THE ABEL LAUREATE SRINIVASA VARADHAN

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On 22 May the King of Norway presented the Abel Prize for 2007 to the distinguished probabilist S.R. Srinivasa Varadhan. The Abel Prize is awarded by the Norwegian Academy of Science and Letters for outstanding work in mathematics, a work of extraordinary depth and influence in the mathematical sciences. Varadhan received the prize for "*his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations*". Kristian Seip, Chairman of the Abel Committee, observes:

> Varadhan's work has great conceptual strength and ageless beauty. His ideas have been hugely influential and will continue to stimulate further research for a long time.

The Abel Prize is considered as the Nobel Prize for mathematics. The city of Calcutta has had the privilege of being associated with most Nobel Laureates from India. Apart from Rabindranath Tagore, one may recall that both Ronald Ross and C.V. Raman had made their crucial scientific discoveries in the city; Mother Teresa had made the city her centre of work; while Amartya Sen had his initial academic career here. Abel Laureate Srinivasa Varadhan too has the Kolkata connection: after graduating from Madras University in 1959, he joined the Indian Statistical Institute (ISI), Calcutta and obtained his PhD from ISI in 1963, when he was just 23 years old. It was in ISI that Varadhan got introduced to areas of probability and mathematics in which he would soon become a leading international figure. For the Indian Statistical Institute — founded by P.C. Mahalanobis in 1932 and blessed by Tagore himself — it is a happy coincidence that the Abel prize was conferred on one of its illustrious alumni in the very year in which the institute is celebrating its Platinum Jubilee.

S.R.S. Varadhan was born in Madras (1940). He completed his B.Sc. with honours in statistics from Presidency College, Madras. Both C.V. Raman and S. Chandrasekhar — the two physicists from India who have won the Nobel Prize — had studied at the same college. Varadhan had excellent academic record — he scored the highest-ever marks in the honours course of Madras University.

Subsequently, Varadhan joined the Research and Training School (RTS) of ISI Calcutta. The RTS was the nucleus of research and teaching activities at ISI during 1949–76. It was the precursor to the present Division of Theoretical Statistics and Mathematics (abbreviated as "Stat-Math Division") formed in 1976. Research activities in probability theory commenced at the RTS in the mid-50s. Some of the results obtained by the ISI school during the 50s and 60s — on weak convergence of probabilities on metric spaces, limit theorems in Hilbert spaces, structure theory of infinitely divisible distributions on locally compact Abelian groups and large deviation results — have now become part of the standard theory in the respective areas. Four ISI youngsters — V.S. Varadarajan, R. Ranga Rao, K.R. Parthasarathy and S.R.S. Varadhan — were at the forefront of the seminal activities during the late 50s and early 60s, with Varadhan being the youngest of the celebrated quartet. Varadhan acknowledges the role of his colleagues in shaping his research interests and the encouragement of his thesis advisor C.R. Rao — a great name in mathematical statistics and in the history of ISI.

While studying Markov processes, Varadhan got interested in partial differential equations (PDE). On the advice of Varadarajan, he went to the Courant Institute of Mathematical Sciences (CIMS), New York, a strong centre of PDE. He found that CIMS had research activities in

diverse areas of mathematics. Since 1963, he has remained in the CIMS enjoying stimulating interactions and collaborations with several colleagues, visiting scientists and graduate students. As he mentions in the Abel Award acceptance speech:

> Courant Institute is a wonderful institution in the way it nurtures its young faculty and encourages them to grow to their full potential.

Varadhan is now Professor of Mathematics and Frank J. Gould Professor of Science at the CIMS. He refers to his collaborator Monroe Donsker as "a wonderful colleague and mentor". Varadhan's theory of "large deviations", which originated in his epoch-making 1966 paper, was worked out during the next decade mostly in collaboration with Donsker. Over the last 40 years, the theory has become a pillar of modern probability theory. It provides a unifying method for deeper understanding of a rich variey of phenomena, in fields as diverse as quantum field theory, statistical physics, population dynamics, econometrics and finance and traffic engineering. It has also expanded our ability to use computers for analysing the occurrence of rare events.

During 1966–72, Varadhan also developed the martingale approach to Markov processes with another distinguished probabilist Daniel W. Stroock (now in MIT). They applied their theory to solve many outstanding problems in fluid mechanics, statistical physics and quantum physics. The American Mathematical Society honoured them with the Steele Prize (1996) in recognition of their work. Later, partly in collaboration with G. Papanicolaou, Varadhan introduced the entropy and the non-gradient system methods to the study of interacting particle systems.

Probability theory has come a long way since the correspondence on "games of chance" between two great mathematicians of the 17th century: Blaise Pascal and Pierre de Fermat. Kolmogorov, one of the greatest mathematicians of 20th century, is regarded as largely responsible for making probability theory a branch of (rigorous) mathematics. (Incidentally, Kolmogorov was one of the thesis examiners of Varadhan.)

Among the early landmarks in probability theory, special mention may be made of the Law of Large Numbers by Jakob Bernoulli (1713) and the Central Limit Theorem of de Moivre (1738). These results led to the formulation of laws governing phenomena that conform to normal behaviour. For instance, when an unbiased coin is tossed a large number of times, say a 1000 times, we expect the number of heads and number of tails to be fairly close. Law of large numbers gives a precise formulation for this regularity.

The study of "Large deviations", on the other hand, is an analysis of rare events, that is, events that have large deviations from normal behaviour. For instance, an event like "100 heads in a row" cannot be completely ruled out even though the chances are extremely remote. Because the probabilities of such phenomena are extremely small, they were overlooked till probabilists like H. Cramer (1893–1985) initiated their study in the late 1930s and found precise estimates of large deviations. Such a study is required to understand (and thereby prepare for) the occurrence of apparently improbable calamities — events like giant waves crashing into oil platforms; or extreme weather conditions; or an extremely "bad year" for an insurance company; or a network breaking down due to overload during peak hours; and so on.

Varadhan went far beyond Cramer and created a subtle modern theory of "large deviations" especially in the context of Brownian motion. He discovered the underlying general principles and brought out their tremedous potential turning the theory into an extremely smooth, powerful and efficient tool in many areas of mathematics and physics.

The botanist Robert Brown (1827) observed that even when water in a container appears to be absolutely still, there is a ceaseless but small motion of tiny particles of matter (like pollen,

dust) suspended in the water. Such motion is called Brownian motion. Theoretical calculations on Brownian motion were made by Einstein (in one of his three great papers of 1905) and Smoluchowski based on which Jean Perrin made the first experiments (for which Perrin got the Nobel Prize for Physics in 1926). These discoveries played a decisive role in the acceptance of the atomic hypothesis. Brownian motion was perceived as being caused by the bombardment of tiny particles by the randomly moving water-molecules.

Subsequently, the mathematical model for Brownian motion was created by Norbert Wiener (middle of 20th century); A. Kolmogorov and Paul Levy continued the work. As mentioned earlier, Brownian motion and the involved techniques — large part of it created by Varadhan — help clarify an immense number of areas in mathematics and physics. Many parts of quantum field theory based on the methods of Feynman can be linked to probability theory and Brownian motion; the famous work of Atiyah-Singer (Abel winners in 2004) can also be envisaged in this framework. Due to all these developments, probability theory has attained a position of extraordinary importance within mathematics.

Varadhan has strongly influenced the modern development of probability theory for several decades and inspired generations of probabilists. His work blends techniques from probability theory, partial differential equations, functional analysis and infinite-dimensional analysis and has applications in statistics, statistical physics, hydrodynamics, partial differential equations, insurance, finance, and, of course, to many parts of probability theory itself. While his areas of research include martingale problems and diffusion theory, large particle systems, hydrodynamical limits, random walks in random media and quantum field theory, the Abel citation laid special emphasis on his monumental work on "large deviations".

Daniel Stroock writes that Varadhan "has had an uncanny ability to understand that large deviations are manifest in all sorts of situations in which nobody else even suspected their presence". Monroe Donsker once told a close friend that Varadhan was the greatest problem-solver he ever came across. Varadarajan remarks that Varadhan combines in him great mathematical power with profound intuition. In a review of a book by Stroock-Varadhan, David Williams writes:

> The Stroock-Varadhan book proceeds on its inexorable way like a massive Bach fugue.

The Abel Prize is named after the Norwegian genius Niels Henrik Abel (1802–29), one of the all-time greats in the history of mathematics. His premature death at the age of 26 is one of the most tragic losses for the world of mathematics; but within this short life-span, he had made great contributions in analysis, algebric geometry and theory of equations whose impact can be felt even today. His most famous result — that the general polynomial of degree 5 (or more) cannot be solved by radicals — is now studied by all mathematics students as part of a more general and deep theory due to Galois.

The first proponent of an international prize in mathematics in honour of Abel was Sophus Lie — Norway's other all-time great mathematician. Towards the end of the 19th century, Lie took the initiative to establish a fund that would award an Abel Prize every 5 years for outstanding work in pure mathematics. Lie's efforts must have been influenced by the fact that Alfreds Nobel's plans for annual prizes, made known in 1897, lacked a prize for mathematics. Lie received overwhelming support, but his efforts ended with his death in 1899. In 1902 (Abel's birth-centenary year), King Oscar II became interested in financing an Abel Prize; the mathematicians Carl Stormer and Ludwig Sylow formulated the statutes and rules for the proposed prize. However, the dissolution of the union between Sweden and Norway in 1905 put an end to further plans. Almost a century

later, the idea of Abel Prize was revived by Abel's biographer Arild Stubhaug in the year 2000 — the "World Mathematical Year". In 2001, the Norway government announced that the bicentenary of Abel would mark the commencement of a new prize for mathematicians — the Abel Prize. The Abel Memorial Fund, for awarding the Abel Prize, was established on 1 January 2002 to commemorate the bicentenary of Abel's birth.

In April 2003, it was announced that Jean-Pierre Serre (College de France) would be the first mathematician to be awarded the Abel Prize. Serre was chosen "for playing a key role in shaping the modern form of many parts of mathematics, including topology, algebraic geometry, and number theory". Serre had played a pioneering role in many of the areas in which Indian mathematicians, especially of the TIFR school, have excelled.

The 2004 Abel Prize was jointly awarded to Michael F. Atiyah (U. Edinburgh) and Isadore M. Singer (MIT) "for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics". It may be mentioned here that a new approach to the Atiyah-Singer Index theorem was made by a brilliant Indian mathematician V.K. Patodi. His death in 1976 at the age of 31 was a cruel blow to Indian mathematics. The approach involved the work (in the 40s) of another gifted Indian mathematician of the previous generation: S. Minakshisundaram.

The 2005 Abel Prize was awarded to Peter D. Lax (also from CIMS) for his work in partial differential equations and the 2006 Abel Prize was awarded to Lennart Carleson (Royal Institute of Sweden) for his work in harmonic analysis and dynamical systems. At the age of 67, Varadhan is the youngest mathematician to win the Abel Prize. The cash amount of the award this year is NOK 6,000,000 (close to 1 million US dollars).

Varadhan is highly regarded not only for his scientific achievements but also for his human qualities — he is friendly and accessible. As Stroock observes:

> Unlike the majority of gifted people whom I have encountered, Varadhan never has used his gifts as a weapon to bludgeon his less gifted colleagues. He was then, and remains, a true gentleman.

A teacher of exceptional patience, Varadhan is regarded with pride and affection by all his acquaintances. He is an inspiring and serene role-model for students. Varadarajan remarks:

> In him are combined transcendent achievement and a rare modesty, an acute awareness of the incremental nature of science and the necessity of standing on the shoulders of others to advance our knowledge. If one combines these attributes with those of a man to whom doing mathematics is effortless, whose creativity and insight always pick out the simplest path to the solution of even the most tangled questions, we begin to get a glimpse of the unique personality of Varadhan.