$

(54) and Assumption 1 imply:

$$\frac{\partial \pi_1}{\partial \alpha_1} \stackrel{s}{=} [q_2 - q_1][\phi_H p_H M_H + \phi_L p_L M_L]. \tag{57}$$

Observation A1. $M \equiv \phi_H p_H M_H + \phi_L p_L M_L > 0$ if $M_H > 0$.

Proof. If $M_H > 0$, then:

$$\phi_H p_H M_H + \phi_L p_L M_L > \phi_L p_L M_H + \phi_L p_L M_L = \phi_L p_L [M_H + M_L]. \tag{58}$$

(55) and (56) imply:

$$\begin{aligned}
M_H + M_L = & 2[(2 - \alpha_1)(2 - \alpha_2) - 1]q_1[1 - q_2] + [(2 - \alpha_1)(2 - \alpha_2) - 1]q_2 q_1 \\
& + 2[(2 - \alpha_1)(2 - \alpha_2) - 1]q_2[1 - q_1] \\
& + [(2 - \alpha_1)(2 - \alpha_2) - 1][1 - q_2][1 - q_1] > 0. \tag{59}
\end{aligned}$$

The inequality in (59) holds because $\alpha_1, \alpha_2 \in [0, 1]$ and $q_1, q_2 \in [\frac{1}{2}, 1]$. The Observation follows from (58) and (59). ■

Observation A2. $M > 0$ if $q_1 > q_2$.

Proof. First suppose $\alpha_2 = 1$. It is apparent from (55) that $M_H > 0$, and so $M > 0$, from Observation A1.

Now suppose $\alpha_2 < 1$. Observe that:

$$\begin{aligned}
1 - 2\alpha_1 + \alpha_1\alpha_2 & = 3 - 2\alpha_1 - 2\alpha_2 + \alpha_1\alpha_2 - 2 + 2\alpha_2 \\
& = [(2 - \alpha_1)(2 - \alpha_2) - 1] - 2[1 - \alpha_2] = Z - 2[1 - \alpha_2], \tag{60}
\end{aligned}$$

where:

$$Z \equiv [2 - \alpha_1][2 - \alpha_2] - 1 \geq 0. \tag{61}$$

(55) and (60) imply that when $q_1 > q_2$:

$$\begin{aligned}
M_H &= 2[1 - \alpha_2][Z - 2(1 - \alpha_2)][q_2 - q_1] + 2Zq_1[1 - q_2] + Zq_2q_1 \\
&= 2[1 - \alpha_2]Z[q_2 - q_1] + 2Zq_1[1 - q_2] + Zq_2q_1 + 4[1 - \alpha_2]^2[q_1 - q_2] \\
&> 2[1 - \alpha_2]Z[q_2 - q_1] + 2Zq_1[1 - q_2] + Zq_2q_1 \\
&> 2Z[q_2 - q_1] + 2Zq_1[1 - q_2] + Zq_2q_1 \\
&= Z[2q_2 - 2q_1 + 2q_1 - 2q_1q_2 + q_2q_1] = Zq_2[2 - q_1] \geq 0.
\end{aligned} \tag{62}$$

The Observation follows from (61), (62), and Observation A1. ■

Observation A3. $M > 0$ if $q_1 < q_2$ and $1 - 2\alpha_1 + \alpha_1\alpha_2 \geq 0$ ($\Leftrightarrow \alpha_1 \leq \frac{1}{2-\alpha_2}$).

Proof. The proof follows immediately from (55) and Observation A1. ■

Observation A4. $M > 0$ if $q_1 < q_2$, $\alpha_1 > \frac{1}{2-\alpha_2}$, and

$$q_2 - q_1 < \frac{[2 - \alpha_1][2 - \alpha_2] - 1}{4[1 - \alpha_2][2\alpha_1 - 1 - \alpha_1\alpha_2]}. \tag{63}$$

Proof.

$$\begin{aligned}
\alpha_1 > \frac{1}{2 - \alpha_2} &\Leftrightarrow 1 - 2\alpha_1 + \alpha_1\alpha_2 = Z - 2[1 - \alpha_2] < 0 \\
&\Leftrightarrow 2[1 - \alpha_2] - Z > 0,
\end{aligned} \tag{64}$$

where Z is defined in (61). From (55), when $q_1 < q_2$ and (64) holds:

$$\begin{aligned}
M_H &= 2[1 - \alpha_2][Z - 2(1 - \alpha_2)][q_2 - q_1] + 2Zq_1[1 - q_2] + Zq_2q_1 \\
&= 2[1 - \alpha_2][Z - 2(1 - \alpha_2)][q_2 - q_1] + Zq_1[2 - q_2] \\
&> 2[1 - \alpha_2][Z - 2(1 - \alpha_2)][q_2 - q_1] + \frac{Z}{2} > 0 \\
&\Leftrightarrow 2[1 - \alpha_2][2(1 - \alpha_2) - Z][q_2 - q_1] < \frac{Z}{2} \\
&\Leftrightarrow q_2 - q_1 < \frac{Z}{4[1 - \alpha_2][2(1 - \alpha_2) - Z]}.
\end{aligned} \tag{65}$$

The proposition follows from (57) and Observations A1 – A4. ■

Proof of Proposition 8.

From (2):

$$\begin{aligned}
\pi_1 &= G_4 + [1 - \gamma_1]V \{ \phi_L p_L q_2 [1 - q_1] + \phi_H p_H q_1 [1 - q_2] \\
&\quad + \phi_L p_L [1 - q_1] [1 - q_2] x_L + \phi_H p_H q_1 q_2 x_H \}
\end{aligned}$$

$$-I [\phi_L (1 - q_1) (1 - q_2) x_L + \phi_H q_1 q_2 x_H], \quad (66)$$

where $G_4 = -I [\phi_L q_2 (1 - q_1) + \phi_H q_1 (1 - q_2)]$ is independent of α_1 , α_2 , γ_1 , and γ_2 . Differentiating (66) provides:

$$\begin{aligned} \frac{\partial \pi_1}{\partial \alpha_2} &= \phi_L [1 - q_1] [1 - q_1] [p_L (1 - \gamma_1) V - I] \frac{\partial x_L}{\partial \alpha_2} + \phi_H q_1 q_2 [p_H (1 - \gamma_1) V - I] \frac{\partial x_H}{\partial \alpha_2} \\ &\quad - \frac{\partial \gamma_1}{\partial \alpha_2} V \{ \phi_L p_L q_2 [1 - q_1] + \phi_H p_H q_1 [1 - q_2] \\ &\quad \quad \quad + \phi_L p_L [1 - q_1] [1 - q_2] x_L + \phi_H p_H q_1 q_2 x_H \}. \end{aligned} \quad (67)$$

From (1) and Lemma 4:

$$\frac{\partial x_L}{\partial \alpha_2} = \left[\frac{p_L V}{2t} \right] \frac{\partial (\gamma_1 - \gamma_2)}{\partial \alpha_2} \begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ as } q_2 \begin{matrix} \geq \\ \leq \end{matrix} q_1, \text{ and} \quad (68)$$

$$\frac{\partial x_H}{\partial \alpha_2} = \left[\frac{p_H V}{2t} \right] \frac{\partial (\gamma_1 - \gamma_2)}{\partial \alpha_2} \begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ as } q_2 \begin{matrix} \geq \\ \leq \end{matrix} q_1. \quad (69)$$

(67), (68), and (69) imply:

$$\begin{aligned} \frac{\partial \pi_1}{\partial \alpha_2} &= \frac{V}{2t} K \frac{\partial (\gamma_1 - \gamma_2)}{\partial \alpha_2} - \frac{\partial \gamma_1}{\partial \alpha_2} V \{ \phi_L p_L q_2 [1 - q_1] + \phi_H p_H q_1 [1 - q_2] \\ &\quad \quad \quad + \phi_L p_L [1 - q_1] [1 - q_2] x_L + \phi_H p_H q_1 q_2 x_H \}, \end{aligned} \quad (70)$$

where, using (10) and (12):

$$\begin{aligned} K &\equiv \phi_L p_L [1 - q_1] [1 - q_2] [p_L (1 - \gamma_1) V - I] + \phi_H p_H q_1 q_2 [p_H (1 - \gamma_1) V - I] \\ &= \phi_L p_L [1 - q_1] [1 - q_2] p_L (1 - \gamma_1) V - \phi_L p_L [1 - q_1] [1 - q_2] I \\ &\quad \quad \quad + \phi_H p_H q_1 q_2 p_H [1 - \gamma_1] V - \phi_H p_H q_1 q_2 I \\ &= [1 - \gamma_1] V [\phi_L p_L^2 (1 - q_1) (1 - q_2) + \phi_H p_H^2 q_1 q_2] \\ &\quad \quad \quad - [\phi_L p_L (1 - q_1) (1 - q_2) + \phi_H p_H q_1 q_2] I = [1 - \gamma_1] \frac{A}{V} - \frac{C}{V} I. \end{aligned} \quad (71)$$

From (29):

$$D_2 - D_1 = -\frac{2t}{A} [B_2 - B_1] \Rightarrow D_2 = D_1 - \frac{2t}{A} [B_2 - B_1]. \quad (72)$$

Using (9) and (72) in (31) provides:

$$\begin{aligned} \gamma_1^* &= \frac{[1 - \alpha_2] [D_1 - \frac{2t}{A} (B_2 - B_1)] + [1 - \alpha_1] [2 - \alpha_2] D_1}{[2 - \alpha_1] [2 - \alpha_2] - 1} \\ &= \frac{D_1 [1 - \alpha_2 + (1 - \alpha_1) (2 - \alpha_2)]}{[2 - \alpha_1] [2 - \alpha_2] - 1} - \frac{2t [1 - \alpha_2] [B_2 - B_1]}{A [(2 - \alpha_1) (2 - \alpha_2) - 1]} \end{aligned}$$

$$= D_1 - \frac{2t[1 - \alpha_2][B_2 - B_1]}{A[(2 - \alpha_1)(2 - \alpha_2) - 1]} \quad (73)$$

$$= 1 - 2t \left[\frac{B_1}{A} \right] - [I + t] \frac{C}{A} - \frac{2t[1 - \alpha_2][B_2 - B_1]}{A[(2 - \alpha_1)(2 - \alpha_2) - 1]}$$

$$\Rightarrow 1 - \gamma_1^* = 2t \left[\frac{B_1}{A} \right] + [I + t] \frac{C}{A} + \frac{2t[1 - \alpha_2][B_2 - B_1]}{A[(2 - \alpha_1)(2 - \alpha_2) - 1]}. \quad (74)$$

(71) and (74) imply that for $\gamma_1 = \gamma_1^*$:

$$\begin{aligned} K &= \left\{ 2t \left[\frac{B_1}{A} \right] + [I + t] \frac{C}{A} + \frac{2t[1 - \alpha_2][B_2 - B_1]}{A[(2 - \alpha_1)(2 - \alpha_2) - 1]} \right\} \frac{A}{V} - \frac{C}{V} I \\ &= \left\{ 2t \left[\frac{B_1}{A} \right] + t \frac{C}{A} + \frac{2t[1 - \alpha_2][B_2 - B_1]}{A[(2 - \alpha_1)(2 - \alpha_2) - 1]} \right\} \frac{A}{V} \\ &= \frac{t}{V} \left\{ 2B_1 + C + \frac{2[1 - \alpha_2][B_2 - B_1]}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} \\ &= \frac{t}{V} \left\{ 2B_1 \left[1 - \frac{1 - \alpha_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right] + C + \frac{2[1 - \alpha_2]B_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} \\ &= \frac{t}{V} \left\{ 2B_1 \left[\frac{3 - 2\alpha_1 - 2\alpha_2 + \alpha_1\alpha_2 - 1 + \alpha_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right] + C + \frac{2[1 - \alpha_2]B_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} \\ &= \frac{t}{V} \left\{ 2B_1 \left[\frac{2 - 2\alpha_1 - \alpha_2 + \alpha_1\alpha_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right] + C + \frac{2[1 - \alpha_2]B_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} \\ &= \frac{t}{V} \left\{ 2B_1 \left[\frac{2(1 - \alpha_1) - \alpha_2(1 - \alpha_1)}{[2 - \alpha_1][2 - \alpha_2] - 1} \right] + C + \frac{2[1 - \alpha_2]B_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} \\ &= \frac{t}{V} \left\{ 2B_1 \left[\frac{(1 - \alpha_1)(2 - \alpha_2)}{[2 - \alpha_1][2 - \alpha_2] - 1} \right] + C + \frac{2[1 - \alpha_2]B_2}{[2 - \alpha_1][2 - \alpha_2] - 1} \right\} > 0. \quad (75) \end{aligned}$$

From Proposition 2:

$$\frac{\partial \gamma_1}{\partial \alpha_2} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad \text{as } q_2 \begin{matrix} \geq \\ \leq \end{matrix} q_1. \quad (76)$$

Since $K > 0$ from (75), (68), (69), (70), and (76) imply: (i) if $q_2 > q_1$, then π_1 decreases as α_2 increases; (ii) if $q_1 > q_2$, then π_1 increases as α_2 increases; and (iii) if $q_1 = q_2$, then π_1 does not change as α_2 increases.

Analogous arguments reveal: (i) if $q_1 > q_2$, then π_2 decreases as α_1 increases; (ii) if $q_2 > q_1$, then π_2 increases as α_1 increases; and (iii) if $q_2 = q_1$, then π_2 does not change as α_1 increases. ■

Appendix B

Conclusion 3 below specifies non-vacuous conditions under which all captive and contested borrowers will undertake their projects and both lenders will serve some contested borrowers in equilibrium. Conclusion 3 reflects the findings presented in Conclusions 1 and 2.²⁰ These Conclusions refer to γ_1^* and γ_2^* as defined in Lemma 3, to $\pi_1(\cdot)$ and $\pi_2(\cdot)$ as defined in (2) and (3), and to $W(\cdot)$ as defined in (6).

Conclusion 1. Welfare is highest when all captive and contested borrowers undertake their projects and both lenders serve some contested borrowers if, for $i, j \in \{1, 2\}$, $j \neq i$:

$$(i) \quad p_H V \gamma_j^* - t \left[\frac{p_H}{p_L} \right] < R < p_H V \gamma_j^* - 2t; \quad (77)$$

$$(ii) \quad t < R; \quad (78)$$

$$(iii) \quad \frac{t}{p_L V} < \gamma_j^* < \frac{t}{p_L V} + \frac{t}{p_H V}; \quad (79)$$

$$(iv) \quad q_j [p_L V - I] + [1 - q_j] \frac{t}{2} \leq 0; \quad (80)$$

$$(v) \quad W(\gamma_1^*, \gamma_2^*) > [1 - q_2] \phi_L \left[p_L V - I - \frac{t}{2} \right] \\ + q_2 \phi_H \left[p_H V - I - \frac{t}{2} \right] + [1 - q_2] \phi_H R; \quad (81)$$

$$(vi) \quad [1 - q_i] q_j \phi_L p_L [p_L V - I] + q_i [1 - q_j] \phi_H p_H [p_H V - R - t - I] \\ > \frac{1}{2} [1 - q_i] \phi_L p_L \frac{p_L}{p_H} [(R + t)(1 + q_j) - p_H V (1 - q_j) \gamma_j^*]; \text{ and} \quad (82)$$

$$(vii) \quad W(\gamma_i^*, \gamma_j^*) > [1 - q_i] q_j \phi_L \frac{p_L}{p_H} \left[1 + \frac{R}{t} \right] \left[p_L V - I - \frac{1}{2} \left(\frac{p_L}{p_H} \right) (R + t) \right] \\ - [1 - q_j] [1 - q_i] \phi_L \left[\frac{t}{4} + \frac{1}{4t} \left(\frac{p_L}{p_H} \right)^2 (p_H V \gamma_j^* - R - t)^2 \right] \\ + q_i [1 - q_j] \phi_H [p_H V - I] + q_j \phi_H \left[p_H V - I - \frac{t}{2} \right] \\ + [1 - q_j] \phi_L [p_L V - I] - q_i [1 - q_j] \frac{t}{2} + [1 - q_j] [1 - q_i] \phi_H R. \quad (83)$$

²⁰The proofs of Conclusions 1 and 2 are lengthy and tedious. The proofs are available upon request from the authors.

Conclusion 2. Each lender maximizes its profit when all captive and contested borrowers undertake their projects and both lenders serve some contested borrowers if one of the following conditions holds for $i, j \in \{1, 2\}$, $j \neq i$:

$$(i) \quad \phi_H p_H^2 [p_H V - I - R] q_i [1 - q_j] + \phi_L p_L^2 \left[p_H V - I \left(\frac{p_H}{p_L} \right) - 2R \right] [1 - q_i] q_j < 0. \quad (84)$$

$$(ii) \quad \frac{V}{p_H t} \{ \phi_H p_H^2 [p_H V - I - R - 2t] q_i [1 - q_j] \\ + \phi_L p_L^2 \left[p_H V - I \left(\frac{p_H}{p_L} \right) - 2R - 2t \right] [1 - q_i] q_j \} > 0, \quad \text{and} \\ \pi_i (\gamma_i^*, \gamma_j^*) > \phi_H q_i [1 - q_j] [p_H V - R - t - I] \\ + \frac{1}{p_H t} \phi_L p_L [1 - q_i] q_j [R + t] \left[p_L V - \frac{p_L}{p_H} (R + t) - I \right]. \quad (85)$$

$$(iii) \quad \frac{V}{p_H t} \{ \phi_H p_H^2 [p_H V - I - R - 2t] q_i [1 - q_j] \\ + \phi_L p_L^2 \left[p_H V - I \left(\frac{p_H}{p_L} \right) - 2R - 2t \right] [1 - q_i] q_j \} < 0, \quad \text{and} \\ \pi_i (\gamma_i^*, \gamma_j^*) > \frac{[\phi_H p_H q_i (1 - q_j) (p_H V + R - I) + \phi_L p_L q_j (1 - q_i) (p_L V - I)]^2}{4t [\phi_L (p_L)^2 (1 - q_i) q_j + \phi_H (p_H)^2 q_i (1 - q_j)]} \\ - q_i [1 - q_j] \phi_H [p_H V - I] \frac{R}{t}. \quad (86)$$

Conclusions 1 and 2 provide:

Conclusion 3. All captive and contested borrowers will undertake their projects and both lenders will serve some contested borrowers in equilibrium if conditions (77) – (83) hold and if at least one of conditions (84), (85), and (86) holds.

Acknowledgements

The first two authors thank the Taft Research Center and the Hewett Kautz Fund at the University of Cincinnati for generous research support. The first author also gratefully acknowledges helpful support through the J. C. Bose National Fellowship, Government of India.

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