Theoretical Statistics and Mathematics Unit, Kolkata
INDIAN STATISTICAL INSTITUTE

SEMINAR

Date: July 01, 2025
Time: 02:00 PM

VENUE:

L- Infinity
(5th Floor, A.N. Kolmogorov Bhavan), ISI Kolkata

TITLE:

The Fundamental Groupoid in Discrete Homotopy Theory

SPEAKER:

Udit Ajit Mavinkurve
Stat-Math Unit, ISI Kolkata

ABSTRACT:

Discrete homotopy theory is a homotopy theory designed for studying graphs, detecting combinatorial (rather than topological) holes. Central to this theory are the discrete homotopy groups, defined using maps out of grids of suitable dimensions. Of these, the discrete fundamental group in particular has found applications in various areas of mathematics, including matroid theory, subspace arrangements, and topological data analysis.

In this talk, based on joint work with Chris Kapulkin, we introduce the discrete fundamental groupoid, a multi-object generalization of the discrete fundamental group, and use it as a starting point to develop some robust computational techniques. A new notion of covering graphs allows us to extend the existing theory of universal covers to all graphs, and to prove a classification theorem for coverings. We also prove a discrete version of the Seifert–van Kampen theorem, generalizing a previous result of H. Barcelo et al. We then use it to solve the realization problem for the discrete fundamental group through a purely combinatorial construction.

Currently, a central open problem in the field is to determine whether the cubical nerve functor, which associates a cubical Kan complex to a graph is a DK-equivalence of relative categories. If true, this would allow the import of results like the Blakers-Massey theorem from classical homotopy theory to the discrete realm. We propose a new line of attack, by breaking it into more tractable problems comparing the homotopy theories of the respective n-types, for each integer $n \geq 0$. We also solve this problem for the first nontrivial case, $n = 1$.

ALL ARE CORDIALLY INVITED