

A Weighted Similarity Measure Between Z-Numbers and Bow-Tie Quantification

Souvik Das , Ashish Garg , Sankar K. Pal , *Life Fellow, IEEE*, and J. Maiti , *Member, IEEE*

Abstract—In this article, we propose a new weighted Z-similarity measure between two Z-numbers, which is able to retain the original information provided by experts in linguistic terms. This measure takes care of the directional aspect of reliability of information by leveraging the Hausdorff distance. Probabilistic-approach-based statistical distance measure is used to characterize the internal relationship between two parts of a Z-number. Some properties of the Z-similarity measure are proved. The measure is capable of quantifying the probability of basic events in bow-tie analysis. It is experimentally shown that the Z-similarity measure is able to overcome the lacuna of the available techniques. Sensitivity analysis is done to verify its feasibility and applicability. We further propose another measure, called Z-similarity-based basic event contribution (Z-BEC), to quantify the contribution of basic events to the occurrence of different accidents. The performance of the Z-BEC measure is compared with that of Fussell–Vesely index and Birnbaum’s structural index.

Index Terms—Bow-tie, fuzzy sets (FSs), Hausdorff distance, safety analytics, Z-numbers, Z-similarity.

I. INTRODUCTION

A. Similarity Measure Between Z-Numbers

INCORPORATION of similarity measure into fuzzy set (FS) theory [1] has the potential of handling impreciseness and uncertainties of experts’ judgments, and information loss, though the assumption of reliable information has been criticized by many researchers. Additionally, information sources for different decision-making problems also vary that adds variability in the reliability of information. For example, an experienced person provides more reliable information than that of a less experienced one. To overcome this, Zadeh [2] further introduced Z-number to solve the issue of unreliability present in information evaluation. A Z-number is an ordered pair of fuzzy numbers, $Z = (A, B)$, where A is an inexplicit value of a random variable X , and B is an inexplicit estimation of the reliability of A [2]. Since then, various studies have been conducted to deal with Z-information [3]–[6], but there is lacuna of research

concerning the similarity measure between Z-numbers. For example, Mohamad and Ibrahim [7] incorporated Jaccard similarity measure with Z-numbers to obtain a reliable and optimal decision. However, they fail to incorporate the relation between two parts of a Z-number and directional change between FSs, causing information loss.

Moreover, the scarcity of literature on similarity measure with Z-numbers can be surrogated by the studies of distance measure between Z-numbers, because similarity measure is the inverse of distance measure. Some key studies on the distance measure between two Z-numbers are highlighted here. Wang *et al.* [8] proposed a distance-based measure of linguistic Z-numbers, but did not take care of the relationship between the two parts, A and B of a Z-number. Aliev *et al.* [9] proposed various distance measures of Z-number for approximate reasoning, but the link between A and B is still missing. Similarly, due to the same structure of type-2 fuzzy number and Z-number, many researchers like Mitrovic and Rusov [10] and De Miguel *et al.* [11] did not consider the relation between A and B . To solve the abovementioned issues, Aliev *et al.* [12], [13] developed a suitable method based on the basic algebraic operations of Z-number. However, the method is cumbersome in evaluating complex decisions. Shen and Wang [14], therefore, developed a new comprehensive weighted distance measure, based on Hellinger distance [15], between two probability distributions. Recently, Shen *et al.* [16] proposed a new similarity measure of Z-number considering random and fuzzy information. These studies utilize the Z-information efficiently but fails to consider the direction between two Z-numbers as well as the λ -cut information of A while calculating the distance measure. From the abovementioned investigations, it is clear that these studies suffer from varying degree of information loss.

To summarize, the study of similarity measure on Z-number is at its infancy. For the sake of reducing the computational complexity, most of the research works on Z-number [17] rely on the conversion process proposed by Kang *et al.* [18]. In practical scenario, this conversion process cannot reflect well the characteristics of the original information. To fully utilize the Z-information in real life scenarios, more deep studies on Z-number are required [9]. In the present article, we have developed a novel similarity measure between two Z-numbers that reduces the information loss by considering the interrelationship between A and B as well as the directional aspects of the reliability of information and the λ -cut information of the possibilistic restriction.

B. Bow-Tie Analysis

Bow-tie is a probabilistic risk evaluation technique that integrates fault tree (FT) and event tree (ET) together and is used

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S. Das, A. Garg, and J. Maiti are with the Department of Industrial and Systems Engineering, Indian Institute of Technology Kharagpur, Kharagpur 721302, India (e-mail: rmdas9@gmail.com; ashish.gargnitt@gmail.com; jhareswar.maiti@hotmail.com).

S. K. Pal is with the Center for Soft Computing Research, Indian Statistical Institute, Kolkata 700108, India (e-mail: sankar@isical.ac.in).

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to identify and assess the root causes of system level failures, their consequences, safety barriers, or risk control systems [19], [20]. Bow-tie has been successfully applied in several safety studies such as in chemical process [21], disaster management [22], unmanned aircraft system [23], submarine pipeline [24], and mining industry [25].

Generally, the inputs to bow-tie are the probabilities of occurrences of basic events in FT and pivotal events in ET. However, often historical and reliable data about the frequencies and failure probabilities of all the basic and pivotal events are not available. The absence of historical data or prior probability information can be handled by obtaining experts' opinion. Often experts vary in their opinion, thereby introduces fuzziness in the data. To overcome it, several researchers like Mon and Cheng [26] and Lin and Wang [27] have integrated the fuzzy concepts in FT analysis to obtain the basic event's probability. Similarly, plethora of cognitive biases are present in experts' opinion while providing the probability values for pivotal events in ETA [28], [29]. Other scholars like Shahriar *et al.* [30], and Aqlan and Mustafa Ali [31] have also applied FS theory for bow-tie quantification and risk assessment.

Although experts' elicitation and FS theory are able to solve the issues of nonavailability and fuzziness of data, respectively, the information provided in terms of experts' opinion sometimes urges the reliability term to incorporate. So the incorporation of reliability parameter in experts' data for obtaining the probability of basic events may add more value in decision-making and provide a trustworthy framework for safety critical decision-making. Moreover, another important aspect is to quantify the contribution of basic events for evaluating design related improvements and criticality analysis. Many researchers utilized various importance measures like Fussell–Vesely (FV) [32], Birnbaum's (BIR) [33], risk reduction worth [34], risk achievement worth [35], etc., to obtain the effect of basic events to the occurrence of the top event in FT analysis. But, these measures fail to consider the importance of basic events to the occurrence of accident consequences, called accident scenario (AS), in a complete bow-tie analysis. The aim of the present research is therefore not only to propose a new weighted similarity measure of Z-number, but also to demonstrate its effectiveness in bow-tie analysis for quantification of basic event probabilities and their contribution to the occurrence of AS. The motivations behind this article and the contributions made are summed up in the next section.

C. Contributions

The research works that have been done so far on the similarity measure between two Z-numbers are mostly based on the conversion method proposed by Kang *et al.* [18], where the Z-number is converted to a regular fuzzy number. However, this conversion leads to information loss that may affect decision-making. To utilize the Z-information properly, a deeper study on similarity measure of Z-number needs to be performed. Second, the relation between A and B parts of a Z-number is very vital to characterize the Z-information, which has been ignored by the researchers. This necessitates the development of a new similarity measure. Third, the direction of change between FSs is also an essential component that enhances the Z-information efficiently, while calculating the similarity measure. The new measure should also consider this directional aspect. In our

study, we propose a new weighted similarity measure that overcomes the abovementioned limitations. The proposed method is applied to quantify bow-tie, which is one of the essential tools and techniques in safety analysis. Sensitivity analysis is performed to gain its insights. Further, a Z-similarity based basic event contribution (Z-BEC) measure has been derived to assess the effect of each basic event of bow-tie on the occurrence of AS.

The rest of this article is structured as follows. Some key concepts required to develop the new similarity measure are reviewed in Section II. The proposed similarity measure and its application in bow-tie quantification are described in Sections III and IV, respectively. Section IV also includes the development of the Z-BEC measure and a case study with comparative and sensitivity analysis. Section V provides a brief discussion. Section VI concludes this article.

II. PRELIMINARIES

In this section, the important definitions needed for further research in FSs, Z-number, and similarity measures are provided.

Definition 1 (see [1] and [36]): A discrete and finite FS A in the universe of discourse X is

$$A = \{\langle x, \mu_A(x) \rangle | x \in X\} = \left\{ \sum_j \mu_A(x_j) / x_j \right\} \quad (1)$$

here the summation stands for "Union."

In case, A is a continuous FS

$$A = \int \mu_A(x) / x$$

here integration stands for "Union."

Definition 2 (see [36]): The fuzzy membership function $\mu_A(x) : R \rightarrow [0, 1]$ is defined as a discrete and finite fuzzy number if $\text{supp}(A) = \{x \in X | \mu_A(x) > 0\}$, and $\text{count}(A) = \sum_{j=1}^n \mu_A(x_j)$, where $x_1 < x_2 < \dots < x_n$ and there exist real indices q, r ($1 \leq q \leq r \leq n$) such that the following conditions hold:

- i) $\mu_A(x_j) = 1 \quad \forall j \in [q, r]$;
- ii) $\mu_A(x_i) \leq \mu_A(x_j) < 1 \quad \forall i, j \in [1, q]$ and $(i \leq j)$;
- iii) $1 > \mu_A(x_i) \geq \mu_A(x_j) \quad \forall i, j \in [r, n]$ and $(i \leq j)$.

Here $\text{supp}(A)$ is the support of A , which is described by a set of elements with positive nonzero membership values and $\text{count}(A)$ represents the summation of all membership values.

Definition 3 (see [13]): An FS A with fuzzy membership function $\mu_A(x) : R \rightarrow [0, 1]$ is defined as a continuous fuzzy number if it satisfies the following properties.

- i) A is a convex FS.
- ii) A is a normal FS.
- iii) $\text{supp}(A)$ is bounded.
- iv) λ -cut of A is a closed interval.

Definition 4 (see [37]): The λ -cut for the FS A , denoted by $A_\lambda \forall \lambda \in [0, 1]$ is defined as $A_\lambda = \{x | \mu_A(x) \geq \lambda, x \in X\}$, where X is a universal set. The λ -cut of A can also be described by $(A_L(\lambda), A_R(\lambda))$, such that

$$A_L(\lambda) = \begin{cases} \inf\{x | x \in A_\lambda\} & \lambda > 0 \\ \inf\{x | x \in \text{supp}(A)\} & \lambda = 0 \end{cases}$$

$$A_R(\lambda) = \begin{cases} \sup\{x | x \in A_\lambda\} & \lambda > 0 \\ \sup\{x | x \in \text{supp}(A)\} & \lambda = 0 \end{cases}$$

where the supremum (sup) represents the minimum value of the upper bound and infimum (inf) is the maximum value of the lower bound. The FS A is the union of all its λ -FSs as

$$A = \left(\bigcup_{\forall \lambda} \lambda A_\lambda \right), \text{ where } \lambda \in [0, 1].$$

Definition 5 (see [2] and [12]): $Z = (A, B)$ is a two-tuple discrete Z-number, where A is a discrete fuzzy number with the following membership function:

$$\begin{aligned} \mu_A : \{\mu_{A1}, \mu_{A2}, \dots, \mu_{An}\} &\rightarrow [0, 1], \\ \{\mu_{A1}, \mu_{A2}, \dots, \mu_{An}\} &\subset R \end{aligned}$$

and B is a discrete fuzzy number with membership function

$$\begin{aligned} \mu_B : \{\mu_{B1}, \mu_{B2}, \dots, \mu_{Bn}\} &\rightarrow [0, 1], \\ \{\mu_{B1}, \mu_{B2}, \dots, \mu_{Bn}\} &\subseteq [0, 1]. \end{aligned}$$

Definition 6 (see [13]): $Z = (A, B)$ is said to be continuous Z-number, when A is a continuous fuzzy number with membership function $\mu_A : R_X \rightarrow [0, 1]$, where R_X is the set of real numbers, and B is also a continuous fuzzy number with the membership function $\mu_B : [0, 1] \rightarrow [0, 1]$.

Definition 7 (see [38]): Given a mapping $D : \text{FS} \times \text{FS} \rightarrow [0, 1]$, $D(A_1, A_2)$ is said to be a distance measure between two fuzzy numbers A_1 and A_2 , if it satisfies the following properties.

- i) $0 \leq D(A_1, A_2) \leq 1$
- ii) $D(A_1, A_2) = 0 \Leftrightarrow A_1 = A_2$.
- iii) $D(A_1, A_2) = D(A_2, A_1)$.
- iv) $D(A_1, A_2) + D(A_2, A_3) \geq D(A_1, A_3)$.
- v) If $A_1 \subseteq A_2 \subseteq A_3$ then $D(A_1, A_3) \geq D(A_1, A_2)$ and $D(A_1, A_3) \geq D(A_2, A_3)$.

Definition 8 (see [39]): Given a mapping $S : \text{FS} \times \text{FS} \rightarrow [0, 1]$, $S(A_1, A_2)$ is said to be a similarity measure between two fuzzy numbers A_1 and A_2 , if it satisfies the following properties:

- i) $0 \leq S(A_1, A_2) \leq 1$;
- ii) $S(A_1, A_2) = 1 \Leftrightarrow A_1 = A_2$;
- iii) $S(A_1, A_2) = S(A_2, A_1)$;
- iv) $T\{S(A_1, A_2), S(A_2, A_3)\} \leq S(A_1, A_3)$;
- v) If $A_1 \subseteq A_2 \subseteq A_3$

then

$$S(A_1, A_3) \leq S(A_1, A_2) \text{ and } S(A_1, A_3) \leq S(A_2, A_3)$$

where, T stands for transitivity.

Definition 9 (see [40]): Let $A = \{a_1, a_2, a_3, a_4; w_A\}$ be a trapezoidal fuzzy number, then the perimeter (P) of A is defined as

$$\begin{aligned} P(A) &= \sqrt{(a_1 - a_2)^2 + w_A^2} + \sqrt{(a_3 - a_4)^2 + w_A^2} \\ &\quad + (a_3 - a_2) + (a_4 - a_1). \end{aligned} \quad (2)$$

And the area of A is defined as

$$\text{Area}(A) = \frac{w_A \times (a_3 - a_2 + a_4 - a_1)}{2}. \quad (3)$$

Definition 10 (see [41]): If distance, $D \rightarrow [0, 1]$ is an ultrametric on an FS, then the fuzzy relation of similarity measure between two points (A_1, A_2) and their distance can be defined as $S(A_1, A_2) = N(D(A_1, A_2))$. $S(A_1, A_2)$ is a T -transitive similarity measure, where t -norm T , t -conorm δ , and a strong negator N form a de-Morgan triplet.

Definition 11 (see [42] and [43]): Let X be a random variable with pdf $p(x)$ and $Z = (A, B)$, where A is a discrete fuzzy number representing the fuzzy constraint on values of X and B plays the role of the fuzzy constraint on the probability measure of A . So the probability of X is A , i.e., $P(X \text{ is } A)$ is B and mathematically be expressed as

$$B = P(X \text{ is } A) = \sum_{i=1}^n \mu_A(x_i) p(x_i).$$

where, $X = \{x_1, x_2, x_3, \dots\}$ is discrete random variable and μ_A is the membership function of A .

When A is continuous fuzzy number, $P(X \text{ is } A)$ is mathematically defined as

$$B = P(X \text{ is } A) = \int \mu_A(x) p(x) dx$$

where μ_A is the membership function of A and $p(x)$ is the probability density function of X .

III. PROPOSED SIMILARITY MEASURE OF Z-NUMBERS

A. Similarity of Possibilistic Restrictions

Z-number, though poses dual-dimensional mathematical attributes, carries different meanings in decision-making and holds a strong relationship between those two attributes. Synthesizing this strong harmony requires more information and further investigations. In this section, a constructivism with various information related to area, height, perimeter, and center of gravity (COG) distance is applied to similarity measure to reveal the underpinning information. Further, the underlying relationship, exhibiting strong correlation between possibilistic restriction and belief measure, can also be manifested by its probability distribution. We define here a similarity measure, which is an improved version of that by Khorshidi and Nikfalazar [44], for quantifying the parallelism of two possibilistic restrictions. Also, a statistical distance measure has been developed for quantifying the similarity between two probability distributions. The sum of the similarity measures of possibilistic restrictions and their underlying probability distributions therefore reflects an efficient utilization of Z-information.

1) *Similarity Measure of Restriction Part (Without Considering the Relationship With Belief Measure)*: Let Z_1 and Z_2 be the two discrete Z-numbers $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ where $A_1 = \{a_{1i}\}$, and $A_2 = \{a_{2i}\}$, $\forall i = (1, 2, \dots, m)$ are the associated possibilistic restrictions, respectively. The similarity measure between these two restrictions can be expressed as (4), shown at the bottom of the page [44], where, $d'(A_1, A_2)$

$$S_Z(A_1, A_2) = \left(1 - \frac{\sum_{i=1}^m |a_{1i} - a_{2i}|}{4} \times d'(A_1, A_2) \right) \left(1 - \frac{(|\text{Area}(A_1) - \text{Area}(A_2)| + |w_{A_1} - w_{A_2}| + \frac{|P(A_1) - P(A_2)|}{\max\{P(A_1), P(A_2)\}})}{3} \right) \quad (4)$$

is the COG distance between the two fuzzy numbers and is expressed as

$$d'(A_1, A_2) = \frac{\sqrt{(\pi_{A_1} - \pi_{A_2})^2 + (\rho_{A_1} - \rho_{A_2})^2}}{\sqrt{1.25}} \quad (5)$$

where, π_{A_q} and ρ_{A_q} determine the COG points of the trapezoidal fuzzy number $A_q (q = 1, 2)$, and are defined as

$$\rho_{A_q} = \begin{cases} \frac{w_{A_q} \times \left(\frac{a_{q3} - a_{q2}}{a_{q4} - a_{q1}} + 2 \right)}{6}, & \text{if } a_{q4} \neq a_{q1} \\ \frac{w_{A_q}}{2}, & \text{if } a_{q4} = a_{q1} \end{cases} \quad (6)$$

$$\pi_{A_q} = \begin{cases} \frac{\rho_{A_q} \times (a_{q3} + a_{q2}) + (a_{q4} + a_{q1}) \times (w_{A_q} - \rho_{A_q})}{2w_{A_q}}, & \text{if } w_{A_q} \neq 0 \\ \frac{a_{q4} + a_{q1}}{2}, & \text{if } w_{A_q} = 0 \end{cases} \quad (7)$$

where $\text{Area}(A_q)$ and $P(A_q)$ are the area and perimeter of the fuzzy number A_q , respectively (see Definition 9).

Proof 1: Let A_1, A_2 , and A_3 be the possibilistic restrictions of Z_1, Z_2 , and Z_3 -numbers, respectively. From the properties of distance measure between two fuzzy numbers, mentioned in Definition 7, we can write

$$d'(A_1, A_3) \leq d'(A_1, A_2) + d'(A_2, A_3). \quad (8)$$

From (4), it is clear that the second term is constant and the first term represents the distance between two fuzzy numbers. Now, from the first term of (4), let us consider

$$\begin{aligned} d_A(A_1, A_3) &= \frac{\sum_{i=1}^m |a_{1i} - a_{3i}|}{4} \\ &= \frac{\sum_{i=1}^m |a_{1i} - a_{2i} + a_{2i} - a_{3i}|}{4} \\ &\leq \frac{\sum_{i=1}^m |a_{1i} - a_{2i}| + \sum_{i=1}^m |a_{2i} - a_{3i}|}{4} \\ &= \frac{\sum_{i=1}^m |a_{1i} - a_{2i}|}{4} + \frac{\sum_{i=1}^m |a_{2i} - a_{3i}|}{4} \\ &= d_A(A_1, A_2) + d_A(A_2, A_3). \end{aligned}$$

Thus, by combining the properties of distance defined in Definition 7, properties of similarity defined in Definition 8, and de-Morgan triplet defined in Definition 10, we have

$$S_Z(A_1, A_3) \geq T\{S_Z(A_1, A_2), S_Z(A_2, A_3)\}. \quad (9)$$

Example 1: Let two discrete Z -numbers be, $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$, whose possibilistic restriction measures, defined by trapezoidal fuzzy numbers, be $A_1 = (0, 0, 0.1, 0.2)$ and $A_2 = (0.1, 0.2, 0.2, 0.3)$, respectively.

From Definition 9, the area and the perimeter of these trapezoidal fuzzy number are $\text{Area}(A_1) = 0.15$ and $\text{Area}(A_2) = 0.1$, and $P(A_1) = 2.305$ and $P(A_2) = 2.21$, respectively.

Thus from (4), the similarity between A_1 and A_2 is

$$S_Z(A_1, A_2) = 0.873.$$

2) *Similarity Measure of Probability Distributions:* Let p_{i_j} be the probability distribution of $Z_i = (A_i, B_i)$, and μ_{A_i} be the membership function of $A_i, i = 1, 2$ and $j = 1, 2, \dots, n$. The restrictions that need to be satisfied are as follows:

- i) $\sum_{j=1}^n p_{i_j} = 1 \quad \forall i$;
- ii) $\sum_{j=1}^n p_{i_j} \mu_{A_{i_j}} \rightarrow B_j$;
- iii) $\sum_{j=1}^n x_{i_j} p_{i_j} = \sum_{j=1}^n x_{i_j} \mu_{A_{i_j}} / \sum_{j=1}^n \mu_{A_{i_j}}$.

The probability distribution is calculated by using maximum entropy method and the similarity between two probability distributions is calculated by using the statistical similarity (variational similarity) measure.

The variational similarity (S_v) between two probability distributions over an FS is defined as (see Definition 10 and [45])

$$S_v(p_{Z_1}, p_{Z_2}) = N \left(\sup \left(\sum_{j=1}^n |p_{Z_{1j}} - p_{Z_{2j}}| \right) \right). \quad (10)$$

Proof 2: Let Z_1, Z_2 , and Z_3 be three Z -numbers and p_{Z_1}, p_{Z_2} , and p_{Z_3} be their corresponding probability distributions, as given in Definition 11. Then

$$\begin{aligned} S_v(p_{Z_1}, p_{Z_3}) &= N \left(\sup \left(\sum_{j=1}^n |p_{Z_{1j}} - p_{Z_{3j}}| \right) \right) \\ &= N \left(\sup \left(\sum_{j=1}^n |p_{Z_{1j}} - p_{Z_{2j}} + p_{Z_{2j}} - p_{Z_{3j}}| \right) \right) \\ &\geq N \left(\sup \left(\sum_{j=1}^n |p_{Z_{1j}} - p_{Z_{2j}}| + \sum_{j=1}^n |p_{Z_{2j}} - p_{Z_{3j}}| \right) \right) \\ &= N \left(\sup \left(\sum_{j=1}^n |p_{Z_{1j}} - p_{Z_{2j}}| \right) \right) \\ &\quad + \inf \left(\sum_{j=1}^n |p_{Z_{2j}} - p_{Z_{3j}}| \right) \\ &= N \left(\sup \left(\sum_{j=1}^n |p_{Z_{1j}} - p_{Z_{2j}}| \right) \right) \\ &\quad + N \left(\inf \left(\sum_{j=1}^n |p_{Z_{2j}} - p_{Z_{3j}}| \right) \right). \end{aligned}$$

Therefore, from the abovementioned equation, we can write

$$S_v(p_{Z_1}, p_{Z_3}) \geq S_v(p_{Z_1}, p_{Z_2}) + S_v(p_{Z_2}, p_{Z_3}). \quad (11)$$

Example 2: Let us consider two discrete Z -numbers, $Z_1 = \{(0, 0, 0.1, 0.2), (0.7, 0.8, 0.9)\}$, and $Z_2 = \{(0.1, 0.2, 0.2, 0.3), (0.5, 0.6, 0.7)\}$.

Then for Z_1 , we can obtain

$$\begin{aligned} p_{Z_{11}} &= \frac{0.061}{0} + \frac{0.18}{0.05} + \frac{0.52}{0.1} + \frac{0.18}{0.15} + \frac{0.061}{0.2} \\ p_{Z_{12}} &= \frac{0.044}{0} + \frac{0.16}{0.05} + \frac{0.59}{0.1} + \frac{0.16}{0.15} + \frac{0.044}{0.2} \end{aligned}$$

$$p_{Z_{13}} = \frac{0.03}{0} + \frac{0.14}{0.05} + \frac{0.66}{0.1} + \frac{0.14}{0.15} + \frac{0.03}{0.2}$$

$$p_{Z_{14}} = \frac{0.017}{0} + \frac{0.114}{0.05} + \frac{0.74}{0.1} + \frac{0.114}{0.15} + \frac{0.017}{0.2}$$

$$p_{Z_{15}} = \frac{0.0084}{0} + \frac{0.083}{0.05} + \frac{0.82}{0.1} + \frac{0.083}{0.15} + \frac{0.0084}{0.2}.$$

Similarly, for Z_2

$$p_{Z_{21}} = \frac{0.15}{0.1} + \frac{0.21}{0.15} + \frac{0.29}{0.2} + \frac{0.21}{0.25} + \frac{0.15}{0.3}$$

$$p_{Z_{22}} = \frac{0.122}{0.1} + \frac{0.21}{0.15} + \frac{0.34}{0.2} + \frac{0.21}{0.25} + \frac{0.122}{0.3}$$

$$p_{Z_{23}} = \frac{0.10}{0.1} + \frac{0.20}{0.15} + \frac{0.40}{0.2} + \frac{0.20}{0.25} + \frac{0.10}{0.3}$$

$$p_{Z_{24}} = \frac{0.079}{0.1} + \frac{0.191}{0.15} + \frac{0.46}{0.2} + \frac{0.191}{0.25} + \frac{0.079}{0.3}$$

$$p_{Z_{25}} = \frac{0.044}{0.1} + \frac{0.161}{0.15} + \frac{0.59}{0.2} + \frac{0.161}{0.25} + \frac{0.044}{0.3}.$$

Therefore, the variational similarity measure between $p_{Z_{11}}$ and $p_{Z_{21}}$ is $S_\nu(p_{Z_{11}}, p_{Z_{21}}) = 0.532$. The Z-similarity measure of the possibilistic restrictions (ZPSM) may be defined as

$$\text{ZPSM}(A_1, A_2) = \frac{1}{2} (S_Z(A_1, A_2) + S_\nu(p_{Z_1}, p_{Z_2})). \quad (12)$$

B. Similarity of Reliability Measures

As reliability is a very important trait of Z-numbers, similarity between two Z-numbers should be computed with consideration of the similarity between their corresponding reliability measures along with their directions. Reconciling this, we define here, leveraging the Hausdorff metric (h) and FS theory, the similarity of reliability measures of Z-numbers (ZRSM). This similarity measure is then used to calculate the distance between two Z-numbers.

Let B_i be the reliability measure of a Z-number with membership function μ_{B_i} and its λ -cut $[B_{iL}, B_{iR}]$, $0 < \lambda < 1$, $i = 1, 2$. Hausdorff metric between two reliability measures B_1 and B_2 is defined by $h(B_1, B_2) = \max\{|B_{1L} - B_{2L}|, |B_{1R} - B_{2R}|\}$. Hausdorff similarity between B_1 and B_2 is then defined as

$$S_h^\lambda(B_1, B_2) = 1 - h(B_1, B_2). \quad (13)$$

Further, the center-center similarity between B_1 and B_2 , can be expressed as

$$S^\lambda(B_1, B_2) = 1 - \left| \frac{B_{1L} + B_{1R}}{2} - \frac{B_{2L} + B_{2R}}{2} \right|. \quad (14)$$

Then, the ZRSM is defined as

$$\text{ZRSM}(B_1, B_2) = 0.5[S_h^\lambda(B_1, B_2) + S^\lambda(B_1, B_2)] \quad (15)$$

It is obvious that $\text{ZRSM}(B_1, B_2)$ satisfies the properties (i)–(iii), and (v) mentioned in Definition 8. Property (iv) can be proved as follows.

Proof 3: Let B_1, B_2 , and B_3 be three fuzzy numbers. By the definitions of absolute value and discrete definite integral of fuzzy numbers, we get the following mathematical lemma:

- 1) $|B_1 + B_2| \leq |B_1| + |B_2|$;
- 2) $\sum_{i=1}^n (B_{1i} + B_{2i}) = \sum_{i=1}^n B_{1i} + \sum_{i=1}^n B_{2i}$;
- 3) If $B_{1i} < B_{2i}$, then $\sum_{i=1}^n B_{1i} < \sum_{i=1}^n B_{2i}$.

So the Hausdorff distance using the aforementioned lemma can be written as

$$h(B_1, B_3) = \max(|B_{1L} - B_{3L}|, |B_{1R} - B_{3R}|)$$

$$= \max(|B_{1L} - B_{2L} + B_{2L} - B_{3L}|$$

$$|B_{1R} - B_{2R} + B_{2R} - B_{3R}|)$$

$$\leq \max(|B_{1L} - B_{2L}| + |B_{2L} - B_{3L}|$$

$$|B_{1R} - B_{2R}| + |B_{2R} - B_{3R}|)$$

$$= \{\max(|B_{1L} - B_{2L}|, |B_{1R} - B_{2R}|)$$

$$+ \max(|B_{2L} - B_{3L}|, |B_{2R} - B_{3R}|)\}$$

$$= h(B_1, B_2) + h(B_2, B_3).$$

We can proceed further by writing

$$h(B_1, B_3) \leq h(B_1, B_2) + h(B_2, B_3)$$

$$h(B_1, B_3) \leq \delta\{h(B_1, B_2), h(B_2, B_3)\}.$$

Applying the negator N both side

$$N[\delta\{h(B_1, B_2), h(B_2, B_3)\}] \leq N[h(B_1, B_3)].$$

Since (T, δ, N) is de-Morgan triplet

$$T\{N[h(B_1, B_2)], N[h(B_2, B_3)]\} \leq N[h(B_1, B_3)]$$

$$T\{S(B_1, B_2), S(B_2, B_3)\} \leq S(B_1, B_3).$$

From the abovementioned proof, it can easily be shown that both S_h^λ and S^λ satisfy all the properties of Definition 8.

Thus

$$\text{ZRSM}(B_1, B_3) \geq T\{\text{ZRSM}(B_1, B_2), \text{ZRSM}(B_2, B_3)\}. \quad (16)$$

In this article, we consider Z_1 and Z_2 as two discrete Z-numbers and B_1, B_2 as two discrete reliability measures of Z_1 and Z_2 , respectively. The λ -cut of B_i can be expressed as follows:

$$B_i = \{b_{ij} \mid \mu_{B_i}(b_{ij}) > \lambda\}, \text{ where } j = 1, 2, \dots, m$$

$$\mu_{B_i}(b_{ij}) : \{\mu_{b_{i1}}, \mu_{b_{i2}}, \dots, \mu_{b_{im}}\} \rightarrow [0, 1], \{b_{ij}\} \subset [0, 1].$$

Here, (B_{iL}, B_{iR}) is defined by

$$B_{iL} = \begin{cases} \inf\{b_{ij} \mid b_{ij} \in B_i^\lambda\}; & \lambda > 0 \\ \inf\{b_{ij} \mid b_{ij} \in \text{supp}(B)\}; & \lambda = 0 \end{cases} \quad (17)$$

$$B_{iR} = \begin{cases} \sup\{b_{ij} \mid b_{ij} \in B_i^\lambda\}; & \lambda > 0 \\ \sup\{b_{ij} \mid b_{ij} \in \text{supp}(B)\}; & \lambda = 0 \end{cases} \quad (18)$$

Similarly, ZRSM between Z_1 and Z_2 with reliability measures B_1 and B_2 , respectively, may be expressed by

$$\text{ZRSM}(B_1, B_2) = 0.5[S_h^\lambda(B_1, B_2) + S^\lambda(B_1, B_2)]. \quad (19)$$

Example 3: Let us consider two discrete Z-numbers, $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$. Let their reliability measures be defined by triangular fuzzy numbers as

$$B_1 = 0/0.7 + 0.6/0.75 + 1/0.8 + 0.4/0.85 + 0/0.9$$

$$B_2 = 0/0.5 + 0.5/0.55 + 1/0.6 + 0.6/0.65 + 0/0.9.$$

First, the proposed method computes $B_{1L}(\lambda)$ and $B_{1R}(\lambda)$ of B_1 and B_2 in every cut set level.

$$B_{1L}(\lambda) = \begin{cases} 0.75 & 0 \leq \lambda \leq 0.6 \\ 0.8 & 0.6 < \lambda \leq 1 \end{cases}$$

$$B_{1R}(\lambda) = \begin{cases} 0.85 & 0 \leq \lambda \leq 0.4 \\ 0.8 & 0.4 < \lambda \leq 1 \end{cases}$$

$$B_{2L}(\lambda) = \begin{cases} 0.55 & 0 \leq \lambda \leq 0.5 \\ 0.6 & 0.5 < \lambda \leq 1 \end{cases}$$

$$B_{2R}(\lambda) = \begin{cases} 0.65 & 0 \leq \lambda \leq 0.6 \\ 0.6 & 0.6 < \lambda \leq 1. \end{cases}$$

Therefore

$$[B_{1L}(\lambda), B_{1R}(\lambda)] = \begin{cases} [0.75, 0.85] & 0 \leq \lambda \leq 0.4 \\ [0.75, 0.8] & 0.4 < \lambda \leq 0.6 \\ [0.8, 0.8] & 0.6 < \lambda \leq 1 \end{cases}$$

$$[B_{2L}(\lambda), B_{2R}(\lambda)] = \begin{cases} [0.55, 0.65] & 0 \leq \lambda \leq 0.5 \\ [0.65, 0.6] & 0.5 < \lambda \leq 0.6 \\ [0.6, 0.6] & 0.6 < \lambda \leq 1. \end{cases}$$

Next, the center to center similarity between B_1 and B_2 can be obtained using (14) as $S^\lambda(B_1, B_2) = 0.8$ and using (1) we get $S_h^\lambda(B_1, B_2) = 0.8$. Finally, the ZRSM between Z_1 and Z_2 can be computed using (19) as

$$\text{ZRSM}(B_1, B_2) = 0.5 \times (0.8 + 0.8) = 0.8.$$

C. Comprehensive Weighted Similarity Measure of Z-Numbers

In the previous section, we have defined the similarity measure of reliability and possibilistic restrictions between two Z-numbers. It is clear that the simultaneous application of similarity measure of both reliability and possibilistic restriction should be taken into account to rationally measure the similarity between two Z-numbers. In different scenarios, preference, related to percentage contribution, of reliability and possibilistic restriction may differ. Therefore, a comprehensive weighted similarity measure needs to be defined to compute the similarity between two Z-numbers and to get better insight in various situations. Such a comprehensive weighted similarity measure may be defined as

$$S_{IM}^w(Z_1, Z_2) = w \times \text{ZRSM}(B_1, B_2) + (1 - w) \times \text{ZPSM}(A_1, A_2) \quad (20)$$

where $0 < w < 1$

The value of w depends on experts' opinions. w lies between 0 to 0.5, when experts think to put more weightage on the similarity value of reliability measure of Z-number, and it lies between 0.5 to 1, when they emphasize on similarity between possibilistic restrictions of Z-information.

Specifically, we can say that, if $w = 0$, the comprehensive similarity value between two Z-numbers depends on possibilistic restriction only, and ignores the contribution of reliability information of the Z-number. Similarly, $w = 1$ signifies that the decision makers are interested only on the reliability component

of Z-information, and not on the effect of possibilistic restriction of Z-number. The proposed measure, with no consideration of similarity measure of restriction part and directional aspects between FSs, boils down to that of [14]. This means that the measure of Shen and Wang is a special case of the proposed measure.

Proof 4: As $\text{ZRSM}(B_1, B_2)$ and $\text{ZPSM}(A_1, A_2)$ follow all the properties (i)–(iv) of Definition 8, S_{IM}^w must follow all the properties given as follows.

As, $0 \leq \text{ZRSM}(B_1, B_2) \leq 1$ and $0 \leq \text{ZPSM}(A_1, A_2) \leq 1$, S_{IM}^w follow the same, therefore $0 \leq S_{IM}^w \leq 1$.

If $B_1 = B_2$, $\text{ZRSM}(B_1, B_2) = 1$, and If $A_1 = A_2$, $\text{ZPSM}(A_1, A_2) = 1$. Therefore

$$S_{IM}^w(Z_1, Z_2) = 1, \text{ if } Z_1 = Z_2.$$

$$S_{IM}^w(Z_1, Z_2) = S_{IM}^w(Z_2, Z_1)$$

$$S_{IM}^w(Z_1, Z_3) \geq T\{S_{IM}^w(Z_1, Z_2), S_{IM}^w(Z_2, Z_3)\}.$$

Example 4: Let two discrete Z-numbers be $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$. Let their possibilistic restriction measures be defined by trapezoidal fuzzy numbers as $A_1 = (0, 0, 0.1, 0.2)$, $A_2 = (0.1, 0.2, 0.2, 0.3)$ and reliability measures by triangular fuzzy numbers $B_1 = (0.7, 0.8, 0.9)$, $B_2 = (0.5, 0.6, 0.7)$. Then, from (12) and (19), we can write

$$\text{ZPSM}(A_1, A_2) = 0.7025, \text{ and } \text{ZRSM}(B_1, B_2) = 0.8.$$

Suppose experts think of equal importance for reliability measure and possibilistic restriction of Z-information, then

$$w = 0.5 \text{ and } S_{IM}^{0.5} = \frac{1}{2}(0.8 + 0.7025) = 0.7513.$$

Examples 5 and 6 are given in supplementary S.1.

IV. APPLICATION IN BOW-TIE QUANTIFICATION

Bow-tie is an analytical technique that combines the FT and ET through a safety critical failure event [i.e., central event (CE)], which depicts the top event for an FT and initiating event for an ET. This way, an efficient why–what relationship of the CE can be demonstrated and delineated in bow-tie. Three fundamental aspects of bow-tie quantification are computing the probabilities of basic events, CE, and ASs. Further, the probabilities of CE and AS are the function of the probability of basic events. Therefore, quantification of the basic events is the most important aspect and many a times experts' opinions are sought for it. The reliability of experts' judgment and its uncertainty are captured and well handled by Zadeh's Z-number in FS theory. However, to utilize the information to the maximum extent, more in-depth study on Z-number is required. Keeping this in mind, we have defined a new weighted similarity measure between two Z-numbers (20) in Section III. In this section, we use this measure in bow-tie quantification in terms of the probability of basic events.

A. Data Collection

To demonstrate the applicability of the proposed method, we have taken an explosive storage house (ESH) of a defense industry in India as an example. The ESH is a very sensitive area in defense organization, where transportation of explosives and ammunition using explosive van, from outside to the storage building is a routine activity. Any type of malfunctioning in the

TABLE I
BASIC EVENTS OF BRAKE FAILURE

Sl No	Basic Events (BE)
1	Worn Out brake shoes (B1)
2	Malfunctioning of brake mechanism (B2)
3	Rupture of hydraulic cylinder (B3)
4	Leakage of oil (B4)

TABLE II
EXPERTS' OPINIONS FOR BASIC EVENT PROBABILITIES

	BE	Decision Maker	Linguistic Z-value (A, B) Probability	
			A	B
1.	Worn out brake shoe (B1)	DM1	VP	VH
		DM2	P	VH
		DM3	VP	VH
2.	Malfunctioning of brake mechanism (B2)	DM1	VP	H
		DM2	MP	H
		DM3	P	VH
3.	Rupture of hydraulic cylinder (B3)	DM1	VP	H
		DM2	P	H
		DM3	P	H
4.	Leakage of brake oil (B4)	DM1	VP	H
		DM2	MP	H
		DM3	VP	VH

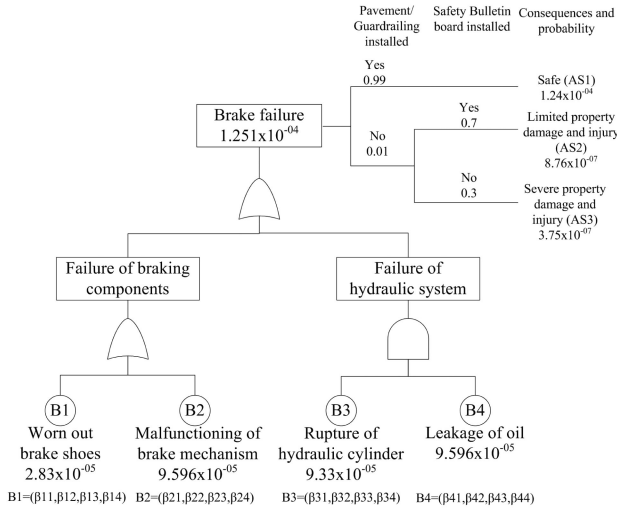


Fig. 1. Bow-tie diagram of brake failure.

explosive van may have severe consequences and high loss to the organization.

A team comprising the authors and experts from the plant has visited the ESH several times to collect data. Past records show that most of the incidents occur due to the brake failure of the explosive van. Brake failure takes place when one or more components such as brake shoe, hydraulic cylinder, etc., of the van fail to perform their intended functions. Here, brake failure has been considered as the CE, and the basic events that may lead to the CE are summarized in Table I. Bow-tie of the brake failure is then constructed, which is shown in Fig. 1.

Next, three experts have provided their opinions about the probability of the occurrence of each of the basic events, B1 to B4 (see Table I) on a linguistic scale (see column 1 of table in Appendix A.1). This probability relates to the A part of the Z-number. The experts have also provided the reliability of their opinions on a linguistic scale (see column 2 of table in Appendix A.1). This relates to the B part of the Z-number. The experts' opinions on A and B for each of the basic events, B1 to B4, are summarized in Table II. Further, the experts differ in their level of expertise based on their position, service time, education, and age factors (see Appendix A.2). These information are used to assign weightage to the experts (see Appendix A.3).

B. Quantification of Basic Event Probability

Let $B_n, n = 1, 2, \dots, N$ represent the n th basic event and $DM_k (k = 1, 2, \dots, K)$ represent the k th decision maker (expert). The following steps are used to calculate the probabilities.

Step 1: Select B_n and the corresponding linguistic opinions given by the experts (DM_1 to DM_k).

Step 2: Convert the linguistic opinions into fuzzy values with trapezoidal and triangular membership functions for A and B, respectively (see Appendix A.1). So for the i th expert, A assumes four values, say $a_{i1}, a_{i2}, a_{i3}, a_{i4}$ and B assumes three values, say b_{i1}, b_{i2}, b_{i3} .

Step 3: Obtain the degree of similarity of the opinions between each pair of experts DM_i and DM_j where $i \neq j$. Here, the opinions of $DM_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; b_{i1}, b_{i2}, b_{i3})$ and $DM_j = (a_{j1}, a_{j2}, a_{j3}, a_{j4}; b_{j1}, b_{j2}, b_{j3})$, are two Z-numbers. Then, the degree of similarity is obtained by the proposed similarity measure S_{IM}^w , given in (20) in Section III

$$S_{IM}^w(DM_i, DM_j) = w \times ZRSM(B_i, B_j) + (1 - w) \times ZPSM(A_i, A_j) \quad (21)$$

where $S_{IM}^w(DM_i, DM_j) \in (0, 1)$, and larger value of $S_{IM}^w(DM_i, DM_j)$ means greater similarity between DM_i and DM_j .

Step 4: Obtain the average agreement (AA) degree, $AA(DM_i)$, for the i th expert

$$AA(DM_i) = \frac{1}{K-1} \sum_{i \neq j}^K S(DM_i, DM_j). \quad (22)$$

Step 5: Calculate the relative agreement (RA) degree, $RA(DM_i)$, for the i th expert

$$RA(DM_i) = \frac{AA(DM_i)}{\sum_{i=1}^K AA(DM_i)}. \quad (23)$$

Step 6: Compute the consensus coefficient (CC) degree for the i th expert by the following function:

$$CC(DM_i) = \beta\psi(DM_i) + (1 - \beta)RA(DM_i) \quad (24)$$

where $\psi(DM_i)$ is normalized weight for the i th expert and β is a constant lying between 0 and 1.

Step 7: Compute the aggregated value of experts' judgments

$$R_{AG_i} = CC(DM_1) \times a_{1i} + CC(DM_2) \times a_{2i} + \dots + CC(DM_k) \times a_{ki} \quad (25)$$

where $i = 1, 2, 3, 4$ for trapezoidal membership function.

TABLE III
SIMILARITY VALUE

BE	Similarity Value	BE	Similarity Value
	DM1,DM2 0.925		DM1,DM2 1
B1	DM2,DM3 1	B2	DM2,DM3 0.925
	DM3,DM1 0.925		DM3,DM1 0.925
	DM1,DM2 1		DM1,DM2 1
B3	DM2,DM3 1	B4	DM2,DM3 0.925
	DM3,DM1 1		DM3,DM1 0.925

TABLE IV
AA, RA, CC

BE	DM	AA	RA	CC
	DM1	0.925	0.324561	0.357281
B1	DM2	0.9625	0.337719	0.29386
	DM3	0.9625	0.337719	0.34886
	DM1	0.9625	0.337719	0.36386
B2	DM2	0.9625	0.337719	0.29386
	DM3	0.925	0.324561	0.342281
	DM1	1	0.333333	0.361667
B3	DM2	1	0.333333	0.291667
	DM3	1	0.333333	0.346667
	DM1	0.9625	0.337719	0.36386
B4	DM2	0.9625	0.337719	0.29386
	DM3	0.925	0.324561	0.342281

Step 8: Convert the aggregated value of experts' judgments to a single value using the defuzzification function D as (26) shown at the bottom of this page.

Step 9: Calculate the probability of occurrence of basic events [27], [46]

$$\Pr = \begin{cases} \frac{1}{10^G} & \text{for } G \neq 0 \\ 0 & \text{for } G = 0 \end{cases} \quad (27)$$

where, $G = \left(\frac{1-D}{D}\right)^{\frac{1}{3}} \times 2.301$.

The linguistic responses (see Table II) of experts on basic events' probabilities are converted into fuzzy values by using Z-linguistic scale (see Appendix A.1). Then, the similarity values between experts are calculated by (21) and are summarized in Table III. AA, RA, and CC among the experts are calculated using (22), (23), and (24), respectively, and are given in Table IV. Aggregated value of the experts' judgments using (25), defuzzification using (26), and probability of each basic event using (27) are reported in Table V.

C. Quantification of CE and AS

The probability of occurrence of the CE can be calculated using either gate by gate method or cut set method [47]. Gate-by-gate method is usually applicable for small FTs, and the probability is calculated as per the logical gates mentioned in an FT. In case of large FT, when all the basic events do not contribute equally for the occurrence of the CE, cut set method is used to calculate the probability of the CE. The minimal cut set (MCS) is obtained by MOCUS [48] or MICSUP [49] algorithms.

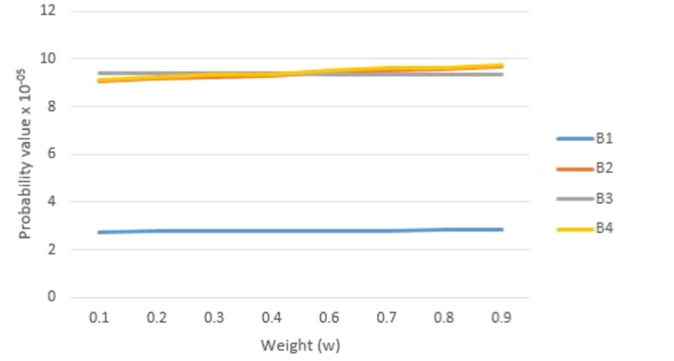


Fig. 2. Sensitivity analysis.

The AS are the end states of an ET. For the case study, there are three AS (see Fig. 1). Given the CE probability and the probabilities of success and failure of pivotal events, usually collected from the experts, the conditional probabilities of different AS of a bow-tie are calculated. In case of independent pivotal events, the probability of an AS is the joint probability of central and pivotal events leading to the AS.

As an example, the probability of brake failure is 1.251×10^{-04} , and the success probability of pavement/guard railing provision is 0.99 (from experts' opinion).

From the bow-tie, it is clear that there will be major safety concerns if the provision of pavement/guard railing is not provided. Under such a situation, if safety bulletin board is displayed, severe property damage and injuries may occur with a probability of 3.75×10^{-07} (see Fig. 1). Using this information, plant management can analyze the safety outcomes and accordingly provide safety interventions. Pseudocode for quantification of basic event probabilities using the proposed similarity measure is provided in supplementary S.2.

D. Sensitivity Analysis

As discussed in Section III, the parameter w depends on experts' thoughts and their preferences about the reliability measure and the probabilistic restriction measure while computing the similarity value. In this section, the effect of the parameter w on the results is discussed. When $w = 0.5$, experts believe that the importance of both the reliability portion and the possibilistic restriction portion is same and carries equivalent amount of information.

Second, $0 < w < 0.5$ signifies that the importance of the probabilistic restriction is more than that of the reliability information. The measure ZPSM then carries more information than the measure ZRSM. Fig. 2 shows the changes in probability value as w changes from 0 to 0.5.

Third, $0.5 < w < 1$ implies that the influence of the ZPSM is less than the ZRSM. Experts are giving more weightage to the reliability measure compared to the probabilistic restriction measure. Fig. 2 shows the changes in probability value as w changes from 0.5 to 1.

From Fig. 2, it is clear that there is a small amount of change in probability with change in w . This proves, in one way, that

$$D = \frac{-R_{AG_1}R_{AG_2} + R_{AG_3}R_{AG_4} + \frac{1}{3}(R_{AG_4} - R_{AG_3})^2 - \frac{1}{3}(R_{AG_2} - R_{AG_1})^2}{-R_{AG_1} - R_{AG_2} + R_{AG_3} + R_{AG_4}} \quad (26)$$

TABLE V
AGGREGATED VALUE, DEFFUZIFICATION VALUE, PROBABILITY

BE		Aggregated value			Defuzzified value		Probability of failure
B1	R_{AG}	0.029386	0.058772	0.129386	0.229386	0.114546	2.82061*10-05
B2	R_{AG}	0.058772	0.088158	0.188158	0.288158	0.158123	9.59578*10-05
B3	R_{AG}	0.063833	0.127667	0.163833	0.263833	0.156882	9.32268*10-05
B4	R_{AG}	0.058772	0.088158	0.188158	0.288158	0.158123	9.59578*10-05

TABLE VI
COMPARATIVE ANALYSIS

BE	Kang <i>et al.</i> [50] Probability	Aliev [9] Probability	Shen & Wang [14] Probability	Proposed Method Probability
B1	2.15912*10-05	2.77643*10-05	2.82061*10-05	2.797751*10-05
B2	6.75955*10-05	9.09053*10-05	9.59578*10-05	9.370906*10-05
B3	7.30126*10-05	9.39893*10-05	9.32268*10-05	9.372926*10-05
B4	6.75955*10-05	9.09053*10-05	9.59578*10-05	9.370906*10-05

the proposed method has relatively good stability and also the fulfillment of the intentions behind the setting of the parameter.

E. Comparison With the Existing Measures

To further validate the feasibility, effectiveness, and benefits of the proposed measures, a comparative analysis has been conducted with the existing measures. The comparing measures based on the following techniques are discussed below.

- 1) Following Kang *et al.* [18], a Z-number is transformed to a classical fuzzy number. The following formula is used to convert the reliability measure to a crisp value:

$$\alpha = \frac{\sum x \mu_B(x)}{\sum \mu_B(x)}. \quad (28)$$

The value of α is then multiplied with the fuzzy restriction A , and usual fuzzy computation is done on A .

- 2) Computing the distance between two Z-numbers based on λ -cut information of both parts of the Z-information [9], [13].

Let $Z = \{(a_1, a_2, a_3, a_4), (b_1, b_2, b_3)\}$ be a Z-number, then the distance between two Z-numbers can be calculated by the following relation:

$$d(Z_1, Z_2) = \frac{1}{D+1} \sum_{k=1}^D \{|a_{1k}^L - a_{2k}^L| + |a_{1k}^R - a_{2k}^R|\} + \frac{1}{D+1} \sum_{k=1}^D \{|b_{1k}^L - b_{2k}^L| + |b_{1k}^R - b_{2k}^R|\} \quad (29)$$

where

$$a_{ik}^L = \min(a_{ik}), \quad a_{ik}^R = \max(a_{ik}) \\ b_{ik}^L = \min(b_{ik}), \quad b_{ik}^R = \max(b_{ik}).$$

- 3) Direct computation of the Z-number based on the distance measure [14].

The probability values obtained by these three approaches and the proposed model are given in Table VI for four different basic events B1, B2, B3, and B4.

The differences in the probabilities, obtained from different methods, are due to the following reasons.

First, the difference in the probabilities of the proposed model and the one provided by Kang *et al.* [18] is due to the deficiencies of the latter in utilizing the information, provided by the experts properly, as it converts a Z-number to a crisp value and fails to emulate the combined Z-information for further processing.

The second comparative measure suffers from the fact of not using Z-information properly as it considers that both parts of the Z-number are independent in nature.

The measure of Shen and Wang [14] is a special case of the proposed measure and, on the other hand, it also fails to utilize the information of the λ -cut of the possibilistic restriction.

The probability values obtained by the proposed method are very close to the standards, given in reliability handbook of brake failure. As per the same standards, the probability values of B2, B3, and B4 should be close enough to each other and the same is evident in our proposed method. As the interrelationship between the two parts of the Z-number as well as λ -cut information of the possibilistic restriction are very important for capturing the Z-information provided in terms of Z-number very efficiently, our measure can be considered to provide better performance than the existing techniques while employing Z-numbers as an information source. This analysis thus provides a basis of feasibility and efficiency of the proposed method.

F. Z-BEC Measure

In FT analysis, to rank the basic events in terms of their contribution to the occurrence of the top event, researchers focus on developing basic event importance measures (e.g., FV index). In a similar line, we propose a new Z-similarity based bow-tie importance measure, called Z-BEC, for identifying the most critical basic event responsible for a particular AS. In order to obtain Z-BEC, the aggregated fuzzy input of the basic events, obtained from (25), are propagated covering the entire bow-tie. Steps to compute Z-BEC are given as follows.

1) *Steps to Compute Z-BEC: Step 1:* Using (21)–(25), obtain the fuzzy number of each of the basic events.

Step 2: Obtain the fuzzy number of each AS by propagating the fuzzy numbers of all the basic events, covering the entire bow-tie.

Step 3: Compute MCSs of the CE, and derive group MCSs for each of the basic events.

Step 4: Using (20), compute fuzzy similarity between group MCS of a basic event and an AS, which is the measure of Z-BEC. Higher the Z-BEC, higher the contribution of a basic event to the occurrence of the AS considered.

2) *Illustrative Example:* Let us consider a bow-tie diagram as in Fig. 1. The experts' opinions about the probabilities of each of the basic events, B1 to B4, is collected as mentioned in Section IV. Now, the aggregated probabilities of each of the basic events in terms of fuzzy numbers are obtained using (21)–(25) and are given in Table V. Next, the fuzzy probabilities of each of the basic events are propagated, covering the entire bow-tie, to

TABLE VII
Z-BEC MEASURE FOR BOW-TIE

BE	AS1	AS2	AS3
B1	0.918872	0.902346	0.901627783
B2	0.945646	0.885466	0.88479364
B3	0.841814	0.96564	0.964812741
B4	0.841814	0.96564	0.964812741

TABLE VIII
COMPARISON OF IMPORTANCE MEASURES

BE	Proposed measure		FV		BIR	
	Importance	Rank	Importance	Rank	Importance	Rank
B1	0.9113039	2	0.225468116	2	0.985	2
B2	0.935822	1	0.767048537	1	0.992	1
B3	0.8382552	3	7.15095 * 10 ⁻⁵	3	9.6 * 10 ⁻⁵	3
B4	0.8382552	3	7.15095 * 10 ⁻⁵	3	9.4 * 10 ⁻⁵	4

obtain the conditional fuzzy probabilities of each AS, as follows:

$$B1 \cup B2 = (\beta_{11} + \beta_{21} - \beta_{11} \times \beta_{21}, \beta_{12} \\ + \beta_{22} - \beta_{12} \times \beta_{22}, \beta_{13} + \beta_{23} - \beta_{13} \\ \times \beta_{23}, \beta_{14} + \beta_{24} - \beta_{14} \times \beta_{24})$$

$$B3 \cap B4 = B3 \times B4 \\ = (\beta_{31} \times \beta_{41}, \beta_{32} \times \beta_{42}, \beta_{33} \times \beta_{43}, \beta_{34} \times \beta_{44}).$$

Therefore, $P(\text{Brake failure}) = (B1 \cup B2) \cup (B2 \cap B3)$
Similarly

$$P(\text{AS1}) = a \times P(\text{Brake failure}) \\ P(\text{AS2}) = (1 - a) \times b \times P(\text{Brake failure}) \\ P(\text{AS3}) = (1 - a) \times (1 - b) \times P(\text{Brake failure}).$$

The MCSs [47] of the CE are $\{B1\}$, $\{B2\}$, and $\{B3, B4\}$.
Now, the group MCS for the basic event B1 is

$$GMCS_{B1} = (MCS_1) = MCS_1 = \{B1\}.$$

Similarly, the group MCS for basic events B2, B3, and B4 are $MCS_2 = B2$, $MCS_3 = B3B4$, and $MCS_4 = B3B4$.

Next, using step 4, the Z-BEC measure of each of the basic events is computed and given in Table VII. Based on the Z-BEC values in Table VII, we infer that the basic events B3 and B4 are the largest contributors to the AS3. Although Table V summarizes that the probability of occurrence of the basic events B2 and B4 are equal but if we consider a particular AS, e.g., AS3, we can say that B4 contributes more compared to B2 to the occurrence of AS3.

3) *Verification of Z-BEC*: For verification purpose, we compared the output of the proposed Z-BEC with FV importance measure and BIR importance measure in terms of contribution of basic events toward the CE. The CE is considered instead of AS as FV and BIR are applicable to CE only and no other measures (other than proposed Z-BEC) do exist in the literature linking to AS. The comparison is given in Table VIII.

The ranks obtained by the Z-BEC measure conform to those of FV index and BIR index. This verifies the applicability of Z-BEC measure. One more example of bow-tie analysis of excessive body heating of engine is provided to validate the developments (see supplementary S.3).

V. DISCUSSION

In this article, a new comprehensive similarity measure between two Z-numbers is proposed to take care of all the relevant information that is inherent in the judgments provided by the experts in terms of Z-numbers. This measure also considers the correlational factor between two parts of the Z-number. Based on the abovementioned analysis, the following advantages of the proposed model are documented.

First, the proposed model explains the randomness and fuzziness of the Z-number efficiently. It is based on the direct computation on Z-number and captures the internal relationship between expert's view (A) and its reliability (B). This similarity measure can portray the differences among different Z-information.

Second, the proposed model can effectively utilize the information contained in Z-number. As compared to the method of Kang *et al.* [18], it reduces the information loss. Moreover, the proposed method also considers the parameter " w " through which it can adapt the different preferences made by decision makers. They can adjust the parameter " w " according to their need and effectively choose the corresponding ideal solution.

Third, the relation of the probability distributions of two parts of Z-number, as well as the directional aspects of the reliability measure of Z-number is also considered while calculating the similarity value between two Z-information. The study therefore provides a very good platform where safety related decisions can be drawn very effectively.

Fourth, the proposed method is applied in bow-tie analysis to prove its flexibility and efficiency. Failure probabilities of the identified basic events were established based on impeccable experts' views and knowledge in terms of linguistic variables. These variables were then converted to fuzzy numbers, and (20) was used to gauge and quantify the risk factors of identified failure events in terms of bow-tie analysis. Closeness of the basic events B2, B3, and B4 in terms of failure probabilities, as obtained from the proposed method, resembles well the one mentioned in the reliability hand book of brake failure. This framework can be applied in industries to measure the performance efficiency and to assess whether the use of specific barrier analysis, possibly in combination with other analysis, is best fit or not to the organization.

Fifth, the developed Z-BEC measure not only helps to quantify the effect of a basic event in the occurrence of AS but also provide an efficient and valid way to compare the basic events' contribution. Another advantage of the Z-BEC measure, as it covers the entire bow-tie, is its ability to rank the basic events' contribution toward the occurrence of intermediate as well as CEs.

The proposed framework simplifies the complex nature of real scenario and provides an illustrative and rather conventional analysis. The FS theory, adopted in this article, is capable of capturing and mitigating the vagueness of experts' judgments. Integration of Z-numbers with fuzzy technique further helps to reduce the unassertive experts' thinking. The new similarity measure also utilizes the information contained in Z-numbers effectively. In a nutshell, it can be inferred that the combination of Z-integrated fuzzy technique and bow-tie analysis may provide an explicit definition of risk factor determination and quantification.

VI. CONCLUSION

Z-number carries the information of experts' opinion with added reliability that is ultimately used for decision-making. It is more practical when expressed in linguistic terms, in which people prefer to express their view. However, due to complex mathematical nature, the research on Z-numbers is very less. In particular, most of the research works depend on the conventional way of converting a Z-number to a crisp value rather than directly computing on it, and most of them convey more or less information loss.

In this article, a new similarity between two reliability parts of two Z-numbers, considering its directional aspects, is defined using the Hausdorff distance. The similarity between the λ -cut values of the restriction measure as well as its probability distribution is also taken into consideration in this article by using a statistical distance measure. Then, a new weighted comprehensive similarity measure is proposed that introduces experts' preference parameter in the study and also correctly describes the core features of the Z-number.

Based on this similarity value, the quantification of the basic events is performed to compute the CE failure probability. In this study, a bow-tie analysis is conducted in ESH for determining the various risk factors associated. A case study with brake failure as the CE is also presented to demonstrate the effectiveness of the proposed methodology. This investigation is expected to enrich the literature in defense safety application as the research in this area is very scarce. It can also be helpful in mitigating the risk and enhancing the safety of organizations as well as for researchers of safety domain. Further, the proposed method is applicable to other application areas where experts' judgments are considered as an input for decision-making, like supplier selection and project selection. The contribution toward the direct computation on Z-numbers, preventing all types of information loss, is also notable.

Consequently, the following benefits are achieved from this research.

- 1) It has sound practical values in risk assessment of ESH in a defense organization.
- 2) Z-number is used to mitigate the reliability issue of experts' opinions.
- 3) The Z-similarity measure helps to retain the information, contained in Z-numbers, provided by the decision maker.
- 4) Application of Z-similarity measure in bow-tie approach overcomes the limitations of the conventional bow-tie analysis, by reducing the vagueness in experts' views. This can also be used in other multicriteria decision making problems.
- 5) The Z-BEC measure enables the identification of the contribution of basic events toward the occurrence of different ASs as well as the intermediate and CEs.
- 6) The investigation, over all, enriches the research on Z-numbers, and opens a new window to deal with problems concerning safety analytics.

The limitations of this study are as follows.

- 1) Proposed model is applicable to discrete Z-numbers only.
- 2) The formula for the similarity measure between constraint parameters of two Z-numbers is developed for generalized trapezoidal fuzzy numbers. However, the concepts can be applied to other fuzzy numbers.

The proposed work can be extended to the following.

- 1) Integration of higher order FS with Z-number for extracting more information and handling higher level uncertainties of experts' evaluations.
- 2) Both objective and subjective evaluations for the reliability component of Z-number.
- 3) Computing the similarity between the constraint parameters of two Z-numbers for other forms of fuzzy numbers.

APPENDIX

A.1

A(restriction component)		B (reliability component)	
Linguistic variable	TrFNs	Linguistic Variable	TFNs
Very Poor (VP)	(0,0,0.1,0.2)	Very Low (VL)	(0,0,0.1)
Poor (P)	(0.1,0.2,0.2,0.3)	Low (L)	(0,0.1,0.3)
Medium Poor (MP)	(0.2,0.3,0.4,0.5)	Medium Low (ML)	(0.1,0.3,0.5)
Medium (M)	(0.4,0.5,0.5,0.6)	Medium (M)	(0.3,0.5,0.7)
Medium Good (MG)	(0.5,0.6,0.7,0.8)	Medium High (MH)	(0.5,0.7,0.9)
Good (G)	(0.7,0.8,0.8,0.9)	High (H)	(0.7,0.9,1)
Very Good (VG)	(0.8,0.9,1,1)	Very High (VH)	(0.9,1,1)

A.2

Constitution	Classification	Score
Profession position	Senior academic	5
	Junior academic	4
	Engineer	3
	Technician	2
Service Time	Worker	1
	>= 30 years	5
	20-29 years	4
	10-19 years	3
	6-9 years	2
Education Level	<=5 years	1
	PhD	5
	Master	4
	Bachelor	3
	School Level 12th	2
	School Level 10th	1
Age	<=50 years	4
	40-49 years	3
	30-39 years	2
	>30 years	1

A.3

Expert (Grade)	Professional Position	Education	Service Time (Years)	Age (Years)	Weight Factor	Normalized weights
S (DM1)	JA	Master	17	42	14	0.39
STO (DM2)	T	Bachelor	8	32	9	0.25
OS (DM3)	T	Secondary	33	55	13	0.36
Total					36	1

JA: Junior Academic; STO: Senior Technical Officer; T: Technician; OS: Office staff; S: Scientist

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