INDIAN STATISTICAL INSTITUTE

Theoretical Statistics and Mathematics Unit, Kolkata

Pre-Thesis Submission Seminar

Date: June 07, 2024
Time: 04:00 PM

VENUE:

L-infinity

(5th Floor, A.N. Kolmogorov Bhavan), ISI Kolkata

TITLE:

Problems in Affine Algebraic Geometry:On triviality and embedding of Linear Hyperplanes and on rigidity of Pham-Brieskorn Surfaces

SPEAKER:

Ananya Pal

Stat-Math Unit, ISI Kolkata

ABSTRACT:

Attached below.

ALL ARE CORDIALLY INVITED

ABSTRACT:

Linear varieties over a field k have been playing a central role in the study of some of the challenging problems on affine spaces like the Zariski Cancellation Problem and the Linearization Problem. Breakthroughs on such problems have occurred by examining two questions on linear polynomials of the form

$$H := \alpha(X_1, \ldots, X_m)Y - F(X_1, \ldots, X_m, Z, T) \in D := k[X_1, \ldots, X_m, Y, Z, T] :$$

(i) Whether the affine variety $\mathbb{V} \in \mathbb{A}_k^{m+3}$ defined by H is isomorphic to $\mathbb{A}_k^{m+2}$.

(ii) If $\mathbb{V}$ is isomorphic to an affine space, then whether H is a coordinate in D.

Neena Gupta and Parnashree Ghosh had addressed the above questions when α is a monomial of the form $\alpha(X_1, \ldots, X_m) = X_1^{r_1} \ldots X_m^{r_m}$; $r_i > 1$, $1 \leqslant i \leqslant m$ and F is of a certain type.

In the first part of this talk we shall address these questions for a wider family of linear varieties. In particular, we show that when the characteristic of k is zero, α is any non-zero polynomial in $k[X_1, \ldots, X_m]$, $F \in k[Z, T]$ and the affine variety $\mathbb{V}$ is an affine space, then H is a coordinate in D along with $X_1, X_2, \ldots, X_m$. Our results yield certain family of higher-dimensional hyperplanes satisfying the Abhyankar–Sathaye Conjecture on the Epimorphism Problem and an infinite family of higher dimensional non-isomorphic varieties which are counterexamples to the Zariski Cancellation Problem in positive characteristic.

This is a joint work with Neena Gupta and Parnashree Ghosh.

In the second portion of the talk, we shall discuss the rigidity of Pham-Brieskorn surfaces of the form

$$B_{(a,b,c)} := \frac{k[X, Y, Z]}{(X^a + Y^b + Z^c)},$$

where a, b, c are positive integers and k is any field of arbitrary characteristic $p \geqslant 0$.

We give some sufficient conditions on (a, b, c) for the rigidity and the stable rigidity of $B_{(a,b,c)}$. The former gives an alternative approach to show that there does not exist any non-trivial exponential map on $k[X, Y, Z, T]/(X^mY + T^{p^r q} + Z^{p^e}) = k[x, y, z, t]$, for $m, q > 1$, $p \nmid mq$ and $e > r \geqslant 1$, which fixes y, a crucial result used by Neena Gupta to show that the Zariski Cancellation Problem does not hold for the affine 3-space in positive characteristic.

This is a joint work with Neena Gupta.