

MATHEMATICS IN ANCIENT INDIA: AN OVERVIEW

[A series of articles on Mathematics in Ancient India were published in *Resonance*, a journal of science education. The first article of the series (April 2002), which featured only a brief overview of some of the basic results of ancient Indian mathematics, is reproduced below with minor modifications (mostly through footnotes).

Several quotations from eminent scholars had been displayed in the margins of the original *Resonance* article. Some of them have now been inserted in the main text, some introduced as footnotes.]

1. Vedic Mathematics: The *Śulba Sūtras*

But the Vedic Hindu, in his great quest of the *Parā-vidyā* ("Supreme knowledge"), *Satyasya Satyam* ("Truth of truths", "Absolute Truth"), made progress in the *Aparā-vidyā* ("inferior knowledge", "relative truths"), including the various arts and sciences, to a considerable extent, and with a completeness which is unparalleled in antiquity. [B. Datta¹ in Preface of ([2]).]

Mathematics, in its early stages, developed mainly along two broad overlapping traditions: (i) the geometric and (ii) the arithmetical and algebraic. Among the pre-Greek ancient civilisations, it is in India that we see a strong emphasis on both these great streams of mathematics. Other ancient civilisations like the Egyptian and the Babylonian had progressed essentially along the computational tradition. A. Seidenberg, an eminent algebraist and historian of mathematics, traced the origin of sophisticated mathematics to the originators of the Ṛg-Vedic rituals ([10, 11]).²

The oldest known mathematics texts in existence are the *Śulba Sūtras* of

1. The scholar-saint Bibhutibhusan Datta (later Swami Vidyaranya) was a doyen among historians of Indian mathematics. *Champaklal's Treasures* (pp. 172–3) contains observations of Sri Aurobindo on B. Datta.

In the Preface of "The Science of the *Śulba*" ([2]), B. Datta exclaimed in a Sanskrit verse alluding to Kālidāsa:

"How great is the science which revealed itself in the *Śulba*, and how meagre is my intellect! I have aspired to cross the unconquerable ocean in a mere raft."

2. The following statements from Seidenberg's paper ([10]) were displayed in the *Resonance* article:

"...nor did he [Thibaut] formulate the obvious conclusion, namely, that the Greeks were not the inventors of plane geometry, rather it was the Indians." (p. 304)

"Anyway, the damage had been done and the *Śulvasūtras* have never taken the position in the history of mathematics that they deserve." (p. 306)

"A common source for the Pythagorean and Vedic mathematics is to be sought either in the Vedic mathematics or in an older mathematics very much like it.... Thus what are regarded as the two main sources of Western mathematics, namely Pythagorean mathematics and Old-Babylonian mathematics, both flow from a still older source. What was this older, common source like? I think its mathematics was very much like what we see in the *Śulvasūtras*." (p. 329)

An article on the origin of geometry and mathematics, in the light of Seidenberg's analyses, is likely to appear in a future issue of *Mother India*.

Baudhāyana, Āpastamba and Kātyāyana which form part of the literature of the *sūtra* period of the later Vedic age. The *Śulba Sūtras* had been estimated to have been composed around 800 BC (some recent researchers are suggesting earlier dates). But the mathematical knowledge recorded in these *sūtras* (aphorisms) is much more ancient; for the *Śulba* authors emphasise that they were merely stating facts already known to the composers of the *Brāhmaṇas* and *Samhitas* of the early Vedic age.

The *Śulba Sūtras* give a compilation of results in mathematics that had been used for designing and constructions of various elegant Vedic fire-altars right from the dawn of civilisation. An altar had rich symbolic significance and had to be constructed with accuracy. The designs of several of these brick-altars are quite involved—for instance, there are constructions depicting a falcon in flight with curved wings, a chariot-wheel complete with spokes or a tortoise with extended head and legs! Constructions of the fire-altars are described in an enormously developed form in the *Śatapatha Brāhmaṇa* (c. 2000 BC; vide [2]); some of them are mentioned in the earlier *Taittirīya Samhita* (c. 3000 BC; vide [2]); but the sacrificial fire-altars are referred—without explicit construction—in the even earlier R̥g-Vedic *Samhitas*, the oldest strata of the extant Vedic literature. The descriptions of the fire-altars from the *Taittirīya Samhita* onwards are exactly the same as those found in the later *Śulba Sūtras*.

The *Śulba* mathematics involves a deep understanding of the geometric and algebraic aspects of the properties of triangles, squares, rectangles, circles, parallelograms, trapezia and similar figures. Plane geometry stands on two important pillars having applications throughout history: (i) the result popularly known as the “Pythagoras theorem” and (ii) the properties of similar figures. In the *Śulba Sūtras*, we see an explicit statement of the Pythagoras theorem³ and its applications in various geometric constructions such as construction of a square equal (in area) to the sum, or difference, of two given squares, or to a rectangle, or to the sum of n squares. These constructions implicitly involve application of algebraic identities such as $(a \pm b)^2 = a^2 + b^2 \pm 2ab$, $a^2 - b^2 = (a+b)(a-b)$, $ab = ((a+b)/2)^2 - ((a-b)/2)^2$ and $na^2 = ((n+1)/2)^2 a^2 - ((n-1)/2)^2 a^2$. They reflect a blending of geometric and subtle algebraic thinking which we associate with Euclid. In fact, the *Śulba* construction of a square equal in area to a given rectangle is exactly the same as given by Euclid several centuries later! There are geometric solutions to what are algebraic and number-theoretic problems. These insights into geometry and geometric algebra show that, during the Vedic age of remote antiquity, Indians had attained mastery over crucial aspects of Euclidean geometry several centuries before Pythagoras (580–495 BC) and Euclid (365–275 BC).

Pythagoras theorem was known in other ancient civilisations like the Babylonian, but the emphasis there was on the numerical and not so much on the proper geometric

3. The result is stated in the *Śulba Sūtras* in the form: “The diagonal of a rectangle produces both [areas] which its length and breadth produce separately.” The original verses are given in ([2], p. 104).

aspect while in the *Śulba Sūtras* one sees depth in both aspects—especially the geometric. This is a subtle point analysed in detail by Seidenberg.⁴ From certain diagrams described in the *Śulba Sūtras*, several historians and mathematicians like Burk, Hankel, Schopenhauer, Seidenberg and van der Waerden have concluded that the *Śulba* authors possessed proofs of geometrical results⁵ including the Pythagoras theorem—some of the details are analysed in the pioneering work ([2]) of Datta. One of the proofs of the Pythagoras theorem, easily deducible from the *Śulba* verses, is later described more explicitly by Bhāskara II (1150 AD).

Apart from the knowledge, skill and ingenuity in geometry and geometric algebra, the Vedic civilisation was strong in the computational aspects of mathematics as well—they handled the arithmetic of fractions as well as surds with ease, found good rational approximations to irrational numbers like the square root of 2, and, of course, used several significant results on mensuration.

An amazing feature of all ancient Indian mathematical literature, beginning with the *Śulba Sūtras*, is that they are composed entirely in verses—an incredible feat! This tradition of composing terse *sūtras*, which could be easily memorised, ensured that, in spite of the paucity and perishability of writing materials, some of the core knowledge got orally transmitted to successive generations.

2. Post-Vedic Mathematics

During the period 600 BC–300 AD, the Greeks made profound contributions to mathematics—they pioneered the axiomatic approach that is characteristic of modern mathematics, created the magnificent edifice of Euclidean geometry, founded trigonometry, made impressive beginnings in number theory, and brought out the intrinsic beauty, elegance and grandeur of pure mathematics. Based on the solid foundation provided by Euclid, Greek geometry soared further into the higher geometry of conic sections developed by Archimedes and Apollonius. Archimedes introduced integration and made several other major contributions in mathematics and physics. But after this brilliant phase of the Greeks, creative mathematics virtually came to a halt in the West till the modern revival.

On the other hand, the Indian contribution, which began from the earliest times, continued vigorously right up to the 16th century AD, especially in arithmetic, algebra and trigonometry. In fact, for several centuries after the decline of the Greeks, it was only in India, and to some extent China, that one could find an abundance of creative and original mathematical activity. Indian mathematics was highly esteemed by

4. Writes Seidenberg ([11], p. 120): "...the basic point is that the dominant aspect of Old Babylonian mathematics is its computational character... The *Śulvasūtras* know both aspects [geometric and computational] and so does the *Śatapatha Brāhmaṇa*."

5. Referring to a verse in the *Āpastamba Śulba Sūtra* on an isosceles trapezoid, Seidenberg writes ([10], p. 332): "The striking thing here is that we have a proof. One will look in vain for such things in Old-Babylonia. The Old-Babylonians, or their predecessors, must have had proofs of their formulae, but one does not find them in Old-Babylonia."

contemporary scholars. A manuscript of 976 AD by Vigila, a monk in a Spanish monastery, records ([6], p. 362; [7], p. 313):

The Indians have an extremely subtle and penetrating intellect, and when it comes to arithmetic, geometry and other such advanced disciplines, other ideas must make way for theirs. The best proof of this is the nine symbols with which they represent each number no matter how large.

That the fame of Indian mathematics had reached the banks of the Euphrates by early 7th century is shown by a passing reference in a passage in the work of the learned Syrian astronomer-monk Severus Sebokht (662 AD):

I shall not now speak of the knowledge of the Hindus, ...of their subtle discoveries in the science of astronomy—discoveries even more ingenious than those of the Greeks and Babylonians—of their rational system of mathematics, or of their method of calculation which no words can praise strongly enough—I mean the system using nine symbols.⁶

2.1. The Decimal Notation and Arithmetic

India gave to the world a priceless gift—the decimal system. This profound anonymous Indian innovation is unsurpassed for sheer brilliance of abstract thought and utility as a practical invention. The decimal notation derives its power mainly from two key strokes of genius: the concept of place-value and the notion of zero as a digit. G. B. Halsted highlighted the power of the place-value of zero with a beautiful imagery ([5], p. 20):

The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local habitation and a name, a picture, a symbol, but helpful power, is the characteristic of the Hindu race whence it sprang. It is like coining the Nirvana into dynamos. No single mathematical creation has been more potent for the general on-go of intelligence and power.

The decimal system has a deceptive simplicity as a result of which children all over the world learn it even at a tender age.⁷ It has an economy in the number of symbols used as well as the space occupied by a written number, an ability to effortlessly express arbitrarily large numbers and, above all, computational facility. Thus the

6. Quoted by A. L. Basham in "The Wonder That Was India" (p. vi); also quoted in [6], p. 366; [7], pp. 311-2.

7. In the perceptive words of the great French mathematician Laplace (1814): "The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculations and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of antiquity: Archimedes and Apollonius." ([6], p. 361).

twelve-digit Roman number (DCCCLXXXVIII) is simply 888 in the decimal system!

Most of the standard results in basic arithmetic are of Indian origin. This includes neat, systematic and straightforward techniques of the fundamental arithmetic operations: addition, subtraction, multiplication, division, taking squares and cubes, and extracting square and cube roots; the rules of operations with fractions and surds; various rules on ratio and proportion like the rule of three; and several commercial and related problems like income and expenditure, profit and loss, simple and compound interest, discount, partnership, computations of the average impurities of gold, speeds and distances, and the mixture and cistern problems similar to those found in modern texts.⁸ The Indian methods of performing long multiplications and divisions were introduced in Europe as late as the 14th century AD. We have become so used to the rules of operations with fractions that we tend to overlook the fact that they contain ideas which were unfamiliar to the Egyptians, who were generally proficient in arithmetic, and the Greeks, who had some of the most brilliant minds in the history of mathematics. The rule of three, brought to Europe via the Arabs, was very highly regarded by merchants during and after the Renaissance. It came to be known as the Golden Rule for its great popularity and utility in commercial computations—much space used to be devoted to this rule by the early European writers on arithmetic.

The excellence and skill attained by the Indians in the foundations of arithmetic was primarily due to the advantage of the early discovery of the decimal notation—the key to all principal ideas in modern arithmetic. For instance, the modern methods for extracting square and cube roots, described by Āryabhaṭa in the 5th century AD, cleverly use the ideas of place-value and zero and the algebraic expansions of $(a + b)^2$ and $(a + b)^3$. These methods were introduced in Europe only in the 16th century AD. Apart from the exact methods, Indians also invented several ingenious methods for determination of approximate square roots of non-square numbers.

Due to the gaps in our knowledge about the early phase of post-Vedic Indian mathematics, the precise details regarding the origin of decimal notation is not known. The concept of zero existed by the time of Piṅgala (3rd century BC or earlier). The idea of place-value had been implicit in ancient Sanskrit terminology⁹—as a result, Indians could effortlessly handle large numbers right from the Vedic Age. There is terminology for all multiples of ten up to 10^{18} in early Vedic literature, the *Rāmāyaṇa* has terms all the way up to 10^{55} , and the Jaina-Buddhist texts show frequent use of large numbers (up to 10^{140} !) for their measurements of space and time. Expressions of such large numbers are not found in contemporary works of other nations. Even the brilliant Greeks had no terminology for denominations above the myriad (10^4) while the Roman terminology stopped with the mille (10^3). The structure of the

8. It may be mentioned here that chess, the most intellectual of all games, originated in India.

9. This point will be elaborated, with quotations from original Vedic verses, in a future issue of *Mother India*.

Sanskrit numeral system and the Indian love for large numbers must have triggered the creation of the decimal system.

We mention here that one of the most brilliant landmarks in ancient Indian mathematics was an algorithm for finding the positive integers satisfying $x^2 - Dy^2 = 1$ (D a fixed natural number)—an important equation in modern number theory.¹⁰ Even the smallest positive integral solution of such equations could be very large; in fact, for $D = 61$, it is (1766319049, 226153980).¹¹ The early Indian solution to this fairly deep problem could be partly attributed to the Indians' traditional fascination for large numbers and ability to play with them.

Due to the absence of good notations, the Greeks were not strong in the computational aspects of mathematics—one of the factors responsible for the eventual decline of Greek mathematics.¹² Archimedes (287–212 BC) did realise the importance of good notation, and made notable progress to evolve one, but failed to anticipate the Indian decimal system.

The decimal system was transmitted to Europe through the Arabs. The Sanskrit word “*sūnya*” was translated into Arabic as “*sifr*” which was introduced into Germany in the 13th century as “*cifra*” from which we have the word “*cipher*”. The word “*zero*” probably comes from the Latinised form “*zephirum*” of the Arabic *sifr*. Leonardo Fibonacci of Pisa (1180-1240), the first major European mathematician of the second millennium, played a major role in the spread of the Indian numeral system in Europe. The Indian notation and arithmetic eventually got standardised in Europe during the 16th–17th century.¹³

The decimal system stimulated and accelerated trade and commerce as well as astronomy and mathematics. It is no coincidence that the mathematical and scientific renaissance began in Europe only after the Indian notation was adopted. Indeed the decimal notation is the very pillar of all modern civilisation.

India has given to antiquity the earliest scientific physicians, and, according to Sir William Hunter, she has even contributed to modern medical science by the discovery of various chemicals and by teaching you how to reform misshapen ears and noses. Even more it has done in mathematics, for algebra, geometry, astronomy, and the triumph of modern science—mixed mathematics—were all invented in India, just so much as the ten numerals, the very cornerstone of all

10. Brahmagupta gave a remarkable partial solution to the problem in 628 AD; Jayadeva gave a complete solution within 11th century. In 1657, Fermat aroused interest in the problem among European mathematicians. André Weil, one of the giants of 20th century mathematics, remarked: “What would have been Fermat's astonishment if some missionary, just back from India, had told him that his problem had been successfully tackled there by native mathematicians almost six centuries earlier!” (*Number Theory: An approach through history*, pp. 81–82.)

11. The case $D = 61$ was specially highlighted by Bhāskara II (1150) and later by Fermat (1657) when discussing the general problem.

12. The February 2002 issue of *Srinvantu* contains a brief discussion on Greek mathematics and its decline.

13. The August 2002 issue of *Srinvantu* contains some details regarding decimal system and arithmetic in ancient India. An expanded version of the article is being planned for *Mother India*.

present civilisation, were discovered in India, and are, in reality, Sanskrit words. [Swami Vivekananda (Collected Works Vol II, pp. 511-12)]

2.2. Algebra

While sophisticated geometry emerged during the origin of the Vedic rituals, its axiomatisation and further development was done by the Greeks. The height reached by the Greeks in geometry by the time of Apollonius (260-170 BC) was not matched by any subsequent ancient or medieval civilisation. But progress in geometry proper soon reached a point of stagnation. Between the times of Pappus (300 AD)—the last big name in Greek geometry—and modern Europe, Brahmagupta's brilliant theorems (628 AD) on cyclic quadrilaterals constitute the solitary gems in the history of geometry. Further progress needed new techniques, in fact a completely new approach in mathematics. This was provided by the emergence and development of a new discipline—algebra. It is only after the establishment of an algebra culture in European mathematics during the 16th century AD that a resurgence began in geometry through its algebraisation by Descartes and Fermat in early 17th century. In fact, the assimilation and refinement of algebra had also set the stage for the remarkable strides in number theory and calculus in Europe from the 17th century.

Algebra was only implicit in the mathematics of several ancient civilisations till it came out in the open with the introduction of literal or symbolic algebra in India.¹⁴ By the time of Āryabhaṭa (499 AD) and Brahmagupta (628 AD), symbolic algebra had evolved in India into a distinct branch of mathematics and became one of its central pillars. After evolution through several stages, algebra has now come to play a key role in modern mathematics both as an independent area in its own right as well as an indispensable tool in other fields. In fact, the 20th century witnessed a vigorous phase of “algebraisation of mathematics”. Algebra provides elegance, simplicity, precision, clarity and technical power in the hands of the mathematicians. It is remarkable how early the Indians had realised the significance of algebra and how strongly the leading Indian mathematicians like Brahmagupta (628 AD) and Bhāskara II (1150 AD) asserted and established the importance of their newly-founded discipline as we shall see in subsequent articles.

Indians began a systematic use of symbols to denote unknown quantities and arithmetic operations. The four arithmetic operations were denoted by “*yu*”, “*kṣa*”,¹⁵

14. In the The Preface of [4] Vol. II, Datta-Singh explains: “The use of symbols—letters of the alphabet to denote unknowns—and equations are the foundations of the science of algebra. The Hindus were the first to make systematic use of the letters of the alphabet to denote unknowns. They were also the first to classify and make a detailed study of equations. Thus they may be said to have given birth to the modern science of algebra.”

H. Hankel wrote ([1], p. 94): “Indeed, if one understands by algebra the application of arithmetical operations to complex magnitudes of all sorts, whether rational or irrational numbers or space-magnitudes, then the learned Brahmins of Hindostan are the real inventors of algebra.”

15. The symbol “+” was sometimes used in ancient India as symbol for subtraction. B. Datta believes that the ancient Indian “+” was a simplified form of the letter *kṣa* ([4] Vol. II, pp. 14–15).

“*gu*” and “*bhā*” which are the first letters of the corresponding Sanskrit words *yuta* (addition), *kṣaya* (subtraction), *guṇa* (multiplication) and *bhāga* (division); similarly “*mū*” or “*ka*” was used for *mūla* or *karaṇī* (root), while the first letters of the names of different colours were used to denote different unknown variables. This introduction of symbolic representation was an important step in the rapid advancement of mathematics. While a rudimentary use of symbols can also be seen in the Greek texts of Diophantus, it is in India that algebraic formalism achieved full development.

The Indians classified and made a detailed study of equations (which were called *samīkaraṇa*), introduced negative numbers together with the rules for arithmetic operations involving zero and negative numbers, discovered results on surds, described solutions of linear and quadratic equations, gave formulae for arithmetic and geometric progression as well as identities involving summation of finite series, and applied several useful results on permutation and combinations including the formulae for ${}^n P_r$ and ${}^n C_r$.

The enlargement of the number system to include negative numbers was a momentous step in the development of mathematics. Thanks to the early recognition of the existence of negative numbers, the Indians could give a unified treatment of the various forms of quadratic equations (with positive coefficients), i.e., $ax^2 + bx = c$, $ax^2 + c = bx$, $bx + c = ax^2$. The Indians were the first to recognise that a quadratic equation has two roots. Śrīdharaçārya (750 AD) gave the well-known method of solving a quadratic equation by completing the square—an idea with far-reaching consequences in mathematics. The Pascal’s triangle for quick computation of ${}^n C_r$ is described by Halāyudha around the 11th century AD as *Meru-Prastāra* six centuries before it was stated by Pascal; and Halāyudha’s *Meru-Prastāra* was only a clarification of a rule invented by Piṅgala more than a millennium earlier!¹⁶

Thus, as in arithmetic, many topics in high-school algebra had been systematically developed in India. This knowledge went to Europe through the Arabs. The word *yava* in *Āryabhaṭīyabhāṣya* of Bhāskara I (around 6th century AD) meaning “to mix” or “to separate” has affinity with that of *aljabr* of al-Khwarizmi (825 AD) from which the word algebra is derived. In his widely acclaimed text on history of mathematics, Cajori ([1], p. 97) concludes the chapter on India with the following remarks:

...it is remarkable to what extent Indian mathematics enters into the science of our time. Both the form and the spirit of the arithmetic and algebra of modern times are essentially Indian. Think of our notation of numbers, brought to perfection by the Hindus, think of the Indian arithmetical operations nearly as

16. Piṅgala (prior to 3rd century BC), in his analysis of *chhandas* (metres), had also introduced the binary representation of numbers two millenniums before the great German philosopher-mathematician Leibniz (1695). Binary numbers is an essential feature in the working of the digital computer. This feat of Piṅgala will be discussed in a future issue of *Mother India*.

also discovered the beautiful formula

$$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \dots,$$

obtained by putting $\theta = \pi/4$ in the Mādhava-Gregory series. This series was rediscovered three centuries later by Leibniz (1674). As one of the first applications of his newly invented calculus, Leibniz was thrilled at the discovery of this series which was the first of the results giving a connection between π and unit fractions. Mādhava also described the series

$$\pi/\sqrt{12} = 1 - 1/3.3 + 1/5.3^2 - 1/7.3^3 + \dots$$

first given in Europe by A. Sharp (1717). Again, three hundred years before Newton (1676 AD), Mādhava had described the well-known power series expansions

$$\sin x = x - x^3/3! + x^5/5! - \dots \text{ and } \cos x = 1 - x^2/2! + x^4/4! - \dots$$

These series were used to construct accurate sine and cosine tables for calculations in astronomy. Mādhava's values are correct, in almost all cases, to the eighth or ninth decimal place—such an accuracy was not to be achieved in Europe within three centuries. Mādhava's results show that calculus and analysis had reached remarkable depth and maturity in India centuries before Newton (1642-1727) and Leibniz (1646-1716). Mādhavācārya might be regarded as the first mathematician who worked in analysis!¹⁹ Unfortunately, the original texts of several outstanding mathematicians like Śrīdhara, Padmanābha, Jayadeva and Mādhava have not been found yet—it is only through the occasional reference to some of their results in subsequent commentaries that we get a glimpse of their work.²⁰ Mādhava's contributions are mentioned in several later texts including the *Tantrasaṅgraha* (1500) of the great astronomer Nīlakaṇṭha (1445-1545) who gave the heliocentric model before Copernicus, the *Yuktibhāṣa* (1540) of Jyeṣṭhadeva (1500-1610) and the *Karaṇapaddhati* of Putumana Somayāji. All these texts themselves were discovered by Charles Whish and published only in 1835. Among ancient mathematicians whose texts have been found, special mention may be made of Āryabhaṭa, Brahmagupta and Bhāskara II.

19. Mādhava also gave correction terms for the remainder in case one breaks off the infinite series after a partial sum of sufficiently high order; e.g., he gave $\pi/4 \sim 1 - 1/3 + 1/5 - \dots \pm 1/n \mp R_n$; his estimates for R_n include $R_n = (n+1)/2$, $R_n = [(n+1)/2]/[(n+1)^2 + 1]$, $R_n = [((n+1)/2)^2 + 1]/[(n+1)^2 + 4 + 1](n+1)/2$.

20. Jayadeva's verses on the brilliant cakravāla method for solving the Diophantine equation $x^2 - Dy^2 = 1$ have been quoted in the text *Sundarī* of Udaydivākara (1073 AD). This text was discovered only in 1954 by K.S. Shukla. The algebraist Jayadeva is not to be confused with the 12th century Vaiṣṇava poet who composed *Gīta-Govinda*.

3. Later Developments

Let us remember... that she [India] was the mother of our philosophy; mother, through the Arabs, of much of our mathematics; mother, through the Buddha, of the ideals embodied in Christianity; mother, through the village community, of self-government and democracy. Mother India is, in many ways, the mother of us all. [Will Durant in *The Case for India* (1930)]

The Indian contributions in arithmetic, algebra and trigonometry were transmitted by the Arabs and Persians to Europe. The Arabs also preserved and transmitted the Greek heritage. After more than a thousand years of slumber, Europe rediscovered its rich Greek heritage and acquired some of the fruits of the phenomenal Indian progress. It is on the foundation formed by the blending of the two great mathematical cultures—the geometric and axiomatic tradition of the Greeks and the algebraic and computational tradition of the Indians—that the mathematical renaissance took place in Europe.

However, Indians virtually took no part in the rapid development of mathematics that took place during 17th–19th century—this period coincided with the general stagnation in the national life. Thus, while high-school mathematics, especially in arithmetic and algebra, is mostly of Indian origin, one rarely comes across Indian names in college and university courses as most of that mathematics was created during the period ranging from late 17th to early 20th century. But should we forget the culture and greatness of India's millenniums because of the ignorance and weakness of a few centuries?

Appendix: 20th century

André Weil predicted in 1936:

The intellectual potentialities of the Indian nation are unlimited and not many years would perhaps be needed before India can take a worthy place in world Mathematics.

Indians made significant contributions in several frontline areas of mathematics during the 20th century, especially during the second half, although this fact is not so well-known even among mathematics students partly because the frontiers of mathematics have expanded far beyond the scope of the university curricula. The two greatest Indian mathematicians of the 20th century were Ramanujan (1887–1920) and Harish-Chandra (1923–1983). As Harish-Chandra's name remains unfamiliar, we quote from a few tributes which could give the general reader some idea regarding the greatness of this outstanding mathematician who was a colossus in his field. (For more details, see December 1993 issue of the journal *Current Science*.) André Weil said that he knew only two mathematicians for whom technical difficulties simply did not exist:

Chevalley and Harish-Chandra. R. P. Langlands, himself a brilliant mathematician, found Harish-Chandra's analytic power and algebraic facility unsurpassed in his experience. In the *Bibliographical Memoirs of FRS 1985*, Langlands described Harish-Chandra's main work as

a Gothic cathedral, heavily buttressed below, but, in spite of its great weight, light and soaring in its upper reaches, coming as close to heaven as mathematics can.

In a lecture (1984) on Harish-Chandra at the Institute of Advanced Study (Princeton) where Harish-Chandra worked, V. S. Varadarajan remarked:

The originality and depth of his work will compel later generations to confer on him that luminous distinction reserved only for the most exalted figures of science. I do not believe that any of us here will ever again come across someone quite like him. In the austere simplicity and uncompromising nature of his approach to life, in his preference for solitary and profound reflection, and in his awesome capacity to discern and persevere after distant goals, he resembled the legendary figures from his country's ancient past.

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Suggested Reading:

1. F. Cajori: *History of Mathematics*, Macmillan (1919); AMS Chelsea (2000).
2. B. Datta: *The Science of the Śulba*, Calcutta University (1932); Cosmo Pub., New Delhi (1993).
3. B. Datta: *Vedic Mathematics*, Chap. 3 of *The Cultural Heritage of India, Vol. VI* (Science and Technology), eds. P. Ray and S. N. Sen, The Ramakrishna Mission Institute of Culture, Calcutta (1986).
4. B. Datta and A. N. Singh: *History of Hindu Mathematics*, Motilal Banarsidass, Lahore (1935–38); Asia Publishing House, Bombay (1962).
5. G. B. Halsted: *On the foundations and techniques of Arithmetic*, Chicago (1912).
6. Georges Ifrah: *The Universal History of Numbers*, John Wiley and Sons (2000).
7. G. G. Joseph: *The Crest of The Peacock: Non-European Roots of Mathematics*, Penguin (1990).
8. John F. Price: *Applied Geometry of the Śulba Sūtras in Geometry at Work*, ed. C. Gorini, MAA, Washington DC (2000).
9. T. A. Sarasvati Amma: *Geometry in Ancient and Medieval India*, Motilal Banarsidass, Delhi (1979; 1999).

10. A. Seidenberg: "The Origin of Mathematics" in *Archive for History of Exact Sciences* (1978).

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As I look back upon the history of my country, I do not find in the whole world another country which has done quite so much for the improvement of the human mind. Therefore I have no words of condemnation for my nation. I tell them, "You have done well; only try to do better." Great things have been done in the past in this land, and there is both time and room for greater things to be done yet ... Our ancestors did great things in the past, but we have to grow into a fuller life and march beyond even their great achievement.

Swami Vivekananda

(*Complete Works*, Vol. III, p. 195)

Mathematics, History and Science

How can mathematics, history or science help me to find you?

They can help in several ways:

1. To become capable of receiving and bearing the light of the Truth, the mind must be made strong, wide and supple. These studies are a very good way to achieve this.

2. If you study science deeply enough, it will teach you the unreality of appearances and thus lead you to the spiritual reality.

3. The study of all the aspects and movements of physical Nature will bring you into contact with the universal Mother, and so you will be closer to me.

The Mother

(*On Education*, CWM, Vol. 12, p. 249)