INDIAN STATISTICAL INSTITUTE

Theoretical Statistics and Mathematics Unit, Kolkata

LECTURE

Date: May 30, 2023, Tuesday
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TITLE:

Characterisation of the affine plane over field of characteristic zero and some remarks in positive characteristics using A^1-homotopy theory

SPEAKER:

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ABSTRACT:

Characterisation of the affine n-space is one of the major problem in affine algebraic geometry. Miyanishi showed an affine complex surface X is isomorphic to $\mathbb{C}^2$ if $\mathcal{O}(X)$ is a U.F.D., $\mathcal{O}(X)^* = \mathbb{C}^*$ and X has a non-trivial $\mathbb{G}_a$-action [3, Theorem 1]. Since the orbits of a $\mathbb{G}_a$-action are affine lines, existence of a non-trivial $\mathbb{G}_a$-action says that there is a non-constant $\mathbb{A}^1$ in X. Ramanujam showed that a smooth complex surface is isomorphic to $\mathbb{C}^2$ if it is topologically contractible and it is simply connected at infinity [5]. Topological contractibility, in particular pathconnectedness says that there are non-constant intervals in X. On the other hand, $\mathbb{A}^1$-homotopy theory has been developed by F.Morel and V.Voevodsky [4] as a connection between algebra and topology. An algebro-geometric analogue of topological connectedness is $\mathbb{A}^1$-connectedness. In this talk, using ghost homotopy techniques [2, Section 3] we will prove that if a complex surface X is $\mathbb{A}^1$-connected, then there is an open dense subset such that through every point there is a non-constant $\mathbb{A}^1$ in X which implies X has negative logarithmic Kodaira dimension. As a consequence using the algebraic characterisation, we will prove that $\mathbb{A}^2$ is the only $\mathbb{A}^1$-contractible smooth affine surface over a field of characteristic zero. This answers the conjecture appeared in [1, Conjecture 5.2.3]. We will also see some other useful consequences of this result. However, we will also see that there are some surfaces over field of positive characteristics which are $\mathbb{A}^1$-connected but has non-negative logarithmic Kodaira dimension. This is a joint work with Prof. Utsav Choudhury.

ALL ARE CORDIALLY INVITED

Characterisation of the affine plane over field of characteristic zero and some remarks in positive characteristics using $\mathbb{A}^1$-homotopy theory

Abstract

Characterisation of the affine n-space is one of the major problem in affine algebraic geometry. Miyanishi showed an affine complex surface X is isomorphic to $\mathbb{C}^2$ if $\mathcal{O}(X)$ is a U.F.D., $\mathcal{O}(X)^* = \mathbb{C}^*$ and X has a non-trivial $\mathbb{G}_a$-action [3, Theorem 1]. Since the orbits of a $\mathbb{G}_a$-action are affine lines, existence of a non-trivial $\mathbb{G}_a$-action says that there is a non-constant $\mathbb{A}^1$ in X. Ramanujam showed that a smooth complex surface is isomorphic to $\mathbb{C}^2$ if it is topologically contractible and it is simply connected at infinity [5]. Topological contractibility, in particular pathconnectedness says that there are non-constant intervals in X. On the other hand, $\mathbb{A}^1$-homotopy theory has been developed by F.Morel and V.Voevodsky [4] as a connection between algebra and topology. An algebro-geometric analogue of topological connectedness is $\mathbb{A}^1$-connectedness. In this talk, using ghost homotopy techniques [2, Section 3] we will prove that if a complex surface X is $\mathbb{A}^1$-connected, then there is an open dense subset such that through every point there is a non-constant $\mathbb{A}^1$ in X which implies X has negative logarithmic Kodaira dimension. As a consequence using the algebraic characterisation, we will prove that $\mathbb{A}^2$ is the only $\mathbb{A}^1$-contractible smooth affine surface over a field of characteristic zero. This answers the conjecture appeared in [1, Conjecture 5.2.3]. We will also see some other useful consequences of this result. However, we will also see that there are some surfaces over field of positive characteristics which are $\mathbb{A}^1$-connected but has non-negative logarithmic Kodaira dimension. This is a joint work with Prof. Utsav Choudhury.

References

[1] A. Asok, P. A. Østvær; $\mathbb{A}^1$-*homotopy theory and contractible varieties: a survey*, Homotopy Theory and Arithmetic Geometry – Motivic and Diophantine Aspects. Lecture Notes in Mathematics, vol 2292. Springer, Cham. https://doi.org/10.1007/978-3-030-78977-05.

[2] C. Balwe, A. Hogadi and A. Sawant; $\mathbb{A}^1$-*connected components of schemes.* Adv Math, Volume 282, 2016.

[3] M. Miyanishi; *An algebraic characterization of the affine plane.* J. Math. Kyoto Univ. 15-1 (1975) 19-184.

[4] F. Morel, V. Voevodsky; $\mathbb{A}^1$-*homotopy theory of schemes*, Publications Mathématiques de l'IHES 90 (1990) p. 45-143.

[5] C. P. Ramanujam; *A topological characterisation of the affine plane as an algebraic variety*, Ann. of Math. (2) 94 (1971) 69–88.