Granulized Z-VIKOR Model for Failure Mode and Effect Analysis

Ashish Garg, Souvik Das, Jhareswar Maiti, *Member, IEEE*, and Sankar K Pal, *Life Fellow, IEEE*

***Abstract*—In this article, we have developed an improved failure mode and effect analysis (FMEA) model by leveraging the concepts of Z-number, rough number (RN), and probabilistic distance measure. Two new concepts, namely, double upper approximated rough number (DUARN) and granulized Z-number (gZN), a new scheme for measuring distance between two gZNs using weighted similarity and average linkage method, a new risk prioritization model, named, granulized Z-VIKOR, and a scheme for uncertainty assessment using box-plot are proposed. DUARN embodies the notion of double sided upper approximation of an ordinal decision class. gZN is developed using Z-number and DUARN. The distance between two gZNs is computed using the maximum entropy principle that captures the relationship between the A (opinion) and B (reliability of A) parts of gZN. The granulized Z-VIKOR involves synergistic integration of gZN and VIKOR. Both objective and subjective risk measures are computed and a combined risk measure is defined, which considers the interactions among the risk criteria using λ-Shapley index. Two case studies are conducted. Sensitivity and comparative analysis is carried out to demonstrate the applicability, effectiveness, and robustness of the proposed model, as well as its superiority to existing models.**

***Index Terms*—Double upper approximated rough number, granulized Z-number, maximum entropy principle, risk assessment, VIKOR.**

I. Introduction

FAILURE mode and effect analysis (FMEA) is a systematic risk assessment technique for identifying, prioritizing, and eliminating potential failure modes (FMs) of a system, process, or service under study [1]. Starting from aerospace, it is extensively used in other industries like chemical, healthcare, automotive, food, and manufacturing [1]–[4]. Three risk criteria, namely probability (P), severity (S), and detectability (D) are generally used in FMEA to quantify risk priority number (RPN) of a failure mode (FM), where RPN = P∗S∗D. FM with higher RPN represents a higher risk. In spite of widespread applicability and acceptability, many researchers [5]–[8] have criticized the conventional RPN based FMEA method, due to the following deficiencies:

i) The risk criteria, P, S, and D are usually assessed in crisp numbers. The experts face difficulty in assessing them accurately because of the impreciseness of human thoughts.

ii) The formula, RPN = P∗S∗D is questionable as P, S, and D are usually measured on an ordinal scale. Multiplication operation on the ordinal scale is not recommended and may produce misleading results.

iii) Equal weight is assigned to each of the three risk criteria, whereas in practice the weights could be different. Further, under equal weight situation, distinct sets of P, S, and D may lead to the same RPN.

iv) Interactions among the three risk criteria are not considered.

To overcome these deficiencies, several extensions using advanced tools and techniques have been reported. These include, for example, fuzzy set theory (FST) [7], [9]–[12] for deficiency (i), multicriteria decision making (MCDM) techniques [5], [6], [13] for deficiency (ii), combined weighting approaches [5], [14] for deficiency iii), and digraph and matrix approach [15], fuzzy measure, and Shapley index [13] for deficiency (iv). However, these advanced FMEA studies also suffer from the issues of ineffective utilization of information such as closeness to true perception of experts, degree of uncertainty, information reliability (experts' opinions reliability), and information loss during computations [5], [16], [17].

To deal with the issues of closeness to true perception of experts and degree of uncertainty, attempts have been made to use rough number (RN), developed following rough set theory (RST), in FMEA [18]. RST [19] provides a mathematical tool to deal with vagueness, arising from granulation in the universe of discourse. A rough set is characterized by a pair of two concepts, namely, lower approximation and upper approximation of a set defined over a granulated domain. The collection of all granules that certainly belong to the set constitutes what is termed as its lower approximation, whereas the upper approximation of the set denotes those granules that definitely as well as possibly belong to it. Zhai *et al.* [20] suggested that the concept of RN can be used to improve the utilization of information. RN moves the expert's opinion to his/her true perception. Further, it also reduces the subjectivity in risk-related decision-making by defining borders of a set with the help of border approximation areas (lower approximation, upper approximation) and

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Ashish Garg, Souvik Das, and Jhareswar Maiti are with the Department of Industrial and Systems Engineering, and Centre of Excellence in Safety Engineering and Analytics, Indian Institute of Technology Kharagpur, Kharagpur 721302, India (e-mail: ashish.gargnitt@gmail.com; rndas9@gmail.com; jhareswar.maiti@hotmail.com).

Sankar K Pal is with the Center for Soft Computing Research, Indian Statistical Institute, Kolkata 700108, India (e-mail: sankar@isical.ac.in).

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the uncertainty involved in between (boundary region). Some researchers [21] suggested the integration of RN with fuzzy sets (FSs) to overcome the issues of ineffective utilization of information. However, the lower and upper approximations, as defined in the aforesaid RN [20], do not obey the characteristics of those of Pawlak's classical RST [19]. Therefore, there is a need to redefine the concept of RN.

Further, FST [22] uses the concept of membership grades to deal with the vagueness and uncertainties of human judgments. In contrast to the crisp numerical models, it assigns a value in the interval [0, 1] to each element of the universe of discourse. In order to incorporate the reliability of information in the traditional FST, Z-number [23] is proposed by Zadeh in 2011. A Z-number is a two-tuple containing fuzzy numbers (A, B), where A represents the experts' opinions (fuzzy restriction) on the values of a risk criteria P, S, or D, and B denotes the reliability of experts' views (certainty measure of A). Recently, Jiang *et al.* [16] described ZRPN to solve the FMEA problem in the context of Z-numbers. However, they fail to capture the relationship between components A and B. Similarly, Mohsen and Fereshteh [5] conducted Z-number based FMEA. In their work, they converted Z-number into a classical fuzzy number for reducing the computational complexity. However, this conversion incurs some degree of information loss. To compute ZRPN, the components A and B cannot be used separately as these are closely linked via a probability distribution. Further, to reduce the information loss during computation, researchers such as Shen and Wang [24] used goal programming [25], and Das *et al.* [26] used the maximum entropy principle [27] to capture the correlation between A and B parts of a Z-number.

From the above discussion, it is clear that the existing FMEA techniques need to be improved so that the issues related to the ineffective utilization of information can be handled. The contributions of the present investigation are as follows:

First, an improved definition of RN, namely, double upper approximated rough number (DUARN), as applicable to ordered classes, is provided which may be treated as an advanced version of the RN of Zhai *et al.* [20]. The proposed DUARN embodies the sense of "definitely belonging," i.e., lower approximation, and "definitely and possibly belonging," i.e., downward upper and upward upper approximations, corresponding to a class. Second, a new concept, called granulized Z-number (gZN), is developed by the judicious integration of Z-number with DUARN to capture the information reliability and the true perceptual thinking of experts. The *gZN* exploits the advantages of both the fuzzy set (FS) and RN. Third, a scheme is proposed for measuring the distance between two gZNs using weighted similarity [26] and the average linkage method, as used in clustering. The correlation between the A and B parts of the *gZN* is captured using the maximum entropy principle. Fourth, a novel risk prioritization model integrating gZN and VIKOR, named as granulized Z-VIKOR, is described. This augmentation overcomes the deficiencies of the conventional FMEA and its recent advancements, as follows: The use of Z-number and VIKOR removes the aforesaid deficiencies (i) and (ii), respectively. Here, VIKOR is preferred over other MCDM techniques as VIKOR has the capability of ranking FMs in the presence of conflicting and noncommensurable risk criteria (P, S, D). Further, VIKOR assumes that conflict can be resolved by providing a compromised solution closest to the ideal, where the compromise refers to the mutual concessions-based agreement. It also does not require any parametric estimation during computation. To overcome deficiencies (iii) and (iv), both subjective and objective measures along with the Shapley index are used. Finally, the integration of *gZN* with VIKOR addresses the issues of true perception of experts, degree of uncertainty, information reliability, and information loss. Fifth, a scheme for uncertainty assessment using box-plot is formulated as a quantitative measure of uncertainty in risk assessment.

The rest of this article is organized as follows: Section II reviews the related literature. Section III provides some basic definitions and the proposed DUARN. Section IV explains the proposed methodology. Two case studies are conducted. One is presented in Section V and another is presented in the Supplementary file. Sensitivity and comparative analysis are presented in Section VI. Finally, Section VII concludes this article. Some related concepts, pseudocode, and results of the proposed model are presented in the Supplementary file.

II. Literature Review

Over the years, several extensions are proposed to overcome the shortcomings of traditional FMEA (see Section I). In practice, experts always prefer to provide opinions in linguistic terms. Bowles and Pelaez [11] and Chang *et al.* [12] introduced FST in FMEA to handle the experts' linguistic evaluations. After that, to circumvent the issue of reliability of the information, Zadeh [23] proposed the Z-number, and researchers like Mohsen and Fereshteh [5], Jiang *et al.*, [16], Ghoushchi *et al.*, [28], and Das *et al.*, [29] extended the concept of Z-number in FMEA. Some major drawbacks of these studies include

i) the borders of fuzzy numbers are generally determined by experience and subjective opinions of experts which may affect the results of FMEA,
ii) the boundary interval of fuzzy numbers may not be similar, as in most of the cases, because the degree of the vagueness of experts' opinions may vary from expert to expert,
iii) conversion of Z-number into convention fuzzy number may bring information loss,
iv) neglecting the relationship between the opinion and its' reliability may also bring information loss during risk computations,
v) avoidance of experts' true perceptual thinking may affect risk ranking results.

Some other efforts, such as application of RN [18], [30], [31], D number [32], and Dempster–Shafer theory [33], have also been made by researchers to handle the vagueness and uncertainty of risk assessment information in FMEA. Among them, RN has continually been gaining popularity as it reduces the subjectivity in risk assessment by modeling the vagueness of human thoughts mathematically. However, RN-based FMEA studies suffer from the following issues: the existing definition of RN does not follow the definitions of classical RST with respect to lower and upper approximations, most of the studies are based on crisp evaluations, and information reliability is not considered.

Further, subjective, objective, and combined weighing approaches have been proposed by researchers to obtain the importance of P, S, and D [5], [14]. In addition, Wang *et al.* [13] and Liu *et al*. [15] used fuzzy digraph and matrix approach and Shapley index, respectively, to consider the interaction among P, S, and D while computing RPN. However, there is a need to investigate the interactions among risk criteria with reliability induced risk assessment information, while computing their importance based on the combined measure.

Moreover, several MCDM techniques, such as VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR) [5], [14], [29], multiobjective optimization by ratio analysis (MOORA) [28], TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [8], [18], [34], TODIM (an acronym in Portuguese for interactive multicriteria decision making) [13], Elimination Et Choix Traduisantla Realité [6], qualitative flexible multiple criteria method [7] have been employed to improve the traditional FMEA. Among them, VIKOR has advantage over other techniques, as mentioned in the previous section.

Accordingly, the present investigation proposes a new FMEA model using Z-number, DUARN, combined measure, Shapley index, and VIKOR to resolve the above-mentioned issues and risk ranking of FMs. Here, DUARN, a modified version of RN, follows the characteristics of lower and upper approximations of classical RST, and the principle of dominance of dominance-based rough set approach (DRSA) [35]. Then, Z-number is integrated with DUARN to obtain the granulized Z-number (*gZN*). Finally, it is used with VIKOR to prioritize FMs.

III. Preliminaries

In this section, some of the basic definitions related to fuzzy set (FS), Z-number, fuzzy measures, Shapely index, and rough set (RS) are provided along with their concepts. A new definition of RN called DUARN, as applicable to ordered classes, is then provided, and its significance in risk assessment is explained. These are used in developing the proposed methodology (see Section IV).

A. FS and Z-Number

Definition 1 [22]*:* A FS, say A, with its finite number of *supports* $x_1, x_2, \ldots, x_j, \ldots, x_n$ in the universe of discourse X is defined as

$$A = \{\langle x, \mu_A(x)\rangle \,|x \in X\} = \bigcup_j \mu_A(x_j)/x_j, j = 1, 2, \ldots, n. \tag{1}$$

Here, $\mu_A(x) : X \to [0, 1]$ denotes the membership function of A. It associates a number for each element *x* of *X* in the interval [0, 1] that represents the grade of membership of *x* in A. ∪ denotes the combination (union) operator. The *support* of *A* is defined as the set of elements in *X* with positive nonzero membership values. The element x_i in *X* with $\mu_A(x_i) = 0.5$ is said to be the crossover point in *A*.

Definition 2: A FS "A," defined on real line R, is called a discrete fuzzy number, if A has a finite number of supports, and there exist real indices q and r $(1 \le q \le r \le n)$such that

$(i)\, \mu_A(x_j) = 1, \forall j \in [q, r];$
$(ii)\, \mu_A(x_i) \le \mu_A(x_j) < 1, \forall i, j \in [1, q]$ and $(i \le j); and$
$(iii)\, 1 > \mu_A(x_i) \ge \mu_A(x_j), \forall i, j \in [r, n]$ and $(i \le j).$

Here, $\mu_A(x) : R \to [0, 1]$ is a π-type function [36], where the membership value is first nondecreasing with x, attains the maximum value of unity, and then it is nonincreasing with x. Triangular and trapezoidal membership functions are some such examples. Count of fuzzy number A is defined as

$$count(A) = \sum_{j=1}^{n} \mu_A(x_j)\,.$$

Definition 3 [23]*:* A Z-number Z = (A, B) is an ordered pair of fuzzy numbers, where the first component A represents the restriction on a real-valued uncertain variable X and second component B is the reliability measure of A. When X is a random variable with pdf p(x), B can be mathematically expressed as

$$B = P(X\, is\, A) = \sum_{j=1}^{n} \mu_A(x_j)p(x_j). \tag{2}$$

B. Fuzzy Measure

Definition 4 [37]*:* A set function $\psi : G \to [0, 1]$**,** defined on a measurable space(Y, G)**,** is a nonadditive and monotonic fuzzy measure if the following axioms are satisfied:

$$i)\ \psi(\varnothing) = 0, \psi(Y) = 1\,(\text{boundary condition});\ and$$
$$ii)\ \text{if}\ E, F \in G(Y)\ \text{and}\ E \subseteq F, then\, \psi(E) \le \psi(F)\,(\text{monotonicity}).$$

Here, $Y = (y_1, y_2, \ldots, y_n)$ is the parent set, and $G(Y)$is the power set of Y.

The set function$\psi$is said to be a λ-fuzzy measure, if it satisfies the following axiom of continuity:

$$\psi_\lambda(E \cup F) = \psi_\lambda(E) + \psi_\lambda(F) + \lambda\psi_\lambda(E)\psi_\lambda(F) \tag{3}$$

where $\lambda > -1$ for all $E, F \in G(Y)\text{with}E \cap F = \varnothing$.

For finite set Y, $\psi_\lambda$can also be expressed as

$$\psi_\lambda(Y) = \psi_\lambda\left(\bigcup_{i=1}^{n} y_i\right) = \begin{cases} \frac{1}{\lambda}\left[\prod_{i=1}^{n}(1 + \lambda\psi_\lambda(y_i)) - 1\right], & \lambda \ne 0 \\ \sum_{i=1}^{n} \psi_\lambda(y_i), & \lambda = 0 \end{cases}. \tag{4}$$

The value of λ is uniquely determined by the following equation:

$$\lambda + 1 = \prod_{i=1}^{n}(1 + \lambda\psi_\lambda(y_i)). \tag{5}$$

C. Shapley Index

Definition 5 [38], [39], [40]*:* Shapely index is one of the payoff indices that is generally used in cooperative game theory. Let *L* represent the set of players, where$L = \{1, 2, \ldots, l\}$ and *S* represent the number of coalitions in a cooperative game, where *S* is a subset of *L*. Shapley index measures the global importance of the *i*th player in the coalition and is given below

$$\phi_i^{Sh}(\psi_\lambda, L) = \sum_{S \subseteq L \setminus i} \frac{(l-s-1)!s!}{(l)!}[\psi_\lambda(i \cup S) - \psi_\lambda(S)],\ \forall i \in L. \tag{6}$$

Here, ψ_λ is a λ-fuzzy measure on *L* and Σ extends over all subsets *S* of *L* that do not contain the player i. l and s are the cardinalities of L and S, respectively.

D. Rough Set and DUARN

Definition 6 [19]*:* If U is a nonempty finite set of objects and R is an equivalence relation on U, then, (U, R) is known as an approximation space. Let V be a subset of U. The lower approximation($\underline{R}$), upper approximation ($\bar{R}$), and boundary region $(BR)$of V are as follows:

$$\left.\begin{aligned} \underline{R}(V) &= \bigcup_{v \in U} \{R(v) : R(v) \subseteq V\};\ \bar{R}(V) \\ &= \bigcup_{v \in U} \{R(v) : R(v) \cap V \neq \varnothing\}; \\ BR(V) &= \bar{R}(V) - \underline{R}(V) \end{aligned}\right\}. \tag{7}$$

Here, $R(v)$is the equivalence class (called, granules) constituted by the relation *R*. Lower ($\underline{R}$) and upper ($\bar{R}$) approximations signify, respectively, the inner and outer approximations of the set *V* in terms of *R(v).* They represent, respectively, all those *R(v)*$_s$ that definitely belong to *V*, and definitely as well as possibly belong to *V*. If $\bar{R}(V) \neq \underline{R}(V)$, i.e., $BR(V) \neq \varnothing$, then the set *V*is said to be rough with respect to R. This inexact (vague) definition of *V* in *U,* in terms of upper and lower approximations, signifies the incompleteness of knowledge about *U* with respect to the relation *R*.

Definition 7: Here, we discuss on RN following certain characteristics of the aforesaid lower and upper approximations, as applicable to ordered classes. Let U be the universe containing objects $\{x\}$, and let these objects belong to n classes, i.e., $D = \{C_1, C_2, \ldots, C_n\}$. Let the classes be ordered as $C_1 < C_2 < \cdots < C_n$. Then, for any class, $C_i \in D$, $1 \leq i \leq n$, the lower and upper approximations are defined as in (8) and (9), respectively

$$\underline{Apr}(C_i) = \cup\{x \in U/D(x) = C_i\} \tag{8}$$

$$\left.\begin{aligned} &\overline{Apr_{\leq}}(C_i) = \text{Downward union of} \\ &\quad C_i = \cup\{x \in U/D(x) \leq C_i\} \text{ and} \\ &\overline{Apr_{\geq}}(C_i) = \text{Upward union of} \\ &\quad C_i = \cup\{x \in U/D(x) \geq C_i\} \end{aligned}\right\}. \tag{9}$$

Here, *D(x)* denotes the class to which object *x* belongs.

Note that the upper approximation has two regions, represented by the downward and upward unions of C_i. These constitute what may be termed as double upper approximations.

The boundary region of $C_i$is

$$Bnd(C_i) = \{\overline{Apr_{\leq}}(C_i) - \underline{Apr}(C_i)\} \cup \{\overline{Apr_{\geq}}(C_i) - \underline{Apr}(C_i)\}. \tag{10}$$

In (10), the boundary region is a nonempty set. For a single class problem, the boundary region becomes empty.

Based on the above definitions, the following properties are obtained:

$i)\underline{Apr}(C_i) \subseteq \overline{Apr_{\leq}}(C_i)\ and \underline{Apr}(C_i) \subseteq \overline{Apr_{\geq}}(C_i)$

$(For\ any C_i)\ and$

$ii)\underline{Apr}(C_i) \cap \overline{Apr_{\leq}}(C_i) \neq \phi$ and $\underline{Apr}(C_i) \cap \overline{Apr_{\geq}}(C_i) \neq \phi$

$(For\ any C_i)$.

Then, the lower and upper limits of $C_i$are determined as

$$\underline{Lim}(C_i) = \frac{1}{M_{\leq}} \sum D(x) | x \in \overline{Apr_{\leq}}(C_i)\ and\ \overline{Lim}(C_i) = \frac{1}{M_{\geq}} \sum D(x) | x \in \overline{Apr_{\geq}}(C_i)\ . \tag{11}$$

Here, $\underline{Lim}(C_i)$ and $\overline{Lim}(C_i)$ are the lower limit and upper limit of C_i, and $M_{\leq}$ and $M_{\geq}$ are the number of objects contained in $\overline{Apr_{\leq}}(C_i)$ and $\overline{Apr_{\geq}}(C_i)$, respectively. The RN of C_i, called DUARN, is expressed as

$$DURAN(C_i) = [\underline{Lim}(C_i),\ \overline{Lim}(C_i)]. \tag{12}$$

From (12), *DUARN* of a class (C_i) is seen to be a range defined by the lower limit and the upper limit of that class. If the lower limit coincides with the upper limit, it means there is no roughness in C_i.

Basic notions of *DUARN* are shown in Fig. 1.

The lower limit ($\underline{Lim}(C_i)$) and upper limit ($\overline{Lim}(C_i)$) of the DUARN are obtained by two elements of an ordered class. These are the upper approximation (downward) ($\overline{Apr_{\leq}}(C_i)$) and upper approximation (upward) ($\overline{Apr_{\geq}}(C_i)$). Further, the lower approximation ($\underline{Apr}(C_i)$), i.e., the exact class along with the lower limit ($\underline{Lim}(C_i)$) and upper limit ($\overline{Lim}(C_i)$) is useful in obtaining the directional bias associated with each of the objects (see Supplementary S.1).

This concept is applicable when the decision classes are ordered, i.e., there exists a dominance relation among the classes [35]. For example, in risk assessment, the risk is ordered as low, medium, or high. A similar concept, viz, double bounded rough sets, has recently been proposed by Kundu and Pal [41] for online social network analysis, where it is used to define the social relation as a string with various forces acting on it.

Example 1: Let five experts be asked to rate a failure event based on its nature of risk using a 10-point scale. Suppose, based on their ratings, the following information (see Table I) is obtained:

Here, D(Ct_i) represents the rating provided by the *i*th expert. The values attained by the decision, called Classes, represent the level of experts' preferences in terms of risk rating of the failure event. Five experts provided three ordered classes (9 > 7 > 5) for the risk rating of the failure event: class "5" is rated

TABLE I
INFORMATION TABLE

Experts	1	2	3	4	5
Decision (Class)	$D(Ct_1) = 5$	$D(Ct_2) = 7$	$D(Ct_3) = 5$	$D(Ct_4) = 9$	$D(Ct_5) = 7$

TABLE II
APPROXIMATION OF CLASSES

Classes	5	7	9
$\underline{Apr}$	{ Ct_1, Ct_3}	{ Ct_2, Ct_5}	{ Ct_4 }
$\overline{Apr_{\leq}}$	{ Ct_1, Ct_3}	{ Ct_1, Ct_2, Ct_3, Ct_5}	{ Ct_1, Ct_2, Ct_3, Ct_4, Ct_5}
$\overline{Apr_{\geq}}$	{ Ct_1, Ct_2, Ct_3, Ct_4, Ct_5}	{ Ct_2, Ct_4, Ct_5}	{ Ct_4}
Bnd	{ Ct_2, Ct_4, Ct_5}	{ Ct_1, Ct_3, Ct_4}	{ Ct_1, Ct_2, Ct_3, Ct_5}

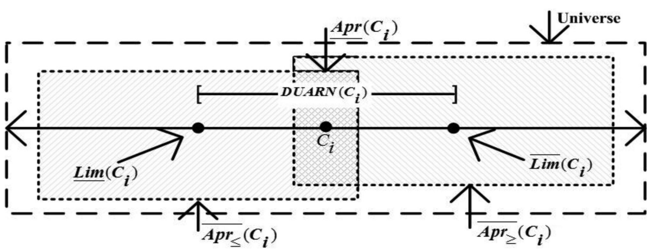

Fig. 1. Basic notions of DUARN.

by experts 1 and 3 (Ct_1, Ct_3); class "7" is rated by experts 2 and 5 (Ct_2, Ct_5); and class "9" is rated by experts 4 (Ct_4). Using (8) to (10), the lower and upper approximations, and the boundary region of each such class are calculated as in Table II.

Using (11) and (12), the lower and upper limits, and the RN of each class can be calculated. For example, for Class "5," these are as follows:

Lower limit: $\underline{Lim}(5) = (D(Ct_1) + D(Ct_3))/2 = (5 + 5)/2 = 5$ and

Upper limit: $\overline{Lim}(5) = (D(Ct_1) + D(Ct_2) + D(Ct_3) + D(Ct_4) + D(Ct_5))/5 = (5 + 7 + 5 + 9 + 7)/5 = 6.6$

$\mathrm{DUARN}(5) = [5,\ 6.6]$.

Similarly, $\mathrm{DUARN}(7) = [6, 7.67]$ and $\mathrm{DUARN}(9) = [6.6,\ 9]$.

DUARN of a decision (class), as expressed in terms of an interval of its lower and upper limits, signifies the true perception of the concerned decision with respect to other decisions (classes). In the aforesaid example, $DUARN(5) = [5,\ 6.6]$ means the rating "5" on the failure event, as decided by two experts (out of five experts), has actual perception value (certainty) that ranges from 5 to 6.6. Similarly, the perception intervals for decisions "7" and "9" are 6 to 7.67 and 6.6 to 9, respectively. These ranges of perception are practically seen to push the concerned decision-values toward the central mean of 6.6 (see Fig. 2), computed over the ratings of five experts. Here, it is to be noted that the properties of *DUARN*, as mentioned above, are satisfied. In addition, the directional bias of each of the expert, obtained through the lower approximation, is provided in the Supplementary S.1. These directional biases help to gain additional insights about the overestimation and underestimation of risks. These insights subsequently may be useful in defining

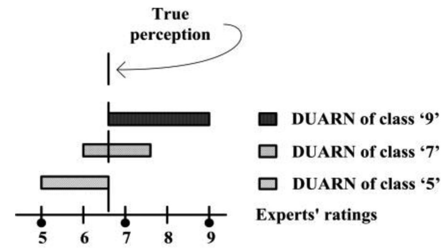

Fig. 2. DUARN of experts' risk ratings.

the risk attitudinal characteristics of each of the experts (see Supplementary S.1).

The union of lower approximation and upper approximation results in the universe (U) when all the experts provide opinions on all the failure modes. However, it is not always true, as sometimes, a few rare-event failure modes may not get judged by some of the experts under consideration. One may note that the aforesaid DUARN (Definition 7) is different from that of Zhai *et al.* [20]. The latter one does not follow Pawlak's rough sets [19] in terms of the characteristics of lower and upper approximations. Further, the proposed DUARN is in line with the development of DRSA [35], as it is applicable to ordinal decision classes. However, DRSA was developed to obtain rule-based decisions, whereas, the proposed DUARN is useful to deal with the issues of ineffective utilization of information such as closeness to true perception of experts. It is also to be noted that additional insights gained from DUARN, as mentioned above, cannot be derived from RN [20].

IV. PROPOSED METHODOLOGY

A. Granulized Z-Number (gZN)

Let the A and B parts of a Z-number be represented by trapezoidal and triangular fuzzy membership functions, respectively. Then $A = (Z^A_{c_1}, Z^A_{c_2}, Z^A_{c_3}, Z^A_{c_4})$ and $B = (Z^B_{c_1}, Z^B_{c_2}, Z^B_{c_3})$, where $Z^A_{c_i}$, $i = 1, 2, 3, 4$, represents the *i*th element of A and $Z^B_{c_i}$, $i = 1, 2, 3$, represents the *i*th element of B.

Let us consider *n* ordered classes of $Z^A_{c_i}$, i.e., $(Z^A_{c_{i1}} < Z^A_{c_{i2}} < \cdots < Z^A_{c_{ir}} < \cdots < Z^A_{c_{in}})$, and $Z^B_{c_i}$, i.e., $(Z^B_{c_{i1}} < Z^B_{c_{i2}} < \cdots < Z^B_{c_{ir}} < \cdots < Z^B_{c_{in}})$, and an empty set of classes, D(v), where v denotes a class and $v \subseteq Z^A_{c_i}$ and $v \subseteq Z^B_{c_i}$. Now, using Definition 7, the lower approximation, upper approximation, and the boundary region of $Z^A_{c_{ir}}$, where $Z^A_{c_{ir}}$ is the *r*th value of $Z^A_{c_i}$, are

$$\left.\begin{array}{l} Z^{A(l_{apr})}_{c_{ir}} = \cup\left\{v \in Z^A_{c_i}/D(v) = Z^A_{c_{ir}}\right\} \\ Z^{A(u_{apr})_{\leq}}_{c_{ir}} = \cup\left\{v \in Z^A_{c_i}/D(v) \leq Z^A_{c_{ir}}\right\} \text{ and } Z^{A(u_{apr})_{\geq}}_{c_{ir}} \\ = \cup\left\{v \in Z^A_{c_i}/D(v) \geq Z^A_{c_{ir}}\right\} \\ Z^{A(bnd)}_{c_{ir}} = \left\{Z^{A(u_{apr})_{\leq}}_{c_{ir}} - Z^{A(l_{apr})}_{c_{ir}}\right\} \\ \cup\left\{Z^{A(u_{apr})_{\geq}}_{c_{ir}} - Z^{A(l_{apr})}_{c_{ir}}\right\} \end{array}\right\}. \quad (13)$$

Similarly, the lower approximation, upper approximation, and the boundary region of $Z^B_{c_{ir}}$, where $Z^B_{c_{ir}}$ is the *r*th value of $Z^B_{c_i}$, are

$$\left.\begin{array}{l} Z_{c_{ir}}^{B(l_{apr})} = \cup\left\{v \in Z_{c_i}^{B}/D(v) = Z_{c_{ir}}^{B}\right\} \\ Z_{c_{ir}}^{B(u_{apr})_{\leq}} = \cup\left\{v \in Z_{c_i}^{B}/D(v) \leq Z_{c_{ir}}^{B}\right\} \text{ and } Z_{c_{ir}}^{B(u_{apr})_{\geq}} \\ = \cup\left\{v \in Z_{c_i}^{B}/D(v) \geq Z_{c_{ir}}^{B}\right\} \\ Z_{c_{ir}}^{B(bnd)} = \left\{Z_{c_{ir}}^{B(u_{apr})_{\leq}} - Z_{c_{ir}}^{B(l_{apr})}\right\} \\ \cup\left\{Z_{c_{ir}}^{B(u_{apr})_{\geq}} - Z_{c_{ir}}^{B(l_{apr})}\right\} \end{array}\right\}. \tag{14}$$

Here, $Z_{c_{ir}}^{A(l_{apr})}$ in (13) depicts the lower approximation of class $Z_{c_{ir}}^{A}$, which includes all the classes which are exactly equal to $Z_{c_{ir}}^{A}$ (definitely belongs to class$Z_{c_{ir}}^{A}$). Similarly, $Z_{c_{ir}}^{A(u_{apr})_{\leq}}$ depicts the upper approximation (downward side) of class $Z_{c_{ir}}^{A}$, which includes all the classes which are less than or equal to $Z_{c_{ir}}^{A}$ and $Z_{c_{ir}}^{A(u_{apr})_{\geq}}$is the upper approximation (upward side) of class $Z_{c_{ir}}^{A}$,which includes all the classes which are greater than or equal to $Z_{c_{ir}}^{A}$. Further, $Z_{c_{ir}}^{A(bnd)}$ represents the boundary region of class $Z_{c_{ir}}^{A}$.

Similarly, (14) can also be explained for class $Z_{c_{ir}}^{B}$. From (13) and (14), the lower and upper limits of $Z_{c_{ir}}^{A}$and $Z_{c_{ir}}^{B}$ are

$$Z_{c_{ir}}^{A(L)} = \frac{1}{M_{\leq}^{A}}\sum D(v)\left|v \in Z_{c_{ir}}^{A(u_{apr})_{\leq}}\right. \quad \text{and}$$

$$Z_{c_{ir}}^{A(U)} = \frac{1}{M_{\geq}^{A}}\sum D(v)\left|v \in Z_{c_{ir}}^{A(u_{apr})_{\geq}}\right. \tag{15}$$

and

$$Z_{c_{ir}}^{B(L)} = \frac{1}{M_{\leq}^{B}}\sum D(v)\left|v \in Z_{c_{ir}}^{B(u_{apr})_{\leq}}\right. \quad \text{and } Z_{c_{ir}}^{B(U)}$$

$$= \frac{1}{M_{\geq}^{B}}\sum D(v)\left|v \in Z_{c_{ir}}^{B(u_{apr})_{\geq}}\right. . \tag{16}$$

Here, *L* denotes the lower limit and *U* denotes the upper limit. $M_{\leq}^{A}$ and $M_{\geq}^{A}$ are the number of classes contained in $Z_{c_{ir}}^{A(u_{apr})_{\leq}}$ and $Z_{c_{ir}}^{A(u_{apr})_{\geq}}$, respectively. Similarly, $M_{\leq}^{B}$ and $M_{\geq}^{B}$ are the number of classes contained in $Z_{c_{ir}}^{B(u_{apr})_{\leq}}$ and $Z_{c_{ir}}^{B(u_{apr})_{\geq}}$, respectively.

The DUARN of $Z_{c_{ir}}^{A}$, say $[DUARN(Z_{c_{ir}}^{A})]$ and $Z_{c_{ir}}^{B}$, say $[DUARN(Z_{c_{ir}}^{B})]$are obtained as

$$DUARN(Z_{c_{ir}}^{A}) = [Z_{c_{ir}}^{A(L)}, Z_{c_{ir}}^{A(U)}] \text{ and } DUARN(Z_{c_{ir}}^{B}) = [Z_{c_{ir}}^{B(L)}, Z_{c_{ir}}^{B(U)}]. \tag{17}$$

Hence, the granulized Z-number (gZN) is $[DUARN(Z_{c_{ir}}^{A}), DUARN(Z_{c_{ir}}^{B})]$.

Example 2: The procedure of computing *gZN* is explained here through an example. Let a FM be evaluated by five experts with respect to the risk criterion P in Z-linguistic terms (see Supplement S.6). Experts' evaluations are shown in Table III. Here, values in each column of the A-part and B-part are denoted as $Z_{c_{ir}}^{A}\,(i = 1,2,3,4)$ and $Z_{c_{ir}}^{B}\,(i = 1,2,3)$; $r = 1,2,3,\ldots,n$, here $n = 5$, respectively, which may also be treated as classes. For example, $Z_{c_{1r}}^{A} = \{0.2,\ 0.5,\ 0.1,\ 0.5,\ 0.2\}$ and $Z_{c_{1r}}^{B} = \{0,\ 0.1,\ 0.3,\ 0.1,\ 0\}$. Next, compute the lower approximation, upper approximation (downward), upper approximation (upward), and boundary region of unique classes of $Z_{c_{ir}}^{A}$ and $Z_{c_{ir}}^{B}$by using (13) and (14), respectively.

TABLE III
EXPERT EVALUATIONS

Experts	A ($Z_{c_{1r}}^{A}$)				B ($Z_{c_{1r}}^{B}$)		
E1	0.2	0.3	0.4	0.5	0	0.1	0.3
E2	0.5	0.6	0.7	0.8	0.1	0.3	0.5
E3	0.1	0.2	0.2	0.3	0.3	0.5	0.7
E4	0.5	0.6	0.7	0.8	0.1	0.3	0.5
E5	0.2	0.3	0.4	0.5	0	0.1	0.3

Next, values of *DUARN* for each class of are obtained using (15) to (17), respectively. The final gZN matrix of experts. evaluations is shown in Table IV.

TABLE IV
GZN OF EXPERTS' EVALUATIONS

Experts	A				B		
E1	[0.167, 0.35]	[0.267, 0.45]	[0.33, 0.55]	[0.43, 0.65]	[0, 0.1]	[0.1, 0.26]	[0.3, 0.46]
E2	[0.3, 0.5]	[0.4, 0.6]	[0.48, 0.7]	[0.58, 0.8]	[0.05, 0.167]	[0.2, 0.367]	[0.4, 0.567]
E3	[0.1, 0.3]	[0.2, 0.4]	[0.2, 0.48]	[0.3, 0.58]	[0.1, 0.3]	[0.26, 0.5]	[0.46, 0.7]
E4	[0.3, 0.5]	[0.4, 0.6]	[0.48, 0.7]	[0.58, 0.8]	[0.05, 0.167]	[0.2, 0.367]	[0.4, 0.567]
E5	[0.167, 0.35]	[0.267, 0.45]	[0.33, 0.55]	[0.43, 0.65]	[0, 0.1]	[0.1, 0.26]	[0.3, 0.46]

TABLE V
FAILURE MODES

Items	Failure modes	Items	Failure modes
FM1	Temperature increase due to external ignition source	FM7	No dissipation of static charge due to absence of non-conducting floor
FM2	Chemical reaction between waste explosive	FM8	Slip-Trip-Fall
FM3	Chemical reactions due to keeping non-compatible A&E together	FM9	No dissipation of static charge due to non-earthing system
FM4	Chemical reactions due to exceeding the authorised limit	FM10	Hit by object/ fall of object
FM5	Spark may come in contact with explosives material	FM11	No dissipation of body static charge
FM6	Moisture absorption	FM12	Excessive heat due to friction

The arithmetic average of modal values of trapezoidal (A-part) and triangular fuzzy numbers (B-part) may be considered as true perception of experts [21]. Arithmetic averaging of the modal values of A-parts results in the interval [0.4, 0.48]. Here, we hope that all of the experts' evaluations would contain this true perception for better decision-making. However, from Table III , we can observe that this value is contained in only two out of five trapezoidal fuzzy numbers. While, in Table IV this true perception can be seen in all the experts' evaluations. Similarly, for B-part, the true perception comes as 0.26, which is not present in the evaluation of E3 (see Table III). However, this true perception is a part of all of the experts' evaluations in Table IV.

B. Relation Between A and B Parts of gZN

From Definition 3 (2), we know that A and B parts of a Z-number are closely linked via an underlying probability distribution (*p*(*x*)). Similar relation can be derived for a gZN as follows: We know, $gZN = [DUARN(Z_{c_{ir}}^{A}), DUARN(Z_{c_{ir}}^{B})]$.

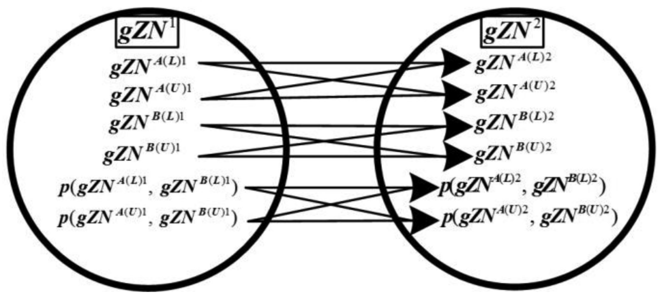

Fig. 3. Average linkage concept to find distance between two *gZNs*.

If A and B are trapezoidal and triangular fuzzy numbers, respectively, then after separating the lower and upper limits, we get

$$\begin{aligned} gZN^L &= [(Z_{c_{1r}}^{A(L)}, Z_{c_{2r}}^{A(L)}, Z_{c_{3r}}^{A(L)}, Z_{c_{4r}}^{A(L)}) \\ &\quad (Z_{c_{1r}}^{B(L)}, Z_{c_{2r}}^{B(L)}, Z_{c_{3r}}^{B(L)})]\, and \\ gZN^U &= [(Z_{c_{1r}}^{A(U)}, Z_{c_{2r}}^{A(U)}, Z_{c_{3r}}^{A(U)}, Z_{c_{4r}}^{A(U)}) \\ &\quad (Z_{c_{1r}}^{B(U)}, Z_{c_{2r}}^{B(U)}, Z_{c_{3r}}^{B(U)})]. \end{aligned} \tag{18}$$

To obtain *p(x)*, the probability distribution linking A and B, we need to do the following:

i) Discretize A and B into *n* equal parts, where *n* is a prime number. Aliev *et al.* [25] recommended $n = 7$ based on the tradeoff between information loss and computational complexity. So, the discretized gZN^L is (19) shown at the bottom of next page. Similarly, discretized gZN^U can also be given.

ii) Obtain the membership value for every discretized element of A, and its associated probability. Following Ross [42], the membership values are obtained using approximate normal distribution. To determine the associated probability, the maximum entropy principle-based optimization problem is formulated. This optimization problem is solved using LINGO software. For more details, see Supplement S.2.

C. Distance Between Two gZNs

In this section, we develop the distance measure between two *gZNs* using the similarity measure between two Z-numbers, proposed by Das *et al.* [26], and average linkage method used in clustering (see Fig. 3).

The weighted distance between two *gZNs,* say gZN^1 and gZN^2,

$$\begin{aligned} D^{\alpha}(gZN^1, gZN^2) &= \alpha * gZNDRM(gZN^{B1}, gZN^{B2}) \\ &+ (1-\alpha) * gZNDPM(gZN^{A1}, gZN^{A2}) \end{aligned} \tag{20}$$

where *gZNDRM* stands for *gZN*-distance of reliability measures (*B1* and *B2*) and *gZNDPM* is the *gZN*-distance of possibilistic restrictions (A1 and A2). The value of α depends on experts' opinions and$0 \le \alpha \le 1$.

For simplicity in representation, we are considering the following (with reference to Fig. 3):

$$\begin{aligned} gZN^{A1} &= A^1, gZN^{A2} = A^2, \\ gZN^{A(L)1} &= A^{L1} = (a_1^{L1}, a_2^{L1}, a_3^{L1}, a_4^{L1}), gZN^{A(U)1} \\ &= A^{U1} = (a_1^{U1}, a_2^{U1}, a_3^{U1}, a_4^{U1}) \\ gZN^{A(L)2} &= A^{L2} = (a_1^{L2}, a_2^{L2}, a_3^{L2}, a_4^{L2}), \text{ and } gZN^{A(U)2} \\ &= A^{U2} = (a_1^{U2}, a_2^{U2}, a_3^{U2}, a_4^{U2}). \end{aligned}$$

Here, *L* and *U* stand for lower and upper values of a gZN, and the superscripts *1* and *2* indicate gZN^1 and gZN^2, respectively.

Similarly, representative notations for *B1* and *B2* can also be given.

The computation of $gZNDRM(B^1, B^2)$ and $gZNDPM(A^1, A^2)$ is shown below.

***C.1** gZN-distance of reliability measures (gZNDRM)*

Considering the concept of average linkage, the distance between the reliability measures (B^1, B^2) of two gZNs is

$$\begin{aligned} &gZNDRM(B^1, B^2) \\ &= \frac{D(B^{L1}, B^{L2}) + D(B^{L1}, B^{U2}) + D(B^{U1}, B^{L2}) + D(B^{U1}, B^{U2})}{4} \end{aligned} \tag{21}$$

where

$$\begin{aligned} D(B^{i1}, B^{j2}) &= 0.5\{D_h^{\lambda}((B^{i1}, B^{j2}) + D^{\lambda}((B^{i1}, B^{j2})\}, i \\ &= L\, or\, U,\ and j = L \,\text{or}\, U. \end{aligned} \tag{22}$$

In (22), $D_h^{\lambda}(B^{i1}, B^{j2}) = max\{|B_{\ell}^{i1} - B_{\ell}^{j2}|, |B_r^{i1} - B_r^{j2}|\}$ is the λ-cut based Hausdorff distance between two reliability measures, and $D^{\lambda}(B^{i1}, B^{j2}) = |0.5(B_{\ell}^{i1} + B_r^{j1}) - 0.5(B_{\ell}^{i2} + B_r^{j2})|$is the center to center distance between two reliability measures.

***C.2** gZN-distance of possibilistic restrictions (gZNPDM)*

gZNDPM is a combination of the distances between the A-parts and between the underlying probability distributions of the two *gZNs*. So,

$$\begin{aligned} gZNPDM(A^1, A^2) &= 0.5\left[D(A^1, A^2)\right. \\ &\quad \left. + D\left\{p(A^1, B^1), p(A^2, B^2)\right\}\right]. \end{aligned} \tag{23}$$

Now, the distance between A-parts of two *gZNs* is

$$gZN^L = \left[\begin{array}{l} \left(Z_{c_{1r}}^{A(L)}, \frac{Z_{c_{1r}}^{A(L)} + Z_{c_{2r}}^{A(L)}}{2}, Z_{c_{2r}}^{A(L)}, \frac{Z_{c_{2r}}^{A(L)} + Z_{c_{3r}}^{A(L)}}{2}, Z_{c_{3r}}^{A(L)}, \frac{Z_{c_{3r}}^{A(L)} + Z_{c_{4r}}^{A(L)}}{2}, Z_{c_{4r}}^{A(L)}\right), \\ \left(Z_{c_{1r}}^{B(L)}, \frac{2Z_{c_{1r}}^{B(L)} + Z_{c_{2r}}^{B(L)}}{3}, \frac{Z_{c_{1r}}^{B(L)} + 2Z_{c_{2r}}^{B(L)}}{3}, Z_{c_{2r}}^{B(L)}, \frac{2Z_{c_{2r}}^{B(L)} + Z_{c_{3r}}^{B(L)}}{3}, \frac{Z_{c_{2r}}^{B(L)} + 2Z_{c_{3r}}^{B(L)}}{3}, Z_{c_{3r}}^{B(L)}\right) \end{array}\right] \tag{19}$$

$$D(A^1, A^2) = \{D(A^{L1}, A^{L2}) + D(A^{L1}, A^{U2}) + D(A^{U1}, A^{L2}) + D(A^{U1}, A^{U2})\}/4 \quad (24)$$

where

$$D(A^{i1}, A^{j2}) = 1 - \left\{ \left(1 - \frac{\sum_{k=1}^{4} \left|a_k^{i1} - a_k^{j2}\right|}{4}\right) \times d'(A^{i1}, A^{j2}) \right\} \times \left\{1 - \frac{\left|Area(A^{i1}) - Area(A^{j2})\right| + \left|W_{A^{i1}} - W_{A^{j2}}\right| + \frac{P(A^{i1}) - P(A^{j2})}{\max[P(A^{i1}), P(A^{j2})]}}{3}\right\}$$
$$i = L\text{or}U, \text{and} j = L\text{or}U. \quad (25)$$

Area (A) and perimeter (P) in (25) are given as

$$\text{Area}(A^{i1}) = \{W_{A^{i1}} * [a_3^{i1} - a_2^{i1} + a_4^{i1} - a_1^{i1}]\}/2 \quad (26)$$

$$P(A^{i1}) = \sqrt{(a_1^{i1} - a_2^{i1})^2 + W_{A^{i1}}^2} + \sqrt{(a_3^{i1} - a_4^{i1})^2 + W_{A^{i1}}^2} + (a_3^{i1} - a_2^{i1}) + (a_4^{i1} - a_1^{i1}). \quad (27)$$

Similarly, Area(A^{j2}) and perimeter $P(A^{j2})$ are computed.

The distance (d') in (25) is defined as follows:

$$d'(A^{i1}, A^{j2}) = \sqrt{(\pi_{A^{i1}} - \pi_{A^{j2}})^2 + (\rho_{A^{i1}} - \rho_{A^{j2}})^2} / \sqrt{1.25} \quad (28)$$

where

$$\rho_{A^{i1}} = \begin{cases} \frac{W_{A^{i1}} \times \left(\frac{a_3^{i1} - a_2^{i1}}{a_4^{i1} - a_1^{i1}}\right)}{6}, & if\, a_4^{i1} \neq a_1^{i1} \\ \frac{W_{A^{i1}}}{2}, & if\, a_4^{i1} = a_1^{i1} \end{cases} \quad (29)$$

$$\pi_{A^{i1}} = \begin{cases} \frac{\rho_{A^{i1}} \times (a_3^{i1} + a_2^{i1}) + (a_4^{i1} + a_1^{i1}) \times (W_{A^{i1}} - \rho_{A^{i1}})}{2W_{A^{i1}}}, & \text{if } W_{A^{i1}} \neq 0 \\ \frac{a_4^{i1} + a_1^{i1}}{2}, & \text{if } W_{A^{i1}} = 0. \end{cases} \quad (30)$$

Similarly, $\rho_{A^{j2}}$ and $\pi_{A^{j2}}$ can be computed.

Finally, the distance between two underlying probability distributions is obtained using the maximum entropy principle, as explained in Section IV-B (Supplement S.2)

$$D\left\{p(A^1, B^1), p(A^2, B^2)\right\} = \text{Average} \begin{bmatrix} D\left\{p(A^{L1}, B^{L1}), p(A^{L2}, B^{L2})\right\}, \\ D\left\{p(A^{L1}, B^{L1}), p(A^{U2}, B^{U2})\right\}, \\ D\left\{p(A^{U1}, B^{U1}), p(A^{L2}, B^{L2})\right\}, \\ D\left\{p(A^{U1}, B^{U1}), p(A^{U2}, B^{U2})\right\} \end{bmatrix} \quad (31)$$

where

$$D\left\{p(A^{i1}, B^{i1}), p(A^{j2}, B^{j2})\right\} = \sup\left\{\sum_{k=1}^{n} \left|p(A^{i1}, B^{i1})_k - p(A^{j2}, B^{j2})_k\right|\right\}$$
$$i = L \text{ or } U, \text{ and } j = L \text{ or } U. \quad (32)$$

D. Granulized Z-VIKOR Model

In this section, we extend the VIKOR model by integrating it with the developed gZN for failure mode and effects analysis (FMEA). VIKOR can be treated as an MCDM problem, where K experts,$DM_k (k = 1, 2, \ldots, K)$, provide their opinions on *n* failure modes, $FM_i (i = 1, 2, \ldots, n)$, based on m risk criteria,$C_j (j = 1, 2, \ldots, m)$. In addition, the experts also provide their opinions on the importance of$C_j (j = 1, 2, \ldots, m)$. The necessary steps to prioritize the *FMs* are as follows.

1) Elicitate the Risk Information By elicitation of risk information, we mean the conversion of experts' Z-linguistic opinions into fuzzy numerical form. For the *kth* expert with *n FMs* and *m* risk criteria, the importance of the *jth* risk criterion and opinion matrix for the *n FMs* can be represented as follows:

$$Z_j^k = \left[(Z^A, Z^B)_j^k\right], \quad \text{where } k = 1, 2, 3, \ldots, K; \forall j. \quad (33)$$

and

$$\tilde{D}_z^k = \begin{bmatrix} (Z^A, Z^B)_{11}^k & (Z^A, Z^B)_{12}^k & \cdots & (Z^A, Z^B)_{1m}^k \\ (Z^A, Z^B)_{21}^k & (Z^A, Z^B)_{22}^k & \cdots & (Z^A, Z^B)_{2m}^k \\ \vdots & \vdots & (Z^A, Z^B)_{ij}^k & \vdots \\ (Z^A, Z^B)_{n1}^k & (Z^A, Z^B)_{n2}^k & \cdots & (Z^A, Z^B)_{nm}^k \end{bmatrix}. \quad (34)$$

Following Section IV-A,

$$(Z^A, Z^B)^k = \left((Z_{c_1}^A, Z_{c_2}^A, Z_{c_3}^A, Z_{c_4}^A)(Z_{c_1}^B, Z_{c_2}^B, Z_{c_3}^B)\right)^k. \quad (35)$$

2) Weight of Risk Criteria The weights of risk criteria are computed using the information provided in (33) and (34). In the existing literature, the weights computed using information (33) are termed as "subjective measure," and the weights computed using information (34) are termed as "objective measure." The computational steps are as follows.

Step 1: Compute subjective measure

First for the *j*th risk criterion, $Z^B = (Z_{c_1}^B, Z_{c_2}^B, Z_{c_3}^B)$ is converted into a crisp number (say, λ) and then, its square-root is multiplied with each element of $Z^A = (Z_{c_1}^A, Z_{c_2}^A, Z_{c_3}^A, Z_{c_4}^A)$. This results in a conventional trapezoidal fuzzy number $[(A_{c_t}^\lambda)_j^k]_S = [(A_{c_1}^\lambda)_j^k, (A_{c_2}^\lambda)_j^k, (A_{c_3}^\lambda)_j^k, (A_{c_4}^\lambda)_j^k]_S$. Here, subscript Sstands for subjective. For more details of the conversion process, see [43].

Next, the crisp importance of the *j*th risk criterion [5] W_j^k, based on the *k*th expert evaluation, is computed as (36) shown at the bottom of this page.

$$[W_j]_S = [W_j^1, W_j^2, \ldots, W_j^k]_S. \quad (37)$$

Then, following Section IV-A, the lower and upper limits, and *DUARN* of W_j^k, i.e., ${W_j}^{kL}$, ${W_j}^{kU}$, $[DUARN({W_j}^k)]_S$ are obtained, where

$$\left[DUARN({W_j}^k)\right]_S = [{W_j}^{kL}, {W_j}^{kU}]_S. \quad (38)$$

$$\left[W_j^k\right]_S = \frac{-(A_{c_1}^\lambda)_j^k (A_{c_2}^\lambda)_j^k + (A_{c_3}^\lambda)_j^k (A_{c_4}^\lambda)_j^k + \frac{1}{3}\left((A_{c_4}^\lambda)_j^k - (A_{c_3}^\lambda)_j^k\right)^2 - \frac{1}{3}\left((A_{c_2}^\lambda)_j^k - (A_{c_1}^\lambda)_j^k\right)^2}{-(A_{c_1}^\lambda)_j^k - (A_{c_2}^\lambda)_j^k + (A_{c_3}^\lambda)_j^k + (A_{c_4}^\lambda)_j^k} \quad (36)$$

Next, average DUARN $[\overline{DUARN(W_j)}]_S$ is computed by considering all K experts, as

$$\left.\begin{array}{l}[W_j{}^L]_S = (W_j{}^{1L} + W_j{}^{2L} + \cdots + W_j{}^{KL})_S/K \ ; [W_j{}^U]_S \\ \quad = (W_j{}^{1U} + W_j{}^{2U} + \cdots + W_j{}^{KU})_S/K \\ \left[\overline{DUARN(W_j)}\right]_S = [W_j{}^L, W_j{}^U]_S \end{array}\right\} \quad (39)$$

and the rough importance vector $[W_R]_S$ for *m* risk criteria is formed as

$$[W_R]_S = \left[[W_1{}^L, W_1{}^U]_S, [W_2{}^L, W_2{}^U]_S, \ldots, [W_m{}^L, W_m{}^U]_S\right]. \quad (40)$$

Finally, the subjective measure $[\omega_j]_S$ for each risk criterion is computed as [17]

$$[\omega_j]_S = \eta\left(1 - \frac{(W_j^U)_S - (W_j^L)_S}{2(\beta-\alpha)}\right) + (1-\eta)\left(\frac{(W_j^U)_S + (W_j^L)_S}{2(\beta-\alpha)}\right). \quad (41)$$

Here, $0 \le [\omega_j]_S \le 1,\ \beta = \max_j (W_j^U)_S, \alpha = \min_j (W_j^L)_S$. η, $0 \le \eta \le 1$, represents the degree to which the subjective measure is affected by uncertainty (denoted by the difference between the upper limit and lower limit of DUARN of the *j*th criterion). Larger η has more effect on the subjective measure.

Step 2: Compute objective measure

The objective measure is computed using $\tilde{D}_z^k$. Following (36)–(38), each element of $\tilde{D}_z^k$ is first converted into DUARN as given below. Here, O stands for objective

$$\left[DUARN(W_{ij}{}^k)\right]_O = [W_{ij}^{kL}, W_{ij}^{kU}]_O. \quad (42)$$

Next, the projection values of $DUARN(W_{ij}{}^k)$, for the *jth* criterion based on the *kth* expert's opinions, are obtained by normalizing $(W_{ij}{}^k)^L$ and $(W_{ij}{}^k)^U$

$$\left.\begin{array}{l}\left[(W_{ij}{}^k)^L\right]_O^N = (W_{ij}{}^k)^L / \sum_{j=1}^m (W_{ij}{}^k)^L \text{ and } \left[(W_{ij}{}^k)^U\right]_O^N \\ = (W_{ij}{}^k)^U / \sum_{j=1}^m (W_{ij}{}^k)^U \\ \left[DUARN(W_{ij}^k)\right]_O^N = [(W_{ij}{}^k)^L, (W_{ij}{}^k)^U]_O^N;\ \forall i = 1, 2, \\ \ldots, n \text{ and}, k = 1, 2, \ldots, K. \end{array}\right\}. \quad (43)$$

So, the average projected DUARN considering all K experts is

$$\left.\begin{array}{l}\left[\overline{DUARN(W_{ij}^k)}\right]_O^N = [W_{ij}{}^L, W_{ij}{}^U]_O, \text{where} \\ [W_{ij}{}^L]_O = \sum_{k=1}^K \left[(W_{ij}{}^k)^L\right]_O^N / K \text{ and } [W_{ij}{}^U]_O \\ = \sum_{k=1}^K \left[(W_{ij}{}^k)^U\right]_O^N / K; \\ \forall i = 1, 2, \ldots, n \ and, \ j = 1, 2, \ldots, m. \end{array}\right\}. \quad (44)$$

Finally, the objective measure of the *j*th risk criterion $[\omega_j]_O$ is computed as

$$[\omega_j]_O = div_j \Big/ \sum_{j=1}^m div_j \quad (45)$$

where $0 \le [\omega_j]_O \le 1$, the degree of divergence (div_j) for criterion *j* is

$$div_j = 1 - e_j \quad (46)$$

and the entropy e_j for criterion *j* is

$$e_j = \theta\left[-\frac{1}{\ln(n)}\sum_{i=1}^n \left\{[W_{ij}{}^L]_O \ln\left([W_{ij}{}^L]_O\right)\right\}\right] + (1-\theta)\left[-\frac{1}{\ln(n)}\sum_{i=1}^n \left\{[W_{ij}{}^U]_O \ln\left([W_{ij}{}^U]_O\right)\right\}\right], 0 \le \theta \le 1. \quad (47)$$

Step 3: Compute combined measure considering interactions

The combined measure $[\omega_j]_C$ is the weighted sum of the subjective and objective measures, given in (41) and (45), respectively. It is defined as

$$[\omega_j]_C = \rho[\omega_j]_S + (1-\rho)[\omega_j]_O, \quad (48)$$

$$0 \le \rho \le 1$$

where $0 \le [\omega_j]_C \le 1$.

Equation (48) holds good for independent risk criteria. In order to incorporate interactions, (48) needs to be modified. We use Shapley index to compute $[\varpi_j]_C$, the combined measure incorporating interactions, as

$$[\varpi_j]_C = \sum_{S_{jk} \subseteq M \backslash j} \frac{(m-1-s_{jk})! s_{jk}!}{(m)!} [\omega_j]_C \prod_{k \in S_j} \times [1 + \lambda[\omega_k]_C], \forall j \in M \quad (49)$$

where $0 \le [\varpi_j]_C \le 1$, m = number of criteria, *M* = power set of m criteria, S_j = all possible subsets of *M* (excluding the subsets containing the *jth* criterion), and s_{jk} = cardinality of the *kth* subset of S_j.

Parameters λ and $[\omega_k]_C$ are computed using (5) and (4), respectively (see Definition 4).

3) Ranking of Failure Modes: VIKOR prioritizes failure modes (FMs) based on their risk values. It evaluates the FMs based on the risk criteria and then obtains the closest ideal solution that the decision-maker wants. The approach takes into account the relative importance of the distances among the compromise solutions, called the best solution, and the worst solution. The necessary steps are as follows.

Step 1: Following the development in Section IV-A, the opinion matrix $\tilde{D}_z^k$ of the *k*th expert is converted into its granulized form ($\tilde{D}_{gZN}^k$), where rows of the matrix represent failure modes and columns represent the risk criteria. Henceforth, we call

$\tilde{D}^k_{gZN}$ as gZN matrix for the *kth* expert and is represented as

$$\tilde{D}^k_{gZN} = \begin{bmatrix} (gZN)^k_{11} & (gZN)^k_{12} & \cdots & (gZN)^k_{1m} \\ (gZN)^k_{21} & (gZN)^k_{22} & \cdots & (gZN)^k_{2m} \\ \vdots & \vdots & (gZN)^k_{ij} & \vdots \\ (gZN)^k_{n1} & (gZN)^k_{n2} & \cdots & (gZN)^k_{nm} \end{bmatrix}. \tag{50}$$

Step 2: Considering all the *k* experts, $\tilde{D}^k_{gZN}$ is aggregated, and the aggregated gZN matrix $\tilde{D}_{gZN}$ is

$$\tilde{D}_{gZN} = \begin{bmatrix} (gZN)_{11} & (gZN)_{12} & \cdots & (gZN)_{1m} \\ (gZN)_{21} & (gZN)_{22} & \cdots & (gZN)_{2m} \\ \vdots & \vdots & (gZN)_{ij} & \vdots \\ (gZN)_{n1} & (gZN)_{n2} & \cdots & (gZN)_{nm} \end{bmatrix}. \tag{51}$$

Separate $(gZN)_{ij}$ into $(gZN)^L_{ij}$ and $(gZN)^U_{ij}$, where $(gZN)^L_{ij}$is

$$\left(\begin{array}{l} \left\{ \begin{array}{l} Z^{A(L)}_{c_{1r}} = Min_k\left(Z^{kA(L)}_{c_{1r}}\right), Z^{A(L)}_{c_{2r}} \\ = \sum_{k=1}^{K}\left(Z^{kA(L)}_{c_{2r}}\right)\Big/K, \\ Z^{A(L)}_{c_{3r}} = \sum_{k=1}^{K}\left(Z^{kA(L)}_{c_{3r}}\right)\Big/K, Z^{A(L)}_{c_{4r}} \\ = Max_k\left(Z^{kA(L)}_{c_{4r}}\right) \end{array} \right\}, \\ \left\{ Z^{B(L)}_{c_{1r}} = Min_k\left(Z^{kB(L)}_{c_{1r}}\right), Z^{B(L)}_{c_{2r}} \right. \\ \left. = \sum_{k=1}^{K}\left(Z^{kB(L)}_{c_{2r}}\right)\Big/K,\ Z^{B(L)}_{c_{3r}} = Max_k\left(Z^{kB(L)}_{c_{3r}}\right) \right\} \end{array} \right). \tag{52}$$

Similarly, $(gZN)^U_{ij}$ is obtained.

Step 3: The best and worst values for each of the risk criteria are considered from $\tilde{D}_{gZN}$. The best and worst values of the *jth* risk criterion are

$$\left. \begin{array}{l} [gZN]^{best}_j = \underset{i}{Min}\,[gZN]_{ij} \\ = \underset{i}{Min}\,(Z^{A(L)}_{c_r}, Z^{A(U)}_{c_r})_{ij},\ \underset{i}{Max}\,(Z^{B(L)}_{c_r}, Z^{B(U)}_{c_r})_{ij} \\ [gZN]^{worst}_j = \underset{i}{Max}\,[gZN]_{ij} \\ = \underset{i}{Max}\,(Z^{A(L)}_{c_r}, Z^{A(U)}_{c_r})_{ij},\ \underset{i}{Min}\,(Z^{B(L)}_{c_r}, Z^{B(U)}_{c_r})_{ij} \end{array} \right\} \tag{53}$$

where $Z^{A(L)}_{c_r} = [Z^{A(L)}_{c_{1r}}, Z^{A(L)}_{c_{2r}}, Z^{A(L)}_{c_{3r}}, Z^{A(L)}_{c_{4r}}]$; $Z^{B(L)}_{c_r} = [Z^{B(L)}_{c_{1r}}, Z^{B(L)}_{c_{2r}}, Z^{B(L)}_{c_{3r}}]$.

Similarly, $Z^{A(U)}_{c_r}$ and $Z^{B(U)}_{c_r}$ are obtained.

Here, the risk criteria are considered as the cost criteria because the higher the risk criterion value, the higher the overall risk.

Step 4: Following the development in Section IV-C, we define the expected relative risk, ERR_i and the expected maximum relative risk, $EMRR_i$ for the *ith* FM as follows:

$$ERR_i = \sum_{j=1}^{m}(\varpi_j D^\alpha[(gZN)_{ij}, (gZN)^{best}_{ij}]/ D^\alpha[(gZN)^{best}_{ij}, (gZN)^{worst}_{ij}]) \tag{54}$$

and

$$EMRR_i = \underset{j}{Max}\left[\varpi_j D^\alpha\{(gZN)_{ij}, (gZN)^{best}_{ij}\}/ D^\alpha\{(gZN)^{best}_{ij}, (gZN)^{worst}_{ij}\}\right]. \tag{55}$$

The risk index (RI) for the *ith* FM, RI_i, $i = 1, 2, \ldots, n$ is computed as

$$RI_i = \phi\frac{ERR_i - ERR^*}{ERR^- - ERR^*} + (1-\phi)\frac{EMRR_i - EMRR^*}{EMRR^- - EMRR^*} \tag{56}$$

where $ERR^* = \min ERR_i$, $ERR^- = \max ERR_i$, $EMRR^* = \min EMRR_i$, $EMRR^- = \max EMRR_i$. $\phi, 0 \le \phi \le 1$,is a policy coefficient for risk based ranking.

Finally, the failure modes are ranked according to the descending values of RI_i, $i = 1, 2, \ldots, n$.

The pseudocode of the proposed granulized Z-VIKOR model is provided in Supplementary file (see S.10).

V. Case Study 1

The proposed methodology is applied for computing the risk indices and ranks of failure modes (FMs) of an ammunition preparation process (APP). The activities performed in the APP are filling of explosives and ammunitions, assembly, and its inspection. A total of 12 FMs were identified (see Table V), verified, and evaluated by a team of five experts with respect to three risk criteria (P, S, and D) in Z-linguistic terms (see Supplements S.3 and S.6). Similarly, the importance of the three-risk criteria P, S, and D were evaluated by the experts (see Supplements S.4, S.5).

Next, following the steps given in Section IV-D.2, subjective and objective measures of risk criteria are computed. The combined measure incorporating the interactions of risk criteria is obtained using (48) (see Supplement S.7). The Z-linguistic information of FMs for each of the risk criteria is then converted into gZN using (13) to (17). For illustrative purpose, gZN of FM3 with respect to P is shown in Supplement S.8. The aggregated gZN matrix is computed using (52). Then, *gZN*^{best} and *gZN*^{worst} for each of the risk criteria are obtained using (53).

After that, the weighted distances between *gZN* of each of the FMs with respect to each of the risk criteria and *gZN*^{best} of the corresponding risk criterion are computed. Similarly, the weighted distances between *gZN*^{worst} and *gZN*^{best} for all the risk criteria are obtained (see Supplement S.9).

The values of expected relative risk (ERR_i) and expected maximum relative risk ($EMRR_i$) for the *i*th FM are then computed using (54) and (55), respectively. Finally, the risk index (RI) is computed using (56), and the FMs are ranked according to the descending values of RI (see Table VI). The overall risk ranking of FMs is FM9 > FM12 > FM3 > FM11 > FM7 > FM8 > FM6 > FM1 > FM5 > FM10 > FM2 > FM4.

One more practical application (Case study 2) to prioritize the risk of supplier selection in a supply chain context is also provided (see Supplement S.11) for demonstrating the effectiveness of the proposed model.

TABLE VI
RANKING OF FAILURE MODES

FMs	RI	Rank	FMs	RI	Rank
FM1	0.364097	8	FM7	0.42425	5
FM2	0.096776	11	FM8	0.400502	6
FM3	0.50919	3	FM9	1	1
FM4	0.042068	12	FM10	0.213255	10
FM5	0.273461	9	FM11	0.47273	4
FM6	0.399408	7	FM12	0.510712	2

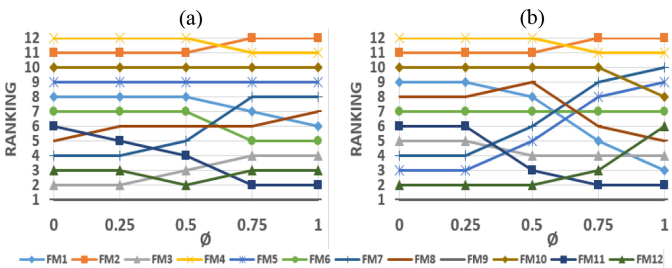

Fig. 4. Sensitivity analysis using (a) proposed model and (b) Z-VIKOR.

VI. SENSITIVITY AND COMPARATIVE ANALYSIS

Here, first we observe the effect of the policy coefficient (ϕ) on the ranks of FMs by the proposed model [see Fig. 4(a)].

In the case of $0< \phi <0.25$, the experts assume that the risk value of FMs depends more on the $EMRR_i$, rather than ERR_i. From Fig. 4(a), it is clear that there is stability in ranking except for FM8 and FM11. Here, the decision-maker gives more weightage to the risk criterion having the maximum contribution to the overall risk. When, $0.25< \phi <0.75$, the experts assume that there is a compromise between ERR_i and $EMRR_i$. From Fig. 4(a), it is clear that the system presents a little unstable framework with respect to the ranking of FMs. The reasons for being unstable may be due to the consideration of the interactions among the risk criteria, which is practically in agreement with the real scenario. Most of the MCDM techniques assume that the criteria are independent, which does not reflect the practical mechanism of failure mode and effect analysis. So, the decision-makers may require further information on the causes of the failure modes. In case of $0.75< \phi <1$, experts assume that the risk value of FMs depends more on the ERR_i. From Fig. 4(a), it is clear that the system presents a stable framework with respect to the ranking of FMs. Here, the rank equally depends on the information of all the risk criteria. Finally, we compare the sensitivity of the proposed model with Z-VIKOR [see Fig.4(b)] to investigate the advantage of introducing RN concepts with Z-number. We found that the proposed model provides better stability in ranking FMs.

To further validate the effectiveness of the proposed model, another comparative evaluation is done with traditional RPN, Z-VIKOR [5], rough VIKOR [17], rough cloud TOPSIS [34], and Z-MOORA [28]. The comparative analysis is given in Fig. 5.

The first comparative analysis is provided with traditional RPN. With different values of risk criteria, FM1 and FM3 have the identical RPN value (= 80). The same is observed for FM8 and FM10. In contrast, the proposed model precisely differentiates the ranks between FM1 and FM3, as well as FM8 and FM10.

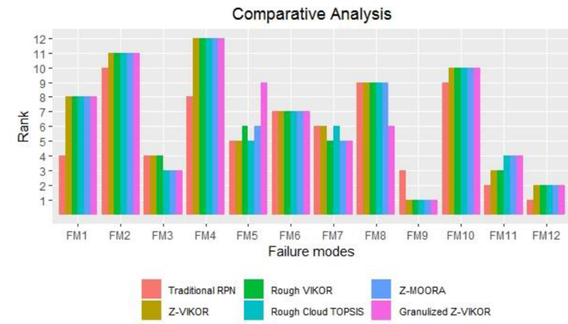

Fig. 5. Comparative analysis.

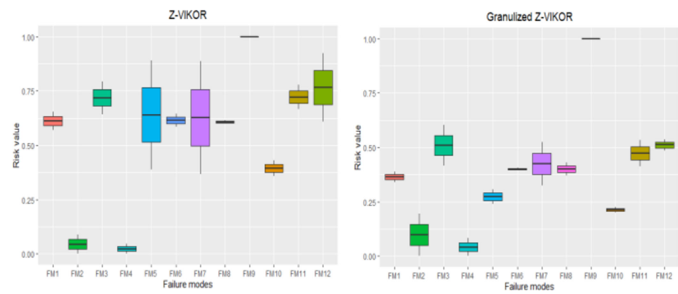

Fig. 6. Box-plot of the risk values of FMs.

Second, the effect of the weight of risk criteria, introduced in the given approach, can clearly be noticed in the prioritization results of FM5 and FM6. From the conventional RPN method, it is clear that FM5 is riskier than FM6. Nevertheless, the risk prioritization of failure modes by the suggested method illustrates that FM6 is riskier than FM5, which is logical from the risk reduction point of view as FM6 has a higher severity than FM5.

In the second comparative analysis, the ranks of the FM3, FM5, FM7, FM8, and FM11 obtained by the Z-VIKOR are different from the rank results derived from the proposed method (see Fig. 5). The reasons behind these inconsistencies are: i) Z-VIKOR does not utilize the concepts of RN, and thus fails to capture the true perception of the experts in the analysis, and ii) interactions among the risk criteria are not considered in Z-VIKOR. For example, FM11 is ranked higher than FM3 by Z-VIKOR. In contrast, the proposed model suggests that FM3 is riskier than FM11. FM3 should be ranked higher than FM11 as it has higher severity and detectability scores.

Further, comparative analysis with rough VIKOR, rough cloud TOPSIS, and Z-MOORA is performed and ranking differences with the proposed model are populated in Fig. 5. The reasons behind these ranking differences can be explained as follows: rough VIKOR and rough cloud TOPSIS based FMEA methods use the crisp evaluations. These methods do not consider the reliability of the risk assessment information and interaction among the risk criteria P, S, and D.

(ii) Z-MOORA incurs some amount of information loss as it converts the Z-number into the traditional fuzzy number to reduce the computational complexity. Interaction among P, S, and D is ignored in this approach. Also, it does not consider

TABLE VII
SPEARMAN'S RANK-CORRELATION TEST

FMs	1	2	3	4	5	6	7	8	9	10	11	12	r_s	Z	
M1-M2	0	0	1	0	4	0	1	3	0	0	1	0	0.903	2.99	>1.645
M1-M3	4	1	1	4	4	0	1	3	2	1	2	1	0.756	2.51	>1.645
M1-M4	0	0	1	0	3	0	0	3	0	0	1	0	0.931	3.09	>1.645
M1-M5	0	0	0	0	4	0	1	3	0	0	0	0	0.909	3.02	>1.645
M1-M6	0	0	0	0	3	0	0	3	0	0	0	0	0.937	3.11	>1.645

the concept of RN in the analysis, hence fails to utilize the true perception of experts.

Moreover, the correctness of the proposed model is also supported by the high correlation coefficients (> 0.90) between the proposed model and the compared models (see Table VII). The suggested approach may, therefore, be treated as the more flexible and reasonable risk evaluation and prioritization model.

In addition, we provide the comparison of uncertainty between Z-VIKOR and the proposed model (see Fig. 6). In view of this, we conduct box-plot analysis of the risk values, computed from Z-VIKOR and the proposed model. Here, risk values are computed by varying the policy coefficient from 0 to 1 with an interval of 0.05. It shows that there is very less variation in risk values while using the proposed model. From this, it can be inferred that the proposed model is able to utilize the information in an effective way with reduced uncertainty.

Further, Spearman's rank-correlation test [44] is conducted to obtain the relationship between rankings obtained from Z-VIKOR (M2), traditional RPN (M3), rough VIKOR (M4), rough cloud TOPSIS (M5), Z-MOORA (M6), and the proposed model (M1). The test statistic (z) for Spearman's rank-correlation (r_s) is

$$z = r_s\sqrt{n-1}, \text{ where } r_s = 1 - 6\sum_{i=1}^{n}(d_i)^2/n(n^2-1).$$

Here, n is the number of failure modes (FMs), and d_i is the difference in the ranking of *FMi*, obtained from different methods. We consider the null (H_0) and alternative (H_1) hypotheses as $H_0 : r_s \leq 0; H_1 : r_s > 0$. As the hypotheses consider one-tail of the population, the critical value of z-statistic for 95% confidence interval is 1.645. The result of Spearman's rank-correlation test is shown in Table VII.

Here, M1-M2 indicates the difference in the rankings obtained from the proposed model and the Z-VIKOR. Similarly, M1-M3, M1-M4, M1-M5, and M1-M6 can be described. The computed z-statistic for all the cases is greater than 1.645 (see Table VII) and hence, the null hypothesis is rejected. So, the rankings obtained from the proposed model are positively correlated with the rankings obtained from M2, M3, M4, M5, and M6. This positive relationship proves the validity of the proposed model.

VII. CONCLUSION

In this article, a new risk prioritization model, named, granulized Z-VIKOR, was developed to analyze and rank FMs in FMEA. The granulized Z-number (gZN), developed by a judicious integration of Z-number and the newly defined DUARN, is used to model the uncertainty and reliability of experts' evaluations. gZN also enriches the quality of the risk assessment information by moving the experts' opinions toward their true perception. DUARN was an improved variant of the existing RN in terms of capturing uncertainty. Augmenting the FMEA through the integration of the proposed *gZN* and VIKOR enables to overcome the deficiencies of the conventional FMEA and its other recent advancements by effective utilization of risk assessment information. Further, capturing the relationship between A and B components of a *gZN* using the maximum-entropy principle could reduce the information loss in risk computations. The model also reflects the importance of risk criteria P, S, and D, considering interactions among them.

A case study with sensitivity and comparative analysis with other existing methods is presented for risk analysis of the APP. Another case study is provided for prioritizing the risks involved in supplier selection.

The results show the applicability, feasibility, effectiveness, and superiority of the proposed model. This new FMEA model can assist safety managers in making safety-related decisions more effectively and efficiently in situations when safety-related information of a system is not readily available or not reliable.

This article can be extended in several ways for future research. First, the additional insights gained from the DUARN using the lower approximation has not been considered with gZN. However, this aspect may be explored in future. Second, lack of consensus among the experts may bring inconsistency in risk assessments. Hence, consensus approaches may be introduced in future. Third, the gZN can be used to obtain risk intervals, which can be another measure of uncertainty in risk assessment. Fourth, the utility of other probabilistic distance measures, e.g., Hellinger and Bhattacharya distances can be explored along with the joining algorithms (e.g., complete or ward linkages) used in clustering. Fourth, instead of using the simple RN, multigranulation rough sets [45] can be explored with Z-numbers and VIKOR for FMEA applications. The other futuristic applications of the proposed model include modeling of different decision-making problems.

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