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Optimal Inference with a Multidimensional Multiscale Statistic

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Abstract: We observe a stochastic process Z on $[0,1]^d$ ($d \geq 1$) satisfying

$$dZ(t) = n^{1/2} f(t)dt + dW(t), \qquad t \in [0,1]^d,$$

where $n \geq 1$ is a given scale parameter ('sample size'), W is a standard Brownian sheet on $[0,1]^d$ and $f \in L_1([0,1]^d)$ is the unknown function of interest. We propose a multivariate multiscale statistic in this setting and prove its almost sure finiteness; this extends the work of Dümbgen and Spokoiny [1] who proposed the analogous statistic for $d = 1$. We use the proposed multiscale statistic to construct optimal tests for testing $f = 0$ versus (i) appropriate Hölder classes of functions, and (ii) alternatives of the form $f = \mu_n \mathbb{I}_{B_n}$, where B_n is a rectangle in $[0,1]^d$ with sides parallel to the coordinate axes and $\mu_n \in \mathbb{R}$; μ_n and B_n unknown.

Utilizing these tests we construct confidence bands for f with guaranteed coverage probability, assuming that the underlying function f is shape-restricted, e.g., (multidimensional) isotonic or convex. These confidence bands are shown to be *adaptive* and asymptotically *optimal* in an appropriate sense.

References

[1] Dümbgen, L. and Spokoiny, V. G. (2001). Multiscale testing of qualitative hypotheses. *Ann. Statist.*, 29(1):124–152.

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