



INDIAN STATISTICAL INSTITUTE

Theoretical Statistics and Mathematics Unit, Kolkata

Number Theory Seminar

Date: April 17, 2024

Time: 02:15 PM

VENUE:

L-infinity

(5th Floor, A.N. Kolmogorov Bhavan), ISI Kolkata

TITLE:

Trace of Powers of Algebraic Numbers

SPEAKER:

Ravindranathan Thangadurai

HRI, Prayagraj

ABSTRACT:

In 1906, Polya proved that an algebraic number α is an algebraic integer if and only if $\text{Tr}(\alpha^n) \in \mathbb{Z}$ for every natural number n . In 1996, B. de Smit proved that if α is an algebraic number of degree d , and $\text{Tr}(\alpha^n) \in \mathbb{Z}$ for all $n = 1, 2, \dots, d + d \lfloor \log_2 d \rfloor$, then α must be an algebraic integer. In 2004, Corvaja and Zannier proved that if $\text{Tr}(\alpha^n) \in \mathbb{Z} \setminus \{0\}$ for infinitely many natural numbers n , then α must be an algebraic integer. In 2021, P. Phillippon and P. Rath proved that if $\text{Tr}(\lambda \alpha^n) \in \mathbb{Z} \setminus \{0\}$ for infinitely many natural number n where λ is a given algebraic number, then α must be an algebraic integer.

In 2023, along with Abhishek Bharatwaj, Aprameyo Pal and Veekesh Kumar, among the other results, we proved the following. Let α be an algebraic number and K be the Galois closure of $\mathbb{Q}(\alpha)$ with Galois group G . Let $\bar{\mathbb{Q}}[G]$ be the group ring. If $f \in K[G]$ is a non-zero element and $f(\alpha^n) \in \mathbb{Z} \setminus \{0\}$ for infinitely many natural number n , then α must be an algebraic integer. In this talk, we shall discuss this work.

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