MANAGERIAL DELEGATION IN MULTIMARKET OLIGOPOLY

ARUP BOSE [*]
Statistics and Mathematics Unit
Indian Statistical Institute
INDIA
bosearu@gmail.com

BARNALI GUPTA [†]
Department of Economics
Miami University, Ohio
USA
guptab@muohio.edu

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Abstract

It is well known that the separation of ownership and control with managerial delegation creates a credible commitment to maximize an incentive different from profit maximization. In two-stage duopoly games, Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987) have shown that if the products are strategic substitutes then a managerial store is more aggressive than an entrepreneurial Cournot duopoly and if the products are strategic complements then a managerial store is more collusive, than an entrepreneurial Bertrand duopoly. We study multimarket competition between chain and independent stores when the firms can choose a priori whether or not to hire a manager. In the three-stage game, we find that the chain store is always less aggressive than independent stores. This is true regardless of whether the chain store is managerial or entrepreneurial in the Nash equilibrium. We also find that if the independent stores have asymmetric SPNE strategies, then with Cournot competition the independent managerial store is more aggressive than its entrepreneurial counterpart while with Bertrand competition, it is exactly the opposite and the independent managerial store is more collusive than its entrepreneurial counterpart.

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1 Introduction

In the strategic managerial delegation literature, it is well established that the separation of ownership and control creates a credible commitment to maximize an incentive different from profit maximization. This literature is pioneered in the work of Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987) (henceforth VFJS). In particular, VFJS have shown

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that if the products are strategic substitutes, then managerial incentives based on profit and sales will shift out a store's reaction function and result in each store being more aggressive in the output market by expanding output. With strategic complements, both managerial stores increase their profit with higher prices and collusive behavior. The literature in this area has subsequently introduced different remuneration schemes for the non-profit component of managerial incentives. While VFJS assumed sales revenue, subsequent work by Salas Fumas (1992) and Miller and Pazgal (2002) modeled managerial bonus as relative profits, and Ritz (2008) and Jansen et al. (2007) modeled market share as the bonus component in managerial compensation. Recently, Jansen et al. (2009, 2012) compared profit, sales revenue, market share and relative profits in the managerial contract to show that if owners can choose, then relative profits will dominate in homogenous and differentiated Cournot duopoly. This literature follows the basic structure of the two stage game in VFJS. In the first stage the owners choose the managerial bonus system. In the second stage, given the bonus system, each manager determines the optimal strategy. The common characteristic in this literature is that the competition is always in a single market.

Now, consider the following example. Pappa's Pizzeria operates in two neighboring towns. In each of them it faces a separate independent competitor who operates only in that town. As a result, although the demands in the two towns are apparently unrelated, all owners have to consider the effects of multimarket (MM) interaction. The actions of the independent store in one market will influence the chain store's actions in both markets and vice-versa. The interplay of strategic managerial delegation and endogenous managerial choice with MM interaction is the focus of this paper.

We adapt the theoretical model of MM oligopoly of Bulow et al. (1985). We also follow Basu's (1995) approach and add a prior stage to the VFJS game where the owner decides whether or not to hire a manager. Managerial delegation as a firm strategy in the presence of MM interaction then raises several natural questions. Do chain and independent owners offer similar managerial incentives following the VFJS finding, or is the chain store more or less aggressive than its independent rivals? Will all independent owners offer similar incentives or can equilibrium strategies in the markets be different? Is the chain store always more profitable than its independent competitors? Since the manager has control over multiple markets and managerial hiring is a fixed cost, one might expect that in equilibrium, the chain store will always choose to be a managerial store.

We find that with strategic substitutes (linear demand and quantity choice), the sub-game perfect Nash equilibrium (SPNE) in a three stage game with MM interaction and endogenous manager choice, run the gamut from all stores hiring a manager, to none of the stores hiring any as the stipulated hiring cost changes. Moreover, the chain store does not hire in a majority of the SPNE. The chain store is always less aggressive than independent stores, with smaller market share in each market. This is true regardless of whether it is a managerial or an entrepreneurial store. It earns the highest profit across both markets when none of the stores have managers. While the chain store's total profit is always more than that of either independent store, this profit decreases in the following rank order in the SPNE: (1) when only one independent store has a manager (2) when both chain and independent stores have managers (3) when only both independent stores have managers. When one independent store has a manager, that market becomes saturated with output from the independent manager who has the incentive to aggressively increase market share. So, the chain owner responds by appropriately acceding market share in that market. When both independent stores have managers, this effect is compounded in both markets, and the chain is unarguably in the weakest position. We also find that when independent stores do not have symmetric hiring strategies, then the independent managerial store is more aggressive and has higher market share than its entrepreneurial counterpart.

With strategic complements (linear demand and differentiated product price competition), and managerial delegation, again all stores do not necessarily hire a manager. We find that the chain store is always more collusive and less aggressive than the independent stores. Depending on the degree of product differentiation, if the independent stores have asymmetric hiring strategies in equilibrium, then the independent managerial store is more collusive than the independent entrepreneurial store.

Our results provide added insight into the effect of managerial delegation as a strategy. With strategic substitutes Bulow et al. (1985) have found that MM competition facilitates aggressive behavior by firms. At the same time, VFJS have shown that managerial delegation in a single market duopoly also facilitates aggressive behavior by the de facto managerial firms. We find that the chain store is more collusive than independent stores with strategic delegation. In other words, while the managerial delegation literature argues that managerial Cournot firms are more aggressive and less collusive than entrepreneurial firms, MM competition facilitates collusive behavior by the chain store, relative to its independent competition. For differentiated product price competition, VFJS and others have shown that managerial firms are more collusive than entrepreneurial firms. Similarly, the main conclusion of Bernheim and Whinston (1990) is that under appropriate conditions, repeated interactions with MM contact between firms facilitates collusive behavior. Bulow et al. (1985) finds that firms are more collusive with strategic complements. This conclusion is reinforced with our results that the chain store is more collusive than independent stores. Thus, in general, managerial delegation can be viewed as a commitment that facilitates collusion under some circumstances. The MM interaction with strategic managerial delegation incentivizes the chain store to be more collusive than the independent stores.

The remainder of the paper is organized as follows: the model is described in Section 2. Sections 3 and 4 discuss results from Cournot competition and differentiated product Bertrand competition respectively. The paper concludes in Section 5. The Appendix has the outline of the proofs.

2 Model

Markets A and B are oligopolistic with chain Store 1 that sells in both markets; Stores 2 and 3 are independent stores that sell only in Markets A and B respectively.

Each store will choose whether to hire a manager. The cost of hiring a manager is Z. The decision of the ith owner, $(i = 1, 2, 3)$, to hire or not to hire a manager, is denoted by $m_i = 1$ and $m_i = 0$ respectively. Recall that "managerial delegation" refers to the separation of ownership and control or decision-making authority. The "manager" is empowered to be the decision maker. Hence every owner can hire, at most one manager.

Following VFJS, the manager is offered a contract to maximize a linear combination of profit and sales, given by:

$$\begin{aligned} g_i &= \lambda_i \pi_i + (1 - \lambda_i) R_i \\ &= R_i - \lambda_i c_i, \quad i = 1, 2, 3. \end{aligned}$$

where for Store i, π_i is the profit, R_i is the sales revenue and c_i is production cost. The parameter $\lambda_i \in [0,\ 1]$ is specified in the contract and chosen by the owner to maximize profit.

Each manager's payoff is of the form

$$I_i = A_i + B_i g_i$$

where A_i and B_i are constants. Hence maximizing I_i and g_i are equivalent when A_i and B_i are constants and output or price are the choice variables. We study Cournot and differentiated product Bertrand competition.

The timing of the game is as follows: In Stage 0, each owner simultaneously decides whether to hire a manager, $m_i \in \{0,1\}, i = 1,2,3$. In Stage 1, the owner offers her manager a contract, specifying λ_i if $m_i = 1$. In Stage 2, the manager simultaneously chooses output or price, to maximize g_i, if $m_i = 1$. Otherwise, the owner chooses output or price to maximize profit. The market clears at the end of Stage 2.

Each store's cost function is

$$c_i = \frac{1}{2}x_i^2, \quad i = 1,2,3.$$

This assumption deviates from the standard assumption of constant marginal cost in VFJS and others, because that is trivial with MM competition. The market demand is assumed to be linear, as is standard in the delegation literature. We assume identical demands across markets in order to isolate strategic effects from market size effects.

In Cournot competition,

$$p = M - X \tag{2.1}$$

in each market where X is the sum of outputs of the stores. In Markets A and B they are respectively,

$$X = x_1^A + x_2 \quad \text{and} \quad X = x_1^B + x_3.$$

With differentiated product Bertrand competition,

$$\begin{array}{rclrcl} x_1^A &=& M - p_1 + bp_2, & x_1^B &=& M - p_1 + bp_3, \\ x_2 &=& M - p_2 + bp_1^A, & x_3 &=& M - p_3 + bp_1^B, \end{array} \tag{2.2}$$

where $0 < b < 1$.

3 Cournot competition

The products are strategic substitutes, with quantity choice and linear demand. The three-stage game is as follows:

Stage 0: Owner chooses whether to hire a manager; $m_i \in \{0,1\}$, $i = 1,2,3$, at cost Z.

Stage 1: If $m_i = 1$, in Stage 0, then owner offers manager an incentive contract that specifies λ_i, the profit and sales share earned by the manager.

Stage 2: If $m_i = 1$ then given λ_i from Stage 1, manager chooses x_i to maximize g_i. If $m_i = 0$, then owner chooses x_i to maximize π_i.

The eight possible choices of m_i in Stage 0 leads to the corresponding eight sub-games SG-1–SG-8 as listed in Table 1.

Table 1.

	Stage 0	Stage 1	Stage 2
SG-1	$m_1 = m_2 = m_3 = 1$	$\lambda_1 \to \pi_1$, $\lambda_2 \to \pi_2$, $\lambda_3 \to \pi_3$	$x_1^A, x_1^B \to g_1$; $x_2 \to g_2$; $x_3 \to g_3$
SG-2	$m_1 = m_2 = 1$, $m_3 = 0$	$\lambda_1 \to \pi$, $\lambda_2 \to \pi_2$, $\lambda_3 = 1$	$x_1^A, x_1^B \to g_1$; $x_2 \to g_2$, $x_3 \to \pi_3$
SG-3	$m_1 = 1$, $m_2 = m_3 = 0$	$\lambda_1 \to \pi_1$, $\lambda_2 = \lambda_3 = 1$	$x_1^A, x_1^B \to g_1$; $x_2 \to \pi_2$, $x_3 \to \pi_3$
SG-4	$m_1 = m_2 = m_3 = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$	$x_1^A, x_1^B \to \pi_1$; $x_2 \to \pi_2$; $x_3 \to \pi_3$
SG-5	$m_1 = m_2 = 0$, $m_3 = 1$	$\lambda_1 = \lambda_2 = 1$, $\lambda_3 \to \pi_3$	$x_1^A, x_1^B \to \pi_1$; $x_2 \to \pi_2$, $x_3 \to \pi_3$
SG-6	$m_1 = 0$, $m_2 = m_3 = 1$	$\lambda_1 = 1$, $\lambda_2 \to \pi_2$, $\lambda_3 \to \pi_3$	$x_1^A, x_1^B \to \pi_1$; $x_2 \to g_2$, $x_3 \to g_3$
SG-7	$m_1 = m_3 = 1$, $m_2 = 0$ (symm. with SG-2)	$\lambda_1 \to \pi_1$, $\lambda_2 = 1$, $\lambda_3 \to \pi_1$	$x_1^A, x_1^B \to g_1$; $x_2 \to \pi_2$, $x_3 \to g_3$
SG-8	$m_1 = m_3 = 0$, $m_2 = 1$ (symm. with SG-5)	$\lambda_1 = \lambda_3 = 1$, $\lambda_2 \to \pi_2$,	$x_1^A, x_1^B \to \pi_1$; $x_2 \to g_2$, $x_3 \to \pi_3$

For example, consider SG-1. Each λ_i is chosen to maximize π_i. Then the outputs x_1 etc. are chosen to maximize g_i. This maximization is solved by standard backward induction. For example in Stage 2, each manager maximizes g_i with respect to x_i, given g_i and $m_i = 1$, $i = 1, 2, 3$. In Stage 1, each owner maximizes π_i with respect to λ_i's. Due to the quadratic cost and concavity in the model, there are unique maximizers explicitly obtainable and computational details are in the Appendix. The solutions to all sub-games, including the SPNE, are given in Tables 2–5 below. Tables 2 and 3 give the optimal λ_i and profit π_i. Tables 4 and 5 give the optimal output x_i and price, p_i. The SPNE outcomes are marked with a (*) and they depend on demand parameter, M and hiring cost, Z.

Table 2. $m_1 = 0$. Values of λ_i, π_i; Si is Store i.

		S2	
		$m_2 = 1$	$m_2 = 0$
S3	$m_3 = 1$	$\lambda_1 = 1, \lambda_2 = \lambda_3 = 0.618034$ **SG-6*** $\pi_1 = 0.116718M^2$ $\pi_2 = \pi_3 = 0.112151M^2 - Z$	$\lambda_1 = \lambda_2 = 1, \lambda_3 = 0.619048$ **SG-5*** $\pi_1 = 0.124611M^2$ $\pi_2 = 0.109769M^2, \pi_3 = 0.113982M^2 - Z$
	$m_3 = 0$	$\lambda_1 = \lambda_3 = 1, \lambda_2 = 0.619048$ **SG-8*** $\pi_1 = 0.124611M^2$ $\pi_2 = 0.113982M^2 - Z, \pi_3 = 0.109769M^2$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$ **SG-4*** $\pi_1 = 0.132231M^2$ $\pi_2 = \pi_3 = 0.11157M^2$

Table 3. $m_1 = 1$. Values of λ_i, π_i

		S2	
		$m_2 = 1$	$m_2 = 0$
S3	$m_3 = 1$	$\lambda_1 = 0.808143, \lambda_2 = \lambda_3 = 0.606107$ **SG-1*** $\pi_1 = 0.117494M^2 - Z$ $\pi_2 = \pi_3 = 0.106681M^2 - Z$	$\lambda_1 = 0.821573, \lambda_2 = 1, \lambda_3 = 0.60789$ SG-7 $\pi_1 = 0.125565M^2 - Z$ $\pi_2 = 0.104671M^2, \pi_3 = 0.108746M^2 - Z$
	$m_3 = 0$	$\lambda_1 = 0.821573, \lambda_2 = 0.60789, \lambda_3 = 1$ SG-2 $\pi_1 = 0.125565M^2 - Z$ $\pi_2 = 0.108746M^2 - Z, \pi_3 = 0.104671M^2$	$\lambda_1 = 0.833333, \lambda_2 = \lambda_3 = 1$ SG-3 $\pi_1 = 0.133333M^2 - Z$ $\pi_2 = \pi_3 = 0.106667M^2$

Table 4. $m_1 = 0$. Values of x_i, p_i.

		S2	
		$m_2 = 1$	$m_2 = 0$
S3	$m_3 = 1$	$x_1^A = x_1^B = 0.1708M$ **SG-6*** $x_1 = 0.3416M, x_2 = x_3 = 0.3167M$ $p_A = p_B = 0.5124M$	$x_1^A = 0.1884M$, $x_1^B = 0.1641M$ **SG-5*** $x_1 = 0.3525M, x_2 = 0.2705M, x_3 = 0.3191M$ $p_A = 0.5410M, p_B = 0.5167M$
	$m_3 = 0$	$x_1^A = 0.1641M$, $x_1^B = 0.188M$ **SG-8*** $x_1 = 0.3525M, x_2 = 0.3191M, x_3 = 0.2705M$ $p_A = 0.5167M, p_B = 0.5410M$	$x_1^A = x_1^B = 0.1818M$ **SG-4*** $x_1 = 0.3636M, x_2 = x_3 = 0.2727M$ $p_A = p_B = 0.5454M$

Table 5. $m_1 = 1$. Values of x_i, p_i.

		S2	
		$m_2 = 1$	$m_2 = 0$
S3	$m_3 = 1$	$x_1^A = x_1^B = 0.1906M$ **SG-1*** $x_1 = 0.3812M, x_2 = x_3 = 0.3105M$ $p_A = p_B = 0.4987M$	$x_1^A = 0.2075M, x_1^B = 0.1829M$ SG-7 $x_1 = 0.3904M, x_2 = 0.2641M, x_3 = 0.3132M$ $p_A = 0.5283M, p_B = 0.5037M$
	$m_3 = 0$	$x_1^A = 0.18295M, x_1^B = 0.20752M$ SG-2 $x_1 = 0.39047M, x_2 = 0.313299M, x_3 = 0.26416M$ $p_A = 0.50375M, p_B = 0.52832M$	$x_1^A = x_1^B = 0.2M$ SG-3 $x_1 = 0.4M, x_2 = x_3 = 0.2666M$ $p_A = p_B = 0.5333M$

Tables 1–5 show the SPNE in SG-1, SG-4, SG-5, SG-6 and SG-8. The findings from the SPNE are as follows:

Finding 1: There is a very low hiring cost $\underline{Z}$ such that for $0 \leq Z < \underline{Z}$, all stores will hire managers and offer them sales incentives (Table 3, SPNE in SG-1, with $m_1 = m_2 = m_3 = 1$). We find that the chain store is more collusive and less aggressive than the independent stores with $x_1^A = x_1^B < x_2 = x_3$. (Table 5, SPNE in SG-1 with $m_1 = m_2 = m_3 = 1$)

VFJS have shown that with managerial delegation based on profit and sales, each store is more aggressive in the output market. With MM competition however, we find that the chain store is less aggressive and has lower market share and profit than the independent store in each market. Although all managerial stores have the incentive to increase output, the chain store also has the opposite incentive to be relatively less aggressive in each market, because of MM interaction. This result is related to Bulow et al. (1985) for strategic substitutes in multiple markets. We see similar incentives here for the chain store, that partially offsets standard managerial incentives to expand output. When all stores hire managers in the SPNE, MM interaction does not give the chain an advantage in market share or profit in either market, over the independent firm. (For SG-1 with $m_1 = m_2 = m_3 = 1$, see the SPNE values of π_1, π_2 and π_3 in Table 3). We should note that this pattern continues in subsequent findings. The chain is less aggressive and has smaller market share and profit.

Comparing π_1, π_2 and π_3 for SG-4 in Table 2 for $m_1 = m_2 = m_3 = 0$, with SG-1 in Table 3 for $m_1 = m_2 = m_3 = 1$, we also find that if hiring cost is zero, each store faces a prisoner's dilemma: each is better off if neither has a manager. For sufficiently small hiring cost, managerial incentives push out each store's reaction function; output increases, price is lower, and profit is lower for all stores.

Finding 2: For low hiring cost Z, such that $Z_1 > Z > \underline{Z}$, *both independent stores* hire managers and the chain store remains entrepreneurial (Table 2, SPNE in SG-6 with $m_1 = 0, m_2 = m_3 = 1$).

The chain store does not hire a manager and we hypothesize that this is due to the effect of MM operations with increasing marginal cost as well as the strategic product substitutability with the independent store. Both independent stores however, find it profitable to make the managerial commitment. Consequently, in both markets, the managerial independent store is more aggressive than the chain store with $x_1^A < x_2, x_1^B < x_3$ (SG-6 in Table 4), because managerial incentives for Stores 2 and 3 push out their reaction functions. Hence, market share is higher for the managerial stores and the chain store compensates by reducing its output in both markets. Consequently, its profit is also lower than the independent managerial store's profit in each market.

Finding 3: There is a "medium" hiring cost Z, such that $\overline{Z} > Z > Z_1 > \underline{Z}$ only one independent store hires a manager. The second independent store and chain store choose not to hire (Table 2, SPNE in SG-5 and SG-8, for $m_1 = m_2 = 0, m_3 = 1$ and $m_1 = m_3 = 0, m_2 = 1$).

Depending on Z, there are SPNE where the chain store faces a managerial independent store in one market and an entrepreneurial independent store in the other. Not surprisingly, we find a more aggressive managerial independent store relative to the entrepreneurial independent and chain stores. Note that from the outputs in SG-8, Table 4, where $x_2 > x_3$ and $\pi_2 > \pi_3$, with $m_1 = m_3 = 0, m_2 = 1$, the managerial independent store sells more and and earns higher profit than its entrepreneurial counterpart. Further, both independent stores sell more than the chain in each market. From Table 4, also note that $x_1^A < x_2$ and $x_1^B < x_3$ in SG-5 and SG-8. The chain store is less aggressive than either of its independent competitors, managerial and entrepreneurial, although it is relatively more aggressive in the entrepreneurial independent market.

Finding 4: For sufficiently high hiring cost $\overline{Z}$, such that $Z > \overline{Z} > Z_1 > \underline{Z}$, no managers are hired by any store, chain or independent. (Table 2, SG-4 with $m_1 = m_2 = m_3 = 0$)

From the SPNE we conclude that if the chain store has a manager, then both independent stores are also managerial. In every SPNE, one or both the independent stores stores is a managerial store while the chain may be managerial or entrepreneurial. Since the commitment to separate ownership and control makes a managerial store more aggressive, the MM chain finds it less profitable to make that commitment, because of marginal cost and demand linkages with strategic substitutes. The managerial delegation incentive for the chain to be aggressive in a specific market is partially mitigated by MM competition and we conclude that the chain is always less aggressive than an independent store, managerial or entrepreneurial.[1]

The summary implications of Findings 1-4 are as follows:[2] When hiring cost is very low, all stores commit to separation of ownership and control and hire a manager. When hiring cost is low, the two independent stores hire managers but the chain store does not; with medium hiring cost, only one independent store hires a manager and when hiring cost is high, managers are not hired by any store.

4 Differentiated product Bertrand competition

With competition between similar stores, it has been demonstrated by VFJS, that managerial Bertrand firms are less aggressive and commit to higher prices. We explore (1) whether we can

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[1] Alternative managerial incentives such as relative profit might change this result. However, we believe that with MM interaction, as long as costs are sufficiently convex, the chain store will be less aggressive than an independent competitor.

[2] It can be checked from Tables 2–5, that $\underline{Z} = 0.000776M^2$, $Z_1 = 0.002382M^2$, $\overline{Z} = 0.002412M^2$

identify a pattern in the hiring strategies of the chain and independent stores and (2) the role of product differentiation in chain store/independent store equilibrium strategies.

Recall that market demand is:

$$\begin{array}{rrclrcl} \text{Market A}: & x_1^A & = & M - p_1 + bp_2, & x_2 & = & M - p_2 + bp_1, \\ \text{Market A}: & x_1^B & = & M - p_1 + bp_3, & x_3 & = & M - p_3 + bp_1, \end{array}$$

where product differentiation is modeled by b, $0 < b < 1$.

The timing of the three-stage game is:

Stage 0: Owner chooses whether to hire a manager: $m_i \in \{0, 1\}$, $i = 1, 2, 3$. The cost of hiring a manager is Z.

Stage 1: If $m_i = 1$, $i = 1, 2, 3$, in Stage 0, then the owner offers the manager an incentive contract that specifies λ_i, the share of profit and revenue.

Stage 2: If $m_i = 1$ and λ_i is given, manager chooses p_i to maximize g_i. If $m_i = 0$, owner chooses x_i to maximize π_i.

Table 6 lists the strategy combinations for the three stores in the eight sub-games.

Table 6

	Stage 0	Stage 1	Stage 2
SG-1	$m_1 = m_2 = m_3 = 1$	$\lambda_1 \to \pi_1$, $\lambda_2 \to \pi_2$, $\lambda_3 \to \pi_3$	$p_1^A, p_1^B \to g_1$; $p_2 \to g_2$, $p_3 \to g_3$
SG-2	$m_1 = m_2 = 1$, $m_3 = 0$	$\lambda_1 \to \pi_1$, $\lambda_2 \to \pi_2$, $\lambda_3 = 1$	$p_1^A, p_1^B \to g_1$; $p_2 \to g_2$, $p_3 \to \pi_3$
SG-3	$m_1 = 1$, $m_2 = m_3 = 0$	$\lambda_1 \to \pi_1$, $\lambda_2 = \lambda_3 = 1$	$p_1^A, p_1^B \to g_1$; $p_2 \to \pi_2$, $p_3 \to \pi_3$
SG-4	$m_1 = m_2 = m_3 = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$	$p_1^A, p_1^B \to \pi_1$; $p_2 \to \pi_2$, $p_3 \to \pi_3$
SG-5	$m_1 = m_2 = 0$, $m_3 = 1$	$\lambda_1 = \lambda_2 = 1$, $\lambda_3 \to \pi_3$	$p_1^A, p_1^B \to \pi_1$; $p_2 \to \pi_2$, $p_3 \to \pi_3$
SG-6	$m_1 = 0$, $m_2 = m_3 = 1$	$\lambda_1 = 1$, $\lambda_2 \to \pi_2$, $\lambda_3 \to \pi_3$	$p_1^A, p_1^B \to \pi_1$; $p_2 \to g_2$, $p_3 \to g_3$
SG-7	$m_1 = m_3 = 1$, $m_2 = 0$ (symmetric with SG-2)	$\lambda_1 = \pi_1$, $\lambda_2 = 1$, $\lambda_3 \to \pi_1$	$p_1^A, p_1^B \to g_1$; $p_2 \to \pi_2$, $p_3 \to g_3$
SG-8	$m_1 = m_3 = 0$, $m_2 = 1$ (symmetric with SG-5)	$\lambda_1 = \lambda_3 = 1$, $\lambda_2 \to \pi_2$,	$p_1^A, p_1^B \to \pi_1$; $p_2 \to g_2$, $p_3 \to \pi_3$

An entrepreneurial owner, one who retains decision making authority and does not hire a manager, will choose $\lambda = 1$. Any $\lambda > 1$ implies that the managerial owner emphasizes profit incentives over revenue and the corresponding prices are higher than with entrepreneurial profit maximization.

Each three-stage sub-game is solved by standard backward induction. With differentiated product Bertrand,the solution will depend on the product differentiation parameter b. While the backward induction solution is conceptually straightforward, its execution is algebraically complicated with unwieldy first-order expressions at each stage. In order to meaningfully comment on the role of product differentiation, we numerically solve sub-games for a small but representative set of values of $b \in (0,\ 1)$. Specifically, we find the SPNE for $b \in \{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}\}$. The solutions to all sub-games, including the SPNE outcomes (in bold) are given in Tables 7–11 below. Based on the solutions we find the following SPNE:

Finding 5: For each $b \in \{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}\}$, there is a (sufficiently small) $\underline{Z}$, such that for $Z < \underline{Z}$, all stores hire managers. Further, the chain store is *less* aggressive than its independent competitors (Tables 7–11, SG-3 where $p_1^A = p_1^B > p_2 = p_3$).

Finding 6: For each $b \in \{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}\}$, there is a (sufficiently large) $\overline{Z} = \overline{Z}(b)$, such that for $Z > \overline{Z} = \overline{Z}(b)$, no managers will be hired (Tables 7–11, SG-6).

Finding 7: For each $b \in \{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}\}$ there is a range of hiring cost $\underline{Z}(b) < Z < \overline{Z}(b)$, where chain and independent stores have asymmetric SPNE strategies as follows:

(i) with $b = \frac{1}{8}$, only the chain store will hire a manager; $m_1 = 1; m_2 = m_3 = 0$ in Table 7, SG-1.

(ii) with $b = \frac{1}{4}$, only both independent stores will hire a manager; $m_1 = 0; m_2 = m_3 = 1$ in Table 8, SG-4.

(iii) with $b = \frac{3}{8}$, only one independent store will hire a manager; $m_1 = m_2 = 0; m_3 = 1$ in SG-8 and $m_1 = m_3 = 0; m_2 = 1$ in Table 9, SG-5.

In VFJS where the firms are independent store duopolists, and where all stores are de facto managerial, both commit their managers to $\lambda > 1$, and are more collusive with higher prices than entrepreneurial stores. In the chain-independent duopoly with strategic managerial delegation, the chain and/or the independent store will commit to be entrepreneurial or managerial, depending on the degree of product differentiation. From the SPNE for each $b \in \{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}\}$, we observe some patterns in the relationships between chain and independent store SPNE outcomes. The following patterns are described with the caveat that they are based on the specified set of product differentiation parameters:

(1) The chain store is more collusive and less aggressive than either independent store, with a higher price in every SPNE. This is true, regardless of whether the chain and/or the independent store is managerial or entrepreneurial. This also holds true, regardless of the degree of product differentiation. When the products are strategic complements, VFJS and others have found that managerial firms are more collusive than entrepreneurial firms. The separation of ownership and decision making authority, can be viewed as a commitment that facilitates collusion under some circumstances. We find that MM interaction with strategic managerial delegation, incentivizes the chain store to be more collusive than the independent stores, in the spirit of the Bulow et al. (1985) finding that MM interaction with strategic complementarity facilitates collusion.

(2) The chain and independent stores have symmetric hiring strategies when the stores' products are relatively better substitutes, $b \in \{\frac{1}{2}, \frac{5}{8}\}$. They have asymmetric hiring strategies when products are relatively more differentiated, $b \in \{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}\}$.

For $b \in \{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}\}$, there are SPNE in Finding 7, where the chain store does not hire and one or both independent stores do, and vice versa. In contrast, for $b \in \{\frac{1}{2}, \frac{5}{8}\}$ the SPNE have symmetric hiring strategies for chain and independent stores in Finding 5 and 6, where all stores either hire or not depending on the hiring cost.

We posit that if products are better substitutes, the effectiveness of leveraging managerial delegation as a strategy is driven by cost of hiring, rather than by chain/independent store differences. Asymmetric hiring strategies between chain and independent and stores occur with greater product differentiation because each store has more "monopoly" power. For $b = \frac{1}{8}$, the chain store leverages its monopoly power and hires a manager while the independent stores' best response in the SPNE is to remain entrepreneurial. For higher b, either one or both independent stores may be managerial while the chain store is entrepreneurial.

(3) The chain store's share of total profit increases with product differentiation. It can be checked that for the same SPNE strategies, its profit share is highest for $b = \frac{1}{8}$ and decreases as b increases.

Table 7. Solution for $b = \frac{1}{8}$.

$m_1 = 0.$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$ **SG-6*** $p_1^A = p_1^B = 0.818898M$ $p_2 = p_3 = 0.734908M$ $\pi_1 = 0.298041M^2$ $\pi_2 = \pi_3 = 0.202534M^2$	$\lambda_1 = \lambda_3 = 1; \lambda_2 = 1.00986$ SG-5 $p_1^A = 0.818992M; p_1^B = 0.818917M$ $p_2 = 0.73612M; p_3 = 0.73491M$ $\pi_1 = 0.298083M^2$, $\pi_2 = 0.202536M^2 - Z$, $\pi_3 = 0.202535M^2$
	$m_3 = 1$	$\lambda_1 = \lambda_2 = 1; \lambda_3 = 1.00986$ SG-8 $p_1^B = 0.81892M, p_1^A = 0.818917M$ $p_2 = 0.73491M; p_3 = 0.73612M$ $\pi_1 = 0.298083M^2$ $\pi_2 = 0.202535M^2, \pi_3 = 0.202536M^2 - Z$	$\lambda_1 = 1; \lambda_2 = \lambda_3 = 1.00986$ SG-4 $p_1^A = p_1^B = 0.819011M$ $p_2 = p_3 = 0.736122M$ $\pi_1 = 0.298124M^2 - Z$ $\pi_2 = \pi_3 = 0.202537M^2 - Z$

$m_1 = 1$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = 1.00526, \lambda_2 = \lambda_3 = 1$ **SG-1*** $p_1^A = p_1^B = 0.81962M$ $p_2 = p_3 = 0.734968M$ $\pi_1 = 0.298044M^2 - Z$ $\pi_2 = \pi_3 = 0.202567M^2$	$\lambda_1 = 1.00527, \lambda_2 = 1.00987, \lambda_3 = 1$ SG-7 $p_1^A = 0.819715M; p_1^B = 0.819639M$ $p_2 = 0.736181M; p_3 = 0.734970M$ $\pi_1 = 0.298085M^2$ $\pi_2 = 0.202569M^2 - Z, \pi_3 = 0.202568M^2$
	$m_3 = 1$	$\lambda_1 = 1.00527, \lambda_2 = 1, \lambda_3 = 1.00987$ SG-2 $p_1^A = 0.819639M; p_1^B = 0.819715M$ $p_2 = 0.734970M; p_3 = 0.736181M$ $\pi_1 = 0.298085M^2$ $\pi_2 = 0.202568M^2$, $\pi_3 = 0.202569M^2 - Z$	$\lambda_1 = 1.00527; \lambda_2 = 1.00987; \lambda_3 = 1.00987$ **SG-3*** $p_1^A = p_1^B = 0.819735M$ $p_2 = p_3 = 0.736183M$ $\pi_1 = 0.298126M^2 - Z$ $\pi_2 = \pi_3$, $\pi_3 = 0.20257M^2 - Z$

Table 8. Solution for $b = \frac{1}{4}$.

$m_1 = 0.$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$ **SG-6*** $p_1^A = p_1^B = 0.903226M$ $p_2 = p_3 = 0.817204M$ $\pi_1 = 0.362585M^2$ $\pi_2 = \pi_3 = 0.25043M^2$	$\lambda_1 = \lambda_3 = 1; \lambda_2 = 1.0407$ SG-5 $p_1^A = 0.904104M; p_1^B = 0.903406M$ $p_2 = 0.82282M; p_3 = 0.817234M$ $\pi_1 = 0.363011M^2$ $\pi_2 = 0.250478M^2 - Z$, $\pi_3 = 0.250452M^2$
	$m_3 = 1$	$\lambda_1 = \lambda_2 = 1; \lambda_3 = 1.0407$ SG-8 $p_1^A = 0.903406M; p_1^B = 0.904104M$ $p_2 = 0.817234M; p_3 = 0.82282M$ $\pi_1 = 0.363011M^2$ $\pi_2 = 0.250452M^2$, $\pi_3 = 0.250478M^2 - Z$	$\lambda_1 = 1; \lambda_2 = \lambda_3 = 1.0407$ **SG-4*** $p_1^A = p_1^B = 0.904284M$ $p_2 = p_3 = 0.822851M$ $\pi_1 = 0.363436M^2$ $\pi_2 = \pi_3 = 0.250497M^2 - Z$

(continued on next page)

Table 8 continued, $b = \frac{1}{4}$, $m_1 = 1$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = 1.02174; \lambda_2 = \lambda_3 = 1$ SG-1 $p_1^A = p_1^B = 0.906568M$ $p_2 = p_3 = 0.817761M$ $\pi_1 = 0.362627M^2 - Z$ $\pi_2 = \pi_3 = 0.250775M^2$	$\lambda_1 = 1.02182; \lambda_2 = 1.04079; \lambda_3 = 1$ SG-7 $p_1^A = 0.907463M; p_1^B = 0.906762M$ $p_2 = 0.823396M; p_3 = 0.817794M$ $\pi_1 = 0.363054M^2 - Z$ $\pi_2 = 0.250821M^2 - Z$, $\pi_3 = 0.250795M^2$
	$m_3 = 1$	$\lambda_1 = 1.02182; \lambda_2 = 1; \lambda_3 = 1.04079$ SG-2 $p_1^A = 0.906762M; p_1^B = 0.907463M$ $p_2 = 0.817794M; p_3 = 0.823396M$ $\pi_1 = 0.363054M^2 - Z$ $\pi_2 = 0.250795M^2$, $\pi_3 = 0.250821M^2 - Z$	$\lambda_1 = 1.02189; \lambda_2 = \lambda_3 = 1.04079$ **SG-3*** $p_1^A = p_1^B = 0.907657M$ $p_2 = p_3 = 0.823429M$ $\pi_1 = 0.363480M^2 - Z$ $\pi_2 = \pi_3 = 0.2508M^2 - Z$

Table 9. Solution for $b = \frac{3}{8}$.

$m_1 = 0$.

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$ **SG-6*** $p_1^A = p_1^B = 1.0084M$ $p_2 = p_3 = 0.918768M$ $\pi_1 = 0.451945M^2$ $\pi_2 = \pi_3 = 0.31655M^2$	$\lambda_1 = 1; \lambda_2 = 1.09662; \lambda_3 = 1$ **SG-5*** $p_1^A = 1.01198M; p_1^B = 1.00916M$ $p_2 = 0.934011M; p_3 = 0.918957M$ $\pi_1 = 0.453897M^2$ $\pi_2 = 0.316859M^2 - Z$, $\pi_3 = 0.316681M^2$
	$m_3 = 1$	$\lambda_1 = \lambda_2 = 1; \lambda_3 = 1.09662$ **SG-8*** $p_1^A = 1.00916M; p_1^B = 1.01198M$ $p_2 = 0.918957M; p_3 = 0.934011M$ $\pi_1 = 0.453897M^2$ $\pi_2 = 0.316681M^2$, $\pi_3 = 0.316859M^2 - Z$	$\lambda_1 = 1; \lambda_2 = \lambda_3 = 1.09663$ SG-4 $p_1^A = p_1^B = 1.01275M$ $p_2 = p_3 = 0.934204M$ $\pi_1 = 0.455846M^2$ $\pi_2 = \pi_3 = 0.31699M^2 - Z$

$m_1 = 1$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = 1.05172, \lambda_2 = \lambda_3 = 1$ SG-1 $p_1^A = p_1^B = 1.01752M$ $p_2 = p_3 = 0.921048M$ $\pi = 0.452233M^2 - Z$ $\pi_2 = \pi_3 = 0.318123M^2$	$\lambda_1 = 1.05217; \lambda_2 = 1.09717; \lambda_3 = 1$ SG-7 $p_1^A = 1.02123M; p_1^B = 1.01839M$ $p_2 = 0.936438M; p_3 = 0.921263M$ $\pi_1 = 0.454201M^2 - Z$ $\pi_2 = 0.318450M^2 - Z$, $\pi_3 = 0.318272M^2$
	$m_3 = 1$	$\lambda_1 = 1.05217; \lambda_2 = 1; \lambda_3 = 1.09717$ SG-2 $p_1^A = 1.01839M; p_1^B = 1.02123M$ $p_2 = 0.921263M; p_3 = 0.936438M$ $\pi_1 = 0.454201M^2 - Z$ $\pi_2 = 0.318272M^2$, $\pi_3 = 0.318450M^2 - Z$	$\lambda_1 = 1.05262; \lambda_2 = \lambda_3 = 1.09718$ **SG-3*** $p_1^A = p_1^B = 1.0221M$ $p_2 = p_3 = 0.936659M$ $\pi_1 = 0.456167M^2 - Z$ $\pi_2 = \pi_3 = 0.318599M^2 - Z$

Table 10. Solution for $b = \frac{1}{2}$.

$m_1 = 0$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$ **SG-6*** $p_1^A = p_1^B = 1.14286M$ $p_2 = p_3 = 1.04762M$ $\pi_1 = 0.580499M^2$ $\pi_2 = \pi_3 = 0.411565M^2$	$\lambda_1 = \lambda_3 = 1; \lambda_2 = 1.18621$ SG-5 $p_1^A = 1.15363M; p_1^B = 1.14525M$ $p_2 = 1.08193M; p_3 = 1.04842M$ $\pi_1 = 0.587240M^2$ $\pi_2 = 0.412975M^2 - Z$, $\pi_3 = 0.412192M^2$
	$m_3 = 1$	$\lambda_1 = \lambda_2 = 1; \lambda_3 = 1.18621$ SG-8 $p_1^A = 1.14525M; p_1^B = 1.15363M$ $p_2 = 1.04842M; p_3 = 1.08193M$ $\pi_1 = 0.587240M^2$ $\pi_2 = 0.412192M^2$, $\pi_3 = 0.412975M^2 - Z$	$\lambda_1 = 1; \lambda_2 = \lambda_3 = 1.18624$ SG-4 $p_1^A = 1.15603M; p_1^B = 1.15603M$ $p_2 = p_3 = 1.0827M$ $\pi_1 = 0.593962M^2$ $\pi_2 = \pi_3 = 0.413605M^2 - Z$

$m_1 = 1$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = 1.1, \lambda_2 = \lambda_3 = 1$ SG-1 $p_1^A = p_1^B = 1.16364M$ $p_2 = p_3 = 1.05455M$ $\pi = 0.581818M^2 - Z$ $\pi_2 = \pi_3 = 0.417025M^2$	$\lambda_1 = 1.1018; \lambda_2 = 1.18844; \lambda_3 = 1$ SG-7 $p_1^A = 1.17508M; p_1^B = 1.16655M$ $p_2 = 1.08964M; p_3 = 1.05552M$ $\pi_1 = 0.588698M^2 - Z$ $\pi_2 = 0.418581M^2 - Z$, $\pi_3 = 0.417794M^2$
	$m_3 = 1$	$\lambda_1 = 1.1018; \lambda_2 = 1; \lambda_3 = 1.18844$ SG-2 $p_1^A = 1.16655M; p_1^B = 1.17508M$ $p_2 = 1.05552M; p_3 = 1.08964M$ $\pi_1 = 0.588698M^2 - Z$ $\pi_2 = 0.418581M^2 - Z$, $\pi_3 = 0.417794M^2$	$\lambda_1 = 1.10357; \lambda_2 = \lambda_3 = 1.18852$ **SG-3*** $p_1^A = p_1^B = 1.17802M$ $p_2 = p_3 = 1.09066M$ $\pi_1 = 0.595562M^2 - Z$ $\pi_2 = \pi_3 = 0.419354M^2 - Z$

Table 11. Solution for $b = \frac{5}{8}$.

$m_1 = 0$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 1$ **SG-6*** $p_1^A = p_1^B = 1.32039M$ $p_2 = p_3 = 1.21683M$ $\pi_1 = 0.774856M^2$ $\pi_2 = \pi_3 = 0.555252M^2$	$\lambda_1 = \lambda_3 = 1; \lambda_2 = 1.32633$ SG-5 $p_1^A = 1.34877M; p_1^B = 1.32712M$ $p_2 = 1.28892M; p_3 = 1.21963M$ $\pi_1 = 0.795830M^2$ $\pi_2 = 0.560648M^2 - Z$, $\pi_3 = 0.557813M^2$
	$m_3 = 1$	$\lambda_1 = \lambda_2 = 1; \lambda_3 = 1.32633$ SG-8 $p_1^A = 1.32712M; p_1^B = 1.34877M$ $p_2 = 1.21963M, p_3 = 1.28892M$ $\pi_1 = 0.795830M^2$ $\pi_2 = 0.557813M^2$, $\pi_3 = 0.560648M^2 - Z$	$\lambda_1 = 1; \lambda_2 = \lambda_3 = 1.32653$ SG-4 $p_1^A = p_1^B = 1.3556M$ $p_2 = p_3 = 1.29194M$ $\pi_1 = 0.81673M^2$ $\pi_2 = \pi_3 = 0.563241M^2 - Z$

(continued on next page)

Table 11 continued, $b = \frac{5}{8}$, $m_1 = 1$

		S2	
		$m_2 = 0$	$m_2 = 1$
S3	$m_3 = 0$	$\lambda_1 = 1.17606, \lambda_2 = \lambda_3 = 1$ SG-1 $p_1^A = p_1^B = 1.36493M$ $p_2 = p_3 = 1.23539M$ $\pi_1 = 0.779961M^2 - Z$ $\pi_2 = \pi_3 = 0.572319M^2$	$\lambda_1 = 1.18224; \lambda_2 = 1.33414; \lambda_3 = 1$ SG-7 $p_1^A = 1.39647M; p_1^B = 1.37399M$ $p_2 = 1.31109M; p_3 = 1.23916M$ $\pi_1 = 0.801885M^2 - Z$ $\pi_2 = 0.578688M^2 - Z$, $\pi_3 = 0.575823M^2$
	$m_3 = 1$	$\lambda_1 = 1.18224; \lambda_2 = 1; \lambda_3 = 1.33414$ SG-2 $p_1^A = 1.37399M; p_1^B = 1.39647M$ $p_2 = 1.23916M; p_3 = 1.31109M$ $\pi_1 = 0.801885M^2 - Z$ $\pi_2 = 0.575823M^2$, $\pi_3 = 0.578688M^2 - Z$	$\lambda_1 = 1.18821; \lambda_2 = \lambda_3 = 1.33462$ **SG-3*** $p_1^A = p_1^B = 1.40568M$ $p_2 = p_3 = 1.3152M$ $\pi_1 = 0.823783M^2 - Z$ $\pi_2 = \pi_3 = 0.582236M^2 - Z$

5 Conclusion

The interaction between chain and independent stores in multiple markets, provides further insight into the role of a strategic separation between ownership and control. We find that with Cournot and differentiated product Bertrand competition, strategic delegation makes the chain store more collusive than independent stores. This finding adds to the existing body of managerial delegation literature in single market competition, which shows that Cournot firms are more aggressive and Bertrand firms more collusive, than their entrepreneurial counterparts. Thus, in general, managerial delegation can be viewed as a commitment that facilitates collusion under some circumstances. It may be useful for future work with MM competition, to explore alternative delegation incentives such as relative profit, but our hypothesis is that these results will persist for strategic substitutes and strategic complements.

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Appendix

We first consider the three stage Cournot competition. As mentioned before, the sub-games are solved using backward induction. Further, SG-2 and SG-7 can be treated symmetrically and so can SG-5 and SG-8. The proof for all the sub-games is similar and we provide the outline and skip the details.

In general, in Stage 0, each owner chooses $m_i = 0$ or 1, $i = 1, 2, 3$. For instance for SG-1, in Stage 0, all $m_i = 1$. In contrast for SG-4, in Stage 0, all $m_i = 0$. If $m_i = 0$ for any i, then in Stage 1, $\lambda_i = 1$. This is what happens for instance in SG-4 for all i. Otherwise, in Stage 1, an owner's best response function is (for instance in SG-1)

$$\lambda_i = \lambda_i(\lambda_j, \lambda_k), i \neq j, k \in \{1, 2, 3\}.$$

Then we consider Stage 2 and the best response function of the manager or owner as the case may be. For instance, for SG-1, the managers' best response functions are of the form

$$\begin{array}{rclrcl} x_1^A & = & x_1^A(x_2, \lambda_1), & x_1^B & = & x_1^B(x_1, \lambda_1), \\ x_2 & = & x_2(x_1^A, \lambda_2), & x_3 & = & x_3(x_1^B, \lambda_3). \end{array}$$

In contrast, for SG-4, the *owner's* best response functions in Stage 2 are of the form

$$\begin{array}{rclrcl} x_1^A & = & x_1^A(x_2), & x_1^B & = & x_1^B(x_3), \\ x_2 & = & x_2(x_1^A), & x_3 & = & x_3(x_1^B). \end{array}$$

Then we maximize g_i if $m_i = 1$ and maximize π_i if $m_i = 0$, with respect to the quantities x. These optimums are functions of the λ_i. We then optimize g_i (or π_i) with respect to those λ_i which are not already equal to 1.

SG-1, $m_1 = m_2 = m_3 = 1$. In Stage 2, manager of each Store $i = 1, 2, 3$ chooses output x_i to maximize g_i, given Stage 1 incentive contract that specifies λ_i and Stage 0 owner choice of $m_i = 1$ for all i.

The manager of independent Store 2 in Market A will choose x_2 to maximize his objective function:

$$g_2 = R_2 - \lambda_2 c_2$$

$$\begin{aligned} &= p_A x_2 - \lambda_2 \frac{1}{2}\, x_2^2 \\ &= (M - x_1^A - x_2)x_2 - \lambda_2 \frac{1}{2}\, x_2^2. \end{aligned}$$

This is clearly a concave function of x_2 and the interior optimum is obtained by solving the first order condition which gives:

$$x_2 = \frac{M - x_1^A}{2 + \lambda_2}. \tag{5.1}$$

Likewise, the profit maximizing output chosen by the manager of Store 3 in Market B is

$$x_3 = \frac{M - x_1^B}{2 + \lambda_3}. \tag{5.2}$$

where x_1^B is the output of Store 1 in Market B.

The manager of Store 1 in Markets A and B will choose x_1^A and x_1^B to maximize

$$\begin{aligned} g_1 &= R_1^A + R_1^B - \lambda_1 \left(c_1^A + c_1^B\right) \\ &= p_A x_1^A + p_B x_1^B - \lambda_1 \frac{1}{2} \left(x_1^A + x_2^B\right)^2 \\ &= \left(M - x_1^A - x_2\right) x_1^A + \left(M x_1^B - x_3\right) x_1^B - \lambda_1 \frac{1}{2} \left(x_1^A + x_1^B\right)^2. \end{aligned}$$

The optimum x_1^A and x_1^B are obtained by the first order conditions

$$\frac{\partial g_1}{\partial x_1^A} = 0 \ \text{ and } \ \frac{\partial g_1}{\partial x_1^B} = 0 \tag{5.3}$$

which imply

$$x_1^A = \frac{M - x_2 - \lambda_1 x_1^B}{2 + \lambda_1}, \quad x_1^B = \frac{M - x_3 - \lambda_1 x_1^A}{2 + \lambda_1}. \tag{5.4}$$

Solving the first-order conditions we find:

$$x_1^A = \frac{M\left[(1+\lambda_2)\{(2+\lambda_1)(2+\lambda_3) - 1\} - (\lambda_1)(2+\lambda_2)(1+\lambda_3)\right]}{\left[(2+\lambda_1)(2+\lambda_2) - 1\right]\left[(2+\lambda_1)(2+\lambda_3) - 1\right]} \tag{5.5}$$

$$x_1^B = \frac{M\left[(1+\lambda_3)\{(2+\lambda_1)(2+\lambda_2) - 1\} - \lambda_1(2+\lambda_3)(1+\lambda_2)\right]}{\left[(2+\lambda_1)(2+\lambda_2) - 1\right]\left[(2+\lambda_1)(2+\lambda_3) - 1\right]}. \tag{5.6}$$

Having obtained x_1^A and x_1^B, x_2 and x_3 are given by (5.1) and (5.2).

In Stage 1 Store i owner chooses λ_i to maximize profit $\pi_i, i = 1, 2, 3$. Owner of Store 1 chooses λ_1 to maximize

$$\begin{aligned} \pi_1 &= p_A x_1^A + p_B x_1^B - \frac{1}{2}\, (x_1^A + x_1^B)^2 \\ &= (M - x_1^A - x_2)x_1^A + (M - x_1^B - x_3)x_1^B - \frac{1}{2}(x_1^A + x_1^B)^2 \end{aligned}$$

where x_2, x_3, x_1^A, x_1^B are as in (5.1), (5.2), (5.5) and (5.6).

Likewise, owner of Store 2 will chose λ_2 to maximize

$$\pi_2 = p_A x_2 - c_2$$

$$= \quad (M - x_1^A - x_2)x_2 - \frac{1}{2}\, x_2^2$$

where x_1^A, x_2 are as above.

Owner of Store 3 will choose λ_3 to maximize

$$\begin{aligned} \pi_3 &= p_A x_1^A + p_B x_1^B - \frac{1}{2}\,(x_1^A + x_1^B)^2 \\ &= (M - x_1^A - x_2)x_1^A + (M - x_1^B - x_3)x_1^B - \frac{1}{2}\,(x_1^A + x_1^B)^2. \end{aligned}$$

Standard first-order maximization gives the optimum values of $\lambda_i, i = 1, 2, 3$ and substituting these values in Stage 2 equations, we obtain the optimum x_2, x_3, x_1^A and x_1^B.

SG-2, $m_1 = m_2 = 1, m_3 = 0$. This is symmetric with SG-7 which is solved later.

SG-3, $m_1 = 1, m_2 = m_3 = 0$. In Stage 2, Manager of chain Store 1 chooses x_1 to maximize g_1; Owners of independent Stores 2 and 3 choose x_i to maximize π_i, $i = 2, 3$, given that in Stage 1, Store 1 owner offers λ_1 and owners of Stores 2 and 3 do nothing and in Stage 0 owner 1 has chosen $m_1 = 1$ and owners 2 and 3 have chosen $m_2 = m_3 = 0$. Hence $\lambda_2 = \lambda_3 = 1$. Using backward induction we find the Stage 2 reaction functions are:

$$x_1^A = x_1^B = \frac{2M}{5 + 6\lambda_1}, \quad x_2 = x_3 = \frac{M}{5 + 6\lambda_1}(1 + 2\lambda_1). \tag{5.7}$$

In Stage 1, Stores 2 and 3 do nothing, given Stage 0 $m_2 = m_3 = 0$. For Store 1, the owner chooses λ_1 to maximize

$$\begin{aligned} \pi_1 &= \pi_1^A + \pi_1^B \\ &= R_1^A + R_1^B - C_1 \\ &= (M - x_1^A - x_2)x_1^A + (M - x_1^B - x_3)x_1^B - \frac{1}{2}(x_1^A + x_1^B)^2. \end{aligned}$$

Substituting from Stage 2 solutions, we obtain $\lambda_1 = 0.8\dot{3}$. Then we can solve for quantities x.

SG-4, $m_1 = m_2 = m_3 = 0$. As a consequence, $\lambda_i = 0$ for all i. In Stage 2, owners choose x_i to maximize π_i given Stage 0, $m_i = 0, i = 1, 2, 3$ by standard arguments.

SG-5, $m_1 = m_2 = 0, and \;\; m_3 = 1$. This is symmetric with SG-8.

SG-6, $m_1 = 0, m_2 = m_3 = 1$. In Stage 2, manager of independent Stores 2 and 3, choose $x_i, i = 2, 3$ to maximize $g_i, i = 2, 3$; Owner of chain Store 1 chooses x_1 to maximize π_1, given that in Stage 1, Store 2 owner offers λ_i and Store 1 does nothing and in Stage 0 Owner 1 has chosen $m_1 = 0$ and Owners 2 and 3 have chosen $m_2 = m_3 = 1$. Using standard backward induction we find the Stage 2 reaction functions:

$$\begin{aligned} x_1^A &= \frac{M\,[(1 + \lambda_2)(5 + 3\lambda_3) - (2 + \lambda_2)(1 + \lambda_3)]}{(5 + 3\lambda_2)(5 + 3\lambda_3) - (2 + \lambda_2)(2 + \lambda_3)}, \\ x_1^B &= \frac{M\,[(1 + \lambda_3)(5 + 3\lambda_2) - (2 + \lambda_3)(1 + \lambda_2)]}{(5 + 3\lambda_3)(5 + 3\lambda_2) - (2 + \lambda_3)(2 + \lambda_2)}, \\ x_2 &= \frac{M(9 + 6\lambda_3)}{(5 + 3\lambda_2)(5 + \lambda_3) - (2 + \lambda_2)(2 + \lambda_3)}, \\ x_3 &= \frac{M(9 + 6\lambda_2)}{(5 + 3\lambda_2)(5 + \lambda_3) - (2 + \lambda_2)(2 + \lambda_3)}. \end{aligned}$$

We now proceed to maximize π_2 and π_3.

$$\begin{aligned}
\pi_2 &= p_A x_2 - \frac{1}{2}x_2^2 \\
&= (M - x_1^A - x_2)x_2 - \frac{1}{2}x_2^2 \\
&= \left[M - \frac{M\left[(1+\lambda_2)(5+3\lambda_3) - (2+\lambda_2)(1+\lambda_3)\right]}{Z} - \frac{3M(3+2\lambda_3)}{Z}\right] \\
&\quad \left[\frac{3M(3+2\lambda_3)}{Z}\right] - \frac{1}{2}\ \frac{9M^2(3+2\lambda_3)^2}{Z^2} \\
&= \frac{M^2}{Z^2}\ \frac{9}{2}(3+2\lambda_3)^2\left(\frac{1}{2}+\lambda_2\right)
\end{aligned}$$

where

$$Z = (5+3\lambda_2)(5+3\lambda_3) - (2+\lambda_2)(2+\lambda_3).$$

By symmetry,

$$\pi_3 = \frac{M^2}{Z^2}\ \frac{9}{2}(3+2\lambda_2)^2\left(\frac{1}{2}+\lambda_3\right).$$

Note that

$$\frac{\partial Z}{\partial \lambda_2} = 3(5+3\lambda_3) - (2+\lambda_3) = 13 + 8\lambda_3.$$

Hence

$$\frac{\partial \pi_2}{\partial \lambda_2} = 0 \Rightarrow 2Z^2 - (1+2\lambda_2)2Z(13+8\lambda_3) = 0. \tag{5.8}$$

Likewise, from the first order condition for λ_3, we have

$$2Z^2 - (1+2\lambda_3)2Z(13+8\lambda_3) = 0. \tag{5.9}$$

From (5.8) and (5.9), we get $\lambda_2 = \lambda_3$. Simplifying and solving we find $\lambda_2 = \lambda_3 = \frac{-1+\sqrt{5}}{2} \approx 0.615$. We can then proceed to find the optimum quantities.

SG-7, $m_1 = 1, m_2 = 0, m_3 = 1$. Owner 2 chooses x_2 to maximize π_2. Managers 1 and 3 choose x_i, to maximize g_i, $i = 1,3$, *given* Stage 1, Owners 1 and 3 choose λ_1 and λ_3 respectively, to maximize π_1 and π_3 and *given* Stage 1 $m_1 = m_3 = 1, m_2 = 0$. Using backward induction, the Stage 2 reaction functions are:

$$\begin{aligned}
x_1^A &= \frac{M[6+4\lambda_3+\lambda_1(1-\lambda_3)]}{15+19\lambda_1+(10+11\lambda_1)\lambda_3}, \\
x_1^B &= \frac{M[5+5\lambda_3-\lambda_1(1-\lambda_3)]}{15+19\lambda_1+(10+11\lambda_1)\lambda_3}, \\
x_2 &= \frac{M(1+2\lambda_1)(3+2\lambda_3)}{15+19\lambda_1+(10+11\lambda_1)\lambda_3}, \\
x_3 &= \frac{5M(1+2\lambda_1)}{15+19\lambda_1+(10+11\lambda_1)\lambda_3}.
\end{aligned}$$

In Stage 1, Owner 1 and Owner 3 choose λ_1 and λ_3 to maximize π_1 and π_3 respectively. Substituting Stage 2 reaction function and maximizing, we obtain, $\lambda_1 = 0.822$ and $\lambda_3 = 0.608$.

SG-8, $m_1 = m_3 = 0$ and $m_2 = 1$. In Stage 2, manager of independent Store 2 chooses x_2 to maximize g_2. Owners of chain Store 1 and independent Store 3 choose x_i to maximize $\pi_i, i = 1,3$;

given Stage 1, Owner 2 chooses λ_2 to maximize π_2 and *given* Stage 1, $m_1 = m_3 = 0$ and $m_2 = 1$. Using standard backward induction, we obtain the Stage 2 reaction functions as:

$$x_1^A = \frac{2M(2+3\lambda_2)}{34+21\lambda_2}, \quad x_1^B = \frac{M(7+3\lambda_2)}{34+21\lambda_2}$$

and

$$x_2 = \frac{M - x_1^A}{2+\lambda_2}, \quad x_3 = \frac{M - x_1^B}{3}.$$

It follows that

$$x_2 = \frac{M - x_1^A}{2+\lambda_2} = \frac{1}{2+\lambda_2}[M - \frac{2M(2+3\lambda_2)}{34+21\lambda_2}] = \frac{15M}{34+21\lambda_2},$$

$$x_3 = \frac{M - x_1^B}{3} = \frac{1}{3}[M - \frac{M(7+3\lambda_2)}{34+21\lambda_2} = \frac{1}{3}\frac{9M(3+2\lambda_2)}{34+21\lambda_2} = \frac{3M(3+2\lambda_2)}{34+21\lambda_2}.$$

In Stage 1, Owner 2 chooses λ_2 to maximize $\pi_2 = p_A x_2 + \frac{1}{2}x_2^2$. Substituting Stage 3 reaction function and simplifying, $\lambda_2 = 0.619048$. Then we proceed to find the quantities.

Now we consider the differentiated product Bertrand competition which is more involved due to the product differentiation parameter b. The SPNE λ_i now depend on b:

$$\lambda_i^* = \begin{cases} \lambda_i^*(b) & \text{if } m_i = 1 \\ 1 & \text{if } m_i = 0. \end{cases}$$

Likewise, if all $m_i = 1$ in Stage 0, then the *managers'* best response functions in Stage 2 are

$$\begin{aligned} p_1^A &= p_1^A(p_2, \lambda_1), \; p_1^B &= p_1^B(p_3, \lambda_1) \\ p_2 &= p_2(p_1^A, \lambda_2), \; p_3 &= p_3(p_1^B, \lambda_3). \end{aligned}$$

If instead, all $m_i = 0$, in Stage 0, then all $\lambda_i = 1$ and the *owner's* best response functions in Stage 2 are

$$\begin{aligned} p_1^A &= p_1^A(p_2), \; p_1^B &= p_1^B(p_3), \\ p_2 &= p_2(p_1^A), \; p_3 &= p_3(p_1^B). \end{aligned}$$

It is difficult to find the closed form solutions for general b and M. We solve the games numerically for a selected but representative set of values for b. The outline of the backward induction approach for SG-1 is illustrated with $b = \frac{1}{4}$ and $M = 10$.

Stage 2: The manager of the chain store will choose p_1^A and p_1^B to maximize

$$\begin{aligned} g_1 &= p_1^A(M - p_1^A + \frac{1}{4}p_2) + p_1^B(M - p_1^B + \frac{1}{4}p_3) \\ &\quad -\lambda_1 \frac{1}{2}(M - p_1^A + \frac{1}{4}p_2 + M - p_1^B + \frac{1}{4}p_3)^2. \end{aligned}$$

The manager of the independent Store 2 will choose p_2 to maximize

$$g_2 = p_2(M - p_2 + \frac{1}{4}p_1^A) - \frac{1}{2}\lambda_2(M - p_2 + \frac{1}{4}p_1^A)^2.$$

The manager of independent Store 3 will choose p_3 to maximize

$$g_3 = p_3(M - p_3 + \frac{1}{4}p_1^B) - \frac{1}{2}\lambda_3(M - p_3 + \frac{1}{4}p_1^B)^2.$$

The first order conditions are

$$\begin{aligned} p_1^A &= p_1^B = \frac{40(567 - 1134\lambda_1 + 315\lambda_2 + 614\lambda_1\lambda_2 + 279\lambda_3 + 574\lambda_1\lambda_3 + 155\lambda_2\lambda_3 + 310\lambda_1\lambda_2\lambda_3)}{3969 + 3906\lambda_1 + 1953\lambda_2 + 1906\lambda_1\lambda_2 + 1953\lambda_3 + 1906\lambda_1\lambda_3 + 961\lambda_2\lambda_3 + 930\lambda_2\lambda_3} \\ p_2 &= p_3 = \frac{40(567 + 630\lambda_1 + 567\lambda_2 + 630\lambda_1\lambda_2 + 279\lambda_3 + 310\lambda_1\lambda_3 + 279\lambda_2\lambda_3 + 310\lambda_1\lambda_2\lambda_3)}{3969 + 3906\lambda_1 + 1953\lambda_2 + 1906\lambda_1\lambda_2 + 1953\lambda_3 + 1906\lambda_1\lambda_3 + 961\lambda_2\lambda_3 + 930\lambda_1\lambda_2\lambda_3}. \end{aligned}$$

Stage 1: The owner of Store 1 chooses λ_1 to maximize

$$\begin{aligned} \pi_1 &= p_1^A(M - p_1^A + \frac{1}{4}p_2) + p_1^B(M - p_1^B + \frac{1}{4}p_3) \\ &\quad - \frac{1}{2}(M - p_1^A + \frac{1}{4}p_2 + M - p_1^B + \frac{1}{4}p_3)^2 - Z. \end{aligned}$$

The owner of Store 2 chooses λ_2 to maximize

$$\pi_2 = p_2(M - p_3 + \frac{1}{4}p_1^A) - \frac{1}{2}(M - p_2 + \frac{1}{4}p_1^A)^2 - Z.$$

The owner of Store 3 chooses λ_3 to maximize

$$\pi_3 = p_3(M - p_3 + \frac{1}{4}p_1^B) - \frac{1}{2}(M - p_3 + \frac{1}{4}p_1^B)^2 - Z.$$

Solving the first order conditions and simplifying, we get

$$\begin{aligned} \lambda_1 &= 1.02189, \\ \lambda_2 &= \lambda_3 = 1.04079. \end{aligned}$$

Then we can obtain the prices and the profit values.