Equal Pay for Unequal Work: Limiting Sabotage in Teams

by

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Abstract

We demonstrate the value of "equal pay" policies in teams, even when team members have distinct abilities and make different contributions to team performance. A commitment to compensate all team members in identical fashion eliminates the incentive that each team member otherwise has to sabotage the activities of teammates in order to induce the team owner to implement a more favorable reward structure. The reduced sabotage benefits the team owner, and can secure Pareto gains under plausible circumstances.

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1 Introduction.

It is well known that participants in a tournament can face strong incentives to sabotage the activities of fellow participants (e.g., Dye, 1984; Lazear, 1989; Chen, 2003). By sabotaging a competitor's activities, a participant can improve his relative performance and thereby increase the probability that he wins the tournament and collects the associated prize.

It is also well known that very different incentives often prevail in team settings, where only the aggregate performance of the team – not the performance of individual team members – is observable. Consider, for example, the simple setting in which the team project either succeeds or fails, each team member (i.e., each "agent") is risk neutral, no agent has any initial wealth, and the probability that the team project succeeds is a strictly increasing function of the contribution of each agent. In this setting, each agent will optimally receive a positive payment only when the project succeeds. Therefore, given the prevailing wage structure, each agent's expected profit increases as the aggregate probability of success increases. In such a setting, each agent will wish to assist, not sabotage, the operations of his teammates.

Although members of a team often will refrain from sabotage, they will not always do so. We show that an agent can gain from sabotaging the operations of his teammates when the sabotage occurs before the terms of the team's compensation structure are finalized. This timing may arise, for example, when the sabotage in question has long-term ramifications (e.g., when an agent can limit a teammate's ability to develop enduring job skills or acquire lasting complementary resources) and when payments to team members are adjusted periodically to reflect changing labor market conditions or skill levels. In such settings, an agent can benefit from sabotage that induces a favorable response from the team owner ("the principal").

To illustrate the nature of the favorable response that sabotage can elicit, suppose one team member – agent A – sabotages the activities of his teammates. The sabotage reduces agent A's relative cost of enhancing the aggregate probability of project success. In response to agent A's lower relative cost, the principal optimally secures a larger contribution to project success from agent A. The principal does so by differentially increasing the payment to agent A when the project succeeds. The prospect of securing this increased payment leads agent A to sabotage the operations of his teammates in the simple, but arguably plausible, team setting that we analyze.

We also show that the principal can eliminate the incentives for such sabotage if she is able to commit herself in advance to implement an "equal pay" policy that promises identical payments to all agents. An equal pay policy does not constrain the principal to specify in advance the exact payment that she will deliver to each agent when the project succeeds. Consequently, the policy affords the principal some ability to adjust payments as the environment changes. However, by eliminating her ability to differentially favor an agent as his relative cost declines, the principal eliminates the incentive that agents would otherwise have to sabotage the activities of their teammates.

Equal pay policies are common in practice. Farrell and Scotchmer (1988) and Encinosa et al. (2007), for example, report that members of legal and medical partnerships often share profits equally. In practice, equal pay policies may not be implemented solely to limit sabotage by members of a team.[1] However, this function of an equal pay policy would seem to be of practical relevance in many settings of interest. Furthermore, to our knowledge, this function of an equal pay policy has not been addressed in the literature.

Our analysis may be most closely related to the work of Itoh (1991), who identifies conditions under which a principal gains by introducing marginal incentives for agents to provide limited assistance to one another. Itoh's work differs from ours in part because each agent in Itoh's model is primarily responsible for a distinct project, and the outcome (success or failure) of each project is observable and contractible, much as in a tournament.[2] In our

[1] For example, Bartling and von Siemens (2007) demonstrate that an equal sharing rule can constitute an optimal compensation structure in a partnership when the partners are averse to inequitable outcomes.

[2] The agents in Itoh's model are not wealth constrained. Consequently, if the agents were risk neutral in Itoh's model as they are in our model, the principal could costlessly resolve the moral hazard problems that she faces. The principal could do so by selling to each agent the individual project for which he is primarily responsible at a price that reflects the expected net return from the project, given the efficient effort supply.

model, only the aggregate performance of the entire team is contractible. Consequently, an agent cannot reduce the performance of a teammate without simultaneously reducing his own performance.

Our analysis proceeds as follows. Section 2 reviews the key elements of our model. Section 3 considers the optimal team size. Section 4 presents our main findings. Section 5 discusses extensions of our basic model. Section 6 concludes and suggests directions for future research. The proofs of all formal conclusions are presented in the Appendix.

2 The Model.

We consider a setting where a risk-neutral principal hires two risk-neutral agents – agent A and agent B – to operate her project.[3] The project either succeeds or fails. A successful project generates value V for the principal. A failed project generates no value.

Agent $i \in \{A, B\}$ contributes success probability p_i to the project. The aggregate probability that the project succeeds (p) is the sum of the success probabilities contributed by the two agents, i.e., $p = p_A + p_B$. This simple formulation is adopted initially for analytic ease. A formulation that admits interactions among the agents' contributions is considered in section 5.

An agent will work for the principal as long as he anticipates non-negative profit from doing so. An agent's profit is the difference between the payment he receives from the principal and the cost he incurs working for the principal. Agent i incurs cost $\frac{k_i}{\theta}[p_i]^\theta$ when he delivers success probability p_i, where $k_i \in [\underline{k}_i, \overline{k}_i]$ and $0 < \underline{k}_i < \overline{k}_i$.[4] We assume that $\theta > 1$, so each agent's cost is an increasing, convex function of the success probability that he contributes.

We will analyze the incentive that each agent has to assist or to hinder the operations of

[3] Lemma 1 in section 3 identifies conditions under which the principal prefers to hire two agents than to hire a single agent.

[4] This class of cost functions admits closed form solutions for all relevant variables, and thereby facilitates direct comparisons of distinct institutional regimes. While not completely general, this cost structure admits a wide variety of plausible cost functions.

a teammate. To model this behavior most simply, we assume that agent i can, at no personal cost, choose his preferred level of $k_j \in [\underline{k}_j, \overline{k}_j]$ for $j \neq i$, $i, j \in \{A, B\}$. Thus, one might view $k_j^o \in [\underline{k}_j, \overline{k}_j]$ as agent j's innate operating cost.[5] Agent i can reduce k_j below k_j^o by providing assistance to agent j. Alternatively, agent i can increase k_j above k_j^o by sabotaging agent j's activities. The bounds on feasible assistance $(k_j^o - \underline{k}_j)$ and sabotage $(\overline{k}_j - k_j^o)$ might reflect, for example, technological considerations or the limited amount of time that agent i has available to assist or to sabotage his teammate's operations. We assume that the agents determine simultaneously and independently the level of assistance (or sabotage) that they will deliver.

Neither the individual contributions (p_i) of the team members nor the final operating costs $(k_i \in [\underline{k}_i, \overline{k}_i])$ of the agents are contractible.[6] Thus, even though she ultimately observes each agent's operating cost, the principal cannot base payments directly on the level of these costs. Payments can only reflect the ultimate success or failure of the project. T_i will denote the payment the principal delivers to agent i when the project succeeds. The principal makes the minimum possible payment (0) to each agent when the project fails.[7] This minimum payment might reflect the agent's limited wealth or legal limits on the agent's liability, for example (Sappington, 1983). The principal seeks to maximize the expected difference between the value (V) she derives from the project and the payments she makes to the agents.

Given the prevailing configuration of the agents' costs (k_A, k_B), the principal's problem, [P], is the following:

$$\underset{\{T_A,\, T_B\}}{Maximize} \quad [\, p_A + p_B \,]\, [\, V \,-\, T_A \,-\, T_B \,] \tag{1}$$

[5] For expositional ease, we will often refer to k_i as agent i's "cost" or "operating cost," even though k_i is actually a parameter of the agent's cost function.

[6] Final operating costs $k_j \in [\underline{k}_j, \overline{k}_j]$ (and thus the level of assistance that each agent provides) may not be contractible, for instance, because of the detailed, idiosyncratic knowledge of the working environment that is required to distinguish among the many factors that affect operating costs. However, sabotage in excess of $\overline{k}_j - k_j^o$ may be so egregious that it is readily observed and documented, and thus contractible.

[7] It is readily shown that it is optimal for the principal to deliver the smallest possible payment to each agent when the project fails. This payment structure minimizes the agents' rent for any given level of incentive they face to increase the probability of project success.

subject to:
$$[\, p_A + p_B \,]\; T_i \;-\; \frac{k_i}{\theta} [p_i]^\theta \;\geq\; 0 \quad \text{for } i \;=\; A,\, B\,; \tag{2}$$

$$p_A \;=\; \arg\max_{p} \left\{ [\, p + p_B \,]\; T_A \;-\; \frac{k_A}{\theta} [p]^\theta \right\}; \tag{3}$$

$$p_B \;=\; \arg\max_{p} \left\{ [\, p_A + p \,]\; T_B \;-\; \frac{k_B}{\theta} [p]^\theta \right\}; \text{ and} \tag{4}$$

$$0 \;\leq\; p_A + p_B \;\leq\; 1. \tag{5}$$

Expression (1) reflects the principal's desire to maximize the expected difference between the value she derives from the project and the payments she makes to the agents. Inequality (2) ensures that the agents receive non-negative (expected) profit, in equilibrium.[8] Equations (3) and (4) identify the equilibrium success probabilities that agents A and B, respectively, contribute to the project (simultaneously and independently). Inequality (5) ensures that the aggregate probability of success is well defined.

3 Team Composition.

The discussion to this point has assumed that the principal always hires two agents to operate her project. Conceivably, the principal might prefer to hire a single agent, and thereby avoid both sabotage and free-rider problems. A free-rider problem can arise when an agent reduces his contribution to the aggregate success probability, knowing that the principal cannot distinguish between his contribution and the contribution of his teammate. On the other hand, the principal might prefer to hire two agents because it is more costly for a single agent to secure a given aggregate success probability than it is for two agents to secure the same success probability. The increased cost when the team consists of a single agent reflects the convex cost structure under which each agent operates (i.e., $\theta > 1$).

Lemma 1 reports that the principal prefers to hire two agents rather than a single agent whenever the diminishing returns that each agent faces in enhancing the probability of project success are sufficiently pronounced (i.e., whenever θ is sufficiently large). The lemma

[8] For expositional ease, the term "profit" often will be employed in place of "expected profit."

refers to $\underline{k} \equiv \min\{\underline{k}_A, \underline{k}_B\}$ and $\overline{k} \equiv \max\{\overline{k}_A, \overline{k}_B\}$.

Lemma 1. *The principal prefers to hire two agents rather than a single agent if* $\theta > 2 + \frac{\ln(\overline{k}/\underline{k})}{\ln 2}$.

The ensuing discussion will restrict attention to settings such as those identified in Lemma 1 to ensure that the presumed operation with two agents is indeed optimal for the principal.[9]

4 Hindering Teammates.

As noted in the Introduction, agents often have strong incentives to sabotage each other's activities when they compete in tournaments. By diminishing a competitor's performance, an agent can improve his own relative performance, and thereby increase the probability that he wins the tournament.

Very different incentives can arise when agents operate in teams. In team settings like the one considered here, the principal cannot reward agents according to their relative performance because individual contributions to aggregate performance are not contractible. Payments can only reflect the observed success or failure of the project. To motivate both agents to contribute to the success of the project, the principal optimally delivers a strictly positive payment to each agent whenever the project succeeds. Therefore, given the prevailing reward structure, each agent's expected net payoff increases as the aggregate probability of success increases. Consequently, when the reward structure is set before the agents decide how much assistance to deliver,[10] each agent will maximize his own profit by assisting his teammate to the greatest extent possible. The assistance reduces the teammate's operating cost (k_j), which leads him to deliver a higher success probability. This conclusion is recorded formally as Proposition 1.

Proposition 1. *Suppose the principal specifies the final terms of the reward structure before*

[9] For analytic ease, we assume the team consists of at most two agents. The possibility of expanded team membership is discussed in section 6.

[10] For expositional ease, the ensuing discussion often will refer to an agent's decision about how much assistance or sabotage to deliver simply as the agent's decision about how much assistance to deliver.

each agent chooses the level of assistance that he will deliver to his teammate. Then each agent will provide the maximum feasible assistance to his teammate, i.e., agent A will set $k_B = \underline{k}_B$ *and agent B will set* $k_A = \underline{k}_A$*.*

There are some settings in which a principal might be able to make a binding commitment to the details of a reward structure before each agent decides how much assistance (or sabotage) he will deliver. This timing might arise, for example, in a short-term employment setting where the employer first specifies the wage structure and each individual employee then decides how much of his scarce time to devote to physically assisting a fellow employee or to distracting the employee from his assigned task.

Alternative timings are plausible in different settings, though. Consider, for example, a long-term employment relationship in which labor market conditions and employee productivities change over time, and where employers can adjust wages to attract or retain valued workers and to induce more efficient configurations of employee effort. Suppose further that each employee can mentor his fellow workers and help them to improve their operating skills. Alternatively, the employee can make a concerted effort to reduce the long-term productivity of fellow workers, perhaps by blocking their efforts to acquire valuable skills or secure complementary resources.

In settings like these, it is reasonable to view the agents as choosing their preferred levels of assistance before the principal commits to the terms of a reward structure. When this timing prevails, rational agents will consider the impact of their actions on the reward structure that ultimately will be implemented. Lemma 2 characterizes this reward structure and the corresponding success probability that each agent will contribute in equilibrium, given prevailing costs (k_i).

Lemma 2. $T_i = V\theta^{-1}\left[1 + \left(\frac{k_i}{k_j}\right)^{\frac{1}{\theta-2}}\right]^{-1}$ *and* $p_i = V^{\frac{1}{\theta-1}}\left(\theta k_i\left[1 + \left(\frac{k_i}{k_j}\right)^{\frac{1}{\theta-2}}\right]\right)^{-\frac{1}{\theta-1}}$ *at the solution to [P], for* $i, j \in \{A, B\}, \ i \neq j$*.*

Lemma 2 reveals that the principal optimally provides a larger reward for success (T_i) to agent i in this team setting (where $\theta > 2$) as the cost (k_j) of agent j increases. Furthermore, the rate at which T_i increases as k_j increases exceeds the corresponding rate at which the success probability that agent j ultimately contributes (p_j) declines. Thus, on balance, each agent gains as he hinders the activities of his teammate because of the resulting impact on the equilibrium reward structure, as Proposition 2 reports.

Proposition 2. *Suppose each agent chooses the level of assistance that he delivers to his teammate before the principal makes any commitment regarding the terms of the reward structure. Then each agent will sabotage his teammate's activities to the greatest extent possible, i.e., agent A will set* $k_B = \overline{k}_B$ *and agent B will set* $k_A = \overline{k}_A$.

Proposition 2 reflects the fact that the principal effectively has two sources from which she can procure an essential input (i.e., success probability) – agent A and agent B. As agent B's operating cost (k_B) increases, agent A becomes a relatively less costly source of the essential input. Consequently, the principal optimally secures more of the input from the less costly source by increasing the payment that she makes to agent A when the project succeeds. The prospect of securing an increased reward for success leads each agent to sabotage his teammate's activities to the maximum extent possible when each agent chooses his preferred level of assistance before the principal commits to the terms of the reward structure.

Propositions 1 and 2 suggest that the principal could benefit from credibly committing to an immutable, long-term reward structure before the agents choose their preferred levels of assistance. In practice, though, long-term binding commitments to all relevant details of a reward structure can be difficult to ensure. Ongoing changes in labor market conditions and worker productivities can compel employers to modify compensation structures in order to retain valued employees and to allocate aggregate effort supplies more efficiently, for example. A question that arises, then, is whether the principal can avoid the losses that sabotage imposes even when she cannot make a credible long-term commitment to all relevant details of a reward structure.

Proposition 3 reports that the principal can avoid these losses if she can commit to implement an egalitarian reward structure. An egalitarian reward structure is one that always compensates the two agents in identical fashion.[11] A commitment to an egalitarian reward structure does not eliminate the principal's ability to alter the compensation schedule as environmental conditions change. The commitment only obligates the principal to treat the two agents symmetrically.

Proposition 3. *Suppose the principal commits to implement an egalitarian reward structure before each agent chooses the level of assistance that he delivers to his teammate. Then each agent will provide the maximum feasible assistance to his teammate, i.e., agent A will set* $k_B = \underline{k}_B$ *and agent B will set* $k_A = \underline{k}_A$.

A commitment to an egalitarian reward structure alters the incentive of an agent (agent A, for example) to sabotage his teammate's operation. Agent A can reduce his relative cost by increasing agent B's cost (k_B). However, when she is committed to an egalitarian reward structure, the principal cannot respond to the reduction in agent A's relative cost by differentially increasing agent A's payment for success, as she would in the absence of the commitment. Consequently, the primary effect of increasing k_B in the presence of an egalitarian reward structure is to reduce the aggregate equilibrium probability of success (p). Because the reduction in p reduces agent A's expected profit, the agent will not sabotage his teammate's activities. Instead, agent A will deliver the maximum possible assistance to agent B in order to increase his own profit by increasing the aggregate probability of success.

The increased assistance induced by a commitment to an egalitarian reward structure benefits the principal by increasing the probability that the project succeeds. This benefit often outweighs any loss that the principal incurs from an inability to tailor payments to the distinct capabilities of the agents. As Proposition 4 reports, the benefit exceeds the loss

[11] Thus, $T_A = T_B$ under an egalitarian reward structure. Using techniques analogous to those employed in the proof of Lemma 1, it is readily shown that when she implements an egalitarian reward structure, the principal prefers to hire two agents rather than one agent if $\theta > 2 + \frac{\ln(\underline{k}_x / \underline{k}_n)}{\ln 2}$, where $\underline{k}_x \equiv \max\{\underline{k}_A, \underline{k}_B\}$ and $\underline{k}_n \equiv \min\{\underline{k}_A, \underline{k}_B\}$.

whenever there is any overlap in the range of possible costs for the two agents.

Proposition 4. *Suppose* $(\underline{k}_A, \overline{k}_A) \cap (\underline{k}_B, \overline{k}_B) \neq \varnothing$ *and suppose each agent chooses his preferred level of assistance before the principal specifies the precise rewards for success. Then the principal's profit is strictly higher when she commits herself ex ante to implement an egalitarian reward structure than when she makes no such commitment.*

To understand the principal's preference for an egalitarian reward structure under the condition specified in Proposition 4, let $\widetilde{k}$ denote the smallest k in $(\underline{k}_A, \overline{k}_A) \cap (\underline{k}_B, \overline{k}_B)$. The principal can ensure that both agents operate with a cost that is below $\widetilde{k}$ by committing to implement an egalitarian reward structure. As Proposition 3 indicates, this commitment induces each agent to deliver the maximum possible assistance to his teammate. Consequently, agent $i \in \{A, B\}$ will operate with cost $\underline{k}_i < \widetilde{k}$. In contrast, if the principal does not commit herself to an egalitarian reward structure, each agent will increase his teammate's cost above $\widetilde{k}$ in an attempt to secure differentially favorable treatment from the principal. The resulting high costs will reduce the principal's profit below the level that she would secure if both agents operated with cost $\widetilde{k}$. This profit, in turn, is less than the profit the principal secures when agent A operates with cost $\underline{k}_A < \widetilde{k}$ and when agent B operates with cost $\underline{k}_B < \widetilde{k}$. The principal can ensure this higher level of profit by committing to an egalitarian reward structure.

The principal's preference for an egalitarian reward structure identified in Proposition 4 is strong in three respects. First, the preference holds whenever there is any overlap in the agent's cost structures, however slight that overlap might be.[12] Second, the preference holds regardless of the magnitudes of the maximum feasible levels of assistance and sabotage. Third, the principal's preference holds for every combination of initial costs (k_A^o, k_B^o) that might arise, not just in expectation. Consequently, the preference identified in Proposition

[12] While the overlap identified in Proposition 4 is sufficient to ensure that the principal prefers to commit to implement an egalitarian reward structure, overlap is not necessary for this preference to arise. This conclusion is illustrated in Table 3 (below).

4 would persist in a setting where the principal is initially uncertain about the agents' costs, but she observes these costs before committing to the final details of a reward structure.

The principal's gain from committing to an egalitarian reward structure is illustrated in Tables 1 and 2 for the setting in which the value of success (V) is 4 and the cost parameter θ is also 4. Table 1 reports the equilibrium contributions of the two agents (p_A and p_B), the aggregate probability of project success (p), the payments to the two agents when the project succeeds (T_A and T_B), and the profits of agent A (R_A), agent B (R_B), and the principal (π) in the absence of any commitment to an egalitarian reward structure. Table 2 provides the corresponding measures when the principal commits *ex ante* to implement an egalitarian reward structure.[13]

Tables 1 and 2 reveal that if $\underline{k}_A = \underline{k}_B = 8$ and $\overline{k}_A = \overline{k}_B = 9$, the principal's profit increases by approximately 4% ($= 100\left[\frac{2.381-2.289}{2.289}\right]\%$) when she commits herself to implement an egalitarian reward structure. The corresponding increase is approximately 7% ($= 100\left[\frac{2.381-2.221}{2.221}\right]\%$) when $\underline{k}_A = \underline{k}_B = 8$ and $\overline{k}_A = \overline{k}_B = 10$. The larger increase in profit in this latter case reflects the more pronounced reduction in sabotage that the commitment to an egalitarian reward structure ensures.

Tables 1 and 2 also reveal that the agents, like the principal, can gain from operating under an egalitarian reward structure. Consider, for example, the setting in which $k_A \in [8, 9]$ and $k_B \in [9, 10]$. In this setting, the commitment to an egalitarian reward structure increases agent A's profit by 1.2% (from .336 to .340) and agent B's profit by 6.5% (from .321 to .342) as it increases the principal's profit. More generally, the commitment to an egalitarian reward structure always secures Pareto gains if the set of feasible costs is the same for the two agents, as Proposition 5 reports.

Proposition 5. *Suppose* $\underline{k}_A = \underline{k}_B$ *and* $\overline{k}_A = \overline{k}_B$. *Then the profit of the principal and the profit of both agents are strictly higher when the principal commits herself ex ante to*

[13] Equilibrium profit levels under the optimal egalitarian reward structure are denoted by the subscript "e" in Table 2 and in subsequent tables.

implement an egalitarian reward structure than when she makes no such commitment.

When the set of feasible costs is the same for the two agents, they both operate with cost $\underline{k}_A$ ($= \underline{k}_B$) when the principal commits to an egalitarian payment structure. In contrast, they both operate with cost $\overline{k}_A$ ($= \overline{k}_B$) when the principal makes no such commitment. It is readily shown that when they have the same final operating cost (i.e., when $k_A = k_B$), the profit that each agent secures in equilibrium increases as his operating cost declines. Therefore, both agents and the principal benefit from the lower symmetric costs that arise when the two agents have the same set of feasible costs and when the principal ensures the lowest feasible cost by committing herself to implement an egalitarian reward structure.

As Tables 1 and 2 reveal, a commitment to an egalitarian reward structure can secure Pareto gains more generally. However, such gains are not guaranteed. A particularly productive agent may be harmed by a commitment to an egalitarian reward structure. The agent benefits from the increased assistance that an egalitarian reward structure engenders. However, he may be harmed when the principal reduces the payment for success that she otherwise would deliver to the more productive agent. The principal optimally reduces this payment when she is committed to deliver the same payment to both agents.

Table 3 illustrates this more general conclusion. The table reports the equilibrium outcomes when $V = \theta = 4$ and when the cost of the more productive agent (agent A) can vary between 8.0 and 8.2 while the cost of the less productive agent (agent B) can vary between 9.8 and 10.0. In the absence of a commitment to an egalitarian reward structure, the principal optimally pays agent A more than she pays agent B when the project succeeds ($T_A = .525 > .475 = T_B$). Doing so induces the more productive agent to deliver a relatively large contribution ($p_A = .4$) to the aggregate probability of success. When a commitment to an egalitarian reward structure compels the principal to deliver the same payments to the two agents, she optimally reduces the payment to agent A (from .525 to .50). This payment reduction reduces agent A's profit (from .348 to .334), despite the cost reduction (from 8.2 to 8.0) that he secures under an egalitarian reward structure.

5 Extensions.

The analysis to this point has focused on settings in which the contributions of the two agents are independent. Before concluding, we briefly consider the extent to which our key qualitative conclusions persist in more general settings.

To do so, suppose the team project succeeds with probability $p = p_A + p_B + \gamma p_A p_B$ when agent A contributes success probability p_A and agent B contributes success probability p_B. γ is a parameter that captures the interactions between the agents' contributions. When γ is positive, the contributions of the two agents are complements. In other words, an increased contribution by one agent increases the rate at which the aggregate probability of success increases as the contribution of the other agent increases. When γ is negative, the contributions of the two agents are substitutes in the sense that an increased contribution by one agent reduces the rate at which the aggregate probability of success increases as the contribution of the other agent increases.

When the contributions of the two agents are substitutes (so $\gamma < 0$), each agent has two reasons to sabotage his teammate's operations. First, as explained above, sabotage of a teammate reduces an agent's relative cost, which helps him to garner more favorable treatment from the principal. Second, sabotage of a teammate reduces the teammate's equilibrium contribution to the aggregate probability of project success. The reduced contribution increases the rate at which the agent's own contribution increases the aggregate success probability. In essence, by sabotaging a teammate's operations, an agent increases both his relative productivity and his absolute productivity. Therefore, as numerical solutions reveal, agents typically will undertake the maximum level of sabotage when $\gamma < 0$ if the principal does not commit to an egalitarian reward structure.[14]

A different conclusion can arise when the contributions of the two agents are complements (i.e., when $\gamma > 0$). In this case, an agent's own (absolute) productivity declines when he sabotages a teammate's operations and thereby reduces the teammate's contribution to the

[14] This is the case, for example, for all values of $\theta \in \{4, 6\}$, $V \in \{3, 4, 5\}$, $\gamma \in \{-0.5, -0.4, -0.3, -0.2, -0.1\}$, and $k_i \in \{2.0, 2.5, 3.0, 3.5, ..., 9.0, 9.5, 10.0\}$ for $i = A, B$.

aggregate probability of project success. Numerical solutions reveal that when γ is sufficiently large, the agents will deliver the maximum feasible level of assistance even if the principal does not commit herself to implement an egalitarian reward structure. Consequently, the principal will not make this commitment when γ is sufficiently large. Such a commitment would only limit the principal's ability to induce a more efficient configuration of agent contributions without fostering increased assistance among team members.

To illustrate this more general conclusion, consider the setting in which $V = \theta = 4$, $\gamma = .5$, $k_A \in [8, 10]$, and $k_B \in [9.5, 10]$. As Table 4 reveals, the principal prefers not to commit herself to an egalitarian reward structure (i.e., $\pi > \pi^e$) in this setting. With no such commitment in place, the principal can tailor payments to the distinct equilibrium costs of the agents. These equilibrium costs ($k_A = \underline{k}_A = 8$ and $k_B = \underline{k}_B = 9.5$) are the same costs that would arise under an egalitarian reward structure because each agent delivers the maximum feasible level of assistance to his teammate in this setting.[15] Sabotage is not advantageous for the agents here because of the complementary nature of their contributions to the aggregate probability of project success.

6 Conclusions.

We have shown that sabotage can arise in teams, just as it can in tournaments. An agent reduces his relative cost when he sabotages the operation of a teammate. In response, the principal secures a larger contribution from the relatively more capable agent by rewarding him more generously when the project succeeds. Each agent finds it profitable to sabotage the operation of his teammate in order to secure this increased payment from the principal.

The principal can eliminate this incentive for sabotage – and thereby preclude sabotage – by committing herself to an egalitarian reward structure. Furthermore, the principal often

[15] In contrast, if γ were equal to 0 in this setting, each agent would deliver the maximum feasible level of sabotage if the principal did not commit herself to implement an egalitarian reward structure. The principal prefers to make this commitment in order to eliminate sabotage. Numerical solutions reveal that in the absence of a commitment to an egalitarian reward structure in the present setting, agent A will undertake sabotage when $\gamma < 0.15$ and refrain from sabotage when $\gamma > 0.15$. Agent B will undertake sabotage when $\gamma < 0.17$ and refrain from sabotage when $\gamma > 0.17$.

gains more from the reduced sabotage than she loses from the inability to tailor the reward structure to the individual capabilities of team members. This is the case, for example, whenever there is any overlap of the feasible operating costs of the two agents. The agents also can gain from the reduced sabotage that an egalitarian reward structure secures.

The structured environment that we considered permitted closed-form solutions for all relevant variables, and thereby facilitated a direct comparison of institutional settings. Future research should consider alternative production functions and cost functions in order to assess the extent to which our qualitative conclusions persist in different environments.

Future research also might consider explicit costs of assistance and sabotage. When it is costly for an agent to sabotage the activities of a teammate, the agent may not implement the maximum feasible level of sabotage. However, an egalitarian reward structure seems likely to be valuable in limiting sabotage whenever the personal cost that an agent incurs in implementing sabotage is sufficiently small.

Our analysis has emphasized the benefit of an egalitarian reward structure. In settings where the productivities of individual agents are subject to substantial idiosyncratic variation, a principal may prefer to retain some flexibility to deliver different rewards for success to different agents. Future research might explore the extent to which a principal optimally constrains her ability to reward agents asymmetrically in such settings in order to limit (but perhaps not eliminate) sabotage within the team.

More varied outcomes (e.g., a continuum of outcomes rather than only success or failure) and expanded team membership also warrant consideration. When a team consists of more than two agents, each agent may have to decide how to allocate among his teammates the total assistance or sabotage that he delivers. An agent might find it profitable to assist some teammates and sabotage the operations of other teammates, for example. These considerations and others (including the selection of agents into teams when some teams promise an egalitarian reward structure and others do not) await future research.

	$k_B = 8.0$	$k_B = 9.0$	$k_B = 10.0$
$k_A = 8.0$	$p_A = 0.397$ $p_B = 0.397$ $p = 0.794$ $T_A = 0.500$ $T_B = 0.500$ $R_A = 0.347$ $R_B = 0.347$ $\pi = 2.381$	$p_A = 0.401$ $p_B = 0.378$ $p = 0.778$ $T_A = 0.515$ $T_B = 0.485$ $R_A = 0.349$ $R_B = 0.332$ $\pi = 2.335$	$p_A = 0.404$ $p_B = 0.361$ $p = 0.766$ $T_A = 0.528$ $T_B = 0.472$ $R_A = 0.351$ $R_B = 0.319$ $\pi = 2.297$
$k_A = 9.0$	$p_A = 0.378$ $p_B = 0.401$ $p = 0.778$ $T_A = 0.485$ $T_B = 0.515$ $R_A = 0.332$ $R_B = 0.349$ $\pi = 2.335$	$p_A = 0.382$ $p_B = 0.382$ $p = 0.763$ $T_A = 0.500$ $T_B = 0.500$ $R_A = 0.334$ $R_B = 0.334$ $\pi = 2.289$	$p_A = 0.385$ $p_B = 0.365$ $p = 0.750$ $T_A = 0.513$ $T_B = 0.487$ $R_A = 0.336$ $R_B = 0.321$ $\pi = 2.250$
$k_A = 10.0$	$p_A = 0.361$ $p_B = 0.404$ $p = 0.766$ $T_A = 0.472$ $T_B = 0.528$ $R_A = 0.319$ $R_B = 0.351$ $\pi = 2.297$	$p_A = 0.365$ $p_B = 0.385$ $p = 0.750$ $T_A = 0.487$ $T_B = 0.513$ $R_A = 0.321$ $R_B = 0.336$ $\pi = 2.250$	$p_A = 0.368$ $p_B = 0.368$ $p = 0.737$ $T_A = 0.500$ $T_B = 0.500$ $R_A = 0.322$ $R_B = 0.322$ $\pi = 2.210$

Table 1. Outcomes With No Commitment to an Egalitarian Reward Structure when $V = \theta = 4$.

	$k_B = 8.0$	$k_B = 9.0$	$k_B = 10.0$
$k_A = 8.0$	$p_A = 0.397$ $p_B = 0.397$ $p = 0.794$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.347$ $R_B^e = 0.347$ $\pi^e = 2.381$	$p_A = 0.397$ $p_B = 0.382$ $p = 0.778$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.340$ $R_B^e = 0.342$ $\pi^e = 2.335$	$p_A = 0.397$ $p_B = 0.368$ $p = 0.765$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.333$ $R_B^e = 0.337$ $\pi^e = 2.296$
$k_A = 9.0$	$p_A = 0.382$ $p_B = 0.397$ $p = 0.778$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.342$ $R_B^e = 0.340$ $\pi^e = 2.335$	$p_A = 0.382$ $p_B = 0.382$ $p = 0.763$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.334$ $R_B^e = 0.334$ $\pi^e = 2.289$	$p_A = 0.382$ $p_B = 0.368$ $p = 0.750$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.327$ $R_B^e = 0.329$ $\pi^e = 2.250$
$k_A = 10.0$	$p_A = 0.368$ $p_B = 0.397$ $p = 0.765$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.337$ $R_B^e = 0.333$ $\pi^e = 2.296$	$p_A = 0.368$ $p_B = 0.382$ $p = 0.750$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.329$ $R_B^e = 0.327$ $\pi^e = 2.250$	$p_A = 0.368$ $p_B = 0.368$ $p = 0.737$ $T_A = 0.500$ $T_B = 0.500$ $R_A^e = 0.322$ $R_B^e = 0.322$ $\pi^e = 2.210$

Table 2. Outcomes With a Commitment to an Egalitarian Reward Structure when $V = \theta = 4$.

No Commitment to an Egalitarian Reward Structure	Commitment to an Egalitarian Reward Structure
$p_A = 0.400$	$p_A = 0.397$
$p_B = 0.362$	$p_B = 0.371$
$p \ = 0.762$	$p \ = 0.768$
$T_A = 0.525$	$T_A = 0.500$
$T_B = 0.475$	$T_B = 0.500$
$R_A = 0.348$	$R_A^e = 0.334$
$R_B = 0.319$	$R_B^e = 0.338$
$\pi \ = 2.287$	$\pi^e = 2.303$

Table 3. Outcomes when $V = \theta = 4$, $k_A \in [8.0,\ 8.2]$, and $k_B \in [9.8,\ 10]$.

No Commitment to an Egalitarian Reward Structure	Commitment to an Egalitarian Reward Structure
$p_A = 0.443$	$p_A = 0.439$
$p_B = 0.412$	$p_B = 0.416$
$p \ = 0.946$	$p \ = 0.946$
$T_A = 0.575$	$T_A = 0.560$
$T_B = 0.544$	$T_B = 0.560$
$R_A = 0.467$	$R_A = 0.455$
$R_B = 0.446$	$R_B = 0.458$
$\pi \ = 2.725$	$\pi^e = 2.724$

Table 4. Outcomes when $V = \theta = 4$**,** $\gamma = 0.5$, $k_A \in [8,\ 10]$, and $k_B \in [9.5,\ 10]$**.**

Appendix

Proof of Lemma 1.

When the principal hires a single agent, the agent's (expected) profit, given success probability p and payment for success T, is $pT - \frac{k}{\theta}(p)^{\theta}$. Therefore, the agent's profit-maximizing choice of p, given T, is determined by:

$$T \;=\; k\,(p)^{\theta-1}\,. \tag{6}$$

(6) implies that the principal's expected profit is:

$$\pi^{s} \;=\; p\,[V-T] \;=\; p\left[V - k\,(p)^{\theta-1}\right] \;=\; pV - k\,(p)^{\theta}\,. \tag{7}$$

Maximizing this function with respect to p provides:

$$\frac{\partial \pi^{s}}{\partial p} \;=\; V - k\theta p^{\theta-1} \;=\; 0 \quad\Rightarrow\quad p = \left[\frac{V}{\theta k}\right]^{\frac{1}{\theta-1}}. \tag{8}$$

(6), (7), and (8) imply that the principal's maximum profit with a single agent is:

$$\pi^{s} \;=\; p\left[V - k\,(p)^{\theta-1}\right] \;=\; \left[\frac{V}{\theta k}\right]^{\frac{1}{\theta-1}}\left[V - \frac{V}{\theta}\right]$$

$$=\; \left[\frac{V}{\theta k}\right]^{\frac{1}{\theta-1}}(V)\left[1-\frac{1}{\theta}\right] \;=\; [\theta-1]\left(\frac{V}{\theta}\right)^{\frac{\theta}{\theta-1}}\left[\frac{1}{(k)^{\frac{1}{\theta-1}}}\right]. \tag{9}$$

Now suppose the principal hires two agents. Since agent i's profit given p_i, p_j, and T_i is $[p_i+p_j]T_i - \frac{k_i}{\theta}(p_i)^{\theta}$, the profit-maximizing choices of p_A and p_B are determined by:

$$T_A \;=\; k_A[p_A]^{\theta-1} \quad\text{and}\quad T_B \;=\; k_B[p_B]^{\theta-1}. \tag{10}$$

(10) implies that the principal's problem can be stated as:

$$\underset{\{p^A,\,p^B\}}{Maximize} \quad [p_A+p_B]\left[V - k_A\,(p_A)^{\theta-1} - k_B\,(p_B)^{\theta-1}\right]. \tag{11}$$

After simplification, the necessary conditions for an interior solution to this problem can be stated as:

$$V - \theta k_A[p_A]^{\theta-1} - k_B[p_B]^{\theta-1} - k_A p_B\,[\theta-1]\,(p_A)^{\theta-2} \;=\; 0; \;\text{ and} \tag{12}$$

$$V - \theta k_B[p_B]^{\theta-1} - k_A[p_A]^{\theta-1} - k_B p^A\,[\theta-1]\,(p_B)^{\theta-2} \;=\; 0. \tag{13}$$

Subtracting (12) from (13), and simplifying, provides:

$$k_A[p_A]^{\theta-2} \;=\; k_B[p_B]^{\theta-2}. \tag{14}$$

(10) and (14) imply:

$$p_B T_A \;=\; p_A T_B. \tag{15}$$

(12) and (15) imply:

$$V - \theta k_A[p_A]^{\theta-1} - k_B[p_B]^{\theta-1} - \left[\frac{p_B\,(\theta-1)}{p_A}\right] k_A\,(p_A)^{\theta-1} \;=\; 0$$

$$\Rightarrow \qquad V p_A - \theta k_A\,(p_A)^{\theta} - p_B \theta k_A\,(p_A)^{\theta-1} \;=\; 0. \tag{16}$$

(14) implies:

$$p_B \;=\; \left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-2}} p_A. \tag{17}$$

Substituting from (17) into (16) provides:

$$V p_A - \theta k_A\,(p_A)^{\theta} - \left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-2}} \theta k_A\,(p_A)^{\theta} \;=\; 0$$

$$\Rightarrow \quad V - \theta k_A\,(p_A)^{\theta-1} - \left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-2}} \theta k_A {p_A}^{A\theta-1} \;=\; 0$$

$$\Rightarrow \qquad (p_A)^{\theta-1} \;=\; \frac{V}{\theta k_A\left\{1+\left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-2}}\right\}}. \tag{18}$$

Similarly, it is readily shown that:

$$(p_B)^{\theta-1} \;=\; \frac{V}{\theta k_B\left\{1+\left[\frac{k_B}{k_A}\right]^{\frac{1}{\theta-2}}\right\}}. \tag{19}$$

(18) and (19) imply:

$$(p_A)^{\theta-1} = \frac{V}{\theta k_A}\left\{\frac{(k_B)^{\frac{1}{\theta-2}}}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}; \quad \text{and} \tag{20}$$

$$(p_B)^{\theta-1} = \frac{V}{\theta k_B}\left\{\frac{(k_A)^{\frac{1}{\theta-2}}}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}. \tag{21}$$

(10), (20), and (21) imply:

$$T_A = \frac{V}{\theta}\left\{\frac{(k_B)^{\frac{1}{\theta-2}}}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}; \quad \text{and} \tag{22}$$

$$T_B = \frac{V}{\theta}\left\{\frac{(k_A)^{\frac{1}{\theta-2}}}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}. \tag{23}$$

(22) and (23) imply:

$$T_A + T_B = \frac{V}{\theta}. \tag{24}$$

(24) implies that the principal's profit is:

$$[p_A + p_B]\left[V - \frac{V}{\theta}\right] = [\theta - 1]\left[\frac{V}{\theta}\right][p_A + p_B]. \tag{25}$$

(20) and (21) imply:

$$\begin{aligned} p_A + p_B &= \left\{\frac{V}{\theta k_A}\left\{\frac{(k_B)^{\frac{1}{\theta-2}}}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}\right\}^{\frac{1}{\theta-1}} + \left\{\frac{V}{\theta k_B}\left\{\frac{(k_A)^{\frac{1}{\theta-2}}}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}\right\}^{\frac{1}{\theta-1}} \\ &= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}}\left\{\frac{(k_B)^{\frac{1}{\theta-2}}}{k_A}\right\}^{\frac{1}{\theta-1}}\left\{\frac{1}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}^{\frac{1}{\theta-1}} \\ &\quad + \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}}\left\{\frac{(k_A)^{\frac{1}{\theta-2}}}{k_B}\right\}^{\frac{1}{\theta-1}}\left\{\frac{1}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}^{\frac{1}{\theta-1}} \\ &= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}}\left\{\frac{1}{(k_A)^{\frac{1}{\theta-2}}+(k_B)^{\frac{1}{\theta-2}}}\right\}^{\frac{1}{\theta-1}}\left[\left\{\frac{(k_B)^{\frac{1}{\theta-2}}}{k_A}\right\}^{\frac{1}{\theta-1}} + \left\{\frac{(k_A)^{\frac{1}{\theta-2}}}{k_B}\right\}^{\frac{1}{\theta-1}}\right] \end{aligned}$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{\frac{1}{(k_A)^{\frac{1}{\theta-2}} + (k_B)^{\frac{1}{\theta-2}}}\right\}^{\frac{1}{\theta-1}} \left[\left\{\frac{(k_B)^{\frac{1}{(\theta-2)(\theta-1)}}}{(k_A)^{\frac{1}{\theta-1}}}\right\} + \left\{\frac{(k_A)^{\frac{1}{(\theta-2)(\theta-1)}}}{(k_B)^{\frac{1}{\theta-1}}}\right\}\right]$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{\frac{1}{(k_A)^{\frac{1}{\theta-2}} + (k_B)^{\frac{1}{\theta-2}}}\right\}^{\frac{1}{\theta-1}} \left[\frac{(k_B)^{\frac{1}{(\theta-2)(\theta-1)}+\frac{1}{\theta-1}} + (k_A)^{\frac{1}{(\theta-2)(\theta-1)}+\frac{1}{\theta-1}}}{(k_A)^{\frac{1}{\theta-1}}\,(k_B)^{\frac{1}{\theta-1}}}\right]$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{\frac{1}{(k_A)^{\frac{1}{\theta-2}} + (k_B)^{\frac{1}{\theta-2}}}\right\}^{\frac{1}{\theta-1}} \left[\frac{(k_B)^{\frac{1}{\theta-2}} + (k_A)^{\frac{1}{\theta-2}}}{(k_A)^{\frac{1}{\theta-1}}\,(k_B)^{\frac{1}{\theta-1}}}\right]$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{(k_A)^{\frac{1}{\theta-2}} + (k_B)^{\frac{1}{\theta-2}}\right\}^{\frac{\theta-2}{\theta-1}} \left[\frac{1}{(k_A)^{\frac{1}{\theta-1}}\,(k_B)^{\frac{1}{\theta-1}}}\right]$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{(k_A)^{\frac{1}{\theta-2}} + (k_B)^{\frac{1}{\theta-2}}\right\}^{\frac{\theta-2}{\theta-1}} \{k_A k_B\}^{-\frac{1}{\theta-1}}$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{(k_A)^{\frac{1}{\theta-2}} + (k_B)^{\frac{1}{\theta-2}}\right\}^{\frac{\theta-2}{\theta-1}} \{k_A k_B\}^{\left(\frac{\theta-2}{\theta-1}\right)\left(-\frac{1}{\theta-2}\right)}$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{(k_B)^{\left(-\frac{1}{\theta-2}\right)} + (k_A)^{\left(-\frac{1}{\theta-2}\right)}\right\}^{\frac{\theta-2}{\theta-1}}$$

$$= \left(\frac{V}{\theta}\right)^{\frac{1}{\theta-1}} \left\{\frac{1}{(k_B)^{\frac{1}{\theta-2}}} + \frac{1}{(k_A)^{\frac{1}{\theta-2}}}\right\}^{\frac{\theta-2}{\theta-1}}. \tag{26}$$

(25) and (26) imply that the principal's profit is:

$$\pi = [\theta-1]\left(\frac{V}{\theta}\right)^{\frac{\theta}{\theta-1}} \left\{\frac{1}{(k_B)^{\frac{1}{\theta-2}}} + \frac{1}{(k_A)^{\frac{1}{\theta-2}}}\right\}^{\frac{\theta-2}{\theta-1}}. \tag{27}$$

As the proof of Proposition 2 reveals, the equilibrium costs of agent A and B will be $\overline{k}_A$ and $\overline{k}_B$, respectively, in this setting. Therefore, (27) implies:

$$\pi = [\theta-1]\left(\frac{V}{\theta}\right)^{\frac{\theta}{\theta-1}} \left\{\frac{1}{(\overline{k}_B)^{\frac{1}{\theta-2}}} + \frac{1}{(\overline{k}_A)^{\frac{1}{\theta-2}}}\right\}^{\frac{\theta-2}{\theta-1}}. \tag{28}$$

Without loss of generality, suppose the initial cost of agent A (k_A^o) is less than the corresponding initial cost of agent B. The the principal will hire agent A if she decides to hire just one agent. (9) and (28) imply that the principal will prefer to hire two agents rather than one agent if and only if:

$$\left\{\frac{1}{\left(\overline{k}_B\right)^{\frac{1}{\theta-2}}}+\frac{1}{\left(\overline{k}_A\right)^{\frac{1}{\theta-2}}}\right\}^{\frac{\theta-2}{\theta-1}} > \left(\frac{1}{\left(k_A^o\right)^{\frac{1}{\theta-1}}}\right)$$

$$\Leftrightarrow \quad \left\{\frac{1}{\left(\overline{k}_B\right)^{\frac{1}{\theta-2}}}+\frac{1}{\left(\overline{k}_A\right)^{\frac{1}{\theta-2}}}\right\}^{\theta-2} > \frac{1}{k_A^o}$$

$$\Leftrightarrow \quad \frac{1}{\left(\overline{k}_B\right)^{\frac{1}{\theta-2}}}+\frac{1}{\left(\overline{k}_A\right)^{\frac{1}{\theta-2}}} > \frac{1}{\left(k_A^o\right)^{\frac{1}{\theta-2}}}. \tag{29}$$

Define $\overline{k} \equiv \max\{\overline{k}_A, \overline{k}_B\}$ and define $\underline{k} \equiv \min\{\underline{k}_A, \underline{k}_B\}$. Then when $\theta > 2$:

$$\frac{1}{\left(\overline{k}_B\right)^{\frac{1}{\theta-2}}}+\frac{1}{\left(\overline{k}_A\right)^{\frac{1}{\theta-2}}} \geq \frac{2}{\left(\overline{k}\right)^{\frac{1}{\theta-2}}} \quad \text{and} \quad \frac{1}{\left(\underline{k}\right)^{\frac{1}{\theta-2}}} \geq \frac{1}{\left(k_A^o\right)^{\frac{1}{\theta-2}}}. \tag{30}$$

(30) implies that (29) will hold when $\theta > 2$ if:

$$\frac{2}{\left(\overline{k}\right)^{\frac{1}{\theta-2}}} > \frac{1}{\left(\underline{k}\right)^{\frac{1}{\theta-2}}} \quad \Leftrightarrow \quad \left(\frac{\overline{k}}{\underline{k}}\right)^{\frac{1}{\theta-2}} < 2$$

$$\Leftrightarrow \quad \left[\frac{1}{\theta-2}\right]\ln\left(\frac{\overline{k}}{\underline{k}}\right) < \ln 2 \quad \Leftrightarrow \quad \theta > 2+\frac{\ln\left(\overline{k}/\underline{k}\right)}{\ln 2}. \quad \blacksquare \tag{31}$$

Proof of Proposition 1.

(20) and (21) imply that p_j is decreasing in k_j for $j = A, B$ at the solution to [P] when $\theta > 2$. Therefore, since agent i's profit is $[p_i + p_j]T_i - \frac{k_i}{\theta}(p_i)^\theta$, agent i will choose k_j to maximize p_j when the reward structure has already been determined. Consequently, agent i will set $k_j = \underline{k}_j$ for $j \neq i$, $i, j \in \{A, B\}$. $\blacksquare$

Proof of Proposition 2.

Substituting from (10) and (15) into (12) provides:

$$V - \theta T_A - T_B - T_A\left[\frac{(\theta-1)\,p_B}{p_A}\right] = 0$$

$$\Rightarrow \quad Vp^A - \theta T_A p_A - T_B p_A - T_A [\theta - 1] p_B = 0$$

$$\Rightarrow \quad Vp_A = \theta T_A [p_A + p_B] \quad \Rightarrow \quad p_A + p_B = \frac{Vp_A}{\theta T_A}. \tag{32}$$

Agent A's equilibrium profit (or rent) is:

$$R_A = [p_A + p_B] T_A - \frac{1}{\theta} k_A [p_A]^\theta. \tag{33}$$

Substituting from (32) into (33) provides:

$$R_A = \frac{Vp_A}{\theta} - \frac{1}{\theta} k_A [p_A]^\theta. \tag{34}$$

(18) implies:

$$\theta k_A \left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} \right\} [p_A]^\theta = Vp_A. \tag{35}$$

Substituting from (35) into (34) provides:

$$R_A = k_A \left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} \right\} [p_A]^\theta - \frac{1}{\theta} k_A [p_A]^\theta$$

$$= k_A [p_A]^\theta \left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} - \frac{1}{\theta} \right\}. \tag{36}$$

From (18):

$$k_A (p_A)^\theta = \left[\frac{V}{\theta \left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} \right\}} \right]^{\frac{\theta}{\theta-1}}. \tag{37}$$

Substituting from (37) into (36) provides:

$$R_A = \left[\frac{V}{\theta \left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} \right\}} \right]^{\frac{\theta}{\theta-1}} \left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} - \frac{1}{\theta} \right\}$$

$$= \left[\frac{V}{\theta} \right]^{\frac{\theta}{\theta-1}} \left[\frac{1}{\left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} \right\}} \right]^{\frac{\theta}{\theta-1}} \left\{ 1 + \left[\frac{k_A}{k_B} \right]^{\frac{1}{\theta-2}} - \frac{1}{\theta} \right\}. \tag{38}$$

Let $\frac{1}{x} = \left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-2}}$. Then (38) implies:

$$R_A \;=\; \left[\frac{V}{\theta}\right]^{\frac{\theta}{\theta-1}} \left[\frac{1}{\left\{1+\frac{1}{x}\right\}}\right]^{\frac{\theta}{\theta-1}} \left\{1+\frac{1}{x}-\frac{1}{\theta}\right\}. \tag{39}$$

(39) implies:

$$\begin{aligned}
\log R_A \;&=\; \left[\frac{\theta}{\theta-1}\right]\log\left[\frac{V}{\theta}\right] + \left[\frac{\theta}{\theta-1}\right]\left[\log(1) - \log\left(1+\frac{1}{x}\right)\right] + \log\left(1+\frac{1}{x}-\frac{1}{\theta}\right) \\
&=\; \left[\frac{\theta}{\theta-1}\right]\log\left[\frac{V}{\theta}\right] - \left[\frac{\theta}{\theta-1}\right]\log\left(\frac{x+1}{x}\right) + \log\left(\frac{x\theta+\theta-x}{x\theta}\right) \\
&=\; \left[\frac{\theta}{\theta-1}\right]\log\left[\frac{V}{\theta}\right] - \left[\frac{\theta}{\theta-1}\right]\left[\log(x+1) - \log(x)\right] \\
&\qquad +\; \log(x\theta+\theta-x) - \log(x\theta).
\end{aligned} \tag{40}$$

(40) implies:

$$\begin{aligned}
\frac{d\log R_A}{dx} \;&=\; -\left[\frac{\theta}{\theta-1}\right]\left[\frac{1}{x+1}-\frac{1}{x}\right] + \left[\frac{\theta-1}{x\theta+\theta-x}\right] - \left[\frac{1}{x}\right] \\
&=\; \left[\frac{\theta}{\theta-1}\right]\left[\frac{1}{x(x+1)}\right] - \left[\frac{\theta}{x(x\theta+\theta-x)}\right] \\
&=\; \frac{\theta}{\theta(\theta-1)(1+x)(x\theta+\theta-x)} \;>\; 0.
\end{aligned} \tag{41}$$

(41) implies $\frac{dR_A}{dx} > 0$. Also, $\frac{dx}{dk_B} > 0$ when $\theta > 2$ because $x = \left[\frac{k_B}{k_A}\right]^{\frac{1}{\theta-2}}$. Therefore:

$$\frac{dR_A}{dk_B} \;=\; \left(\frac{dR_A}{dx}\right)\left(\frac{dx}{dk_B}\right) > 0 \;\text{ for all } k_A \in \left[\underline{k}_A,\; \overline{k}_A\right] \text{ and } k_B \in \left[\underline{k}_B,\; \overline{k}_B\right]. \tag{42}$$

(42) implies that each agent will undertake the maximum feasible level of sabotage, i.e., $k_A = \overline{k}_A$ and $k_B = \overline{k}_B$. ■

<u>Proof of Proposition 3</u>.

Let $T_A = T_B = T$. Then (10) implies:

$$T \;=\; k_A[p_A]^{\theta-1} \;=\; k_B[p^B]^{\theta-1} \quad\Rightarrow\quad p_B \;=\; \left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-1}} p_A. \tag{43}$$

Recall that the principal maximizes:

$$\pi^e \;=\; [p_A(\cdot)+p_B(\cdot)]\,[V_S - T - T]\,. \tag{44}$$

Substituting from (43) into (44) provides:

$$\pi^e \;=\; \left[p_A + \left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-1}}(p_A)\right]\left\{V - k_A[p_A]^{\theta-1} - k_A[p_A]^{\theta-1}\right\}$$

$$=\; p_A\left[1+\left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-1}}\right]\left\{V - 2k_A[p_A]^{\theta-1}\right\}. \tag{45}$$

(45) implies:

$$\frac{\partial \pi^e}{\partial p^A} \;=\; \left[1+\left[\frac{k_A}{k_B}\right]^{\frac{1}{\theta-1}}\right]\left\{V - 2\theta k_A[p_A]^{\theta-1}\right\}. \tag{46}$$

(46) implies:

$$\frac{\partial \pi^e}{\partial p_A} \;=\; 0 \quad\Leftrightarrow\quad V - 2\theta k_A[p_A]^{\theta-1} \;=\; 0 \quad\Leftrightarrow\quad p_A \;=\; \left[\frac{V}{2\theta k_A}\right]^{\frac{1}{\theta-1}}. \tag{47}$$

Note that $\frac{\partial^2 \pi^e}{\partial (p_A)^2} < 0$ since $\theta > 1$.

Similarly:

$$\frac{\partial \pi^e}{\partial p_B} \;=\; 0 \quad\Leftrightarrow\quad p_B \;=\; \left[\frac{V}{2\theta k_B}\right]^{\frac{1}{\theta-1}}. \tag{48}$$

(43) and (47) imply:

$$T \;=\; k_A[p_A]^{\theta-1} \;=\; k_A\left[\frac{V}{2\theta k_A}\right] \;=\; \frac{V}{2\theta}. \tag{49}$$

(47) and (48) imply:

$$\frac{\partial p_A}{\partial k_B} \;=\; 0 \quad\text{and}\quad \frac{\partial p_B}{\partial k_B} \;<\; 0. \tag{50}$$

From (49), the rate at which agent A's profit increases with k_B is:

$$\frac{\partial R_A}{\partial k_B} \;=\; \frac{\partial}{\partial k_B}\left\{[p_A+p_B]\left[\frac{V}{2\theta}\right] - \frac{1}{\theta}k_A[p_A]^{\theta}\right\}$$

$$= \left[\frac{\partial p_A}{\partial k_B} + \frac{\partial p_B}{\partial k_B}\right] T - k_A [p_A]^{\theta - 1} \left[\frac{\partial p_A}{\partial k_B}\right]. \tag{51}$$

(50) and (51) imply:

$$\frac{\partial R_A}{\partial k_B} = \left[\frac{\partial p_B}{\partial k_B}\right] T < 0. \tag{52}$$

(52) implies that if the principal commits to implement an egalitarian reward structure, the agents will choose $k_A = \underline{k}_A$ and $k_B = \underline{k}_B$. ■

Proof of Proposition 4.

Let $\pi^e(k_A, k_B)$ denote the principal's (maximized) profit when she commits *ex ante* to an egalitarian reward structure, and let $\pi(k_A, k_B)$ denote the corresponding profit in the absence of this commitment, given (k_A, k_B). Proposition 3 implies that $(k_A, k_B) = (\underline{k}_A, \underline{k}_B)$ in equilibrium under an egalitarian reward structure. Proposition 2 implies that $(k_A, k_B) = (\overline{k}_A, \overline{k}_B)$ in equilibrium in the absence of a commitment to an egalitarian reward structure. We seek to show that:

$$\pi\left(\overline{k}_A, \overline{k}_B\right) < \pi^e\left(\underline{k}_A, \underline{k}_B\right).$$

From (11) and the envelope theorem:

$$\frac{\partial \pi(k_A, k_B)}{\partial k_i} = -[p_A + p_B](p_i)^{\theta - 1} < 0 \quad \text{for} \quad i = A, B. \tag{53}$$

(47) - (49) imply that under an egalitarian reward structure:

$$p_A = \left[\frac{V}{2\theta k_A}\right]^{\frac{1}{\theta - 1}}; \quad p_B = \left[\frac{V}{2\theta k_B}\right]^{\frac{1}{\theta - 1}}; \quad \text{and} \quad T = \frac{V}{2\theta}. \tag{54}$$

(54) implies:

$$\pi^e(k_A, k_B) = [p_A + p_B][V - 2T] = \left\{\left[\frac{V}{2\theta k_A}\right]^{\frac{1}{\theta - 1}} + \left[\frac{V}{2\theta k_B}\right]^{\frac{1}{\theta - 1}}\right\} V \left[1 - \frac{1}{\theta}\right]. \tag{55}$$

It is apparent from (55) that when $\theta > 2$:

$$\frac{d\pi^e(k_A, k_B)}{dk_i} < 0 \quad \text{for} \quad i = A, B. \tag{56}$$

Consider $\left(\widehat{k}, \widehat{k}\right) \in (\underline{k}_A, \overline{k}_A) \cap (\underline{k}_B, \overline{k}_B)$. Then, (53) implies:

$$\pi\left(\overline{k}_A,\overline{k}_B\right) \;<\; \pi\left(\widehat{k},\,\overline{k}_B\right) \;<\; \pi\left(\widehat{k},\,\widehat{k}\right). \tag{57}$$

(56) implies:

$$\pi^e\left(\widehat{k},\,\widehat{k}\right) \;<\; \pi^e\left(\underline{k}_A,\,\widehat{k}\right) \;<\; \pi^e\left(\underline{k}_A,\underline{k}_B\right). \tag{58}$$

(15) and (17) imply that $T_A = T_B$ when $k_A = k_B$. Therefore, $\pi\left(\widehat{k},\,\widehat{k}\right) = \pi^e\left(\widehat{k},\,\widehat{k}\right)$. Consequently, (57) and (58) imply:

$$\pi\left(\overline{k}_A,\overline{k}_B\right) \;<\; \pi\left(\widehat{k},\,\widehat{k}\right) \;=\; \pi^e\left(\widehat{k},\,\widehat{k}\right) \;<\; \pi^e\left(\underline{k}_A,\underline{k}_B\right). \quad \blacksquare \tag{59}$$

Proof of Proposition 5.

Proposition 3 implies that $(k_A, k_B) = (\underline{k}_A, \underline{k}_B)$ in equilibrium when the principal commits *ex ante* to implement an egalitarian reward structure. Proposition 2 implies that $(k_A, k_B) = \left(\overline{k}_A, \overline{k}_B\right)$ in equilibrium in the absence of a such a commitment. Since $\overline{k}_A = \overline{k}_B$, the same equilibrium costs arise absent a commitment to an egalitarian reward structure that arise in the presence of such a commitment when the equilibrium costs for agent A and agent B are $k_A = \overline{k}_A$ and $k_B = \overline{k}_B$, respectively. Therefore, it suffices to show that:

$$\pi^e\left(\underline{k}_A,\underline{k}_B\right) \;>\; \pi^e\left(\overline{k}_A,\overline{k}_B\right) \quad \text{and} \quad R_i^e\left(\underline{k}_A,\underline{k}_B\right) \;>\; R_i^e\left(\overline{k}_A,\overline{k}_B\right) \quad \text{for } i = A, B, \tag{60}$$

$$\text{where} \qquad R_i^e\left(k_A, k_B\right) \;=\; \left[p_A + p_B\right] T \;-\; \frac{k_i}{\theta}\left(p_i\right)^{\theta} \quad \text{for } i = A, B. \tag{61}$$

(56) implies that $\pi^e\left(\underline{k}_A,\underline{k}_B\right) > \pi^e\left(\overline{k}_A,\overline{k}_B\right)$. (49) and (61) imply:

$$R_A^e\left(k_A, k_B\right) \;=\; \left[p_A + p_B\right]\left[\frac{V}{2\theta}\right] - \frac{1}{\theta}\left(p_A\right)\left[\frac{V}{2\theta}\right] \;=\; \left[\frac{V}{2\theta}\right]\left\{p_A\left[1 - \frac{1}{\theta}\right] + p_B\right\}. \tag{62}$$

(54) and (62) imply that $R_A^e\left(k_A, k_B\right)$ is decreasing in both k_A and k_B when $\theta > 2$. Therefore, $R_A^e\left(\underline{k}_A,\underline{k}_B\right) > R_A^e\left(\overline{k}_A,\overline{k}_B\right)$. Analogous arguments reveal that $R_B^e\left(\underline{k}_A,\underline{k}_B\right) > R_B^e\left(\overline{k}_A,\overline{k}_B\right)$. $\blacksquare$

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