

**HIGHER ORDER ELLIPTIC PROBLEMS WITH CRITICAL
SOBOLEV GROWTH ON A COMPACT RIEMANNIAN
MANIFOLD: BEST CONSTANTS AND EXISTENCE**

SAIKAT MAZUMDAR

ABSTRACT

We investigate the existence of solutions to a nonlinear elliptic problem involving the critical Sobolev exponent for a Polyharmonic operator on a Riemannian manifold M . We first show that the best constant of the Sobolev embedding on a manifold can be chosen as close as one wants to the Euclidean one, and as a consequence derive the existence of minimizers when the energy functional goes below a quantified threshold. Next, higher energy solutions are obtained by Coron's topological method, provided that the minimizing solution does not exist and the manifold satisfies a certain topological assumption. To perform the topological argument, we obtain a decomposition of Palais-Smale sequences as a sum of bubbles and adapt Lions's concentration-compactness lemma.

INSTITUT ELIE CARTAN DE LORRAINE, UNIVERSITÉ DE LORRAINE, FRANCE
E-mail address: saikat.mazumdar@univ-lorraine.fr