Theoretical Statistics and Mathematics Unit, Kolkata

INDIAN STATISTICAL INSTITUTE

Thesis Defence Seminar

Date: April 11, 2025
Time: 12:00 Noon

VENUE:

L - Infinity

(5th Floor, A.N. Kolmogorov Bhavan), ISI Kolkata

TITLE:

Problems in Affine Algebraic Geometry: On triviality and embedding of Linear Hyperplanes and on rigidity of Pham-Brieskorn Surfaces

SPEAKER:

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ABSTRACT:

Linear hypersurfaces over a field k have been playing a central role in the study of some of the challenging problems on affine spaces. Breakthroughs on such problems have occurred by examining two difficult questions on linear polynomials of the form

$$H := \alpha(X_1, \ldots, X_m)Y - F(X_1, \ldots, X_m, Z, T) \in D := k[X_1, \ldots, X_m, Y, Z, T] :$$

(i) Whether H defines a closed embedding of $\mathbb{A}^{m+2} \hookrightarrow \mathbb{A}^{m+3}$, i.e., whether the affine variety $\mathbb{V} \subseteq \mathbb{A}^{m+3}$ defined by H is isomorphic to $\mathbb{A}^{m+2}$.

(ii) If H defines a closed embedding $\mathbb{A}^{m+2} \hookrightarrow \mathbb{A}^{m+3}$ then whether H is a coordinate in D.

Question (i) connects to the Characterization Problem of identifying affine spaces among affine varieties; Question (ii) is a special case of the formidable Epimorphism/Embedding Problem for affine spaces.

In the first part of the talk we shall address some recent developments on the above two questions.

We shall exhibit several families of linear hypersurfaces of the form H, which satisfies the Abhyankar–Sathaye Conjecture on the Epimorphism/Embedding Problem.

We will also provide some criterion which enables one to recognise a large family of affine varieties non-isomorphic to an affine space.

Our results in positive characteristic yield new infinite family of counterexamples in higher dimensions ($\geqslant 3$) to the Zariski Cancellation Problem.

This is a joint work with Neena Gupta and Parnashree Ghosh.

In the second portion of the talk, we shall discuss the rigidity of Pham-Brieskorn surfaces of the form

$$B_{(a,b,c)} := \frac{k[X,Y,Z]}{(X^a + Y^b + Z^c)},$$

where a, b, c are positive integers and k is a field of arbitrary characteristic.

We shall give some sufficient conditions on (a, b, c) for the rigidity and the stable rigidity of $B_{(a,b,c)}$.

The former gives an alternative approach to show that there does not exist any non-trivial exponential map on

$$\frac{k[X,Y,Z,T]}{(X^mY + T^{p^r q} + Z^{p^e})} = k[x,y,z,t], \quad \text{for} \quad m, q > 1, p \nmid mq \text{ and } e > r \geqslant 1,$$

which fixes y, a crucial result used by Neena Gupta to show that some Asanuma's threefolds provide counterexample to the Zariski Cancellation Problem for the affine 3-space in positive characteristic.

This is a joint work with Neena Gupta.

ALL ARE CORDIALLY INVITED