



**Theoretical Statistics and Mathematics Unit, Kolkata**  
**INDIAN STATISTICAL INSTITUTE**

**Seminar**

**Date: May 16, 2025**  
**Time: 4:15 PM**

**VENUE:**

**L-infinity**

**(5<sup>th</sup> Floor, A.N. Kolmogorov Bhavan), ISI Kolkata**

**TITLE:**

**Absence of  $L^p$  eigenfunctions for diffusion operators in regions with cavities**

**SPEAKER:**

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**ABSTRACT:**

In his seminal 1943 paper F. Rellich proved that, in the complement of a cavity  $\Omega = \{x \in \mathbb{R}^n \mid |x| > R_0\}$ , there exist no nontrivial solution  $f$  of the Helmholtz equation  $\Delta f = -\lambda f$ , when  $\lambda > 0$ , such that  $\int_{\Omega} |f|^2 dx < \infty$ . It turns out quite remarkably that from an estimate in that same paper of Rellich, it follows that there are no  $L^p$  eigenfunctions for all  $p \in (0, \frac{2n}{n-1}]$ . This result is sharp since for any  $p > \frac{2n}{n-1}$ ,  $L^p$  eigenfunctions do exist in  $\Omega$ . In this talk, I plan to describe the original proof of Rellich based on spherical harmonics and then present an alternate approach which extends to a class of variable coefficient diffusion operators. This is based on some recent joint works with Nicola Garofalo.

**ALL ARE CORDIALLY INVITED**