On the Design and Performance of Piece-Rate Contracts

by

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Abstract

We characterize optimal piece-rate contracts and examine their ability to replicate the performance of fully optimal contracts in the canonical moral hazard setting with a wealth-constrained, risk averse agent. We find that optimal piece-rate contracts have a simple, intuitive characterization in a structured, but broad class of settings. We also show that piece-rate contracts can perform well when the productivity of the agent's effort is limited, but can perform relatively poorly when this productivity is pronounced.

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1 Introduction.

A piece-rate contract is a compensation contract under which the total payment delivered to an agent is the product of a fixed piece-rate and the number of units of output that he produces. Piece-rate contracts are often employed in practice, even though they are seldom optimal contracts.[1] For example, tenant farmers and salesmen frequently operate under piece-rate contracts.[2] The common use of piece-rate contracts likely reflects in part their simple structure. As Schmalensee (1989, p. 418) notes, "piece-rate schemes ... seem to be more readily understood and administered in practice than nonpiece-rate regimes."

Because piece-rate contracts are prevalent in practice, it is important to understand their optimal design and performance. We address these issues in the canonical moral hazard setting (e.g., Holmstrom, 1979) where a principal designs a reward structure to motivate a risk-averse agent to deliver productive effort. We show that the optimal piece-rate contract often has a simple and intuitive characterization in a structured but broad class of environments. We also show that the use of a piece-rate contract need not be very constraining for the principal. The principal can secure with a piece-rate contract a large fraction of the profit that she obtains under a fully optimal contract when the productivity of the agent's effort is limited, when the agent is not very averse to risk, and when effort is quite onerous for the agent to supply. More generally, though, the principal may sacrifice considerable profit if she employs a piece-rate contract rather than the fully optimal contract.

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[1] Linear contracts – but not necessarily piece-rate contracts – can be optimal contracts in some special moral hazard settings. This is the case, for example, when the agent is risk neutral and is not wealth constrained (e.g., Shavell, 1979) or when a risk averse agent with a negative exponential utility function controls the drift of a Brownian motion process (e.g., Holmstrom and Milgrom, 1987; Sung, 1995). Linear contracts also can be an optimal means to resolve double moral hazard problems (e.g., Bhattacharyya and Lafontaine, 1995; Kim and Wang, 1998; Corbett et al., 2005) or to motivate efficient investment (e.g., Pfeiffer and Velthius, 2005). Menus of linear contracts can constitute optimal contracts in adverse selection settings (e.g., Laffont and Tirole, 1986; Reichelstein, 1992) and in settings with both adverse selection and moral hazard (e.g., Guesnerie et al., 1989), when agents are risk neutral.

[2] Milgrom and Roberts (1992, p. 216) note that "Linear compensation formulas are commonly observed in the form of commissions paid to sales agents, contingency fees paid to attorneys, piece rates paid to tree planters or knitters, crop shares paid to sharecropping farmers, and so on." Bhattacharyya and Lafontaine (1995, pp. 763-4) observe that "Linear pricing rules have been found in a number of diverse areas such as, but not limited to, sales force compensation, sharecropping, leasing arrangements, author's fees, legal fees, licensing agreements, commercial real estate rental fees, and franchising."

Our analysis complements the works of other authors who derive explicit expressions for the optimal piece-rate contract in related but distinct settings.[3] Our analysis also complements other studies of factors that influence the value and performance of piece-rate contracts. These factors include the costs of monitoring performance (e.g., Cheung, 1969; Brown, 1990), worker heterogeneity (e.g., Lazear, 1986), and worker tenure on the job (e.g., Goldin, 1986).[4] Our study emphasizes the influence of risk aversion, effort aversion, worker productivity, and the alternative opportunities available to workers. Our theoretical analysis also complements empirical work that estimates losses from employing linear contracts in particular real-world settings of interest.[5]

We develop and present our findings as follows. Section 2 describes the key elements of our model. Sections 3 and 4 characterize the fully optimal contract and the optimal piece-rate contract, respectively. Section 5 presents our findings regarding the relative performance of the optimal piece-rate contract. Section 6 summarizes our key findings and suggests directions for future research.

2 Elements of the Model.

A risk neutral principal contracts with a single risk averse agent to produce output in our model. Increased effort (a) by the agent stochastically increases the realized output ($x \geq 0$). We assume that the density function for output is a two parameter gamma with with mean pa:

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[3] Paarsch and Shearer (2000) derive an expression for the optimal piece rate in a setting where risk neutral workers choose both the quantity and quality of output. The authors focus on the choice between piece-rate contracts and fixed-wage contracts. Haubrich (1994) characterizes the optimal contract in a moral hazard setting where realized output is binary. Holmstrom and Milgrom (1987) provide an explicit expression for the optimal contact (which is linear) in a setting where an agent with structured preferences controls the drift of a Brownian motion process over time.

[4] These studies do not compare in detail the performance of piece-rate contracts and fully optimal contracts. The particular effects considered in these studies do not arise in the canonical moral hazard setting that we examine because the agent's performance is observed perfectly and costlessly, the principal knows the agent's capabilities (but cannot observe his effort supply), and the interaction between the principal and agent is not repeated.

[5] Ferrall and Shearer (1999) estimate that a coal mining firm that implemented a linear bonus scheme sacrificed approximately one-half of the profit that is could have secured with the optimal bonus scheme.

$$f(x|a) \;=\; \frac{x^{p-1}\, e^{-x/a}}{a^p\, \Gamma(p)} \quad \text{for} \;\; x \in [0, \infty), \quad \text{where} \;\; \Gamma(p) \;=\; \int_0^\infty e^{-u}\, u^{p-1}\, du. \tag{1}$$

The parameter p can be viewed as the productivity of the agent's effort (a) because the mean of the density increases with a at rate p.

The gamma density is drawn in Figure 1 for selected values of the parameter p. As Figure 1 suggests, this density admits a wide variety of stochastic relationships between effort and output.[6] As explained further below, the gamma density also facilitates the identification of conditions under which the tractable "first-order approach" can be employed to solve the principal's problem (e.g., Rogerson, 1985; Jewitt, 1988; Mirrlees, 1999).

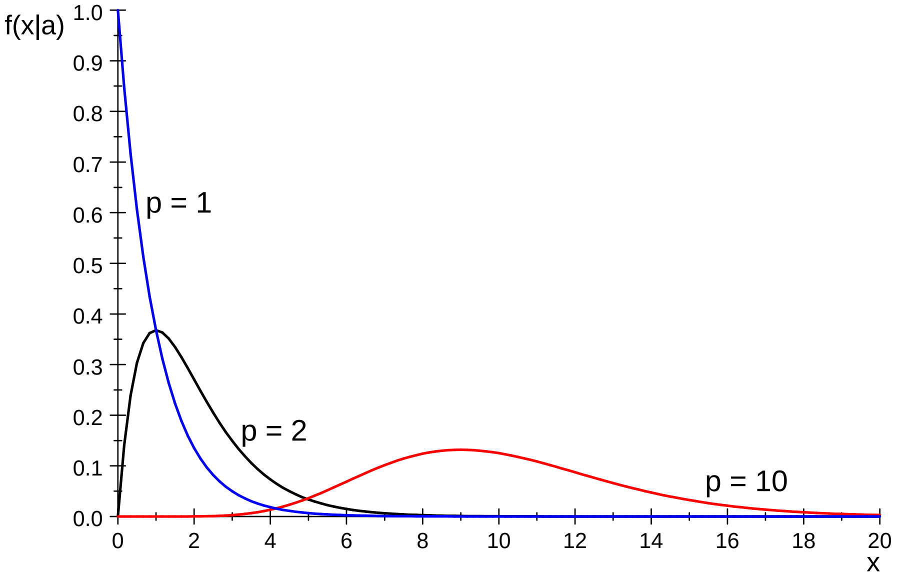

Figure 1. The Gamma Density for $p = 1$, 2, and 10 $(a = 1)$.

The agent's utility when he delivers effort a and is paid (wage) w is $U^A(w, a) = 2w^\theta - a^\delta$, where $\theta \in (0, \frac{1}{2}]$ and $\delta \geq 1$.[7] The agent has no wealth initially. Because θ is less than 1, the

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[6] When $p = 1$, the gamma density is the exponential density with mean a. The normal distribution can be approximated by an appropriately centered gamma density with a large value for p. The gamma distribution is used extensively in applied work in many disciplines, including meteorology, ecology, and economics (Johnson et al., 1994).

[7] As explained further below, θ is assumed to be no greater than $\frac{1}{2}$ to ensure that the first-order approach can be employed to derive the properties of optimal contracts.

agent is averse to risk. His degree of relative risk aversion $(1-\theta)$ declines as θ increases.[8] This parameterization of the agent's preferences facilitates either closed-form analytic solutions to the principal's problem or systems of simultaneous equations that can be solved numerically to characterize optimal contracts and the performance they induce.[9]

The principal designs a payment schedule, $w(x)$, that specifies the payment she will deliver to the agent for each level of output that he produces. The principal seeks to maximize her expected profit, which is the expected difference between the output the agent produces and the payment the principal makes to the agent.[10] The agent will work for the principal if and only if doing so provides an expected utility of at least $\overline{U}$.

The timing in the model is as follows. First, the principal specifies the payment schedule, or contract, $w(x)$. Second, the agent decides whether he will accept the specified contract and work for the principal. If the agent declines to do so, his interaction with the principal is forever terminated. If the agent accepts the contract, he next chooses his effort supply. Finally, realized output is observed and the principal delivers the promised payment to the agent. This interaction is not repeated.

This timing, the stochastic relationship between x and a, and the agent's preferences are all common knowledge. The output produced by the agent is observed publicly, but the agent's effort supply is not. The principal's problem in this setting, labeled $[P]$, is the following:

$$\underset{w(x)\,\geq\,0,\;a}{\text{Maximize}} \quad \int_x [x - w(x)]\, f(x|a)\; dx \tag{2}$$

$$\text{subject to:} \quad \int_x U^A(w(x), a)\, f(x|a)\; dx \;\geq\; \overline{U}\,, \;\text{ and} \tag{3}$$

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[8] The agent's degree of relative risk aversion is $-w[\frac{\partial^2}{\partial w^2}U^A(w,a)]/[\frac{\partial}{\partial w}U^A(w,a)] = -w[2\theta(\theta-1)w^{\theta-2}]/[2\theta w^{\theta-1}] = 1-\theta$.

[9] As Guo and Yang (2006, p. 150) note, "Using general utility functions as well as general output processes, it is usually not possible to characterize explicitly optimal contracts and other properties that can be tested empirically."

[10] Thus, for simplicity, we normalize the unit value of output to 1, so x denotes both the amount of output produced and the value of that output to the principal.

$$a \in \arg\max_{\widehat{a}} \{ \int_x U^A(w(x), \widehat{a}) \, f(x|\widehat{a}) \; dx \}. \tag{4}$$

Expression (2) reflects the principal's desire to maximize her expected profit. Inequality (3), the agent's participation constraint, ensures that he anticipates at least $\overline{U}$ in expected utility when he works for the principal. Expression (4), the agent's effort selection constraint, reflects the fact that the agent chooses his effort to maximize his expected utility, given the prevailing contract, $w(x)$.

It is readily verified that when $\theta \leq \frac{1}{2}$, $\delta \geq 1$, and $f(\cdot)$ is as specified in equation (1), the first-order approach to solving $[P]$ is valid (Jewitt, 1988, Theorem 1). Under these conditions (which are maintained throughout the ensuing analysis), the agent's effort selection constraint, (4), can be replaced by the following first-order condition for the agent's self-interested choice of a in deriving the solution to $[P]$:

$$\int_x \{ \, U^A(w(x), \, a) \, f_a(x|a) \; + \; \frac{\partial U^A(.)}{\partial a} \, f(x|a) \} \, dx \;\; = \;\; 0. \tag{5}$$

Problem [P-R] is identical to problem [P] except that the following restriction is imposed:

$$w(x) \;\; = \;\; \beta \, x \quad \text{for all } \; x \geq 0. \tag{6}$$

Thus, the solution to [P-R] identifies the optimal piece-rate contract for the principal.

3 The Fully Optimal Contract.

The solution to [P] is readily characterized using pointwise optimization. For completeness, this standard characterization is presented in Lemma 1. The lemma refers to λ and μ, which are the Lagrange multipliers associated with constraints (3) and (5), respectively.

Lemma 1. *The fully optimal contract (i.e., solution to [P]) is characterized by the solution to the following relations (and the corresponding complementary slackness conditions):*

$$w(x) \;\; = \;\; \begin{cases} 0 & \textit{if} \;\; x \; < \; \widehat{x} \\ \left[2\theta \left(\lambda + \mu \left[\frac{f_a(x|a)}{f(x|a)} \right] \right) \right]^{\frac{1}{1-\theta}} & \textit{if} \;\; x \; \geq \; \widehat{x} \; ; \end{cases}$$

$$\int_{\widehat{x}}^{\infty} 2[w\,(x)]^{\theta}\, f_a(x|a)dx \;-\; \delta a^{\delta-1} \;\geq\; \overline{U}\,;$$

$$\int_{\widehat{x}}^{\infty} 2[w\,(x)]^{\theta}\; f(x|a)dx \;-\; a^{\delta} \;=\; 0\,; \qquad \textit{and}$$

$$\int_{0}^{\widehat{x}} x\; f_a(x|a)dx \;+\; \int_{\widehat{x}}^{\infty} [x - w(x)]\;\; f_a(x|a)dx$$

$$+\;\mu\;\left[\int_{\widehat{x}}^{\infty} 2[w\,(x)]^{\theta}\, f_{aa}(x|a)dx \;-\; \delta\,[\delta-1]\,a^{\delta-2}\right] \;=\; 0\,,$$

$$\textit{where}\quad \widehat{x} \;=\; \min\left\{x \geq 0 \;|\; \lambda + \mu\left[\frac{f_a(x|a)}{f(x|a)}\right] \geq 0\right\}.$$

We will denote by π the principal's expected profit (i.e., the value of her objective function) at the solution to [P].[11] The functional forms that we employ admit numerical solutions to the relations identified in Lemma 1. These solutions are considered in section 5, after the optimal piece-rate contract is characterized in section 4.

4 The Optimal Piece-Rate Contract.

The optimal piece-rate contract (i.e., the solution to [P-R]) is characterized most readily in settings where the agent's reservation utility ($\overline{U}$) is sufficiently small that the agent's participation constraint, (3), does not bind at the solution to [P-R]. As the proof of Proposition 1 reveals, this will be the case when $\overline{U}$ is less than $\overline{U}_1 \equiv \left[\frac{\delta-\theta}{\theta}\right]\left[\frac{2\,\Gamma(p+\theta)}{\Gamma(p)}\left(\frac{\theta}{\delta}\right)^{\theta+1}\right]^{\frac{\delta}{\delta-\theta}}$.

Proposition 1. *Suppose* $\overline{U} \leq \overline{U}_1$. *Then* $\beta = \frac{\theta}{\delta}$ *at the solution to [P-R].*

Proof. Given $w(x) = \beta\, x$, the agent chooses his effort, a, to maximize:

$$H\,(a) \;=\; \int_{0}^{\infty} 2\,(\beta x)^{\theta}\, f(x|a)dx - a^{\delta}$$

$$=\; 2\,(\beta)^{\theta}\int_{0}^{\infty} x^{\theta}\frac{1}{a^{p}\Gamma\,(p)}x^{p-1}e^{-x/a}dx - a^{\delta} \;=\; 2\,(\beta)^{\theta}\left[a^{\theta}\right]\frac{\Gamma\,(p+\theta)}{\Gamma\,(p)} - a^{\delta}. \tag{7}$$

Maximization of $H(a)$ with respect to a provides:

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[11] For expositional simplicity, we will often refer to "expected profit" as "profit" in the ensuing discussion.

$$H'(a) = 2(\beta)^{\theta}\left[\frac{\Gamma(p+\theta)}{\Gamma(p)}\right]\theta a^{\theta-1} - \delta a^{\delta-1} \; = \; 0 \quad \Rightarrow \quad \widetilde{a} \; = \; \left[\frac{2\theta(\beta)^{\theta}}{\delta}\left(\frac{\Gamma(p+\theta)}{\Gamma(p)}\right)\right]^{\frac{1}{\delta-\theta}}. \qquad (8)$$

Given effort supply $\widetilde{a}$, expected output is $p\,\widetilde{a}$, and so the principal's expected profit is $[1-\beta]\,p\,\widetilde{a}$. Therefore, equation (8) implies that when constraint (3) does not bind at the solution to [P-R], the principal's problem is to choose β to maximize:

$$L \; = \; [1-\beta]\,p\left[\frac{2\,\theta\,(\beta)^{\theta}}{\delta}\left(\frac{\Gamma(p+\theta)}{\Gamma(p)}\right)\right]^{\frac{1}{\delta-\theta}} \; = \; K\,(\beta)^{\frac{\theta}{\delta-\theta}}\,[1-\beta], \qquad (9)$$

$$\text{where} \quad K \; \equiv \; p\left[\frac{2\theta}{\delta}\left(\frac{\Gamma(p+\theta)}{\Gamma(p)}\right)\right]^{\frac{1}{\delta-\theta}}.$$

From equation (9): $\ln(L) \; = \; \ln(K) + \ln(1-\beta) + \left[\frac{\theta}{\delta-\theta}\right]\ln(\beta)$

$$\Rightarrow \quad \frac{\partial \ln(L)}{\partial\beta} \; = \; -\,\frac{1}{1-\beta} + \frac{\theta}{\beta\,[\delta-\theta]} \qquad (10)$$

$$\Rightarrow \quad \frac{\partial^2 \ln(L)}{\partial\beta^2} \; = \; -\,\frac{1}{[1-\beta]^2} - \frac{\theta}{\beta^2\,[\delta-\theta]} \; < \; 0. \qquad (11)$$

Equations (10) and (11) imply that the value of β that maximizes $\ln(L)$ (and thus L) is given by:

$$-\,\frac{1}{1-\beta} + \frac{\theta}{\beta\,[\delta-\theta]} \; = \; 0 \quad \Rightarrow \quad [1-\beta]\,\theta \; = \; \beta\,[\delta-\theta] \quad \Rightarrow \quad \beta \; = \; \frac{\theta}{\delta}. \qquad (12)$$

It remains to prove that (3) does not bind at the identified solution to [P-R] when $\overline{U} \leq \overline{U}_1$. Equations (7), (8), and (12) imply that constraint (3) will not bind if and only if:

$$2(\beta)^{\theta}\left[\widetilde{a}^{\theta}\right]\frac{\Gamma(p+\theta)}{\Gamma(p)} - \widetilde{a}^{\delta} \; \geq \; \overline{U} \quad \Leftrightarrow \quad \widetilde{a}^{\delta}\left[2\left(\frac{\theta}{\delta}\right)^{\theta}\left[\widetilde{a}^{\theta-\delta}\right]\frac{\Gamma(p+\theta)}{\Gamma(p)} - 1\right] \; \geq \; \overline{U}$$

$$\Leftrightarrow \quad \widetilde{a}^{\delta}\left[\frac{2\left(\frac{\theta}{\delta}\right)^{\theta}\left(\frac{\Gamma(p+\theta)}{\Gamma(p)}\right)}{2\left(\frac{\theta}{\delta}\right)^{\theta+1}\left(\frac{\Gamma(p+\theta)}{\Gamma(p)}\right)} - 1\right] \; \geq \; \overline{U} \quad \Leftrightarrow \quad \widetilde{a}^{\delta}\left[\frac{\delta}{\theta} - 1\right] \; \geq \; \overline{U}$$

$$\Leftrightarrow \quad \left[\frac{2\,\Gamma(p+\theta)}{\Gamma(p)}\left(\frac{\theta}{\delta}\right)^{\theta+1}\right]^{\frac{\delta}{\delta-\theta}}\left[\frac{\delta-\theta}{\theta}\right] \; \geq \; \overline{U} \quad \Leftrightarrow \quad \overline{U}_1 \; \geq \; \overline{U}. \qquad \blacksquare$$

Proposition 1 reports that when the agent's reservation utility is sufficiently small (i.e., when $\overline{U} \leq \overline{U}_1$), the optimal piece rate is simply the ratio of θ to δ. This piece rate is the one that maximizes the principal's expected profit when she is not constrained to deliver a certain level of expected utility to the agent in order to ensure his participation. In this case, the principal chooses β to maximize $a\,p\,[1-\beta]$, and so β is determined by:

$$\frac{d}{d\beta}\left\{a\,p\,[1-\beta]\right\} \;=\; 0 \quad\Leftrightarrow\quad [1-\beta]\,\frac{\partial a}{\partial \beta} - a \;=\; 0 \quad\Leftrightarrow\quad \frac{\partial a}{\partial \beta}\left[\frac{\beta}{a}\right] \;=\; \frac{\beta}{1-\beta}\,. \tag{13}$$

Equation (13) indicates that the principal's preferred piece rate equates the elasticity of the agent's effort with respect to β ($\frac{\partial a}{\partial \beta}\frac{\beta}{a}$) to the ratio of the agent's share of output to the principal's share ($\frac{\beta}{1-\beta}$).[12] This policy constitutes the resolution of the fundamental trade-off that the principal faces in designing a piece-rate contract: a higher piece-rate induces the agent to deliver more effort, but reduces the share of (the increased) output that the principal receives. The principal optimally resolves this trade-off by increasing the piece rate up to the point at which a further increase in β would increase the expected payment to the agent more rapidly than it would increase the principal's expected payoff by increasing the agent's effort supply.

Notice from equation (8) that at the solution to [P-R]:

$$\ln(a) \;=\; \left[\frac{1}{\delta-\theta}\right]\left[\theta\ln(\beta) + \ln\left(\frac{2\theta}{\delta}\left[\frac{\Gamma(p+\theta)}{\Gamma(p)}\right]\right)\right]$$

$$\Rightarrow\quad \left[\frac{1}{a}\right]\frac{\partial a}{\partial \beta} \;=\; \left[\frac{\theta}{\delta-\theta}\right]\frac{1}{\beta} \quad\Rightarrow\quad \frac{\partial a}{\partial \beta}\left[\frac{\beta}{a}\right] \;=\; \frac{\theta}{\delta-\theta}\,. \tag{14}$$

Equation (14) implies that the elasticity of the agent's effort with respect to β increases with θ, decreases with δ, and is independent of p. Therefore, given the profit-maximizing choice of β identified in equation (13), the principal optimally increases β as θ increases, reduces β as δ increases, and leaves β unchanged as p increases in the setting of Proposition 1.

The elasticity of the agent's effort with respect to β increases with θ and decreases with

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[12] As is evident from equation (13), this is a general conclusion that holds even if the functional forms presently under consideration are not satisfied.

δ for the following reasons. As θ increases, the agent's marginal valuation of the smallest payments declines while his marginal valuation of the higher payments increases.[13] Therefore, a given increase in the rate at which payments increase with output (i.e., a given increase in β) induces the agent to expand his effort supply more rapidly in order to increase the likelihood of the higher outputs and the corresponding higher payments. In contrast, as δ increases, the agent's marginal disutility of effort declines for the lowest effort levels but increases for the higher effort levels.[14] The resulting increased aversion to expanding effort diminishes the rate at which a given increase in β will induce expanded effort.

The proof of Proposition 1 (equations (8), (9), and (12), in particular) provides the following additional information about the optimal piece-rate contract and its effects. Corollary 1 refers to $\widetilde{\pi}$, which is the principal's profit at the solution to [P-R].

Corollary 1. *Suppose* $\overline{U} \leq \overline{U}_1$. *Then* $a = \left[2 \left(\frac{\theta}{\delta} \right)^{\theta+1} \left(\frac{\Gamma(p+\theta)}{\Gamma(p)} \right) \right]^{\frac{1}{\delta-\theta}}$, $\widetilde{\pi} = a\, p \left[1 - \frac{\theta}{\delta} \right]$, *and the agent secures strictly positive rent at the solution to [P-R].*

Proposition 2 now explains how the principal adjusts the optimal piece rate when she must do so in order to ensure the agent's participation. The proposition refers to $\overline{U}_2 = \overline{U}_1 \left(\frac{\delta}{\theta} \right)^{\frac{\theta\delta}{\delta-\theta}}$, which is the largest value of $\overline{U}$ for which the principal's expected payoff at the solution to [P-R] is non-negative.[15]

Proposition 2. *Suppose* $\overline{U} \in (\overline{U}_1, \overline{U}_2]$. *Then* $\beta = \frac{\theta}{\delta} \left[\frac{\overline{U}}{\overline{U}_1} \right]^{\frac{\delta-\theta}{\theta\delta}}$ *at the solution to [P-R].*

Proof. The proof of Proposition 1 reveals that constraint (3) binds at the solution to [P-R] when $\overline{U} > \overline{U}_1$. From equations (7) and (8), the principal's problem in this case is to:

$$\underset{\beta,\, \widetilde{a}}{Maximize} \quad \int_0^{\infty} [x - \beta x]\, f\,(x|\widetilde{a})\, dx \;\; = \;\; [1 - \beta]\, \widetilde{a}\, p\,, \tag{15}$$

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[13] Since $\frac{\partial}{\partial w} U^A(w,a) = 2\,\theta w^{\theta-1}$, $\frac{\partial}{\partial \theta} \left\{ \frac{\partial}{\partial w} U^A(\cdot) \right\} = 2\, w^{\theta-1} [1 + \theta \ln w] \lesseqgtr 0$ as $w \lesseqgtr e^{-\frac{1}{\theta}}$.

[14] Since $\frac{\partial}{\partial a} U^A(w,a) = -\delta\, a^{\delta-1}$, $\frac{\partial}{\partial \delta} \left| \frac{\partial}{\partial a} U^A(\cdot) \right| = a^{\delta-1} [1 + \delta \ln a] \lesseqgtr 0$ as $a \lesseqgtr e^{-\frac{1}{\delta}}$.

[15] Proposition 2 reveals that $\beta = \frac{\theta}{\delta} \left[\frac{\overline{U}}{\overline{U}_1} \right]^{\frac{\delta-\theta}{\theta\delta}}$ when constraint (3) binds at the solution to [P-R]. It is never profitable for the principal to increase β above 1. Therefore, $\left(\frac{\theta}{\delta} \right) \left[\frac{\overline{U}_2}{\overline{U}_1} \right]^{\frac{\delta-\theta}{\theta\delta}} = 1 \;\Rightarrow\; \overline{U}_2 = \overline{U}_1 \left(\frac{\delta}{\theta} \right)^{\frac{\theta\delta}{\delta-\theta}}$.

$$\text{where} \quad \widetilde{a} \;=\; \left[\frac{2\,\theta\,(\beta)^{\theta}}{\delta}\left(\frac{\Gamma\,(p+\theta)}{\Gamma\,(p)}\right)\right]^{\frac{1}{\delta-\theta}}, \quad \text{and} \tag{16}$$

$$2\,(\beta)^{\theta}\left[\widetilde{a}^{\theta}\right]\frac{\Gamma\,(p+\theta)}{\Gamma\,(p)} \;-\; \widetilde{a}^{\delta} \;=\; \overline{U}. \tag{17}$$

Equations (16) and (17) imply that at the solution to [P-R]:

$$\widetilde{a}^{\delta}\left[2\,(\beta)^{\theta}\left[\widetilde{a}^{\theta-\delta}\right]\frac{\Gamma\,(p+\theta)}{\Gamma\,(p)}-1\right] \;=\; \overline{U} \quad\Rightarrow\quad \widetilde{a}^{\delta}\left[\frac{2\,(\beta)^{\theta}\left(\frac{\Gamma(p+\theta)}{\Gamma(p)}\right)}{\frac{2\,\theta(\beta)^{\theta}}{\delta}\left(\frac{\Gamma(p+\theta)}{\Gamma(p)}\right)}-1\right] \;=\; \overline{U}$$

$$\Rightarrow\quad \widetilde{a}^{\delta}\left[\frac{\delta}{\theta}-1\right] \;=\; \overline{U} \quad\Rightarrow\quad \widetilde{a} \;=\; \left[\frac{\theta\,\overline{U}}{\delta-\theta}\right]^{\frac{1}{\delta}}. \tag{18}$$

Equations (16) and (18) imply:

$$\left[\frac{2\,\theta\,(\beta)^{\theta}}{\delta}\left(\frac{\Gamma\,(p+\theta)}{\Gamma\,(p)}\right)\right]^{\frac{1}{\delta-\theta}} \;=\; \left[\frac{\theta\,\overline{U}}{\delta-\theta}\right]^{\frac{1}{\delta}} \quad\Rightarrow\quad \frac{2\theta\,(\beta)^{\theta}}{\delta}\left(\frac{\Gamma\,(p+\theta)}{\Gamma\,(p)}\right) \;=\; \left[\frac{\theta\,\overline{U}}{\delta-\theta}\right]^{\frac{\delta-\theta}{\delta}}$$

$$\Rightarrow\quad (\beta)^{\theta} \;=\; \frac{\delta\,\Gamma\,(p)}{2\,\theta\,\Gamma\,(p+\theta)}\left[\frac{\theta}{\delta-\theta}\right]^{\frac{\delta-\theta}{\delta}}\left[\,\overline{U}\,\right]^{\frac{\delta-\theta}{\delta}}. \tag{19}$$

From the definition of $\overline{U}_1$:

$$\left[\overline{U}_1\right]^{\frac{\theta-\delta}{\delta}} \;=\; \left[\frac{\delta-\theta}{\theta}\right]^{\frac{\theta-\delta}{\delta}}\left[\frac{\Gamma(p)}{2\,\Gamma(p+\theta)}\left(\frac{\delta}{\theta}\right)\right]\left(\frac{\delta}{\theta}\right)^{\theta}$$

$$\Rightarrow\quad \left[\frac{1}{\overline{U}_1}\right]^{\frac{\delta-\theta}{\delta}} \;=\; \left[\frac{\theta}{\delta-\theta}\right]^{\frac{\delta-\theta}{\delta}}\left[\frac{\delta\,\Gamma(p)}{2\,\theta\,\Gamma(p+\theta)}\right]\left(\frac{\delta}{\theta}\right)^{\theta}$$

$$\Rightarrow\quad \left[\frac{\theta}{\delta-\theta}\right]^{\frac{\delta-\theta}{\delta}}\left[\frac{\delta\,\Gamma(p)}{2\,\theta\,\Gamma(p+\theta)}\right] \;=\; \left(\frac{\theta}{\delta}\right)^{\theta}\left[\frac{1}{\overline{U}_1}\right]^{\frac{\delta-\theta}{\delta}}. \tag{20}$$

Equations (19) and (20) imply:

$$(\beta)^{\theta} \;=\; \left(\frac{\theta}{\delta}\right)^{\theta}\left[\frac{1}{\overline{U}_1}\right]^{\frac{\delta-\theta}{\delta}}\left[\,\overline{U}\,\right]^{\frac{\delta-\theta}{\delta}} \quad\Rightarrow\quad \beta \;=\; \left(\frac{\theta}{\delta}\right)\left[\frac{\overline{U}}{\overline{U}_1}\right]^{\frac{\delta-\theta}{\theta\delta}}. \quad \blacksquare \tag{21}$$

Propositions 1 and 2 together indicate that the principal increases β when her preferred piece rate (i.e., the one for which $\frac{\partial a}{\partial \beta}\frac{\beta}{a} = \frac{\beta}{1-\beta}$) does not ensure the agent's participation.[16] The optimal increase in β is more pronounced the larger is $\overline{U}$. Furthermore, the optimal proportionate increase in β declines as p increases.[17] This is the case because as the productivity of the agent's effort increases, a given increase in β will generate a larger increase in the agent's expected utility. Consequently, the principal can ensure the agent's participation with a smaller proportionate increase in β.

The proof of Proposition 2 (equations (15), (18), and (21), in particular) provides the following additional information about the optimal piece-rate contract.

Corollary 2. *Suppose* $\overline{U} \in (\overline{U}_1, \overline{U}_2]$. *Then* $a = \left[\frac{\theta \overline{U}}{\delta-\theta}\right]^{\frac{1}{\delta}}$, $\widetilde{\pi} = a\, p \left[1 - \frac{\theta}{\delta}\left(\frac{\overline{U}}{\overline{U}_1}\right)^{\frac{\delta-\theta}{\theta\delta}}\right]$, *and the agent secures no rent at the solution to [P-R].*

The formulae in Corollaries 1 and 2 facilitate an assessment of the performance of optimal piece-rate contracts. This assessment is presented in section 5.

5 The Performance of Piece-Rate Contracts.

To assess the reduction in profit that the principal experiences when she employs a piece-rate contract, we calculate $\frac{\widetilde{\pi}}{\pi}$, the ratio of the principal's profit under the optimal piece-rate contract to her profit under the fully optimal contract. For given values of the parameters p, δ, θ, and $\overline{U}$, the relevant value of $\widetilde{\pi}$ can be derived from the formulae presented in Corollaries 1 and 2. The relations identified in Lemma 1 can be solved numerically to derive the relevant value of π.

First consider the following benchmark setting. Suppose the mean of the output distribution is $2a$ when the agent supplies effort a (so $p = 2$), the agent's utility from wealth w is $2\sqrt{w}$ (so $\theta = \frac{1}{2}$), his disutility from supplying effort a is a^2 (so $\delta = 2$), and the agent will work for the principal whenever he anticipates non-negative utility from doing so (so

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[16] Notice that $\left[\frac{\overline{U}}{\overline{U}_1}\right]^{\frac{\delta-\theta}{\theta\delta}} > 1$ when $\overline{U} > \overline{U}_1$.

[17] $\left[\frac{\overline{U}}{\overline{U}_1}\right]^{\frac{\delta-\theta}{\theta\delta}}$ is a decreasing function of p because $\overline{U}_1$ is an increasing function of p.

$\overline{U} = 0$). Figure 2 presents the fully optimal contract and the optimal piece-rate contract in this benchmark setting.

Under the fully optimal contract, the payment to the agent is an increasing, convex function of his performance. The payment to the agent is strictly positive if and only if $x > 1.22$. The agent delivers .863 units of effort under this contract, and the principal's profit is 1.29.

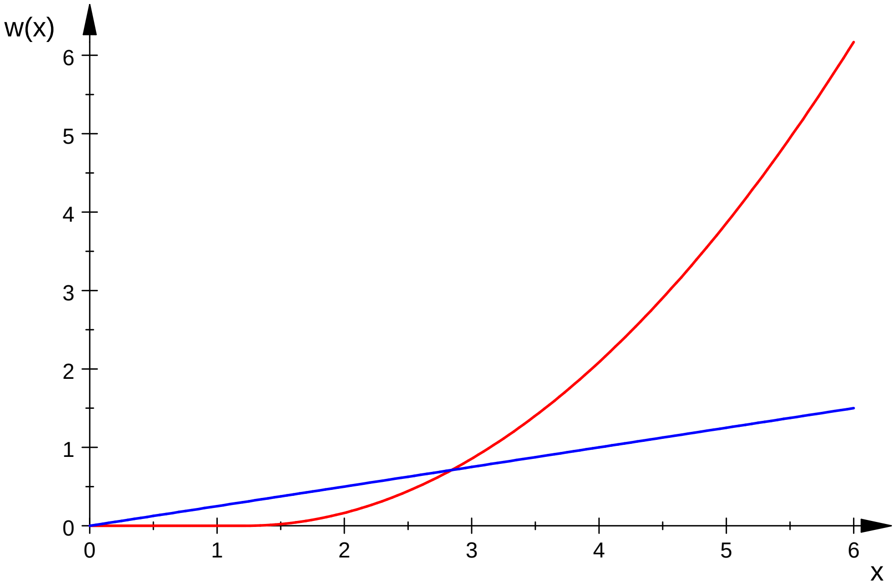

Figure 2. The Fully Optimal and Optimal Piece-Rate Contracts when $p = 2$, $\theta = \frac{1}{2}$, $\delta = 2$, and $\overline{U} = 0$.

Under the optimal piece-rate contract in this benchmark setting, the agent receives one-fourth of the output he produces (i.e., $\beta = .25$). The agent delivers .48 units of effort under this contract, and the principal's profit is .72. Thus, the optimal linear contract secures for the principal approximately 56% of the profit she achieves under the fully optimal contract.

This modest performance of the optimal piece-rate contract in the benchmark setting reflects its limited ability to replicate the incentives created by the fully optimal contract. The single blunt instrument (β) that is available under the optimal piece-rate contract forces the principal to deliver too much compensation for the smallest outputs and too little compensation for the largest outputs relative to the compensation under the fully optimal contract.

Consequently, the agent delivers considerably less effort under the optimal piece-rate contract than under the fully optimal contract, which causes a substantial proportionate reduction in the principal's profit.

Now consider how the nature and performance of these optimal contracts change as the environment in which they are implemented changes. Tables 1 – 4 illustrate the effects of changes in p, θ, δ, and $\overline{U}$, respectively. The optimal piece rate (β), the agent's effort supply ($\widetilde{a}$), and the principal's profit ($\widetilde{\pi}$) under the optimal piece-rate contract are presented in columns 2, 3, and 6 of the tables, respectively. The agent's effort supply (a) and the principal's profit (π) under the fully optimal contract appear in columns 4 and 5 of the tables, respectively. The relative performance of the optimal piece-rate contract ($\widetilde{\pi}/\pi$) is recorded in the last column of the tables.

Tables 1 – 4 reveal that the principal can sometimes secure with a piece-rate contract a substantial fraction of the profit that she secures with a fully optimal contract. However, the relative performance of the optimal piece-rate contract declines monotonically in the identified settings as p increases, as θ decreases, and as δ decreases, *ceteris paribus.* In addition, the optimal piece-rate contract performs relatively poorly when the agent's reservation utility ($\overline{U}$) is either quite high or quite low.[18] These findings reflect the following considerations.

As p increases, the productivity of the agent's effort increases. Therefore, the principal would like to induce the agent to deliver substantial effort. She does so under a fully optimal contract by specifying a threshold level of output, $x_T > 0$, and withholding all payment from the agent when realized output is below the threshold (i.e., by setting $w(x) = 0$ for all $x \leq x_T$). However, the principal is constrained to deliver the same per-unit reward for all $x \geq 0$ under a piece rate contract, and so can only induce the agent to deliver substantial effort by allowing the agent to earn substantial rent. Consequently, when constrained to implement a piece-rate contract, the principal induces relatively little effort from the agent, which reduces total surplus and the principal's profit.[19]

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[18] These qualitative conclusions can be shown to arise more generally, not solely in the benchmark setting.

[19] As p increases, the variance of $f(\cdot)$ also increases, *ceteris paribus*, which can render the agent's observed

As θ declines, the agent's degree of relative risk aversion increases, and so the principal optimally reduces the payment variation that the agent faces. The principal can limit payment variation without diminishing unduly the agent's incentive to deliver effort under the fully optimal contract. In contrast, a piece-rate contract forces the principal to induce a substantial reduction in the agent's effort supply when she reduces payment variation. Consequently, total surplus and the principal's profit under a piece-rate contract decline relatively rapidly as θ declines.

When δ is sufficiently small that the principal optimally induces the agent to deliver more than one unit of effort, the agent becomes less averse to delivering equilibrium effort as δ declines. Therefore, the principal would like to induce substantial effort from the agent. When constrained to implement a piece-rate contract, the principal is forced to concede considerable rent to the agent if she induces him to deliver substantial effort (via increasing β). Consequently, the principal induces less than the ideal level of effort from the agent. The result is reduced total surplus and profit for the principal.

When δ is sufficiently large that the principal optimally induces the agent to deliver less than one unit of effort, the agent becomes more averse to delivering equilibrium effort as δ declines. This increased effort aversion is particularly constraining for the principal when her policy instruments are restricted by the requirement to implement a piece-rate contract. The principal must increase the agent's compensation systematically for all levels of output in order to induce him to deliver more effort. This systematic increase in payments to the agent substantially reduces the principal's profit.

A piece-rate contract also can secure relatively little profit for the principal when $\overline{U}$ is small. When $\overline{U}$ is substantially below $\overline{U}_1$, the principal's preferred piece rate generates considerable rent for the agent, which reduces the principal's profit. As Table 4 reveals, an optimal piece-rate contract also can generate for the principal only a small fraction of the

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performance less informative about his unobserved effort. The resulting increased difficulty in motivating the agent to work diligently can be particularly constraining for the principal when she is limited to employing a piece-rate contract.

profit that she secures under a fully optimal contract when $\overline{U}$ is close to $\overline{U}_2$. When $\overline{U}$ is so large that the principal must promise the agent almost all of the output that he produces (so $\beta \to 1$) in order to ensure his participation, the principal's profit ($\widetilde{\pi}$) under the optimal piece-rate contract becomes very small (i.e., $\widetilde{\pi} \to 0$ and so $\widetilde{\pi}/\pi \to 0$).

6 Conclusions.

We have examined the optimal design of piece-rate contracts and their performance in a structured but fairly broad class of economic environments. We found that the optimal piece-rate contract assumes a particularly simple form when the principal's preferred piece rate ensures the agent's participation. We also found that use of a piece-rate contract may not entail the loss of substantial profit for the principal when the agent's reservation utility ($\overline{U}$) is moderately large, when the productivity of the agent's effort is limited (so p is small), when the agent's aversion to risk is limited (so θ is relatively large), and when the agent's equilibrium effort is relatively limited because the agent's aversion to substantial effort supply is pronounced (i.e., when δ is large).

For brevity, we have only illustrated these effects for particular parameter values. However, we have solved problems [P] and [P-R] for many additional parameter values and have found the effects to persist more generally. We have also found these effects to persist for alternative density functions.[20]

In addition to exploring alternative preferences for the agent, future research might examine the optimal design and performance of different types of "simple" contracts. The principal's loss from employing a simple contract will decline as the contract admits additional instruments. To illustrate, suppose the principal must pay the agent a constant piece rate for all output above a specified threshold, $\underline{x}$, but can withhold all payment from the agent if realized output is below the threshold (so $w(x) = \max\{0,\, \beta[x - \underline{x}]\}$). The additional

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[20] The alternative densities that we have examined include the beta density, $\frac{\Gamma(\alpha+\gamma)}{\Gamma(\alpha)\Gamma(\gamma)}\,[1-x]^{1-\gamma}\,[x]^{\alpha-1}$ for $x \in [0,1]$, where $\alpha > 0$ and $\gamma > 0$ are parameters. When $\alpha < 1$ and $\gamma < 1$, this density approaches ∞ as x approaches 0 and as x approaches 1, and so is bimodal.

flexibility to withhold payment for poor performance can be particularly advantageous for the principal when the agent's reservation utility is small and when the smallest outputs are unlikely if the agent works diligently.[21,22]

The canonical moral hazard setting analyzed here also might be extended to admit adverse selection concerns.[23] The adverse selection literature has identified circumstances under which a small number of contracts and/or simple contracts can perform nearly as well as the complete menu of comparatively complex contracts that are fully optimal.[24] It would be useful to determine whether similar conclusions arise in the presence of adverse selection and moral hazard concerns with risk averse agents.[25]

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[21] It can be shown, for example, that in the benchmark setting where $p = 2,\ \theta = \frac{1}{2},\ \delta = 2,$ and $\overline{U} = 0$, the principal can secure with a contract of this form 99% of the profit that she can secure with a fully optimal contract. See Bose et al. (2009).

[22] See Gjesdal (1988), Reichelstein (1992), and Zhou and Swan (2003) for analyses of the design and performance of alternative "simple" contracts.

[23] Gibbons (1987) shows that piece-rate contracts typically are not optimal contracts in the presence of adverse selection concerns. Zabojnik (1996) examines a moral hazard setting in which $x = ba + \varepsilon$, where the agent learns the value of b after signing a contract but before choosing his effort, a. Zabojnik finds that the slope of the optimal linear contract declines as the variance of the normally distributed random variable ε increases, but may increase as the variance of the random variable b increases.

[24] See Bower (1993), McAfee (2002), Rogerson (2003), and Chu and Sappington (2007), for example.

[25] Guesnerie et al. (1989) demonstrate that optimal contracts may consist of menus of linear contracts in settings with adverse selection, moral hazard, and risk neutral agents.

p	β	$\widetilde{a}$	a	π	$\widetilde{\pi}$	$\widetilde{\pi}/\pi$
0.001	0.25	0.0058	0.0063	4.723×10^{-6}	4.355×10^{-6}	0.9221
0.01	0.25	0.0267	0.0291	2.186×10^{-4}	2.005×10^{-4}	0.9171
0.1	0.25	0.1153	0.1324	0.0099	0.0086	0.8732
0.25	0.25	0.1926	0.2381	0.0446	0.0361	0.8095
0.50	0.25	0.2710	0.3686	0.1382	0.1016	0.7353
0.75	0.25	0.3246	0.4751	0.2672	0.1826	0.6833
1.0	0.25	0.3661	0.5687	0.4265	0.2746	0.6439
2.0	0.25	0.4798	0.8627	1.2940	0.7197	0.5562
4.0	0.25	0.6170	1.2553	3.7659	1.8510	0.4915
10.0	0.25	0.8479	1.9253	14.4400	6.3592	0.4404
20.0	0.25	1.0727	2.5543	38.3150	16.0911	0.4200
50.0	0.25	1.4596	3.5907	134.6522	54.7340	0.4065

Table 1. Relative Performance of the Optimal Piece-Rate Contract as p Changes ($\delta = 2,\ \theta = \frac{1}{2},\ \overline{U} = 0$).

θ	β	$\widetilde{a}$	a	π	$\widetilde{\pi}$	$\widetilde{\pi}/\pi$
0.01	0.005	0.0965	0.0177	1.7523	0.1920	0.1095
0.1	0.050	0.2604	0.6875	1.3061	0.4947	0.3788
0.2	0.100	0.3342	0.6914	1.2444	0.6015	0.4834
0.3	0.150	0.3859	0.7217	1.2268	0.6559	0.5347
0.4	0.200	0.4319	0.7794	1.2468	0.6911	0.5543
0.5	0.250	0.4798	0.8627	1.2940	0.7197	0.5562

Table 2. Relative Performance of the Optimal Piece-Rate Contract as θ Changes ($\delta = 2,\ p = 2,\ \overline{U} = 0$).

δ	β	$\widetilde{a}$	a	π	$\widetilde{\pi}$	$\widetilde{\pi}/\pi$
1.0	0.5000	0.8836	2.6667	2.6667	0.8836	0.3313
1.5	0.3333	0.5117	1.1176	1.4902	0.6822	0.4578
2.0	0.2500	0.4798	0.8627	1.2940	0.7197	0.5562
2.5	0.2000	0.4876	0.7672	1.2275	0.7802	0.6356
3.0	0.1667	0.5046	0.7251	1.2086	0.8411	0.6959
3.5	0.1429	0.5236	0.7083	1.2142	0.8976	0.7393
4.0	0.1250	0.5424	0.7027	1.2297	0.9491	0.7719
4.5	0.1111	0.5602	0.7026	1.2491	0.9959	0.7973
5.0	0.1000	0.5768	0.7055	1.2699	1.0383	0.8176
10.0	0.0500	0.6907	0.7598	1.4437	1.3123	0.9090
20.0	0.0250	0.7917	0.8293	1.6172	1.5438	0.9546

Table 3. Relative Performance of the Optimal Piece-Rate Contract as δ Changes ($p = 2,\ \ \theta = \frac{1}{2},\ \ \overline{U} = 0$).

$\overline{U}$	β	$\widetilde{a}$	a	π	$\widetilde{\pi}$	$\widetilde{\pi}/\pi$
0.0	0.2500	0.4798	0.8627	1.294	0.7197	0.5562
0.2	0.2500	0.4798	0.8440	1.223	0.7197	0.5885
0.4	0.2500	0.4798	0.8220	1.126	0.7197	0.6392
0.6	0.2500	0.4798	0.7975	1.011	0.7197	0.7119
0.8	0.3117	0.5164	0.7723	0.8792	0.7109	0.8085
1.0	0.4356	0.5774	0.7474	0.7315	0.6517	0.8909
1.2	0.5726	0.6325	0.7228	0.5674	0.5406	0.9527
1.4	0.7216	0.6831	0.6985	0.3870	0.3804	0.9829
1.5	0.8003	0.7071	0.6864	0.2905	0.2824	0.9723
1.6	0.8816	0.7303	0.6745	0.1898	0.1729	0.9109
1.7	0.9656	0.7528	0.6628	0.0850	0.0519	0.6104
1.74	0.9998	0.7616	0.6581	0.0418	0.0002	0.0059

Table 4. Relative Performance of the Optimal Piece-Rate Contract as $\overline{U}$ Changes ($p = 2, \;\; \theta = \frac{1}{2}, \;\; \delta = 2$).

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