STRINGY GEOMETRY OF ORBIFOLDS: AN INTRODUCTION

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Abstract. Orbifolds are generalizations of manifolds as well as finite groups. In this brief survey I will first describe how orbifolds appear naturally in some important areas of Mathematics and Physics. I will then describe some elementary features of the stringy geometry of orbifolds. In particular I will describe some invariants of orbifolds that specialize to invariants of manifolds as well as groups. My aim is to give the uninitiated reader a feeling for the subject including recent developments and provide adequate references for further exploration. These notes are an expanded version of my talk at a conference at the Tripura University.

1. Introduction

The notion of a differentiable orbifold was introduced by Satake [Sa] in the fifties under the name V-manifold, as a natural generalization of the notion of differentiable manifold. The term orbifold was first coined by Thurston [Th] in his work on the geometry of three dimensional manifolds. In this sense an orbifold is, roughly speaking, a space which is locally the quotient space of a smooth manifold by the effective action of a finite group. Note that an action of a group is effective if its kernel, as defined in Definition 1.2 below, is trivial. In algebraic geometry the analogous notion is that of a complex variety with quotient singularities. However there is a more general notion of a smooth Deligne-Mumford stack. This notion incorporates the example of a group acting on a point. In this example, the structure of the group is the only data since the action is trivial. In this sense finite groups are stacks. The current notion of an orbifold, as opposed to Satake, is analogous to the one of a smooth Deligne-Mumford stack. I will define differentiable orbifolds in this general sense. Orbifolds in the sense of Satake are now known as effective or reduced orbifolds.

The definition of a differentiable orbifold resembles to some extent the definition of a differentiable manifold. A differentiable orbifold X of dimension n is a topological space, assumed to be Hausdorff and second countable, with some additional structure. The extra structure may be given in terms of an atlas of local charts. Each local chart describes the structure locally, that is on some open subset U of X. Then there is a notion of compatibility between these local structures on the intersection of open sets.

Local charts are also called uniformizing systems by various authors. The notations I use here are slightly different from the usual notations. This is done to make the meaning of the notation more explicit.

Definition 1.1. *A local chart on X consists of data (U, V, G, ρ, π) where*

(1) *U is an open subset of X,*
(2) *V is a differentiable manifold of dimension n,*
(3) *G is a finite group,*
(4) *ρ is an action of G on V by diffeomorphisms,*
(5) *$\pi : V \to U$ is a continuous map that induces homeomorphism between V/G and U.*

To describe the compatibility between local charts we need to introduce the notion of injection of local charts. This is a generalization of the notion of smooth embedding of differentiable manifolds.

Definition 1.2. *An injection of local charts $(i, f, \phi) : (U_1, V_1, G_1, \rho_1, \pi_1) \to (U_2, V_2, G_2, \rho_2, \pi_2)$ consists of*

(a) *$i : U_1 \to U_2$ to be an inclusion,*
(b) *an embedding of smooth manifolds $f : V_1 \to V_2$*
(c) *an injective group homomorphism $\phi : G_1 \to G_2$ inducing isomorphism $ker(\rho_1) \to ker(\rho_2)$ where $ker(\rho_i) := \{g \in G_i \,:\, g \cdot x = x \,\forall x \in V_i\}$.*

For (i, f, ϕ) to be an injection the various maps must also satisfy the following conditions.

- $f \circ \rho_1 = (\rho_2 \circ \phi) \circ f$
- $i \circ \pi_1 = \pi_2 \circ f$

Definition 1.3. *Two charts $(U_j, V_j, G_j, \rho_j, \pi_j)$ $j = 1, 2$ are declared to be compatible if for every point $x \in U_1 \cap U_2$ there exists a chart $(U_3, V_3, G_3, \rho_3, \pi_3)$ with $x \in U_3 \subset U_1 \cap U_2$ and injections $(U_3, V_3, G_3, \rho_3, \pi_3) \to (U_j, V_j, G_j, \rho_j, \pi_j)$ for $j = 1, 2$.*

Intuitively the above means that the orbifold structure on U_3 given by the three charts are equivalent. More generally compatible charts induce same orbifold structure on their 'intersection'.

Definition 1.4. *An orbifold atlas on X is a collection $\mathcal{U} := \{(U_j, V_j, G_j, \rho_j, \pi_j)\}$ of pairwise compatible local charts such that $\bigcup U_j = X$.*

An orbifold atlas determines an orbifold structure on X. However different atlases may correspond to the same structure. For instance, suppose X is the quotient of a manifold M by the smooth action ρ of a finite group G. Then the chart $(X, M, G, \rho, -)$ constitutes an atlas (I will use '–' when an entry is obvious from the context). However we can easily construct another atlas that should give the same orbifold structure as follows. Let $M \sqcup M = \{(x, \epsilon) \,:\, x \in M, \epsilon = 1 \,\text{or}\, -1\}$ be the disjoint union of two copies of M. Let $\mathbf{Z}_2 = \{1, -1\}$ be the multiplicative group of order 2. Define an action $\tilde{\rho}$ of $G \times \mathbf{Z_2}$ on $M \sqcup M$ by $(g, a) \cdot (x, \epsilon) = (g \cdot x, a\epsilon)$ where $g \cdot x = \rho(g, x)$ and $a \in \mathbf{Z_2}$. That is, the action of $\mathbf{Z_2}$ just interchanges the two copies of M. Then the atlas $\{(X, M \sqcup M, G \times \mathbf{Z_2}, \tilde{\rho}, -)\}$ should give the same orbifold structure on X as the atlas $\{(X, M, G, \rho, -)\}$. To make this sort of requirement precise we need to define equivalence of orbifold atlases.

Definition 1.5. *An atlas $\mathcal{U}$ is said to refine another atlas $\mathcal{V}$ if every chart of $\mathcal{U}$ has an injection into some chart of $\mathcal{V}$. Two atlases are said to be equivalent if they have a common refinement.*

Definition 1.6. *An orbifold structure on X is an equivalence class of n-dimensional orbifold atlases on X.*

One can also define orbifold in the topological, that is C^0, category. It is a good exercise now to put additional structures on a differentiable orbifold such as an almost complex, complex or symplectic structure by assuming such structures for the spaces, actions and maps in the charts and injections. The notion of algebraic structure is more involved and will not be explicitly described in these notes. This is due to the coarseness of Zariski topology. One needs to introduce the notion of étale topology. A fairly easy description can be given in the language of (étale) groupoids. Moreover the language of groupoids is very general and incorporates he notion of orbifolds in all the categories mentioned above. It is also a convenient tool for constructions and proofs. The reader is referred to [Mo] for a nice treatment of orbifolds in the language of groupoids.

1.1. **Examples.**

(1) Consider an orbifold given by a single chart $(M/G, M, G, \rho, \pi)$ where M is a manifold, G is finite group. If G is trivial, then the orbifold is just the manifold M. On the other hand if M is a point, then the orbifold structure is same as the group structure of G. An orbifold defined by a single chart is called a global quotient.

(2) More generally we can consider the action of a Lie group H on a manifold M with finite isotropy groups. The quotient space M/H has a natural structure of an orbifold in this case. Conversely any effective orbifold may be obtained in this way. This is an extremely useful fact. Proofs of these facts and an excellent account of the foundations of the theory of orbifolds can be found in [MM].

(3) It is worth noting that the same topological space may have different orbifold structures. For instance the quotient of the complex plane by the antipodal action of $\mathbf{Z_2}$ (i.e. $z \mapsto -z$ by the action of the nontrivial element)is homeomorphic to the complex plane. So the complex plane can be given at least two distinct orbifold structures, one corresponding to its natural manifold structure and the other via the atlas $\{(\mathbb{C}, \mathbb{C}, \mathbf{Z}_2, -, -)\}$. Similarly a Riemann surface can have many orbifold structures. See the article by Scott [Sc] for a nice classification and a very interesting invariant, due to Thurston, called the orbifold fundamental group which is often useful in determining if an orbifold can be a global quotient.

(4) Examples of an orbifold algebraic variety may be constructed easily. For instance consider the action of $\mathbf{Z_2}$ on $\mathbb{C}^2$ by $-1 \cdot (z_1, z_2) = (-z_1, -z_2)$. The coordinate ring of the quotient space, by definition, is generated by the invariant polynomials of the action. The invariant polynomials are generated

by $u = z_1^2$, $v = z_2^2$ and $w = z_1 z_2$ with a single relation $uv = w^2$. Thus the quotient space $(\mathbb{C}^2/\mathbf{Z}_2)$ and the variety $\{uv - w^2 = 0\} \subset \mathbb{C}^3$ are isomorphic.

More involved examples of orbifolds in algebraic geometry are simplicial toric varieties and their generic hypersurfaces. The reader is referred to chapter 3 of [CK] for a detailed description starting from the basics as well as applications.

(5) Two main sources of orbifolds in geometry are moduli spaces and quotients of physical space such as phase space by group of symmetries that preserve mechanics. We describe these briefly in the next two sections.

2. Orbifolds as moduli stacks

A moduli space is a parameter space for families of objects of a category. The category could have, to give an example, curves of a fixed genus as objects and isomorphisms of such curves as morphisms. Note that an object can be considered as a family over a point.

Roughly speaking, $\mathcal{M}$ has to satisfy two conditions to be a moduli space:

(i) There is exactly one point in $\mathcal{M}$ for each isomorphism class of allowable fiber (object).

(ii) The base of any family can be mapped into $\mathcal{M}$ such that the fibers above each point correspond to the point in M they are mapped to.

A coarse moduli space is the set of the isomorphism classes of objects. It satisfies the two conditions above. It can usually be given a natural topology. Let us consider an example.

2.1. Moduli space of triangles.

Define a triangle to be an ordered 3-tuple i.e. a point (x_1, x_2, x_3) in $\mathbb{R}^3$ with positive coordinates which satisfy the three inequalities that require the sum of any coordinates to exceed the third. Define two triangles to be isomorphic if one is a permutation of the other. In this way we can give a natural topology to the parameter space $\mathcal{P}$ of triangles as a subspace of $\mathbb{R}^3$. The coarse moduli space of triangles is the quotient space $\mathcal{P}/S_3$, where the symmetric group S_3 acts naturally on $\mathcal{P}$ by permuting coordinates. Note that for the notion of coarse moduli it suffices to consider the quotient topology on $\mathcal{P}/S_3$, unless we want to impose a differentiable structure. However the orbifold structure becomes indispensable if we want to construct a fine moduli space. We illustrate this next.

2.2. Fine moduli space.

A fine moduli space is a space $\mathcal{M}$, together with a family $\mathcal{F}$ called the universal family, such that in addition to condition (i) and (ii) the following holds.

(iii) Given any family $\mathcal{G}$ over a base space B, there is a unique map $f : B \to \mathcal{M}$ such that the pullback of $\mathcal{F}$ along f is $\mathcal{G}$.

The pullback family is defined by specifying that its fiber over any point $b \in B$ is same as the fiber of $\mathcal{F}$ over the point $f(b) \in \mathcal{M}$.

It is evident that a fine moduli space has much more information than a coarse one. However there is often no space which satisfies all the three conditions. Rather we have to settle for something more general than a space, called a stack. A stack is roughly speaking a space together with an equivalence relation. Deatiled notes on stacks are available on the homepages of William Fulton and Andrew Kresch.

Accordingly, we also have to modify the condition (i) for moduli to the following altered condition.

(i′) There is exactly one point in $\mathcal{M}$ up to equivalence for each isomorphism class of allowable fiber (object).

2.3. **Moduli stack of triangles.** Let us revisit the moduli problem for triangles. The fine moduli stack is the orbifold $(\mathcal{P}/S_3, \mathcal{P}, S_3, -, -)$ with the tautological family with induced S_3 action as the universal family.

Consider the family of triangles

$$\Delta_t = \{(\frac{1}{4}+\frac{t}{2}, \frac{3}{4}-\frac{t}{2}, 0.9) \,:\, 0 \le t \le 1\}.$$

Observe that Δ_t can be recovered from fine moduli space by pullback, but not as pullback of any family over the coarse moduli space.

3. Orbifolds in mechanics

3.1. **Hamiltonian mechanics.** Recall the Hamiltonian formulation of Newtonian mechanics. For a finite dimensional classical system, the configuration space is defined to be $Q := \{\mathbf{q} = (q_1, \ldots, q_n) : q_i \text{ is a position coordinate}\}$. The phase space for such a system is defined to be $M := \{(\mathbf{q}, \mathbf{p}) : \mathbf{q} = \text{position},\ \mathbf{p} = \text{momentum}\} = T^*Q$. If we consider a system of n particles in $\mathbb{R}^3$, then the configuration space is $\mathbb{R}^{3n}$ and the phase space is $\mathbb{R}^{6n}$.

A (Time-independent) Hamiltonian $H : M \to \mathbb{R}$ is a smooth function (energy) that controls dynamics via Hamilton's equations:

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$$

For instance, if we consider the motion of a single particle of unit mass in $\mathbb{R}$ with position coordinate $x(t)$, and potential energy $V(x)$ we have

$$H = \frac{1}{2}(x')^2 + V(x), \quad q = x,\ p = x' = \frac{dx}{dt}.$$

Then Hamilton's equations yield Newton's laws of motion:

$$\frac{\partial H}{\partial p} = x' = \frac{dq}{dt} \quad \text{and} \quad -\frac{\partial H}{\partial q} = -\frac{\partial V}{\partial x} = \text{Force} = p' = x''.$$

3.2. **Geometric formulation.** A symplectic form ω is a closed non-degenerate skew-symmetric bilinear form

$$\omega : \Gamma(TM) \times \Gamma(TM) \to C^\infty M.$$

Non-degeneracy implies that ω induces an isomorphism $\phi : T^*M \to TM$, defined by $(\eta)(Y) = \omega(\phi(\eta), Y)$ for any $\eta \in T^*M$ and $Y \in TM$. For any smooth function h on M, the vector field X_h is defined to be $\phi(dh)$.

There is a natural ω that provides a geometric formulation of Hamilton's equations (dynamics) as a correspondence between the Hamiltonian H and a vector field $X_H := \phi(dH)$ on the phase space. Dynamics is the flow generated by X_H, see [Ar] or [Bu] for details. In the example of a single particle in $\mathbb{R}$,

$$\omega = dq \wedge dp \quad \text{and} \quad X_H = (\frac{\partial H}{\partial p}, -\frac{\partial H}{\partial q}).$$

The great advantage of this formulation is that it gives us the optimal method to study mechanics in any manifold.

3.3. **Symmetries and constants of motion.** The structure (H, ω) admit groups of symmetries. For example in the case of n particles in $\mathbb{R}^3$,

$$H(\mathbf{q}, \mathbf{p}) = \sum_i \frac{\mathbf{p}_i^2}{2m_i} + G \sum_{i<j} \frac{m_i m_j}{||\mathbf{q}_i - \mathbf{q}_j||},$$

and each element of the Euclidean group of rotations and translations on $\mathbb{R}^3$ is a symmetry. Dynamics is invariant under symmetry. That is dynamics is qualitatively indistinguishable at different translates of the symmetry. This gives rise to an interesting debate about nature of physical space, see [Bu]. The following theorem of Emmy Noether is of crucial importance.

Theorem 3.1. *(Noether) Every symmetry corresponds to a constant of motion.*

Proof. We will give an idea of the proof. Define the Poisson bracket $\{f, h\} := \omega(X_f, X_h)$ for any smooth functions f and h on M. It is skew-symmetric since ω is so. Moreover $X_f(h) = dh(X_f) = \omega(\phi(dh), X_f) = \omega(X_h, X_f) = \{h, f\}$. Consider a symmetry to be a vector field X whose flow preserves both the Hamiltonian H and the symplectic form ω. It is then possible to show using basic differential geometry that X is locally of the form X_F for some locally defined function F. Now it follows from the calculation below that F is a constant under dynamics.

$$X_F(H) = \{H, F\} = 0 \Longleftrightarrow 0 = \{F, H\} = X_H(F)$$

□

3.4. **Symplectic reduction.** For simplification it is desirable to consider the level set (resp. intersection of level sets) of a conserved function (resp. a group worth of conserved functions) instead of the entire phase space. However such a level set may not be symplectic, ω restricted to such subset may not be non-degenerate. However a subgroup of the full symmetry group may act on the level set so that the quotient space is symplectic. The quotient space is often an orbifold. This is how orbifolds often arise as a physical space.

4. Stringy invariants

The basic idea in string theory is that the fundamental constituents of physical objects are strings of extremely small scale which vibrate at specific frequencies. Thus, any particle should be thought of as a tiny vibrating string, rather than as a point. This object can vibrate in different modes, with every mode appearing as a different particle. Motivated by the discussion on reduction in the last section, we consider strings on orbifolds in this section following the seminal approach of [DHVW].

4.1. **Closed strings.** For simplicity we will only consider a global quotient orbifold (X, M, G, ρ, π). Note that the group G is finite. Consider closed strings in $X = M/G$. Mathematically these are maps $\gamma : [0,1] \to X$ with $\gamma(0) = \gamma(1)$. A lift $\tilde{\gamma}$ of γ is a map $\tilde{\gamma} : [0,1] \to M$ such that $\pi(\tilde{\gamma}) = \gamma$. A lift need not be a loop. But its endpoints project down to the same point in X and therefore must satisfy $\tilde{\gamma}(1) = g \cdot \tilde{\gamma}(0)$ for some $g \in G$.

Two lifts of γ to M can be considered equivalent if they differ by the action of a group element,

$$\tilde{\gamma}_1 \sim \tilde{\gamma}_2 \text{ if } \tilde{\gamma}_2(t) = h \cdot \tilde{\gamma}_1(t).$$

Note that if $\tilde{\gamma}_1(1) = g \cdot \tilde{\gamma}_1(0)$ then

$$\tilde{\gamma}_2(1) = h \cdot \tilde{\gamma}_1(1) = hg \cdot \tilde{\gamma}_1(0) = hgh^{-1} \cdot \tilde{\gamma}_2(0)$$

4.2. **Inertia orbifold.** In the classical limit, the strings become points(particles). So we consider constant maps $\gamma(t) \equiv x$ and a lift satisfies $\tilde{x} = g \cdot \tilde{x}$ for some g. Hence $\tilde{x}$ is a fixed point of g. An equivalent lift $h \cdot \tilde{x}$ then satisfies $hgh^{-1} \cdot (h \cdot \tilde{x}) = h \cdot \tilde{x}$.

Thus from string theory perspective, we are led to consider the space $\bigsqcup_G M^g$ with an equivalence relation induced by the G action on M. In other words, we obtain the so-called inertia orbifold $(-, \bigsqcup_G M^g, G, -, -)$. Note that a group element h maps M^g to $M^{hgh^{-1}}$. So a different representation for the inertia orbifold is

$$\bigsqcup_{|G|} (-, M^g, C(g), -, -),$$

where $|G|$ denotes the set of conjugacy classes of G and $C(g)$ denotes the centralizer of g in G.

The inertia orbifold can also be constructed from other viewpoints such as equivariant K-theory. It was discovered by Kawasaki [Ka] long before the work of string theorists.

4.3. **Invariants.** The more interesting "classical" invariants of orbifolds are invariants of the inertia orbifold or its underlying topological space. For example the (ungraded) orbifold (Chen-Ruan) cohomology group of a global quotient orbifold $(-, M, G, -, -)$ is $\bigoplus_{|G|} H^*(M^g/C(g))$. When the orbifold has almost complex structure it is possible to introduce a grading and define Poincaré duality. This invariant was known for global quotients due to physicists. But the general construction and a ring structure was introduced by Chen-Ruan in [CR1].

When M is a point, the orbifold cohomology group is identified with the set of class functions of the group G and the Chen-Ruan ring is the center of the group ring of G. When M is a differentiable manifold and G is trivial, Chen-Ruan cohomology is just the singular cohomology of the manifold.

The orbifold K-theory of $(-, M, G, -, -)$ may be defined as the G-equivariant K-theory of M. This may also be related to the inertia orbifold by the following localization result of Atiyah-Segal [AS],

$$K^*_G(M) \otimes \mathbb{C} = \oplus_{|G|} K^*(M^g/C(g)) \otimes \mathbb{C}.$$

A more general construction of orbifold K-theory and analogous characterization in terms of inertia orbifold is given by Adem-Ruan [AR].

4.4. **McKay correspondence.** We refer the reader to [Re] for a comprehensive review of McKay correspondence. In our context, quite remarkably, it means that orbifold cohomology group is graded isomorphic to the cohomology group of a crepant resolution of an algebraic variety with $SL(n, \mathbb{C})$ quotient singularities. A resolution (birational map) $\rho : Y \to X$ of singularities is called crepant if $\rho^*(K_X) = K_Y$. This can be proved directly in case of toric hypersurfaces, see [Po, BM]. However there is a more general, beautiful and intrinsic method called motivic integration. It was introduced by Kontsevich and developed by Batyrev [Ba] and Denef-Loeser [DL]. See [Cr] for a very readable introduction. Using this technique, the result was obtained for general complete algebraic varieties with SL quotient singularities in [LP] and [Ya] independently. The Chen-Ruan cohomology ring is not usually invariant under crepant resolution. However it is so if the orbifold and the resolution has holomorphic symplectic structure. Results in this direction have been obtained by [FG], [Ur] and [GK].

The "quantum" invariants of orbifolds are at a higher level of subtlety than the inertia orbifold. Main invariants are orbifold Gromov-Witten invariants due to Chen-Ruan [CR2]. Since crepant resolutions do not always exist, these invariants are expected to play an important role in mirror symmetry in dimensions greater than three.

It is not possible to describe all the invariants and directions of research in these brief notes. The reader is encouraged to look at [ALR] for a more comprehensive overview, many more exciting results and conjectures.

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