INDIAN STATISTICAL INSTITUTE

Theoretical Statistics and Mathematics Unit, Kolkata

Two Random Matrix Days

Venue: L-infinity, A N Kolmogorov Bhavan, 5th floor, ISI Kolkata
Date: 8th-9th August 2023

TIME	08.08.2023		09.08.2023	
	SPEAKER	TITLE	SPEAKER	TITLE
10:30 - 11:30	**Arup Bose** ISI Kolkata	Introduction to Random Matrices and some questions	**Shambhu Nath Maurya** ISI Kolkata	Convergence of high dimensional Toeplitz and Hankel matrices with correlated inputs
11:30 - 11:40	Coffee Break		Coffee Break	
11:40 - 12:40	**Soumendu S Mukherjee** ISI Kolkata	Some characterization results on classical and free Poisson thinning	**Bishakh Bhattacharya** ISI Kolkata	Fluctuations of Extreme Eigenvalues and Eigenvectors in Perturbed Wigner-Type Matrices
12:40 - 14:30	Lunch		Lunch	
14:30 - 15:30	**Debashis Paul** ISI Kolkata	Spectral behaviour of partial sum processes associated with large sample covariance matrices	**Sayan Das** University of Chicago	A new theory of extreme diffusion
15:30 - 15:45	Coffee Break		Coffee Break	
15:45 - 16:45	**Arijit Chakrabarty** ISI Kolkata	Inhomogeneous Erdős-Rényi random graphs: bulk and edge of the spectrum	**Rajat Subhra Hazra** University of Leiden	*Large deviations of the largest eigenvalue of the adjacency and Laplacian matrices in dense graphs.*

ALL ARE CORDIALLY INVITED

8-Aug-23

Arup Bose (ISI, Kolkata)

Title: Introduction to Random Matrices and some questions

Abstract: We shall give a brief introduction to Random Matrices, its connection to cumulants and free cumulants, and list a few open questions.

Soumendu Sundar Mukherjee (ISI, Kolkata)

Title: Some characterization results on classical and free Poisson thinning

Abstract: Poisson thinning is an elementary result in probability, which is of great importance in the theory of Poisson point processes. In this talk, we will discuss a couple of characterization results on Poisson thinning. We will also consider several free probability analogues of Poisson thinning, collectively dubbed as free Poisson thinning, and discuss characterization results for them, similar to the classical case. One of these free Poisson thinning procedures arises naturally as a high-dimensional asymptotic analogue of Cochran's theorem from multivariate statistics on the "Wishart-ness" of quadratic functions of Gaussian random matrices. We will note the implications of our characterization results in the context of Cochran's theorem. We will also discuss a free probability analogue of Craig's theorem, another well-known result in multivariate statistics on the independence of quadratic functions of Gaussian random matrices.

Debashis Paul (ISI, Kolkata)

Title: Spectral behaviour of partial sum processes associated with large sample covariance matrices

Abstract: In this talk we look at some recent developments in terms of analyzing the spectral behaviour of large sample covariance-type matrices. However, unlike in the classical framework, we consider a sequence of sample covariance matrices that is computed based on the data {X1,…,Xt}, for t=1,…,n. The corresponding sequential sample covariance matrices, denoted St, therefore, form a matrix-valued stochastic process indexed by time t. It is of interest to study the spectral statistics of such matrices as stochastic processes, when the dimension of the data vectors is comparable to the sample size. A first result of this kind has recently been proved by Dörnemann and Dette (2023), who established the asymptotic distribution of linear spectral statistics associated with the process St. We study a related problem, that of characterizing the behaviour of the normalized spiked eigenvalues of the process St when the population covariance matrix is assumed to have the form of a finite rank perturbation of the identity matrix, and the spiked eigenvalues are sufficiently big. We establish the existence of a limiting process associated with these multivariate processes. We also consider a potential application of the results to constructing rotation-invariant tests for detecting change points in the structure of the population covariance matrix when such changes only affect the sizes of the population spikes.

(This is a joint work with Nina Dörnemann from Univ. of Bochum and Univ. of California, Davis).

Arijit Chakrabarty (ISI, Kolkata)

Title: Inhomogeneous Erdős-Rényi random graphs: bulk and edge of the spectrum

Abstract: The talk is on inhomogeneous Erdős-Rényi random graphs in the non-dense regime. The eigenvalues of the adjacency matrix of the graph are studied. The empirical spectral distribution of the matrix after suitable scaling and centering is shown to have a deterministic limit in probability. Depending on the rank of the inhomogeneity kernel generating the random graph, the largest few eigenvalues have a much higher magnitude than that of the bulk. Assuming the rank to be finite, the second order behaviour of those few eigenvalues, after suitable centring and scaling, is shown to be multivariate Gaussian. The asymptotic behaviour of the corresponding eigenvectors is also studied.
The talk is based on joint works with Sukrit Chakraborty, Rajat Hazra, Frank den Hollander and Matteo Sfragara.

9-Aug-23

Shambhu Nath Maurya (ISI, Kolkata)

Title: Convergence of high dimensional Toeplitz and Hankel matrices with correlated inputs

Abstract: It is known that symmetric, independent copies of high dimensional Toeplitz and Hankel matrices converge jointly under the state expected average trace, when the input sequence is real-valued i.i.d.~variables with all moments finite

Let T_n be the random Toeplitz matrix, D_n be the deterministic Toeplitz matrix and P_n be the backward identity permutation matrix. In this work, we investigate the joint convergence of several copies of these matrices when the entries are complex random variables and have a pair-correlation structure. We also extend the above results for generalized Toeplitz matrices. The limits depend on the correlation structure but are universal in that they do not depend on the underlying distributions of the entries. In particular, these results provide the joint convergence of Hankel matrices. Moreover, the earlier results on joint convergence of such matrices follow as special cases. This talk is part of a joint work with Kartick Adhikari and Arup Bose.

Bishakh Bhattacharya (ISI, Kolkata)

Title: Fluctuations of Extreme Eigenvalues and Eigenvectors in Perturbed Wigner-Type Matrices

Abstract: In this talk, we will study perturbations of Wigner matrices with a variance profile, where the entries have the form X_(ij)f(i/N, j/N). Here, X_(ij)'s are normalized variables satisfying specific moment conditions, and f is a bounded, measurable, and Riemann integrable function. We will analyze the fluctuations of the largest eigenvalue of this perturbed matrix, as well as the asymptotic alignment of the corresponding normalized eigenvector. This is an ongoing joint work with Arijit Chakrabarty and Rajat Subhra Hazra.

Sayan Das (University of Chicago)

Title: A new theory of extreme diffusion

Abstract: In Einstein's diffusion model (i.e., the model with independent random walks), the bulk behavior is Gaussian, and extreme value statistics are asymptotically Gumbel. However, this model overlooks the influence of a common background environment. In this talk, we shall focus on a Random Walk in Random Environment (RWRE) model that takes the background environment into account. While the bulk behavior of this model matches Einstein's model, the extreme behavior differs significantly. Our recent work focuses on a continuous version of the RWRE model and shows that extrema are asymptotically distributed as Gumbel shifted by a KPZ equation in a particular regime. Joint work with Hindy Drillick and Shalin Parekh.

Rajat Subhra Hazra (Leiden University)

Title: Large deviations of the largest eigenvalue of the adjacency and Laplacian matrices in dense graphs.

Abstract: We consider an inhomogeneous Erdős-Rényi random graph for which the pair of vertices i,j , i≠j, is connected by an edge with probability r_N(i/N,j/N), independently of other pairs of vertices. Here r plays the role of a reference graphon. Let λ_N be the maximal eigenvalue of the and Adjacency and Laplacian matrix then λ_N/N satisfies a downward LDP and an upward LDP. We identify the rate function and describe its basic properties. The talk is based on two joint works, one with Arijit Chakrabarty, Frank den Hollander, Matteo Sfragara and another with Frank den Hollander and Maarten Markering.

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