Extreme Screening Policies

by

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Abstract

We show that a lender often experiences increasing marginal returns to screening in a standard setting where the lender decides how intensively to screen prospective borrowers. The increasing marginal returns imply that even small changes in industry parameters can produce large changes in equilibrium screening intensity. In particular, a small reduction in the expected return from borrowers' projects can produce a pronounced increase in the screening of prospective borrowers, with substantial corresponding welfare effects.

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1 Introduction.

Although loans and venture capital may have been plentiful before the recent financial crisis, they were relatively scarce during and immediately after the crisis. Many lenders increased their minimum credit standards. In addition, lenders and venture capitalists closely scrutinized requests for funding from all potential borrowers, even those with solid credentials.[1] There are several possible explanations for this behavior, including relatively meager expected returns from business ventures[2] and a limited supply of loanable funds, due in part to a diminished ability to sell securitized loans.[3] We suggest one additional factor that may have contributed to the pronounced variation in the screening of loan applicants – a fundamental convexity in a lender's return from screening.

We analyze a standard model of lending in which a lender chooses the probability, q, with which she discerns the true merits of projects proposed by potential borrowers. We demonstrate that the lender in this model often experiences increasing marginal returns to improved screening accuracy, q. The increasing marginal returns imply that as long as the marginal cost of increasing q does not increase too rapidly in the relevant range, the lender's expected profit will be a convex function of q. Consequently, the lender will optimally choose either a particularly limited or a particularly pronounced screening accuracy. Furthermore, small changes in industry parameters can induce large changes in the equilibrium screening accuracy. In particular, a small reduction in the expected return from borrowers' projects can trigger a pronounced increase in the screening of these projects.

The increasing marginal returns from improved screening accuracy (q) reflect the following interplay between q and β, which is the fraction of the financial return from a successful

[1] Dell'Ariccia et al. (2008) and Keys et al. (2010) present evidence of relatively lenient lending standards before the financial crisis. Avery et al. (2009) document the increased standards during and after the crisis. Avery et al. (1996) review the substantial discretion that lenders employ in determining the allocation of available credit.

[2] See Ruckes (2004), for example.

[3] Dell'Ariccia et al. (2008) examine the relationship between the supply of loans and a lender's ability to sell securitized loans to investors. Also see Mian and Sufi (2008). Rajan (1994) explains how career concerns can promote severe credit contraction during economic downturns.

project that the lender promises to a borrower whose project is approved. As q increases, borrowers with profitable projects realize that their projects are more likely to be funded whereas borrowers with unprofitable projects realize that their projects are less likely to be funded. Consequently, as q increases, a given increase in β attracts a more profitable mix of projects. Therefore, the lender optimally sets a higher β as q increases. The increased sharing rate, in turn, can increase the marginal return to a higher q. This is the case because with a more substantial payment at stake, the lender's incremental return from distinguishing more accurately between profitable and unprofitable projects increases.

When increasing marginal returns to q render the lender's profit a convex function of q, a small reduction in the expected return from an unscreened project can induce the lender to increase her screening substantially. This increased screening can reduce dramatically the welfare of borrowers with unprofitable projects. Consequently, even mild reductions in economic activity can cause these borrowers to experience substantial losses.

We document the increasing marginal returns to screening accuracy and develop the implications of these increasing returns as follows. Section 2 describes the key elements of our model. Section 3 provides a relatively simple sufficient condition for the increasing marginal returns to arise in a benchmark setting where all screening costs are fixed costs. Section 4 extends the analysis to more general settings with variable screening costs. Section 4 also illustrates how small changes in industry parameters (e.g., the expected profitability of borrowers' projects) can induce pronounced changes in project screening, with substantial welfare effects. Section 5 provides concluding observations and discusses some extensions of the analysis. The proofs of all formal conclusions are presented in the Appendix.

Before proceeding, we explain the key differences between our analysis and related analyses in the literature. Some authors (e.g., Inderst and Mueller, 2006; Bose et al., 2010) analyze models with screening technologies similar to ours but take the level of screening accuracy to be exogenous. Other authors (e.g., Thakor, 1996; Wang, 1998; Fulghieri and Lukin, 2001; Manove et al. 2001) admit endogenous screening but focus on a binary technology in which a lender either incurs a fixed cost to acquire a specified amount of new information or incurs no cost and receives no new information.

Most models with "inside lenders" and "outside lenders" (e.g., Sharpe, 1990; von Thadden, 2004; Dell'Ariccia and Marquez, 2006) incorporate a corresponding binary feature. Inside lenders automatically acquire a specified amount of information about the likely returns from funding certain projects whereas outside lenders acquire no such information. Hauswald and Marquez (2003) is an exception. In their model, an inside lender can choose the accuracy of the information she acquires about certain projects. Furthermore, some of the acquired information may become available to outside lenders. The authors focus on the welfare implications of changes in the prevailing information technology and demonstrate that no pure strategy equilibrium may arise in the competition between informed and uninformed lenders.[4] None of these studies (and, to our knowledge, no other study) identifies the increasing marginal returns to screening accuracy on which we focus.

2 Elements of the Model.

We consider a standard setting in which risk neutral borrowers ("entrepreneurs") have no wealth and so rely on a risk neutral lender (e.g., a venture capitalist) to finance their projects. Each project either succeeds or fails. An entrepreneur has either a high quality project or a low quality project. A high quality project has a higher probability of success ($p_H \in (0,1)$) than does a low quality project ($p_L \in (0, p_H)$). A project generates payoff $V > 0$ when it succeeds and 0 when it fails. Each project requires a fixed investment of $I > 0$. A high quality project generates positive net surplus, whereas a low quality project generates negative net surplus, i.e., $p_L V - I < 0 < p_H V - I$.

Each entrepreneur knows the quality of his project. The lender does not share this knowledge. Initially, the lender knows only that the fraction $\phi_H \in (0,1)$ of entrepreneurs have high quality projects and the complementary fraction $\phi_L = 1 - \phi_H$ have low quality

[4]See Sharpe (1990) for a related conclusion. Cao and Shi (2001) demonstrate that increased competition among lenders to finance a borrower's project can reduce the equilibrium level of project screening. See Petersen and Rajan (1995) for a related observation.

projects. The lender subsequently observes a signal ($s \in \{p_L, p_H\}$) about the quality (i.e., the success probability) of the project of each entrepreneur that applies for funding. The signal reveals the true project quality with probability $q \in [\frac{1}{2}, \overline{q}]$ and reports the incorrect project quality with probability $1-q$. Unless otherwise noted, we will consider the outcomes that arise when the lender funds an entrepreneur's project if and only if the project produces a favorable signal ($s = p_H$). This policy will be called the "selective approval" (SA) policy.[5]

We will refer to q as the lender's screening accuracy. This accuracy might reflect the screening procedures the lender institutes and the experience and ability of the screening personnel the lender hires, for example. $K(q, n)$ will denote the lender's cost of implementing screening accuracy q and screening the projects of $n \geq 0$ entrepreneurs. $K(\cdot)$ is a non-decreasing function of each of its arguments (so $\frac{\partial K(\cdot)}{\partial q} \geq 0$ and $\frac{\partial K(\cdot)}{\partial n} \geq 0$ for all $q \in [\frac{1}{2}, \overline{q}]$). For simplicity, the cost of an uninformative signal is normalized to 0 (so $K(\frac{1}{2}, n) = 0$ for all $n \geq 0$).[6] $\overline{q} \leq 1$ denotes the highest possible level of screening accuracy. A non-trivial maximum screening accuracy ($\overline{q} < 1$) allows for the possibility that certain elements of project success may be impossible to assess perfectly, regardless of how sophisticated and thorough the screening process might be.

Each entrepreneur experiences a transactions cost when applying for funding. This cost could reflect the time required to develop a compelling project description and associated business plan, for example. Variation in total transactions costs among entrepreneurs is captured in standard Hotelling fashion. Entrepreneurs with low quality projects ("L entrepreneurs") and entrepreneurs with high quality projects ("H entrepreneurs") are both distributed uniformly on the $[0, 1]$ interval. The lender is located at 0. The total number of entrepreneurs is normalized to 1. An L entrepreneur located at point x incurs transactions cost $t_L x$ in applying for funding. The corresponding cost of the H entrepreneur is $t_H x$,

[5]Section 3 identifies the conditions under which the SA policy is optimal for the lender.

[6]When $q = \frac{1}{2}$, the signal is just as likely to indicate the project is of high quality as it is to indicate the project is of low quality. This is the case whether the project truly is of high quality or of low quality. Consequently, the signal is uninformative about the true project quality when $q = \frac{1}{2}$.

where t_L and t_H are strictly positive parameters. We will denote by $x_i \in \{x_L, x_H\}$ the location of the $i \in \{L, H\}$ entrepreneur farthest from the lender that applies for funding.

Entrepreneurs decide whether to apply for funding after observing the lender's screening accuracy (q) and the share of the payoff from a successful project (β) the lender offers to approved entrepreneurs. An entrepreneur whose approved project fails receives a payoff of 0.[7] An entrepreneur will apply for funding if and only if his expected return from doing so exceeds his transactions costs. Thus, when the lender only approves projects for which a favorable signal is observed, an L entrepreneur located at x will apply for funding if and only if $\beta\, p_L V\, [1-q] \,\geq\, t_L\, x$. An H entrepreneur located at x will apply for funding in this case if and only if $\beta\, p_H V\, q \,\geq\, t_H\, x$. These observations provide:

Lemma 1. $x_L \,=\, \min\left\{\beta\, p_L V \left[\frac{1-q}{t_L}\right], 1\right\}$ *and* $x_H \,=\, \min\left\{\beta\, p_H V \left[\frac{q}{t_H}\right], 1\right\}$.

As is apparent from Lemma 1, more entrepreneurs will apply for funding as the expected payoff from their project ($p_i V$) increases, as the sharing rate (β) increases, as the probability that their project will be funded increases, and as their transactions costs (t_i) decline.

The timing in the model is as follows. First, each entrepreneur privately learns the quality of his project. Second, the lender chooses her preferred screening accuracy (q) and sharing rate (β).[8] Third, entrepreneurs decide whether to seek funding for their project from the lender. Fourth, the lender observes a signal about the project quality of each entrepreneur that applies for funding.[9] Finally, the outcome of each funded project is observed publicly. When an entrepreneur's project succeeds, he receives $\beta\, V$ and the lender receives $[1-\beta]\, V$.

We will denote by $\Pi(\beta, q)$ the lender's (expected) profit when she sets screening accuracy

[7] Among all feasible payment structures, this payment structure creates the strongest differential incentive for H entrepreneurs to apply for funding. Consequently, this payment structure is optimal for the profit maximizing lender.

[8] We assume the lender can make a binding commitment to deliver the promised sharing rate. In doing so, we abstract from the possibility that the lender might try to expropriate entrepreneurs by reducing β after they apply for funding. Reputation concerns can promote such commitment, for example.

[9] Entrepreneurs whose request for funding is denied, like entrepreneurs who do not seek funding, earn 0 (in the wage sector of the economy, for example).

q and sharing rate β. This profit is the difference between the lender's (expected) revenue, $\pi(\cdot)$, and her screening costs, $K(\cdot)$. Formally:

$$\Pi(\beta, q) \ = \ \pi(\beta, q) - K(q, \, \phi_L \, x_L + \phi_H \, x_H), \qquad \text{where} \tag{1}$$

$$\pi(\beta, q) \ = \ \phi_L \, x_L \, [1 - q] \, [p_L V \, (1 - \beta) - I] + \phi_H \, x_H \, q \, [p_H V \, (1 - \beta) - I] \, . \tag{2}$$

The second argument of $K(\cdot)$ in equation (1) is the number of projects the lender screens when she sets sharing rate β and screening accuracy q.[10] The first term to the right of the equality in equation (2) reflects the net revenue the lender anticipates from financing the projects of L entrepreneurs. This net revenue is the product of the number of L entrepreneurs that apply for funding ($\phi_L \, x_L$) and the difference between the lender's expected payoff from a low quality project ($p_L V \, [1 - \beta]$) and the cost of financing the project (I). The last term in equation (2) reflects the corresponding net revenue the lender anticipates from funding the projects of H entrepreneurs.

To avoid the uninteresting outcome in which the lender sets $\beta = 0$ and thereby avoids funding any projects, Assumption 1 is presumed to hold throughout the ensuing analysis for all $q \in [\frac{1}{2}, 1]$.

Assumption 1.[11] $\phi_H \, p_H \, q^2 \, t_L \, [p_H V - I] \ + \ \phi_L \, p_L \, t_H \, [1 - q]^2 \, [p_L V - I] \ > \ 0 \, .$ (3)

Having described the key elements of our model, we now proceed to demonstrate the increasing marginal returns to screening accuracy that can arise in this setting.

[10] The values of x_L and x_H in equations (1) and (2) are as specified in Lemma 1.

[11] The proof of Lemma 2 reveals that when $K(q, \, n) \ = \ \widetilde{K}(q)$, so all screening costs are fixed costs, Assumption 1 ensures that the lender's profit is strictly increasing in β at $\beta = 0$ for all $q \in [\frac{1}{2}, 1]$. It also is readily verified using equation (2) that $\pi(\beta, q)$ is bounded above by the product of $\frac{\beta V}{t_L \, t_H}$ and the expression in inequality (3). Therefore, the expression must be strictly positive if the lender is to be able to secure strictly positive revenue.

3 The Benchmark Setting with Fixed Screening Costs.

The conditions under which the lender experiences increasing marginal returns to screening accuracy are demonstrated most readily in the benchmark setting where all screening costs are fixed costs, so $K(q, n) = \widetilde{K}(q)$. In some instances, screening accuracy may be improved primarily by developing new methodologies or improved procedures for analyzing standard information that is readily collected from potential borrowers at little cost to the lender. The costs of improved screening accuracy can be largely independent of the number of funding applications in such instances. We consider this benchmark case first in order to identify particularly tractable conditions under which the lender experiences increasing marginal returns to screening accuracy. Section 4 derives corresponding conditions in more general settings.

It is helpful to begin by identifying the profit-maximizing sharing rate for the lender, given her choice of screening accuracy. This rate is determined by substituting the values of x_L and x_H identified in Lemma 1 into equation (2) and maximizing the resulting expression with respect to β. Doing so provides the following conclusion.

Lemma 2. *Suppose $K(q, n) = \widetilde{K}(q)$, so all screening costs are fixed costs. Then the sharing rate that maximizes the lender's profit when she adopts the SA policy and implements screening accuracy q is:*

$$\widetilde{\beta}(q) = \frac{\phi_L \, p_L \, t_H \, [1-q]^2 \, [p_L V - I] + \phi_H \, p_H \, t_L \, q^2 \, [p_H V - I]}{2 \, V \left[\phi_L \, p_L^2 \, t_H \, (1-q)^2 + \phi_H \, p_H^2 \, t_L \, q^2 \right]}. \tag{4}$$

Straightforward differentiation of equation (4) reveals that the lender's preferred sharing rate increases with the screening accuracy she implements.

Lemma 3. $\widetilde{\beta}\,'(q) > 0$ *for all* $q \in [\frac{1}{2}, 1]$.

As q increases, a given increase in β attracts a more profitable mix of projects. This is the case because as q increases, H entrepreneurs know that their project is now more likely to be financed whereas L entrepreneurs know that their project is now less likely to be financed. Consequently, relative to L entrepreneurs, H entrepreneurs become more willing to incur the transactions costs associated with applying for funding. Therefore, as q increases in the present setting, the lender offers a higher sharing rate in order to attract this relatively profitable mix of projects.

A higher sharing rate, in turn, increases the marginal return from increased screening accuracy as long as a key selection effect outweighs a potentially countervailing scale effect. The selection effect of an increase in q refers to the rate at which the mix of projects the lender faces improves as q increases. This mix of projects is the difference between the number of high quality projects (N_H) and the number of low quality projects (N_L) that are presented for funding. From Lemma 1:

$$N_H - N_L = \phi_H x_H - \phi_L x_L = \beta V \left[\frac{\phi_H p_H q}{t_H} - \frac{\phi_L p_L (1-q)}{t_L} \right] \Rightarrow$$

$$\frac{\partial (N_H - N_L)}{\partial q} = \beta V \left[\frac{\phi_H p_H}{t_H} + \frac{\phi_L p_L}{t_L} \right] \text{ and } \frac{\partial^2 (N_H - N_L)}{\partial \beta \, \partial q} = V \left[\frac{\phi_H p_H}{t_H} + \frac{\phi_L p_L}{t_L} \right]. \quad (5)$$

Expression (5) reveals that the selection effect of an increase in q is positive (i.e., $\frac{\partial (N_H - N_L)}{\partial q} > 0$) and the magnitude of this effect increases with β (i.e., $\frac{\partial^2 (N_H - N_L)}{\partial \beta \, \partial q} > 0$). The increased magnitude arises because the higher is β, the greater is an entrepreneur's incremental expected payoff from having his project approved. Therefore, an entrepreneur's decision about whether to apply for funding becomes more sensitive to q as β increases. Consequently, the higher is β, the more effective is an increase in q at attracting a more profitable mix of projects, and so the lender's maximum revenue $\widetilde{\pi}(q) = \widetilde{\pi}(\beta(q), q)$ can increase more rapidly with q when $K(q, n) = \widetilde{K}(q)$.

$\widetilde{\pi}(q)$ will increase more rapidly with q as β increases as long as this favorable selection effect is not offset by a potentially countervailing scale effect. The scale effect of an increase in q is the rate at which the number of projects that are presented for funding ($N_H + N_L$) increases as q increases. From Lemma 1:

$$N_H + N_L \;=\; \phi_H \, x_H + \phi_L \, x_L \;=\; \beta \, V \left[\frac{\phi_H \, p_H \, q}{t_H} + \frac{\phi_L \, p_L \, (1-q)}{t_L} \right] \quad \Rightarrow$$

$$\frac{\partial \, (N_H + N_L)}{\partial q} \;=\; \beta \, V \left[\frac{\phi_H \, p_H}{t_H} - \frac{\phi_L \, p_L}{t_L} \right] \text{ and } \frac{\partial^2 \, (N_H + N_L)}{\partial \beta \, \partial q} \;=\; V \left[\frac{\phi_H \, p_H}{t_H} - \frac{\phi_L \, p_L}{t_L} \right]. \tag{6}$$

Expression (6) indicates that the scale effect of an increase in q is positive and increases with β (so $\frac{\partial (N_H + N_L)}{\partial q} > 0$ and $\frac{\partial^2 (N_H + N_L)}{\partial \beta \, \partial q} > 0$) as long as the number of H entrepreneurs in the population (ϕ_H) is sufficiently large. An increase in q encourages H entrepreneurs to apply for funding and discourages L entrepreneurs from doing so. Therefore, when ϕ_H is large (and so ϕ_L is small), an increase in q serves to increase the number of entrepreneurs that apply for funding. This increase is larger when β is higher because an entrepreneur's response to increased screening accuracy is more pronounced when his payoff from project success (βV) is larger. Expressions (5) and (6) together indicate that the combined selection and scale effects of an increase in q will be positive and increasing in β when the proportion of H entrepreneurs in the population is sufficiently pronounced.

In summary, in the benchmark setting where all screening costs are fixed costs, the profit-maximizing sharing rate increases as q increases. The higher sharing rate, in turn, enhances the marginal return from increasing q as long as the proportion of H entrepreneurs in the population is sufficiently pronounced. Consequently, the lender's maximum revenue increases with q at an increasing rate under these conditions. This conclusion is recorded formally as Proposition 1, which refers to:

$$\widetilde{\phi}_L \;\equiv\; \frac{p_H^2 \, t_L \, [p_H V - I]}{[p_H V - I] \, [p_L^2 \, t_H + p_H^2 \, t_L] - 2 \, p_H \, p_L \, t_H \, [p_L V - I]} \, .$$

Proposition 1. *Suppose $K(q, n) = \widetilde{K}(q)$ and $\phi_L \leq \widetilde{\phi}_L$.*[12] *Then $\widetilde{\pi}'(q) > 0$ and $\widetilde{\pi}''(q) > 0$ for all $q \in [\frac{1}{2}, 1]$, where:*

$$\widetilde{\pi}(q) \;=\; \frac{\left[\phi_L \, p_L \, t_H \, (1-q)^2 \, (p_L V - I) + \phi_H \, p_H \, t_L \, q^2 \, (p_H V - I) \right]^2}{4 \, t_L \, t_H \left[\phi_L \, p_L^2 \, t_H \, (1-q)^2 + \phi_H \, p_H^2 \, t_L \, q^2 \right]} \, . \tag{7}$$

[12] It is readily verified that the $\phi_L \leq \widetilde{\phi}_L$ requirement is consistent with, but more restrictive than, Assumption 1.

To this point, we have presumed the lender employs the selective approval (SA) policy. Alternatively, the lender might simply finance the projects of all entrepreneurs that apply for funding, thereby avoiding all screening costs.[13] The lender's revenue under this "always approve" (AA) policy, π_A, is specified in Lemma 4.

Lemma 4. $$\pi_A \;=\; \frac{\left[\phi_L\, p_L\, t_H\, (p_L V - I) + \phi_H\, p_H\, t_L\, (p_H V - I)\right]^2}{4\, t_L\, t_H \left[\phi_L\, p_L^2\, t_H + \phi_H\, p_H^2\, t_L\right]}. \tag{8}$$

A comparison of equations (7) and (8) provides:

Lemma 5. $\widetilde{\pi}(\frac{1}{2}) \,<\, \pi_A \,<\, \widetilde{\pi}(1)\,.$

Lemma 5 implies that the lender secures greater revenue by financing the projects of all entrepreneurs who apply for funding than by funding only projects that receive a favorable signal when the signal is uninformative (so $q = \frac{1}{2}$). This is the case because the former policy results in twice as many funded projects, and Assumption 1 ensures that an unscreened project (like a project that is screened with an uninformative signal) produces positive expected revenue for the lender. Lemma 5 also implies that the lender secures greater revenue under the SA policy than under the AA policy when $q = 1$. The increased revenue under the SA policy arises because screening that identifies project quality perfectly eliminates the negative (net) revenue generated by low quality projects.

It is apparent from equation (8) that the lender's revenue when she finances the projects of all entrepreneurs that apply for funding (π_A) does not vary with q. Therefore, Lemma 5 implies that when the lender's revenue under the SA policy increases with her screening accuracy (so $\widetilde{\pi}\,'(q) > 0$), there exists a critical level of screening accuracy, $q^c \in (\frac{1}{2}, 1)$, such that the lender secures the most revenue by: (i) approving all requests for funding if her screening accuracy is less than q^c; and (ii) funding only those projects that produce a favorable signal if her screening accuracy exceeds q^c. Furthermore, Proposition 1 indicates

[13] Assumption 1 ensures that as long as $\widetilde{K}\,'(q)|_{q=\frac{1}{2}}$ is sufficiently small, the lender will always fund some projects.

that under the identified conditions, when the lender adopts the SA policy, she maximizes her revenue by implementing the maximum feasible screening accuracy, $\overline{q}$. These conclusions are stated formally in Proposition 2.

Proposition 2. *Under the conditions specified in Proposition 1, the lender maximizes her revenue by implementing the AA policy when* $\overline{q} < q^c$ *and by implementing the SA policy and setting* $q = \overline{q}$ *when* $\overline{q} > q^c$*, where* $q^c \in (\frac{1}{2}, 1)$ *is defined by* $\widetilde{\pi}(q^c) = \pi_A$.

Recall from Proposition 1 that the lender's revenue under the SA policy ($\widetilde{\pi}(q)$) is an increasing, convex function of q in this benchmark setting where all screening costs are fixed costs. Consequently, the lender's profit under this policy ($\widetilde{\Pi}(q) = \widetilde{\pi}(q) - \widetilde{K}(q)$) will be an increasing function of q as long as $\widetilde{K}(q)$ is not too pronounced and convex. Consequently, as suggested by Proposition 2, the lender will maximize her profit either by financing the projects of all entrepreneurs that apply for funding or by adopting the SA policy and implementing the maximum possible screening accuracy, $\overline{q}$. The lender will find the AA policy to be most profitable if $\overline{q} < \widehat{q}$ whereas she will find the SA policy to be the most profitable if $\overline{q} > \widehat{q}$, where $\widetilde{\Pi}(\widehat{q}) = \pi_A$.

This observation implies that small changes in industry parameters can induce large changes in the equilibrium level of screening accuracy. In particular, consider a change that reduces slightly the expected net revenue from an unscreened project. The change might arise from a small reduction in ϕ_H, V, or p_H, for example. By increasing the relative attraction of screening projects, such a change can reduce the critical screening accuracy ($\widehat{q}$) at which the lender is indifferent between the AA and the SA policies from just above to just below $\overline{q}$. Such a reduction in $\widehat{q}$ will induce the lender to switch from financing the projects of all entrepreneurs that apply for funding to implementing the maximum feasible screening accuracy ($\overline{q}$) and funding only those projects that produce a favorable signal. The implications of such pronounced changes in screening policy in response to small changes in industry parameters are explored in the more general setting considered in Section 4.

4 The Setting with Variable Screening Costs.

We now consider a more general setting in which the lender's cost of securing any desired level of screening accuracy can vary with the number of projects she screens. Such variable costs of screening can arise, for example, when additional personnel must be hired to process an increased number of applications for funding. In order to derive relatively tractable conditions under which the lender continues to experience increasing marginal returns to improved screening accuracy in this more general setting, we assume variable screening costs entail a constant marginal cost of screening each applicant, $c(q)$. Therefore, the lender's cost of screening n applicants with screening accuracy q is $K(q,n) = F(q) + c(q)\,n$, where $F(\cdot)$ represents a fixed cost of screening.

In principle, a lender's marginal cost of screening can either increase or decrease with q. To illustrate, a lender might increase her screening accuracy (q) by devoting more time and effort to her evaluation of each entrepreneur's project. The lender's marginal cost of screening, $c(q)$, typically would increase with q in such a setting. In contrast, screening accuracy might be improved by developing more sophisticated procedures for predicting project success without analyzing the data provided by prospective borrowers as carefully and without devoting as much time to discerning each entrepreneur's strengths and weaknesses. In this case, $c(q)$ could decline with q (whereas $F(q)$ would increase with q).

$\pi^v(\beta, q)$ will denote the lender's variable profit in this setting with variable screening costs when she offers sharing rate β and implements screening accuracy q. This variable profit is the difference between the lender's revenue and her variable screening costs. Formally:

$$\begin{aligned}\pi^v(\beta, q) \;=\; & \phi_L\, x_L\, [1-q]\, [p_L V\,(1-\beta) - I] + \phi_H\, x_H\, q\, [p_H V\,(1-\beta) - I] \\ & -\; c\,(q)\, [\phi_L\, x_L + \phi_H\, x_H\,]\,. \end{aligned} \tag{9}$$

Lemma 6 identifies the lender's profit-maximizing sharing rate, $\beta(q)$, when she adopts the SA policy in this setting. This rate is derived by substituting the values of x_L and x_H identified in Lemma 1 into equation (9) and maximizing the resulting expression with respect to β. Lemma 6 refers to Condition 1, which is the natural counterpart to Assumption 1.

When Condition 1 holds for all $q \in [\frac{1}{2}, 1]$, the lender will optimally implement a strictly positive sharing rate in the present setting with variable screening costs.[14] Consequently, the condition precludes the trivial outcome in which the lender finances no projects.

<u>Condition 1</u>. $\phi_H \, p_H \, t_L \, [p_H V - I] \, q^2 + \phi_L \, p_L \, t_H \, [p_L V - I] \, [1-q]^2$

$$> \; c\,(q) \left[\phi_L \, p_L \, t_H \, (1-q) + \phi_H \, p_H \, t_L \, q \right].$$

Lemma 6. *Suppose* $K(q,n) = F(q) + c(q)\, n$ *and Condition 1 holds for all* $q \in [\frac{1}{2}, 1]$*. Then the sharing rate that maximizes the lender's profit when she adopts the SA policy and implements screening accuracy* q *is:*

$$\beta(q) \;=\; \frac{1}{2\,V\left[\phi_L \, p_L^2 \, t_H \, (1-q)^2 + \phi_H \, p_H^2 \, t_L \, q^2\right]} \{ \phi_L \, p_L \, t_H \, (1-q)^2 \, [p_L V - I]$$

$$+ \;\; \phi_H \, p_H \, t_L \, q^2 \, [p_H V - I] - c\,(q) \left[\phi_L \, p_L \, t_H \, (1-q) + \phi_H \, p_H \, t_L \, q \right] \}. \quad (10)$$

Substituting the expression for $\beta(q)$ in equation (10) into the expression for $\pi^v(\beta, q)$ in equation (9) provides an expression for $\pi^v(q) = \max_{\beta} \pi^v(\beta, q)$, the lender's maximum variable profit in this setting when she implements screening accuracy q.

Lemma 7. *Suppose* $K(q,n) = F(q) + c(q)\, n$ *and Condition 1 holds for all* $q \in [\frac{1}{2}, 1]$*. Then the lender's maximum variable profit when she adopts the SA policy and implements screening accuracy* q *is:*

$$\pi^v(q) \;=\; \frac{1}{4\, t_L \, t_H \left[\phi_L \, p_L^2 \, t_H \, (1-q)^2 + \phi_H \, p_H^2 \, t_L \, q^2\right]} \{ \phi_L \, [1-q]^2 \, p_L \, t_H \, [p_L V - I]$$

$$+ \;\; \phi_H \, q^2 p_H \, t_L \, [p_H V - I] - c\,(q) \left[\phi_L \, p_L \, t_H \, (1-q) + \phi_H \, p_H \, t_L \, q \right] \}^2. \quad (11)$$

Proposition 3 now explains when $\pi^v(q)$ will be a convex function of q. The proposition refers to:

[14] See the proof of Lemma 6 in the Appendix.

Condition 2. $\left[\phi_H\, p_H^2\, t_L\, q^2 + \phi_L\, p_L^2\, t_H\, (1-q)^2\right] \{2\left[\phi_L\, p_L\, t_H\, (p_L V - I) + \phi_H\, p_H\, t_L\, (p_H V - I)\right]$

$$- \; 2\, c'(q) \left[\phi_H\, p_H\, t_L - \phi_L\, p_L\, t_H\right] - c''(q) \left[\phi_L\, p_L\, t_H\, (1-q) + \phi_H\, p_H\, t_L\, q\right]\}$$

$$> \; \left[\phi_H\, p_H^2\, t_L + \phi_L\, p_L^2\, t_H\right] \{\phi_H\, p_H\, t_L\, q^2 \left[p_H V - I\right] + \phi_L\, p_L\, t_H \left[1-q\right]^2 \left[p_L V - I\right]$$

$$- \; c(q) \left[\phi_L\, p_L\, t_H\, (1-q) + \phi_H\, p_H\, t_L\, q\right]\}.$$

Proposition 3. *Suppose* $K(q,n) = F(q) + c(q)\, n$. *Also suppose Conditions 1 and 2 hold for all* $q \in [\frac{1}{2}, 1]$. *Then the lender's maximum variable profit is a convex function of* q, *i.e.,* $\pi^{v\,\prime\prime}(q) > 0$ *for all* $q \in [\, \frac{1}{2}, \overline{q}\,]$.[15]

Condition 2 indicates that $\pi^v(\cdot)$ is more likely to be a convex function of q if the lender's marginal cost of screening does not increase too rapidly with q (so $c''(q)$ is small), *ceteris paribus*. Condition 2 also indicates that $\pi^v(q)$ is more likely to be convex if the lender's marginal cost declines with q (so $c'(q) < 0$) and ϕ_H / t_H is large relative to ϕ_L / t_L, *ceteris paribus*. Under these conditions, an increase in q reduces the marginal cost of screening, induces more entrepreneurs to seek funding for any given sharing rate,[16] and increases the fraction of H entrepreneurs that apply for funding.

When the lender's variable profit is a convex function of q, the lender will optimally implement either the lowest or the highest feasible screening accuracy, provided the fixed cost of screening is not too convex (i.e., provided $F''(q)$ is not too large). Furthermore, small changes in model parameters can induce large changes in the equilibrium level of screening accuracy. To illustrate this point, suppose $K(q,n) = \frac{k_1}{2}\left[q - \frac{1}{2}\right]^2 + \frac{k_2}{2}\left[q - \frac{1}{2}\right]^2 n$ for $q \in [\, \frac{1}{2}, \overline{q}\,]$, where $\overline{q} \in (\frac{1}{2}, 1]$. Under this cost structure, the lender's fixed cost $(\frac{k_1}{2}\left[q - \frac{1}{2}\right]^2)$ and constant unit cost $(\frac{k_2}{2}\left[q - \frac{1}{2}\right]^2)$ of screening are both increasing, quadratic functions of

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[15] Furthermore, it can be shown that $\pi^{v\,\prime}(q) > 0$ for all $q \in [\frac{1}{2}, \overline{q}]$ if $c'(\frac{1}{2}) < [A_1 + A_2]/B$, where $A_1 = 3\,\phi_L\, p_L\, \phi_H\, p_H\, t_L\, t_H\, I\, [p_H - p_L]$, $A_2 = \phi_H^2\, p_H^3\, t_L^2\, [p_H V - I] - \phi_L^2\, p_L^3\, t_H^2\, [p_L V - I]$, and $B = [\phi_L\, p_L\, t_H + \phi_H\, p_H\, t_L]\left[\phi_L\, p_L^2\, t_H + \phi_H\, p_H^2\, t_L\right]$.

[16] Recall from expression (6) that, holding β constant, the number of entrepreneurs who apply for funding increases with q at the rate $\beta\, V \left[\frac{\phi_H\, p_H}{t_H} - \frac{\phi_L\, p_L}{t_L}\right]$.

the screening accuracy, q.

This explicit formulation of the lender's screening costs allows us to solve numerically for the lender's optimal choice of q as model parameters change. One parameter of interest is ϕ_H, the proportion of high quality projects in the economy. When ϕ_H is close to 1, the lender will be fairly sure that any entrepreneur who requests funding has a high quality project. Consequently, when k_1 and k_2 are sufficiently large, the lender will avoid all screening costs by financing the project of every lender who applies for funding.

As ϕ_H declines (perhaps due to a general decline in economic activity and an associated reduction in consumer purchasing power which reduces the likelihood that a new business venture will succeed), the lender's expected return from financing an unscreened project declines. When ϕ_H is sufficiently far below 1 and when k_1 and k_2 are sufficiently small that the lender's profit is a convex function of q, the lender will find it most profitable to implement the maximum feasible level of screening (i.e., she will set $q = \overline{q}$). In particular, there is a critical value of ϕ_H, denoted $\widehat{\phi}_H$, such that the lender will optimally approve all funding applications if $\phi_H > \widehat{\phi}_H$ whereas she will adopt the selective approval policy and set $q = \overline{q}$ if $\phi_H < \widehat{\phi}_H$.

To illustrate, suppose $k_1 = 1$, $k_2 = 2$, and $\overline{q} = 0.9$. Further suppose $V = 40$, $I = 20$, $p_L = .25$, $p_H = .75$, and $t_L = t_H = 5$. Figure 1 depicts the lender's profit in this setting as ϕ_H changes. The solid line labeled $\Pi(SA)$ in Figure 1 represents the lender's profit when she implements the selective approval (SA) policy and sets the maximum screening accuracy, $\overline{q} = 0.9$. The dashed line labeled $\Pi(AA)$ in the figure presents the lender's profit when she adopts the AA policy.

As Figure 1 indicates, $\widehat{\phi}_H = 0.746$ in this setting, so the lender optimally finances the projects of all entrepreneurs that apply for funding when $\phi_H > 0.746$ whereas she adopts the SA policy and sets $q = \overline{q}$ when $\phi_H < 0.746$.[17] Thus, a very small reduction in the proportion of high quality projects in the economy (from, say, 0.75 to 0.74) can produce a

[17] This conclusion and the ensuing conclusions refect numerical solutions produced by *Mathematica*. The details of the analysis are available from the authors.

pronounced change in the equilibrium screening policy.[18]

As Table 1 reveals, this pronounced policy change that arises from a small reduction in the proportion of high quality projects in the economy can have substantial welfare effects. The first column in Table 1 lists selected values of ϕ_H. The second column identifies the corresponding profit-maximizing policy for the lender in the setting under consideration. $SA(.9)$ indicates that the lender adopts the selective approval policy and implements the maximum feasible screening accuracy ($q = \overline{q} = 0.9$). AA implies that the lender finances the projects of all entrepreneurs that apply for funding. The third and fourth columns in Table 1 report the number of L and H entrepreneurs that apply for funding (x_L and x_H), respectively. The fifth column identifies the profit maximizing sharing rate (β) for the lender. The sixth and seventh columns provide the welfare of L and H entrepreneurs (W_L and W_H), respectively.[19] The lender's profit ($\Pi^v(q) = \pi^v(q) - F(q)$) appears in the eighth column of Table 1. The last column in the table presents the level of aggregate welfare (W), which is the sum of $\Pi^v(q)$, W_L, and W_H.

Table 1 reveals that as ϕ_H declines from 0.75 to 0.74, the number of L entrepreneurs who apply for funding declines by nearly 90% (from 0.28571 to 0.03267). The reduced participation of L entrepreneurs, which arises despite an increase in the sharing rate, causes their welfare to decline by more than 98% (from 0.05102 to 0.00069). In contrast, the increased participation of H entrepreneurs induced by the higher sharing rate increases their welfare by approximately 4% (from 1.37755 to 1.43937). The reduction in the proportion of high quality projects reduces the lender's profit by nearly 2% (from 2.85714 to 2.80013) and reduces aggregate welfare by approximately 1% (from 4.28571 to 4.24020).

[18]The same is true if perfect screening of project quality is feasible. If $\overline{q} = 1$ but all other parameter values are as specified above, then $\widehat{\phi}_H = 0.908$. Thus, the lender approves all requests for funding if $\phi_H > 0.908$ but adopts the selective approval strategy and sets $q = 1$ if $\phi_H < 0.908$ in this setting.

[19]The welfare of entrepreneurs is the difference between their expected return when they apply for financing and their associated transactions costs. Formally, $W_L = \phi_L \{ p_L V \beta \, x_L \, [1-q] - \int_0^{x_L} t_L \, x \, dx \}$ and $W_H = \phi_H \{ p_H V \beta \, x_H \, q - \int_0^{x_H} t_H \, x \, dx \}$.

ϕ_H	*Policy*	x_L	x_H	β	W_L	W_H	$\Pi^v(q)$	W
0.70	$SA(.9)$	0.03265	0.88164	0.16327	0.00080	1.36026	2.64211	4.00317
0.71	$SA(.9)$	0.03266	0.88175	0.16329	0.00077	1.38004	2.68162	4.06243
0.72	$SA(.9)$	0.03266	0.88186	0.16331	0.00075	1.39981	2.72112	4.12168
0.73	$SA(.9)$	0.03267	0.88196	0.16333	0.00072	1.41959	2.76063	4.18094
0.74	$SA(.9)$	0.03267	0.88207	0.16335	0.00069	1.43937	2.80013	4.24020
0.75	AA	0.28571	0.85714	0.14286	0.05102	1.37755	2.85714	4.28571
0.76	AA	0.28814	0.86441	0.14407	0.04981	1.41968	2.93898	4.40847
0.77	AA	0.29050	0.87151	0.14525	0.04853	1.46209	3.02123	4.53184
0.78	AA	0.29282	0.87845	0.14641	0.04716	1.50478	3.10387	4.65580
0.79	AA	0.29508	0.88525	0.14754	0.04571	1.54773	3.18689	4.78033
0.80	AA	0.29730	0.89189	0.14865	0.04419	1.59094	3.27027	4.90541

Table 1. The Effects of Changes in ϕ_H in the Example.

Small changes in other model parameters also can produce large changes in the equilibrium level of project screening. To illustrate, consider changes in V, the payoff from a successful project. Such changes might reflect changes in the profitability of a successful new business venture caused by changes in general economic conditions in the economy, for example. As in the setting considered above, suppose $I = 20$, $p_L = .25$, $p_H = .75$, $t_L = t_H = 5$, $k_1 = 1$, $k_2 = 2$, and $\overline{q} = 0.9$. When V is sufficiently pronounced in this setting (i.e., when $V > 40$), the lender will finance the projects of all entrepreneurs that apply for funding without screening the projects. In contrast, the lender will implement the maximum feasible level of screening ($\overline{q} = 0.9$) when $V < 40$.

V	*Policy*	x_L	x_H	β	W_L	W_H	$\Pi^v(q)$	W
35	$SA(.9)$	0.02017	0.54467	0.11527	0.00025	0.55624	1.03298	1.58947
36	$SA(.9)$	0.02267	0.61217	0.12596	0.00032	0.70265	1.32594	2.02891
37	$SA(.9)$	0.02517	0.67967	0.13607	0.00040	0.86615	1.65308	2.51962
38	$SA(.9)$	0.02767	0.74717	0.14565	0.00048	1.04673	2.01441	3.06162
39	$SA(.9)$	0.03017	0.81467	0.15473	0.00057	1.24440	2.40993	3.65490
40	AA	0.28571	0.85714	0.14286	0.05102	1.37755	2.85714	4.28571
41	AA	0.31071	0.93214	0.15157	0.06034	1.62917	3.37902	5.06853

Table 2. The Effects of Changes in V in the Example.

Table 2 presents the corresponding welfare effects. The lender's decision to screen projects fairly intensively (i.e., adopt the selective approval policy and set $q = \overline{q} = 0.9$) as V declines from 40 to 39 reduces by almost 90% (from 0.28571 to 0.03017) the number of L entrepreneurs who apply for funding, despite the increase in the sharing rate (from 0.14286 to 0.15473). The reduced participation of the L entrepreneurs reduces their welfare by nearly 99% (from 0.05102 to 0.00057). Fewer H entrepreneurs apply for funding also. Their reduced participation and the reduction in the payoff from a successful project serve to reduce the welfare of H entrepreneurs by almost 10% (from 1.37755 to 1.24440). The modest (2.5%) decline in the payoff from a successful project causes the lender's profit to decline by more than 15% (from 2.85714 to 2.40993) and aggregate welfare to decline by nearly 15% (from 4.28571 to 3.65490).

These outcomes in the example illustrate the more general conclusion that increasing marginal returns to improved screening accuracy can render a lender's screening policy quite sensitive to changes in industry parameters. Furthermore, the pronounced changes in screening accuracy that can arise from modest changes in industry conditions can have substantial welfare implications.

5 Extensions and Conclusions.

We have explored a standard model of lending with endogenous screening accuracy and demonstrated that a lender often experiences increasing marginal returns to improved screening accuracy. The resulting convexity of the lender's profit function can lead her to implement either a very high or a very low level of screening accuracy. Furthermore, small changes in industry parameters can lead to pronounced changes in equilibrium screening accuracy. For example, even a mild downturn in economic activity that reduces expected project returns slightly can produce a substantial increase in screening accuracy, with corresponding pronounced welfare effects.

The rationale for pronounced changes in screening accuracy that we have identified is certainly not the only explanation for the increased scrutiny of loan applications that arose during the recent financial crisis. However, this rationale may complement other important explanations for recent lending policies and for variation in lending practices more generally (e.g., Rajan, 1994).

Further research is required to assess the practical relevance of the increasing marginal returns to screening accuracy that we have identified. For instance, it is important to assess the extent to which these increasing marginal returns persist when lenders compete to serve borrowers (e.g., Petersen and Rajan, 1995; Cao and Shi, 2005) and in the presence of richer project payoff structures and richer variation in project scale and quality.

Alternative screening technologies and strategies also merit investigation. For example, lenders might screen the applications of some potential borrowers more carefully than others, perhaps based upon past experience with borrowers (e.g., Dell'Ariccia and Marquez, 2006) or upon the varying levels of collateral that borrowers put at risk (e.g., Manove et al., 2001; Jiminez, 2006). A lender might also adjust the intensity with which she screens one borrower in response to the results of her screening of other borrowers. Future research might also consider correlated project qualities and increasing marginal costs of loanable funds. While these extensions might add some useful practical considerations to the analysis, the extensions would not seem to eliminate the basic forces that produce the increasing returns to improved screening accuracy in the streamlined model we have explored.

Appendix

Proof of Lemma 1.

An L entrepreneur's expected payoff from applying for funding is $[1-q]\,p_L\,\beta\,V$. If this expected payoff is less than t_L, then the L entrepreneur located farthest from the lender that will apply for funding is the one for whom this expected payoff equals his transactions cost, i.e., $[1-q]\,p_L\,\beta\,V \;=\; t_L\,x_L$. Consequently, $x_L \;=\; \min\left\{\beta\,p_L V\left[\frac{1-q}{t_L}\right],\,1\right\}$.

The analysis for the H entrepreneur is analogous, and so is omitted. ■

Proof of Lemma 2.

Substituting from Lemma 1 into (2) provides:

$$\pi(\beta,q) \;=\; \left[\frac{\beta\,V}{t_L}\right]\phi_L\,p_L\,[1-q]^2\,[p_L\,(1-\beta)\,V-I]+\left[\frac{\beta\,V}{t_H}\right]\phi_H\,p_H\,q^2\,[p_H\,(1-\beta)\,V-I]\,. \tag{12}$$

Differentiating (12) provides:

$$\begin{aligned}\frac{\partial\pi(\cdot)}{\partial\beta} \;&=\; \frac{V}{t_L}\,\phi_L\,p_L\,[1-q]^2\,[p_LV-I]+\frac{V}{t_H}\,\phi_H\,p_H\,q^2\,[p_HV-I]\\ &\quad -\;2\,\beta\,V^2\left[\phi_L\,p_L^2\,(1-q)^2\,\frac{1}{t_L}+\phi_H\,p_H^2\,q^2\,\frac{1}{t_H}\right].\end{aligned} \tag{13}$$

It is readily verified that $\pi(\cdot)$ is a strictly concave function of β, that $\left.\frac{\partial\pi(\cdot)}{\partial\beta}\right|_{\beta=1} < 0$, and that $\left.\frac{\partial\pi(\cdot)}{\partial\beta}\right|_{\beta=0} > 0$ when Assumption 1 holds. Therefore, (4) follows directly from (13). ■

Proof of Lemma 3.

Differentiating (4) provides:

$$\begin{aligned}\widetilde{\beta}'(q) \;&\stackrel{s}{=}\; \left[\phi_L\,p_L^2\,(1-q)^2\,t_H+\phi_H\,p_H^2\,q^2\,t_L\right]2\{\phi_H\,p_H\,q\,[p_HV-I]\,t_L-\phi_L\,p_L\,[1-q]\,[p_LV-I]\,t_H\}\\ &\quad -\;\left\{\phi_L\,p_L\,[1-q]^2\,[p_LV-I]\,t_H+\phi_H\,p_H\,q^2\,[p_HV-I]\,t_L\right\}\\ &\qquad \cdot 2\left[\phi_H\,p_H^2\,q\,t_L-\phi_L\,p_L^2\,(1-q)\,t_H\right]\\ &\stackrel{s}{=}\; [p_HV-I]\,\{\,\phi_H^2\,p_H^3\,q^3\,t_L^2+\phi_L\,p_L^2\,\phi_H\,p_H\,q\,[1-q]^2\,t_L\,t_H\\ &\qquad\qquad -\phi_H^2\,p_H^3\,q^3\,t_L^2\;+\phi_L\,p_L^2\,\phi_H\,p_H\,q^2\,[1-q]\,t_L\,t_H\,\}\\ &\quad -\;[p_LV-I]\,\{\,\phi_L^2\,p_L^3\,[1-q]^3\,t_H^2+\phi_L\,p_L\,\phi_H\,p_H^2\,q^2\,[1-q]\,t_L\,t_H\\ &\qquad\qquad -\;\phi_L\,p_L\,\phi_H\,p_H^2\,q\,[1-q]^2\,t_L\,t_H-\phi_L^2\,p_L^3\,[1-q]^3\,t_H^2\,\}\\ &=\; [p_HV-I]\,\phi_L\,p_L^2\,\phi_H\,p_H\,q\,[1-q]\,t_L\,t_H\,[1-q+q]\\ &\quad -\;[p_LV-I]\,\phi_L\,p_L\,\phi_H\,p_H^2\,q\,[1-q]\,t_L\,t_H\,[1-q+q]\end{aligned}$$

$$= \phi_L\, p_L\, \phi_H\, p_H\, q\,[1-q]\, t_L\, t_H\, \{p_L\,[p_H V - I] - p_H\,[p_L V - I]\}$$

$$= \phi_L\, p_L\, \phi_H\, p_H\, q\,[1-q]\, t_L\, t_H\,[p_H - p_L]\, I \;\; > \;\; 0. \quad \blacksquare$$

Proof of Proposition 1.

Substituting from Lemma 1 and from (4) into (2) provides:

$$\widetilde{\pi}(q) \;=\; \frac{\left[\phi_H\, p_H\, q^2\, t_L\,(p_H V - I) + \phi_L\, p_L\,(1-q)^2\, t_H\,(p_L V - I)\right]^2}{4\, t_H\, t_L\left[\phi_H\, p_H^2\, q^2\, t_L + \phi_L\, p_L^2\,(1-q)^2\, t_H\right]}$$

$$= \frac{[\phi_H\, p_H\, t_L\,(p_H V - I)]^2}{4\, t_H\, t_L\, \phi_H\, p_H^2\, t_L} \frac{\left[q^2 + \left[\frac{\phi_L\, p_L\, t_H (p_L V - I)}{\phi_H\, p_H\, t_L (p_H V - I)}\right](1-q)^2\right]^2}{\left[q^2 + \left[\frac{\phi_L\, p_L^2\, t_H}{\phi_H\, p_H^2\, t_L}\right](1-q)^2\right]}$$

$$= M\, \frac{\left[q^2 + \delta_2\,(1-q)^2\right]^2}{\left[q^2 + \delta_1\,(1-q)^2\right]}, \tag{14}$$

where:

$$\delta_1 \;=\; \frac{\phi_L\, p_L^2\, t_H}{\phi_H\, p_H^2\, t_L} \;>\; 0\,, \quad \delta_2 \;=\; \frac{\phi_L\, p_L\, t_H\,[p_L V - I]}{\phi_H\, p_H\, t_L\,[p_H V - I]} \;<\; 0\,, \quad \text{and} \tag{15}$$

$$M \;=\; \frac{[\phi_H\, p_H\, t_L\,(p_H V - I)]^2}{4\, t_H\, t_L\, \phi_H\, p_H^2\, t_L}\,.$$

Define

$$\widehat{\pi} \;=\; \frac{\left[q^2 + \delta_2\,(1-q)^2\right]^2}{q^2 + \delta_1\,[1-q]^2}\,. \tag{16}$$

Since δ_1, δ_2 and M are independent of q, (14) and (16) provide:

Result A1. $\widetilde{\pi}$ is convex in q if and only if $\widehat{\pi}$ is convex in q.

From (16):

$$\left[q^2 + \delta_1\,(1-q)^2\right]\widehat{\pi} \;=\; \left[q^2 + \delta_2\,(1-q)^2\right]^2\,. \tag{17}$$

Define:

$$g_1 \;=\; q^2 + \delta_1\,[1-q]^2 \quad \text{and} \quad g_2 \;=\; q^2 + \delta_2\,[1-q]^2\,. \tag{18}$$

(17) and (18) provide:

$$g_1\,\widehat{\pi} \;=\; (g_2)^2\,. \tag{19}$$

Differentiating (19) with respect to q provides:

$$g_1\,\widehat{\pi}' + g_1'\,\widehat{\pi} \;=\; 2\,g_2\,g_2'\,. \tag{20}$$

Differentiating (20) with respect to q provides:

$$g_1'\,\widehat{\pi}' + g_1\,\widehat{\pi}'' \;+\; g_1''\,\widehat{\pi} + g_1'\,\widehat{\pi}' \;=\; 2\,(g_2')^2 + 2\,g_2\,g_2''$$

$$\Leftrightarrow \quad g_1\,\widehat{\pi}'' + \,g_1''\,\widehat{\pi} + 2\,g_1'\,\widehat{\pi}' \;=\; 2\,(g_2')^2 + 2\,g_2\,g_2''$$

$$\Leftrightarrow \quad g_1\,\widehat{\pi}'' \;=\; 2\,(g_2')^2 + 2\,g_2\,g_2'' - \,g_1''\,\widehat{\pi} - 2\,g_1'\,\widehat{\pi}'\,. \tag{21}$$

Since $g_1 > 0$, (21) implies that $\widehat{\pi}$ is convex in q if the expression to the right of the equality in (21) is positive.

From (18):

$$g_1' \;=\; 2\,q - 2\,\delta_1\,[1-q] \quad \Rightarrow \quad g_1'' \;=\; 2 + 2\,\delta_1 \;=\; 2\,[1+\delta_1]\,, \quad \text{and} \tag{22}$$

$$g_2' \;=\; 2\,q - 2\,\delta_2\,[1-q] \quad \Rightarrow \quad g_2'' \;=\; 2 + 2\,\delta_2 \;=\; 2\,[1+\delta_2]\,. \tag{23}$$

Using (22) and (23) in (21) provides:

$$g_1\,\widehat{\pi}'' \;=\; 2\,(g_2')^2 + 4\,g_2\,[1+\delta_2] - 2\,[1+\delta_1]\,\widehat{\pi} - 2\,g_1'\,\widehat{\pi}'$$

$$\Leftrightarrow \quad \left[\frac{g_1}{2}\right]\widehat{\pi}'' \;=\; (g_2')^2 + 2\,g_2\,[1+\delta_2] - [1+\delta_1]\,\widehat{\pi}'' - g_1'\,\widehat{\pi}'\,. \tag{24}$$

From (20):

$$\widehat{\pi}' \;=\; \frac{2\,g_2\,g_2' - g_1'\,\widehat{\pi}}{g_1}\,. \tag{25}$$

Relations (19), (24), and (25) provide:

$$\left[\frac{g_1}{2}\right]\widehat{\pi}'' \;=\; (g_2')^2 + 2\,g_2\,[1+\delta_2] - [1+\delta_1]\,\widetilde{\pi} - g_1'\left[\frac{2\,g_2\,g_2' - g_1'\,\widetilde{\pi}}{g_1}\right] \quad \Leftrightarrow$$

$$\begin{aligned}
\left[\frac{g_1^2}{2}\right]\widehat{\pi}'' \;&=\; g_1\,(g_2')^2 + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi} - g_1'\,[2\,g_2\,g_2' - g_1'\,\widehat{\pi}] \\
&=\; g_1\,(g_2')^2 + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi} - g_1'\,g_2\,g_2' + g_1'\,[\,g_1'\,\widehat{\pi} - g_2\,g_2'] \\
&=\; g_1\,(g_2')^2 + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi} - g_1'\,g_2\,g_2' + g_1'\left[g_1'\,\frac{(g_2)^2}{g_1} - g_2\,g_2'\right] \\
&=\; g_1\,(g_2')^2 + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi} - g_1'\,g_2\,g_2' + g_1'\,g_2\left[g_1'\,\frac{g_2}{g_1} - g_2'\right] \\
&=\; g_1\,(g_2')^2 + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi} - g_1'\,g_2\,g_2' + \frac{g_1'\,g_2}{g_1}\,[g_1'\,g_2 - g_1\,g_2']\,.
\end{aligned} \tag{26}$$

From (18), (22), and (23):

$$\begin{aligned}
g_1'\,g_2 - g_1\,g_2' \;&=\; [2\,q - 2\,\delta_1\,(1-q)]\left[q^2 + \delta_2\,(1-q)^2\right] \\
&\quad - \left[q^2 + \delta_1\,(1-q)^2\right][2\,q - 2\,\delta_2\,(1-q)] \\
&=\; 2\,q^3 - 2\,\delta_1\,q^2\,[1-q] + 2\,q\,\delta_2\,[1-q]^2 - 2\,\delta_1\,\delta_2\,[1-q]^3 \\
&\quad - \left[2\,q^3 + 2\,q\,\delta_1\,(1-q)^2 - 2\,\delta_2\,q^2\,(1-q) - 2\,\delta_1\,\delta_2\,(1-q)^3\right]
\end{aligned}$$

$$= \; - \; 2\,\delta_1\,q^2\,[1-q] - 2\,q\,\delta_1\,[1-q]^2 + 2\,q\,\delta_2\,[1-q]^2 + 2\,\delta_2\,q^2\,[1-q]$$

$$= \; - \; 2\,\delta_1\,q\,[1-q]\,[q+1-q] + 2\,q\,\delta_2\,[1-q]\,[1-q+q]$$

$$= \; - \; 2\,\delta_1 q\,[1-q] + 2\,q\,\delta_2\,[1-q] \;\; = \;\; 2\,q\,[1-q]\,[\delta_2 - \delta_1]\;. \qquad (27)$$

Relation (27) implies:

$$g_1\,(g_2')^2 - g_1'\,g_2\,g_2' \;=\; g_2'\,[g_1\,g_2' - g_1'\,g_2] \;\;=\;\; - \; 2\,g_2'\,q\,[1-q]\,[\delta_2 - \delta_1]\;. \qquad (28)$$

From (26), (27), and (28):

$$\left[\frac{g_1^2}{2}\right] \widehat{\pi}'' \;=\; g_1\,(g_2')^2 - g_1'\,g_2\,g_2' + \frac{g_1'\,g_2}{g_1}\,[g_1'\,g_2 - g_1\,g_2'] + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi}$$

$$= \; - \; 2\,g_2'\,q\,[1-q]\,[\delta_2 - \delta_1] + \frac{g_1'\,g_2}{g_1}\,2\,q\,[1-q]\,[\delta_2 - \delta_1] + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi}$$

$$= \; 2\,q\,[1-q]\,[\delta_2 - \delta_1]\left[\frac{g_1'\,g_2}{g_1} - g_2'\right] + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi}$$

$$= \; \frac{2\,q\,[1-q]\,[\delta_2 - \delta_1]}{g_1}\,[g_1'\,g_2 - g_1\,g_2'] + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi}$$

$$= \; \frac{4\,q^2\,[1-q]^2\,[\delta_2 - \delta_1]^2}{g_1} + 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi}\,. \qquad (29)$$

From (18) and (19):

$$2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\widehat{\pi} \;=\; 2\,g_1\,g_2\,[1+\delta_2] - g_1\,[1+\delta_1]\,\frac{(g_2)^2}{g_1}$$

$$= \; 2\,g_1\,g_2\,[1+\delta_2] - [1+\delta_1]\,(g_2)^2 \;\;=\;\; g_2\,[2\,g_1\,(1+\delta_2) - g_2\,(1+\delta_1)]$$

$$= \; g_2\left\{2\left[q^2 + \delta_1\,(1-q)^2\right][1+\delta_2] - \left[q^2 + \delta_2\,(1-q)^2\right][1+\delta_1]\right\}$$

$$= \; g_2\left\{q^2\,[2\,(1+\delta_2) - (1+\delta_1)] + [1-q]^2\,[2\,\delta_1\,(1+\delta_2) - \delta_2\,(1+\delta_1)]\right\}$$

$$= \; g_2\left\{q^2\,[2 + 2\,\delta_2 - 1 - \delta_1] + [1-q]^2\,[2\,\delta_1 + 2\,\delta_1\,\delta_2 - \delta_2 - \delta_1\,\delta_2]\right\}$$

$$= \; g_2\left\{q^2\,[1 + 2\,\delta_2 - \delta_1] + [1-q]^2\,[2\,\delta_1 + \delta_1\,\delta_2 - \delta_2]\right\}\;. \qquad (30)$$

Using (30) in (29) provides:

$$\left[\frac{g_1^2}{2}\right] \widehat{\pi}'' \;=\; \frac{4\,q^2\,[1-q]^2\,[\delta_2 - \delta_1]^2}{g_1} + g_2\left\{q^2\,[1 + 2\,\delta_2 - \delta_1] + [1-q]^2\,[2\,\delta_1 + \delta_1\,\delta_2 - \delta_2]\right\}$$

$$\Leftrightarrow \quad \left[\frac{g_1^3}{2}\right] \widehat{\pi}'' \;=\; 4\,q^2\,[1-q]^2\,[\delta_2 - \delta_1]^2 + g_1\,g_2\,\{\,q^2\,[1 + 2\,\delta_2 - \delta_1]$$

$$+ \;\; [1-q]^2\,[2\,\delta_1 + \delta_1\,\delta_2 - \delta_2]\,\}\,. \qquad (31)$$

Notice from (15) that $\delta_2 > -1$ since $\phi_H \, p_H \, t_L \, [p_H V - I] + \phi_L \, p_L \, t_H \, [p_L V - I] > 0$. Therefore, from (18):

$$g_2 = q^2 + \delta_2 \, [1-q]^2 > q^2 - [1-q]^2 \geq 0, \text{ since } q \geq \frac{1}{2}. \tag{32}$$

Since $\delta_1 > 0$ and $\delta_2 \in (-1, 0)$:

$$2 \, \delta_1 + \delta_1 \, \delta_2 - \delta_2 = \delta_1 \, [2 + \delta_2] - \delta_2 > 0. \tag{33}$$

Using (32) and (33) in (31) provides:

Result A2. $\widehat{\pi}'' > 0$ if $1 + 2 \, \delta_2 - \delta_1 \geq 0$.

Now, suppose $\widehat{\pi}'' > 0$ for all $q \in \left[\frac{1}{2}, 1\right]$. Then, $\widehat{\pi}'' \geq 0$ when $q = 1$. Using $q = 1$ in (31) provides:

$$\left[\frac{g_1^3}{2}\right] \widehat{\pi}'' = g_1 \, g_2 \, [1 + 2 \, \delta_2 - \delta_1] \, . \tag{34}$$

Result A3 follows from (34).

Result A3. If $\widehat{\pi}'' \geq 0$ for all $q \in \left[\frac{1}{2}, 1\right]$, then $1 + 2 \, \delta_2 - \delta_1 \geq 0$.

Result A4 follows from Results A1, A2, and A3.

Result A4. $\widetilde{\pi}'' \geq 0$ for all $q \in \left[\frac{1}{2}, 1\right]$ if and only if $1 + 2 \, \delta_2 - \delta_1 \geq 0$.

To simplify the condition $1 + 2 \, \delta_2 - \delta_1 \geq 0$, notice from (15) that:

$$\delta_2 = \frac{\phi_L \, p_L \, t_H \, [p_L V - I]}{\phi_H \, p_H \, t_L \, [p_H V - I]} = \delta_1 \, \frac{p_H}{p_L} \left[\frac{p_L V - I}{p_H V - I}\right]. \tag{35}$$

From (15) and (35):

$$1 + 2 \, \delta_2 - \delta_1 = 1 + 2 \, \delta_1 \frac{p_H}{p_L} \left[\frac{p_L V - I}{p_H V - I}\right] - \delta_1 = 1 + \delta_1 \left[2 \, \frac{p_H}{p_L} \left(\frac{p_L V - I}{p_H V - I}\right) - 1\right]$$

$$= 1 + \delta_1 \left[\frac{2 \, p_H \, (p_L V - I) - p_L \, (p_H V - I)}{p_L \, (p_H V - I)}\right] \geq 0$$

$$\Leftrightarrow \delta_1 \left[\frac{2 \, p_H \, (p_L V - I) - p_L \, (p_H V - I)}{p_L \, (p_H V - I)}\right] \geq -1$$

$$\Leftrightarrow \delta_1 \, [2 \, p_H \, (p_L V - I) - p_L \, (p_H V - I)] \geq -p_L \, [p_H V - I]$$

$$\Leftrightarrow \delta_1 \leq \frac{p_L \, [p_H V - I]}{p_L \, [p_H V - I] - 2 \, p_H \, [p_L V - I]} \tag{36}$$

$$\Leftrightarrow \frac{\phi_L \, p_L^2 \, t_H}{\phi_H \, p_H^2 \, t_L} \leq \frac{p_L \, [p_H V - I]}{p_L \, [p_H V - I] - 2 \, p_H \, [p_L V - I]}$$

$$\Leftrightarrow \quad \frac{\phi_L}{\phi_H} \;\le\; \frac{p_L\,[p_H V - I]}{p_L\,[p_H V - I] - 2\,p_H\,[p_L V - I]}\left[\frac{p_H^2\,t_L}{p_L^2\,t_H}\right]$$

$$\Leftrightarrow \quad \frac{\phi_H}{\phi_L} \;\ge\; \frac{p_L\,[p_H V - I] - 2\,p_H\,[p_L V - I]}{p_L\,[p_H V - I]}\left[\frac{p_L^2\,t_H}{p_H^2\,t_L}\right]$$

$$\Leftrightarrow \quad \frac{1-\phi_L}{\phi_L} \;\ge\; \frac{p_L^2\,t_H\,[p_H V - I] - 2\,p_H\,p_L\,t_H\,[p_L V - I]}{p_H^2\,t_L\,[p_H V - I]}$$

$$\Leftrightarrow \quad \frac{1}{\phi_L} \;\ge\; 1 + \frac{p_L^2\,t_H\,[p_H V - I] - 2\,p_H\,p_L\,t_H\,[p_L V - I]}{p_H^2 t_L\,[p_H V - I]}$$

$$\Leftrightarrow \quad \frac{1}{\phi_L} \;\ge\; \frac{[p_H V - I]\,[p_L^2\,t_H + p_H^2\,t_L] - 2\,p_H\,p_L\,t_H\,[p_L V - I]}{p_H^2\,t_L\,[p_H V - I]}$$

$$\Leftrightarrow \quad \phi_L \;\le\; \frac{p_H^2\,t_L\,[p_H V - I]}{[p_H V - I]\,[p_L^2\,t_H + p_H^2\,t_L] - 2\,p_H\,p_L\,t_H\,[p_L V - I]} \;\equiv\; \widetilde{\phi}_L\,. \tag{37}$$

Relation (37) and Result A4 ensure that $\widetilde{\pi}$ is convex in q for all $q \in \left[\frac{1}{2}, 1\right]$ if and only if $\phi_L \le \widetilde{\phi}_L$.

Finally, to prove that $\widetilde{\pi}'(q) > 0$ for all $q \in \left[\frac{1}{2}, 1\right]$, notice from (7) that:

$$\begin{aligned}
\widetilde{\pi}'(q) \;\overset{s}{=}\; & \left[\phi_H\,p_H^2\,q^2\,t_L + \phi_L\,p_L^2\,(1-q)^2\,t_H\right] \\
& \quad 2\left[\phi_H\,p_H\,q^2\,t_L\,(p_H V - I) + \phi_L\,p_L\,(1-q)^2\,t_H\,(p_L V - I)\right] \\
& \quad \cdot 2\,\left[\phi_H\,p_H\,q\,t_L\,(p_H V - I) - \phi_L\,p_L\,(1-q)\,t_H\,(p_L V - I)\right] \\
& \quad - \left[\phi_H\,p_H\,q^2\,t_L\,(p_H V - I) + \phi_L\,p_L\,(1-q)^2\,t_H\,(p_L V - I)\right]^2 \\
& \quad \cdot 2\left[\phi_H\,p_H^2\,q\,t_L - \phi_L\,p_L^2\,(1-q)\,t_H\right]
\end{aligned}$$

$$\begin{aligned}
\overset{s}{=}\; & 2\left[\phi_H\,p_H^2\,q^2\,t_L + \phi_L\,p_L^2\,(1-q)^2\,t_H\right]\left[\phi_H\,p_H\,q\,t_L\,(p_H V - I) - \phi_L\,p_L\,(1-q)\,t_H\,(p_L V - I)\right] \\
& - \left[\phi_H\,p_H\,q^2\,t_L\,(p_H V - I) + \phi_L\,p_L\,(1-q)^2\,t_H\,(p_L V - I)\right]\left[\phi_H\,p_H^2\,q\,t_L - \phi_L\,p_L^2\,(1-q)\,t_H\right]
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad \widetilde{\pi}'(q)\big|_{q=\frac{1}{2}} \;\overset{s}{=}\; & \\
& 2\left[\phi_H\,p_H^2\,t_L + \phi_L\,p_L^2\,t_H\right]\left[\phi_H\,p_H\,t_L\,(p_H V - I) - \phi_L\,p_L\,t_H\,(p_L V - I)\right] \\
& - \left[\phi_H\,p_H\,t_L\,(p_H V - I) + \phi_L\,p_L\,t_H\,(p_L V - I)\right]\left[\phi_H\,p_H^2\,t_L - \phi_L\,p_L^2\,t_H\right]
\end{aligned}$$

$$\begin{aligned}
=\; & 2\,\phi_H^2\,p_H^3\,t_L^2\,(p_H V - I) - 2\,\phi_L\,p_L\,\phi_H\,p_H^2\,t_L\,t_H\,(p_L V - I) \\
& + 2\,\phi_L\,p_L^2\,\phi_H\,p_H\,t_L\,t_H\,(p_H V - I) - 2\,\phi_L^2\,p_L^3\,t_H^2\,(p_L V - I) \\
& - \phi_H^2\,p_H^3\,t_L^2\,(p_H V - I) + \phi_L\,p_L^2\,\phi_H\,p_H\,t_L\,t_H\,(p_H V - I) \\
& - \phi_L\,p_L\,\phi_H\,p_H^2\,t_L\,t_H\,(p_L V - I) + \phi_L^2\,p_L^3\,t_H^2\,(p_L V - I)
\end{aligned}$$

$$= \phi_H^2 \, p_H^3 \, t_L^2 \, (p_H V - I) - \phi_L^2 \, p_L^3 \, t_H^2 \, (p_L V - I)$$
$$+ 3 \, \phi_L \, p_L^2 \, \phi_H \, p_H \, t_L \, t_H \, (p_H V - I) - 3 \, \phi_L \, p_L \, \phi_H \, p_H^2 \, t_L \, t_H \, (p_L V - I) \; > \; 0. \qquad (38)$$

The inequality in (38) holds because $p_L V - I \; < \; 0 \; < \; p_H V - I$.

Because $\widetilde{\pi}\,'(q)|_{q=\frac{1}{2}} > 0$ and $\widetilde{\pi}\,''(q) > 0$ for all $q \in [\frac{1}{2}, 1]$, it follows that $\widetilde{\pi}\,'(q) > 0$ for all $q \in [\frac{1}{2}, 1]$. ■

Proof of Lemma 4.

As in the proof of Lemma 1, it is readily verified that when the lender approves every request for funding, the L entrepreneur that is indifferent between working on the project with sharing rate β and not working on the project is located at $\widetilde{x}_L \; = \; \beta \, p_L V / t_L$. The corresponding location of the marginal H entrepreneur is $\widetilde{x}_H \; = \; \beta \, p_H V / t_H$. Therefore, the lender's expected profit when she approves every request for funding is (using (2)):

$$\pi_A(\beta) \;=\; \phi_L \, [p_L \, (1-\beta) \, V - I] \, \widetilde{x}_L + \phi_H \, [p_H \, (1-\beta) \, V - I] \, \widetilde{x}_H$$
$$= \; \phi_L \, [p_L \, (1-\beta) \, V - I] \, \beta \left[\frac{p_L V}{t_L} \right] + \phi_H \, [p_H \, (1-\beta) \, V - I] \, \beta \left[\frac{p_H V}{t_H} \right] . \qquad (39)$$

Differentiating (39) provides:

$$\pi_A'(\beta) \;=\; \phi_L \, p_L V \, [p_L \, (1 - 2\,\beta) \, V - I] \, \frac{1}{t_L} + \phi_H \, p_H V \, [p_H \, (1 - 2\,\beta) \, V - I] \, \frac{1}{t_H} . \qquad (40)$$

Since Assumption 1 holds for $q = \frac{1}{2}$:

$$\pi_A'(\beta)|_{\beta=0} \;=\; \phi_L \, p_L V \, [p_L V - I] \, \frac{1}{t_L} + \phi_H \, p_H V \, [p_H V - I] \, \frac{1}{t_H} \; > \; 0 \, .$$

Consequently, (40) implies that the profit-maximizing sharing rate when the lender approves every request for funding is determined by:

$$2\,\beta \left[\phi_L \, p_L^2 V^2 \frac{1}{t_L} + \phi_H \, p_H^2 V^2 \frac{1}{t_H} \right] \;=\; \phi_L \, p_L V \, [p_L V - I] \, \frac{1}{t_L} + \phi_H \, p_H V \, [p_H V - I] \, \frac{1}{t_H}$$

$$\Rightarrow \quad \beta \;=\; \frac{\phi_L \, p_L \, [p_L V - I] \, t_H + \phi_H \, p_H \, [p_H V - I] \, t_L}{2\, V \, [\phi_L \, p_L^2 \, t_H + \phi_H \, p_H^2 \, t_L]} \, . \qquad (41)$$

(41) implies:

$$1 - \beta \;=\; \frac{1}{2\, V \, [\phi_L \, p_L^2 \, t_H + \phi_H \, p_H^2 \, t_L]} \{ 2\, V \, [\phi_L \, p_L^2 \, t_H + \phi_H \, p_H^2 \, t_L]$$
$$- \; \phi_L \, p_L \, [p_L V - I] \, t_H - \phi_H \, p_H \, [p_H V - I] \, t_L \}$$
$$= \; \frac{V \, [\phi_L \, p_L^2 \, t_H + \phi_H \, p_H^2 \, t_L] + I \, [\phi_L \, p_L \, t_H + \phi_H \, p_H \, t_L]}{2\, V \, [\phi_L \, p_L^2 \, t_H + \phi_H \, p_H^2 \, t_L]} . \qquad (42)$$

(42) implies:

$$[1-\beta]\,V\left[\phi_L\,p_L^2\,\frac{1}{t_L}+\phi_H\,p_H^2\,\frac{1}{t_H}\right]$$
$$=\ \frac{[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]\,\{V\,[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]+I\,[\phi_L\,p_L\,t_H+\phi_H\,p_H\,t_L]\}}{2\,t_L\,t_H\,[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]}. \tag{43}$$

(39) and (43) imply:

$$\begin{aligned}
\pi_A(\beta) &= \beta\,V\left\{\frac{\phi_L\,p_L}{t_L}\,[p_L\,(1-\beta)\,V-I]+\frac{\phi_H\,p_H}{t_H}\,[p_H\,(1-\beta)\,V-I]\right\}\\
&= \beta\,V\left\{[1-\beta]\,V\left[\phi_L\,p_L^2\,\frac{1}{t_L}+\phi_H\,p_H^2\,\frac{1}{t_H}\right]-\ I\left[\frac{\phi_L\,p_L}{t_L}+\frac{\phi_H\,p_H}{t_H}\right]\right\}\\
&= \beta\,V\left\{\frac{[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]\,\{V\,[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]+I\,[\phi_L\,p_L\,t_H+\phi_H\,p_H\,t_L]\}}{2\,t_L\,t_H\,[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]}\right.\\
&\qquad\left.-\ I\left[\frac{\phi_L\,p_L}{t_L}+\frac{\phi_H\,p_H}{t_H}\right]\right\}\\
&= \beta\,V\,\frac{1}{2\,t_L\,t_H}\left\{V\left[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L\right]+I\,[\phi_L\,p_L\,t_H+\phi_H\,p_H\,t_L]\right\}\\
&\qquad-\ \beta\,V\,\frac{2\,I\,[\phi_L\,p_L\,t_H+\phi_H\,p_H\,t_L]}{2\,t_L\,t_H}\\
&= \frac{\beta\,V}{2\,t_L\,t_H}\left\{V\left[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L\right]-I\,[\phi_L\,p_L\,t_H+\phi_H\,p_H\,t_L]\right\}\\
&= \frac{\beta\,V}{2\,t_L\,t_H}\,\{\phi_L\,p_L\,t_H\,[p_L V-I]+\phi_H\,p_H\,t_L\,[p_H V-I]\}\,.
\end{aligned} \tag{44}$$

(41) and (44) imply that the lender's profit when she approves every request for funding and sets the profit-maximizing sharing rate is:

$$\begin{aligned}
\pi_A &= \frac{V}{2\,t_L\,t_H}\,\{\phi_L\,p_L\,t_H\,[p_L V-I]+\phi_H\,p_H\,t_L\,[p_H V-I]\}\\
&\qquad\cdot\left\{\frac{\phi_L\,p_L\,t_H\,[p_L V-I]+\phi_H\,p_H\,t_L\,[p_H V-I]}{2\,V\,[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]}\right\}\\
&= \frac{[\phi_L\,p_L\,t_H\,(p_L V-I)+\phi_H\,p_H\,t_L\,(p_H V-I)]^2}{4\,t_L\,t_H\,[\phi_L\,p_L^2\,t_H+\phi_H\,p_H^2\,t_L]}. \qquad \blacksquare
\end{aligned} \tag{45}$$

<u>Proof of Lemma 5</u>.

From (7) and (45):

$$\widetilde{\pi}(\frac{1}{2}) \;=\; \frac{\left[\phi_L\, p_L\, t_H\left(\frac{1}{2}\right)^2 (p_L V - I) + \phi_H\, p_H\, t_L\left(\frac{1}{2}\right)^2 (p_H V - I)\right]^2}{4\, t_L\, t_H\left[\phi_L\, p_L^2\, t_H\left(\frac{1}{2}\right)^2 + \phi_H\, p_H^2\, t_L\left(\frac{1}{2}\right)^2\right]}$$

$$=\; \frac{\left[\phi_L\, p_L\, t_H\,(p_L V - I) + \phi_H\, p_H\, t_L\,(p_H V - I)\right]^2}{16\, t_L\, t_H\left[\phi_L\, p_L^2\, t_H + \phi_H\, p_H^2\, t_L\right]} \;=\; \frac{1}{2}\,\pi_A\,.$$

From (45):

$$\pi_A \;=\; \frac{\left[\phi_L\, p_L\, t_H\,(p_L V - I) + \phi_H\, p_H\, t_L\,(p_H V - I)\right]^2}{8\, t_L\, t_H\left[\phi_L\, p_L^2\, t_H + \phi_H\, p_H^2\, t_L\right]} \;<\; \frac{\left[\phi_H\, p_H\, t_L\,(p_H V - I)\right]^2}{8\, t_L\, t_H\left[\phi_L\, p_L^2\, t_H + \phi_H\, p_H^2\, t_L\right]} \tag{46}$$

$$<\; \frac{\left[\phi_H\, p_H\, t_L\,(p_H V - I)\right]^2}{8\, t_L\, t_H\, \phi_H\, p_H^2\, t_L} \;=\; \frac{\phi_H\left[p_H V - I\right]^2}{8\, t_H} \;<\; \frac{\phi_H\left[p_H V - I\right]^2}{4\, t_H} \;=\; \widetilde{\pi}(1)\,. \tag{47}$$

The first inequality in (46) holds because:

$$0 \;<\; \phi_L\, p_L\, t_H\left[p_L V - I\right] + \phi_H\, p_H\, t_L\left[p_H V - I\right] \;<\; \phi_H\, p_H\, t_L\left[p_H V - I\right]. \tag{48}$$

The first inequality in (48) reflects Assumption 1. The second inequality in (48) holds because $p_L V - I < 0$. The last equality in (47) follows from (7). ■

Proof of Proposition 2.

The proof follows immediately from Lemma 5, since $\widetilde{\pi}'(q) > 0$ for all $q \in \left[\frac{1}{2}, 1\right]$, from Proposition 1. ■

Proof of Lemma 6.

From (9) and Lemma 1:

$$\pi^v(\beta, q) \;=\; \phi_L\left[1 - q\right]\left[p_L V\,(1-\beta) - I\right]\left[\frac{p_L V\,(1-q)\,\beta}{t_L}\right] + \phi_H\, q\left[p_H V\,(1-\beta) - I\right]\left[\frac{p_H V\, q\,\beta}{t_H}\right]$$

$$-\; F\,(q) - c\,(q)\left[\frac{\phi_L\, p_L V\,(1-q)\,\beta}{t_L} + \frac{\phi_H\, p_H\, V\, q\,\beta}{t_H}\right] \tag{49}$$

$$\Rightarrow \quad \frac{\partial \pi^v(\cdot)}{\partial \beta} \;=\; \frac{\phi_L\, p_L V\,(1-q)^2\left[p_L V\,(1-2\,\beta) - I\right]}{t_L} + \frac{\phi_H\, p_H V\, q^2\left[p_H V\,(1-2\beta) - I\right]}{t_H}$$

$$-\, c\,(q)\left[\frac{\phi_L\, p_L V\,(1-q)}{t_L} + \frac{\phi_H\, p_H V\, q}{t_H}\right] \;=\; 0$$

$$\Leftrightarrow \quad \phi_L\, p_L\, t_H\left[1-q\right]^2\left[p_L V\,(1-2\,\beta) - I\right]$$

$$+\; \phi_H\, p_H\, t_L\, q^2\left[p_H V\,(1-2\beta) - I\right] - c\,(q)\left[\phi_L\, p_L\, t_H\,(1-q) + \phi_H\, p_H\, t_L\, q\right] \;=\; 0$$

$$\Leftrightarrow \quad \phi_L\, p_L\, t_H\,(1-q)^2\left[p_L V - I\right] + \phi_H\, p_H\, t_L\, q^2\left[p_H V - I\right]$$

$$-\; 2\,\beta\left[\phi_L\, p_L^2 V\, t_H\,(1-q)^2 + \phi_H\, p_H^2 V\, t_L\, q^2\right] -\; c\,(q)\left[\phi_L\, p_L\, t_H\,(1-q) + \phi_H\, p_H\, t_L\, q\right] \;=\; 0$$

$$\Rightarrow \quad \beta \;=\; \frac{1}{2\,V\left[\phi_L\,p_L^2\,t_H\,(1-q)^2+\phi_H\,p_H^2\,t_L\,q^2\right]}\{\phi_L\,p_L\,t_H\,(1-q)^2\,[p_L V - I]$$

$$+\;\phi_H\,p_H\,t_L\,q^2\,[p_H V - I] - c\,(q)\,[\phi_L\,p_L\,t_H\,(1-q)+\phi_H\,p_H\,t_L\,q]\}. \quad (50)$$

It is readily verified that $\pi^v(\cdot)$ is a strictly concave function of β, that $\left.\frac{\partial^v \pi(\cdot)}{\partial \beta}\right|_{\beta=1} < 0$, and that $\left.\frac{\partial \pi^v(\cdot)}{\partial \beta}\right|_{\beta=0} > 0$ when Condition 1 holds. Therefore, when Condition 1 holds, (50) identifies the sharing rate that maximizes the lender's profit given screening accuracy q. ■

Proof of Lemma 7.

Substituting from (50) into (49) provides:

$$\Pi^v(q) \;=\; \pi^v - F\,(q)\,, \qquad \text{where}$$

$$\begin{aligned}
\pi^v \;=\;& \phi_L\,[1-q]\,[p_L V\,(1-\beta)-I]\left[\frac{p_L V\,(1-q)\,\beta}{t_L}\right] + \phi_H\,q\,[p_H V\,(1-\beta)-I]\left[\frac{p_H V\,q\,\beta}{t_H}\right] \\
& -\;c\,(q)\left[\frac{\phi_L\,p_L V\,(1-q)\,\beta}{t_L}+\frac{\phi_H\,p_H V\,q\,\beta}{t_H}\right] \\
=\;& \frac{V\beta}{t_L\,t_H}\left\{\phi_L\,(1-q)^2\,p_L\,t_H\,[p_L V\,(1-\beta)-I] + \phi_H\,q^2\,p_H\,t_L\,[p_H V\,(1-\beta)-I]\right. \\
& \left. -\,c\,(q)\,[\phi_L\,p_L\,t_H\,(1-q)+\phi_H\,p_H\,t_L\,q]\right\} \\
=\;& \frac{V\beta}{t_L\,t_H}\left\{\phi_L\,(1-q)^2\,p_L\,t_H\,[p_L V - I] + \phi_H\,q^2\,p_H\,t_L\,[p_H V - I]\right. \\
& \left. -\;c\,(q)\,[\phi_L\,p_L\,t_H\,(1-q)+\phi_H\,p_H\,t_L\,q] - \beta V\left[\phi_L\,(1-q)^2\,p_L^2\,t_H + \phi_H\,q^2\,p_H^2\,t_L\right]\right\} \\
=\;& \frac{V\beta}{t_L\,t_H}\left[\left\{\phi_L\,[1-q]^2\,p_L\,t_H\,[p_L V - I] + \phi_H\,q^2 p_H\,t_L\,[p_H V - I]\right.\right. \\
& \left. -\;c\,(q)\,[\phi_L\,p_L\,t_L\,(1-q)+\phi_H\,p_H\,t_H\,q]\right\} \\
& -\frac{1}{2}\left\{\phi_L\,(1-q)^2\,p_L\,t_H\,[p_L V - I] + \phi_H\,q^2 p_H\,t_L\,[p_H V - I]\right. \\
& \left.\left. -\,c\,(q)\,[\phi_L\,p_L\,t_H\,(1-q)+\phi_H\,p_H\,t_L\,q]\right\}\right] \\
=\;& \frac{V\beta}{2\,t_L\,t_H}\left\{\phi_L\,[1-q]^2\,p_L\,t_H\,[p_L V - I] + \phi_H\,q^2 p_H\,t_L\,[p_H V - I]\right. \\
& \left. -\;c\,(q)\,[\phi_L\,p_L\,t_H\,(1-q)+\phi_H\,p_H\,t_L\,q]\right\}. \qquad (51)
\end{aligned}$$

Substituting $V\beta$ from (50) into (51) provides:

$$\pi^v(q) \;=\; \frac{1}{4\,t_L\,t_H\left[\phi_L\,p_L^2\,t_H\,(1-q)^2+\phi_H\,p_H^2\,t_L\,q^2\right]}\{\phi_L\,[1-q]^2\,p_L\,t_H\,[p_L V - I]$$

$$+\;\phi_H\,q^2 p_H\,t_L\,[p_H V - I] - c\,(q)\,[\phi_L\,p_L\,t_H\,(1-q)+\phi_H\,p_H\,t_L\,q]\}^2. \;\blacksquare \qquad (52)$$

Proof of Proposition 3.

To determine whether $\pi^v(\cdot)$ is a convex function of q, let $p = 1 - q$. Then from (8):

$$\pi^v(q) = \frac{\{\phi_L\, p^2\, p_L\, t_H\, [p_L V - I] + \phi_H\, q^2 p_H\, t_L\, [p_H V - I] - c\,(q)\, [\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q]\}^2}{4\, t_L\, t_H\, [\phi_L\, p_L^2\, t_H\, p^2 + \phi_H\, p_H^2\, t_L\, q^2]}$$

$$= \frac{\{\phi_L\, p^2\, p_L\, t_H\, [p_L V - I] + \phi_H\, q^2\, p_H\, t_L\, [p_H V - I] - c\,(q)\, [\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q]\}^2}{4\, t_L\, t_H\, \phi_H\, p_H^2\, t_L \left[\frac{\phi_L\, p_L^2\, t_H}{\phi_H\, p_H^2\, t_L} p^2 + q^2\right]}$$

$$= \frac{[\phi_H\, p_H\, t_L\, (p_H V - I)]^2 \left[\frac{\phi_L\, p_L\, t_H [p_L V - I] p^2}{\phi_H\, p_H\, t_L [p_H V - I]} + q^2 - \frac{c(q)}{\phi_H\, p_H\, t_L [p_H V - I]}\, [\phi_L\, p_L\, t_L\, p + \phi_H\, p_H\, t_H\, q]\right]^2}{4\, t_L\, t_H\, \phi_H\, p_H^2\, t_L \left[\frac{\phi_L\, p_L^2\, t_H}{\phi_H\, p_H^2\, t_L} p^2 + q^2\right]}$$

$$= \frac{[\phi_H\, p_H\, t_L\, (p_H V - I)]^2}{4\, t_L\, t_H\, \phi_H\, p_H^2\, t_L} \cdot \left[\frac{\left[\frac{\phi_L\, p_L\, t_H [p_L V - I] p^2}{\phi_H\, p_H\, t_L [p_H V - I]} + q^2 - \frac{c(q)}{\phi_H\, p_H\, t_L [p_H V - I]}\, [\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q]\right]^2}{\left[\frac{\phi_L\, p_L^2\, t_H}{\phi_H\, p_H^2\, t_L} p^2 + q^2\right]}\right]. \tag{53}$$

Define:

$$\delta_1 = \frac{\phi_L\, p_L^2\, t_H}{\phi_H\, p_H^2\, t_L} > 0\,; \quad \delta_2 = \frac{\phi_L\, p_L\, t_H\, [p_L V - I]}{\phi_H\, p_H\, t_L\, [p_H V - I]} < 0\,; \quad \text{and}$$

$$\delta_3 = -\frac{c\,(q)}{\phi_H\, p_H\, t_L\, [p_H V - I]} < 0\,. \tag{54}$$

(53) and (54) provide:

$$\pi^v(q) = \frac{[\phi_H\, p_H\, t_L\, (p_H V - I)]^2}{4\, t_L\, t_H\, \phi_H\, p_H^2\, t_L}\, P \tag{55}$$

where:

$$P = \frac{[q^2 + \delta_2\, p^2 + \delta_3\, (\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q)]^2}{q^2 + \delta_1\, p^2}\,. \tag{56}$$

(55) and (56) imply that $\pi^v(\cdot)$ is convex in q if and only if P is convex in q. To determine whether P is convex in q, define:

$$g_1 = q^2 + \delta_1\, p^2; \qquad g_2 = q^2 + \delta_2\, p^2;$$

$$g_3 = \delta_3\, [\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q]\,; \quad \text{and} \quad G = g_2 + g_3. \tag{57}$$

Using (57) in (56) and differentiating provides:

$$P = \frac{G^2}{g_1} \;\Rightarrow\; P\, g_1 = G^2 \;\Rightarrow\; P\, g_1' + P'\, g_1 = 2\, G\, (G') \tag{58}$$

$$\Rightarrow\; P\, g_1'' + P'\, g_1' + P''\, g_1 + P'\, g_1' = 2\, (G')^2 + 2\, G\, (G'')$$

$$\Leftrightarrow \quad P\,g_1'' + 2\,P'\,g_1' + P''\,g_1 \;=\; 2\,(G')^2 + 2\,G\;(G'')$$
$$\Leftrightarrow \quad P''\,g_1 = 2\,(G')^2 + 2\,G\,(G'') - P\,g_1'' - 2\,P'\,g_1' \,. \tag{59}$$

From (57):

$$g_1 \;=\; q^2 + \delta_1\,p^2 \;=\; q^2 + \delta_1\,[1-q]^2$$
$$\Rightarrow \quad g_1' \;=\; 2\,q - 2\,\delta_1\,[1-q] \quad \text{and} \quad g_1'' \;=\; 2\,[1+\delta_1]\,. \tag{60}$$

$$g_2 \;=\; q^2 + \delta_2\,p^2 \;=\; q^2 + \delta_2\,[1-q]^2$$
$$\Rightarrow \quad g_2' \;=\; 2\,q - 2\,\delta_2\,[1-q] \quad \text{and} \quad g_2'' \;=\; 2\,[1+\delta_2]\;. \tag{61}$$

$$G \;=\; g_2 + g_3$$
$$\Rightarrow \quad G' \;=\; g_2' + g_3' \quad \text{and} \quad G'' \;=\; g_2'' + g_3''\,. \tag{62}$$

(61) and (62) provides:

$$G'' \;=\; 2\,[1+\delta_2] + g_3''\,. \tag{63}$$

Using (60) and (63) in (59) provides:

$$P''g_1 \;=\; 2\,(G')^2 + 2\,G\,[g_2'' + g_3''] - P\,[2\,(1+\delta_1)] - 2\,P'\,g_1'$$
$$\Rightarrow \quad \frac{P''g_1}{2} \;=\; (G')^2 + G\,[g_2'' + g_3''] - P\,[1+\delta_1] - P'\,g_1'\,. \tag{64}$$

Also, from (58):

$$P\,g_1' + P'\,g_1 \;=\; 2\,G\,(G') \quad \Rightarrow \quad P' \;=\; \frac{2\,G\,(G') - P\,g_1'}{g_1}\,. \tag{65}$$

Using (58) and (65) in (64) provides:

$$\frac{P''\,g_1}{2} \;=\; (G')^2 + G\,[g_2'' + g_3''] - \left[\frac{G^2}{g_1}\right][1+\delta_1] - \left[\frac{2\,G\,(G') - P\,g_1'}{g_1}\right] g_1'$$
$$\Rightarrow \quad \frac{P''\,(g_1)^2}{2} \;=\; (G')^2\,g_1 + G\,[g_2'' + g_3'']\,g_1 - G^2\,[1+\delta_1] - 2\,G\,(G')\,g_1' + P\;(g_1')^2\;. \tag{66}$$

Using (58) and (61) in (66) provides:

$$\frac{P''\,(g_1)^2}{2} \;=\; (G')^2\,g_1 + G\,[2\,(1+\delta_2) + g_3'']\,g_1 - G^2\,[1+\delta_1] - 2\,G\,(G')\,g_1' + \left[\frac{G^2}{g_1}\right](g_1')^2$$
$$=\; (G')^2\,g_1 - G\,(G')\,g_1' + \left[\frac{G^2}{g_1}\right](g_1')^2 - G\,(G')\,g_1' + G\,[2\,(1+\delta_2) + g_3'']\,g_1 - G^2\,[1+\delta_1]$$
$$=\; G'\,[G'\,g_1 - G\,g_1'] + \frac{G\,g_1'}{g_1}\,[G\,g_1' - G'g_1] + G\,[2\,(1+\delta_2) + g_3'']\,g_1 - G^2\,[1+\delta_1]$$
$$=\; G'\,[G'g_1 - Gg_1'] - \frac{G\,g_1'}{g_1}\,[G'g_1 - Gg_1'] + G\,[2\,(1+\delta_2) + g_3'']\,g_1 - G^2\,[1+\delta_1]$$

$$= \; [G' g_1 - G\, g_1'] \left[G' - \frac{G g_1'}{g_1} \right] + G \left[2\,(1+\delta_2) + g_3'' \right] g_1 - G^2 \left[1+\delta_1 \right]$$

$$= \; \frac{1}{g_1} \left[G' g_1 - G\, g_1' \right]^2 + G \left[(2\,(1+\delta_2) + g_3'')\, g_1 - G\,(1+\delta_1) \right] . \tag{67}$$

Since $g_1 = q^2 + \delta_1 (1-q)^2 > 0$, the first term on the right hand side of (67) is non-negative. Therefore, a sufficient condition for $P'' > 0$ is

$$G \left[(2\,(1+\delta_2) + g_3'')\, g_1 - G\,(1+\delta_1) \right] \; > \; 0 \,. \tag{68}$$

The inequality in (68) will hold if:

$$G \; > \; 0 \quad \text{and} \quad \left[2\,(1+\delta_2) + g_3'' \right] g_1 - G \left[1+\delta_1 \right] \; > \; 0 \,. \tag{69}$$

From (54) and (57):

$$\begin{aligned} G \; &= \; g_2 + g_3 \; = \; q^2 + \delta_2 \left[1-q \right]^2 + \delta_3 \left[\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q \right] \\ &= \; q^2 + \left[\frac{\phi_L\, p_L\, t_H\, (p_L V - I)}{\phi_H\, p_H\, t_L\, (p_H V - I)} \right] \left[1-q \right]^2 - \frac{c\,(q) \left[\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q \right]}{\phi_H\, p_H\, t_L \left[p_H V - I \right]} \\ &= \; \frac{1}{\phi_H\, p_H\, t_L \left[p_H V - I \right]} \{ \phi_H\, p_H\, t_L \left[p_H V - I \right] q^2 \\ &\qquad + \; \phi_L\, p_L\, t_H \left[p_L V - I \right] \left[1-q \right]^2 - c\,(q) \left[\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q \right] \}. \end{aligned} \tag{70}$$

(70) implies:

$$\begin{aligned} G \; > \; 0 \quad \Leftrightarrow \quad & \phi_H\, p_H\, t_L \left[p_H V - I \right] q^2 + \phi_L\, p_L\, t_H \left[p_L V - I \right] \left[1-q \right]^2 \\ & - \; c\,(q) \left[\phi_L\, p_L\, t_H\, (1-q) + \phi_H\, p_H\, t_L\, q \right] \; > \; 0 \,. \end{aligned} \tag{71}$$

Notice from (50) that the inequality in (71), which is Condition 1, ensures $\beta > 0$.

To analyze the other component of the sufficient condition in (69), notice from (54) and (57) that:

$$g_3 \; = \; \delta_3 \left[\phi_L\, p_L\, t_H\, p + \phi_H\, p_H\, t_L\, q \right] \; = \; - \, \frac{c\,(q) \left[\phi_L\, p_L\, t_H\, (1-q) + \phi_H\, p_H\, t_L\, q \right]}{\phi_H\, p_H\, t_L \left[p_H V - I \right]} \tag{72}$$

$$\begin{aligned} \Rightarrow \quad g_3' \; &= \; - \, \frac{c'\,(q) \left[\phi_L\, p_L\, t_H\, (1-q) + \phi_H\, p_H\, t_L\, q \right]}{\phi_H\, p_H\, t_L \left[p_H V - I \right]} - \frac{c\,(q) \left[-\, \phi_L\, p_L\, t_H + \phi_H\, p_H\, t_L \right]}{\phi_H\, p_H\, t_L \left[p_H V - I \right]} \\ \Rightarrow \quad g_3'' \; &= \; - \, \frac{c''\,(q) \left[\phi_L\, p_L\, t_H\, (1-q) + \phi_H\, p_H\, t_L\, q \right]}{\phi_H\, p_H\, t_L \left[p_H V - I \right]} - \frac{2\, c'\,(q) \left[-\phi_L\, p_L\, t_H + \phi_H\, p_H\, t_L \right]}{\phi_H\, p_H\, t_L \left[p_H V - I \right]} \,. \end{aligned} \tag{73}$$

Also:

$$g_1 \; = \; q^2 + \delta_1 \left[1-q \right]^2 \; = \; q^2 + \frac{\phi_L\, p_L^2\, t_H}{\phi_H p_H^2 t_L} \left[1-q \right]^2 = \; \frac{\phi_H\, p_H^2\, t_L\, q^2 + \phi_L\, p_L^2\, t_H \left[1-q \right]^2}{\phi_H\, p_H^2\, t_L} \,. \tag{74}$$

(54) and (73) imply:

$$2\left[1+\delta_{2}\right]+g_{3}'' \;=\; \frac{1}{\phi_{H}\, p_{H}\, t_{L}\left[p_{H}V-I\right]}\{2\left[\phi_{L}\, p_{L}\, t_{H}\left(p_{L}V-I\right)+\phi_{H}\, p_{H}\, t_{L}\left(p_{H}V-I\right)\right]$$

$$-\;2\, c'\left(q\right)\left[\phi_{H}\, p_{H}\, t_{L}-\phi_{L}\, p_{L}\, t_{H}\right]-c''\left(q\right)\left[\phi_{L}\, p_{L}\, t_{H}\left(1-q\right)+\phi_{H}\, p_{H}\, t_{L}\, q\right]\}\,. \quad (75)$$

From (54) and (57):

$$1+\delta_{1} \;=\; \frac{\phi_{H}\, p_{H}^{2}\, t_{L}\, +\phi_{L}\, p_{L}^{2}\, t_{H}}{\phi_{H}\, p_{H}^{2}\, t_{L}}\,. \quad (76)$$

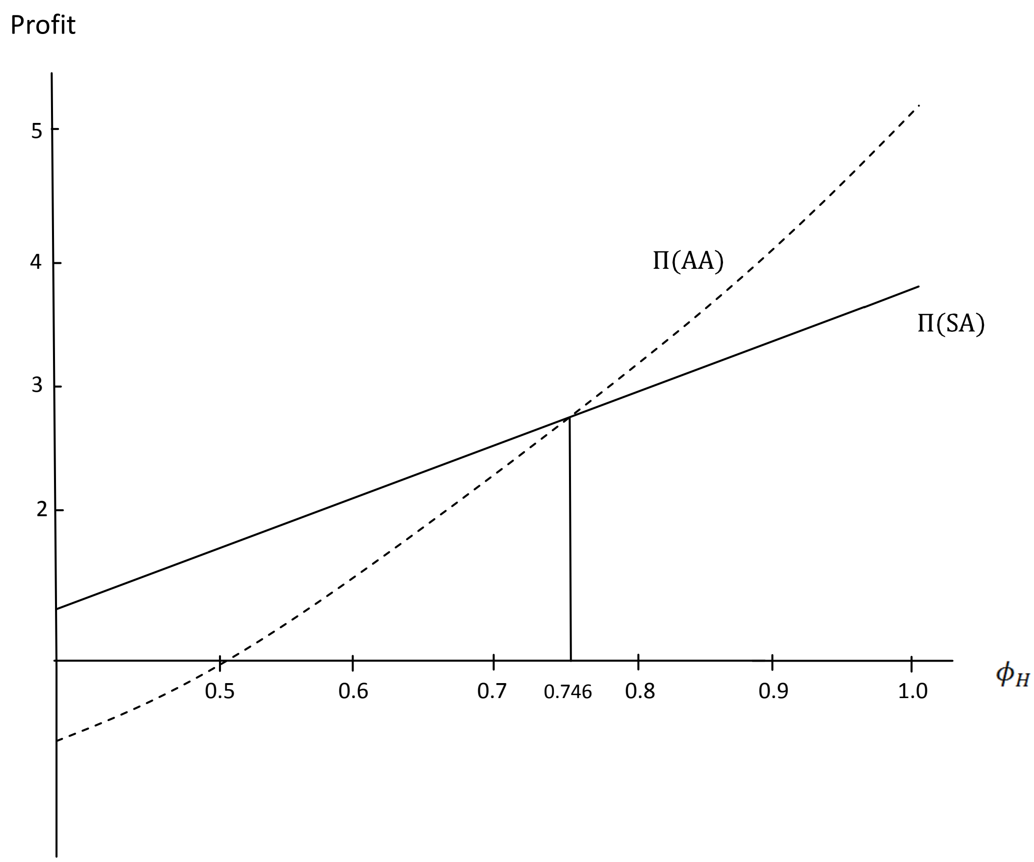
(70), (74), (75), and (76) imply that the second inequality in (69) holds if and only if Condition 2 holds, i.e.:

$$\left[\phi_{H}\, p_{H}^{2}\, t_{L}\, q^{2}+\phi_{L}\, p_{L}^{2}\, t_{H}\left(1-q\right)^{2}\right]\{2\left[\phi_{L}\, p_{L}\, t_{H}\left(p_{L}V-I\right)+\phi_{H}\, p_{H}\, t_{L}\left(p_{H}V-I\right)\right]$$

$$-\;2\, c'\left(q\right)\left[\phi_{H}\, p_{H}\, t_{L}-\phi_{L}\, p_{L}\, t_{H}\right]-c''\left(q\right)\left[\phi_{L}\, p_{L}\, t_{H}\left(1-q\right)+\phi_{H}\, p_{H}\, t_{L}\, q\right]\}$$

$$>\;\left[\phi_{H}\, p_{H}^{2}\, t_{L}\, +\phi_{L}\, p_{L}^{2}\, t_{H}\right]\{\phi_{H}\, p_{H}\, t_{L}\, q^{2}\left[p_{H}V-I\right]+\phi_{L}\, p_{L}\, t_{H}\left[1-q\right]^{2}\left[p_{L}V-I\right]$$

$$-\;c\left(q\right)\left[\phi_{L}\, p_{L}\, t_{H}\left(1-q\right)+\phi_{H}\, p_{H}\, t_{L}\, q\right]\}\,. \quad \blacksquare \quad (77)$$



<u>Figure 1</u>. Profit Under the SA and AA Policies in the Example.

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