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Uncertainty Management in Space Station Autonomous Research: Pattern Recognition Perspective

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ABSTRACT

Various space station autonomous operations where research is being conducted to support unmanned missions are discussed. The problem of representation and management of uncertainties involved in image analysis and recognition tasks of those operations is particularly addressed. Various tools based on fuzzy set theory and probability theory that reflect different kinds of ambiguity, uncertainty, and information in an image are listed. Their usefulness in providing soft decision and quantitative indices for autonomous operations is described along with the uncertainty in membership function evaluation. The merits of incorporating fuzzy set theory in neural networks and in genetic algorithms for efficient handling of uncertainties are also addressed.

1. INTRODUCTION

Real life problems are rarely free from uncertainty, which usually emerges from the deficiencies of information available from a situation. The deficiencies may result from incomplete, imprecise, not fully reliable, vague, or contradictory information, depending on the problem. Management of uncertainty in a decision-making system has been an important research problem in recent years.

Until the inception of the concept of fuzzy set theory in 1965 through a classic paper of Professor Zadeh [1], the theory of probability and statistics was the primary mathematical tool for modelling uncertainty in a system/situation. Fuzzy set theory has shown enormous promise in handling uncertainties of a reasonable extent in various applications, particularly in decision-making models under different kinds of risks, subjective judgment, vagueness, and ambiguity. This theory provides an approximate,

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and yet effective and more flexible means of describing the behavior of systems that are too complex or too ill-defined to admit of precise mathematical analysis by classical methods and tools. Because this theory is a generalization of the classical set theory, it has greater flexibility to capture faithfully the various aspects of incompleteness or imperfection in information about a situation. The flexibility of fuzzy set theory is associated with the elasticity property of the concept of its membership function. The grade of membership is a measure of the compatibility of an object with the concept represented by a fuzzy set. The higher the value of membership, the lesser will be the amount (or extent) to which the concept represented by a set needs to be stretched to fit an object. (It is to be mentioned, in this connection, that the Dempster-Shafer theory [2] and rough set theory [3, 4] have also gained popularity recently in modelling uncertainty in problems related to knowledge-based expert systems.)

At the Johnson Space Center, investigations in control and decision-making systems using fuzzy logic started about seven years ago. Research activities have been continuing in various space autonomous operation problems, and the feasibility of several applications along with their success has been demonstrated [5–12]. Because the techniques of image processing and pattern recognition interact with and support a large percentage of space control problems (e.g., docking, proximity operation, camera tracking, and collision avoidance), the types of uncertainties and ambiguities involved in those problems may be categorized, broadly, in two groups, namely, uncertainty in processing and interpreting of a gray image pattern, and uncertainty in the subsequent control operations.

The purpose of this paper is to explore the effectiveness of fuzzy set theory in representing/describing various uncertainties that might arise in the aforesaid space autonomous operations and the ways these can be managed in order to have successful unmanned missions. Particular attention has been paid from a pattern analysis/recognition point of view. Although the paper is mostly devoted to fuzzy set theoretic applications, some tools and approaches based on probability theory, neural network, and genetic algorithms are also discussed. Section 2 describes various space station problems where research is being conducted for making these tasks automatic. The key features and the role of fuzzy logic in representing various uncertainties in those problems are explained in Section 3. Section 4 describes how the uncertainty arising from subjective evaluation of membership function by the operators can be represented in terms of spectral fuzzy sets [13] and bound functions [14]. Various fuzzy set theoretic and probabilistic tools for measuring information on grayness ambiguity and spatial ambiguity in an image are explained in Section 5. Their applications to low-level vision operations (e.g., segmentation, skeleton extraction, and edge detection), whose outputs are crucial and responsible for the overall performance of a vision system, are then presented for demonstrating the effectiveness of these tools in managing uncertainties by providing both soft and hard decisions. Their usefulness in providing quantitative indices for autonomous operations is also explained. Section 6 describes, in brief, the uncertainties in higher level vision operations, i.e., in feature/primitive extraction, knowledge acquisition, and syntactic classification, and the features of Dempster-Shafer theory and rough set theory in this context. Some of the recent attempts on fusion of the theories of fuzzy sets and neural networks for efficient handling of uncertainty (in the sense of parallel processing, robustness and also performance) are mentioned in Section 7. The concept of genetic algorithms [15] and its possible use are explained in Section 8.

2. AUTONOMOUS SPACE STATION PROBLEMS

Some typical space station problems where research is being conducted at the Software Technology Branch, NASA Johnson Space Center for automating various tasks are explained below. These include autonomous orbital operations such as docking for repair and servicing, camera tracking for moving objects and proximity operation, Mars rover path planning and collision avoidance, and tether control problems. Some of these operations that are related to the theme of this article are explained below.

2.1. *AUTONOMOUS ORBITAL OPERATIONS AND THE ASSOCIATED TASKS*

A typical rendezvous mission scenario for satellite servicing is shown in Figure 1. It requires orbit transfers, rendezvous planning, phasing maneuvers, and guidance and targeting for proximity operations [12]. These tasks are required to approach and capture a satellite for repair or maintenance or to return it to the space station or the earth. Repair and maintenance of satellites also require control of robotic manipulator arms if such repairs are to be performed at the satellite location as opposed to returning it to a permanently manned site such as the space station or the earth. Sometimes a satellite may require only an inspection to determine if there is damage or not. In this case, only station keeping or fly-around maneuvers are necessary.

In the problem of rendezvous of two space vehicles, it is typically assumed that the target vehicle can maintain a stable orbit during the time required for the rendezvous to take place. Ideally, it will also have a stable

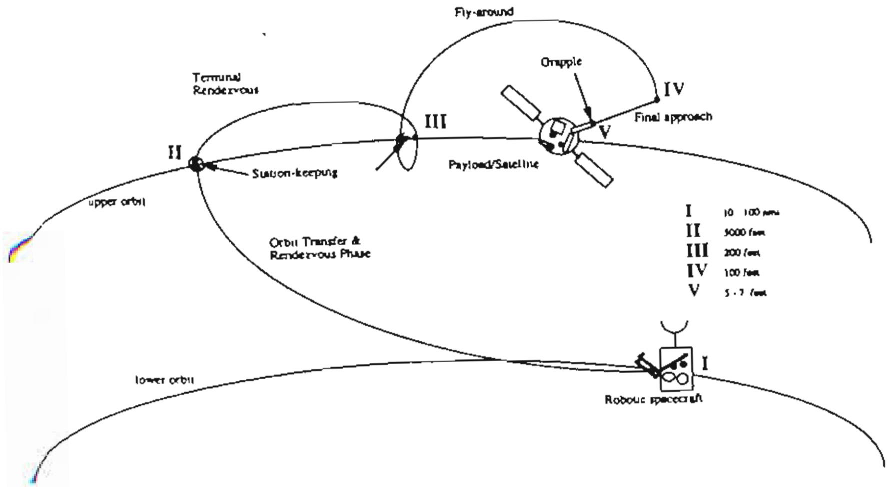


Fig. 1. Typical satellite servicing mission scenario.

attitude, although for vehicles in distress this may not be possible. For severely distressed vehicles, the actual orbit may also be affected. In either case, the problem of rendezvous and capture may be necessary.

The target vehicle is assumed to be at the origin of a coordinate system, known as the local vertical local horizontal (LVLH), where positive z is directed from the target to the center of the earth (or, in general to the center of whatever body it is orbiting), positive y is along the negative of the angular momentum vector, and positive x completes the right-handed coordinate system, as shown in Figure 2. The chasing vehicle will be the only vehicle assumed to be able to intentionally modify its trajectory and attitude in this relative coordinate system. The performance of the aforesaid tasks requires trajectory control of the active vehicle relative to the target vehicle, including not only relative positions of the two vehicles, but also the attitude of the active vehicle.

The performance of a rendezvous requires many tasks to be performed. Mission planning based on mission goals and constraints is at the highest level. For example, a scenario for the capture of a satellite needs to be developed that will incorporate time requirements, fuel constraints, and lighting and communications requirements based on the best assessment of the current and projected situation. The system will have to be intelligent enough to continually evaluate the status of the rendezvous and to be able to adapt (learning) to the unexpected occurrences through contingency planning or real time tuning of control algorithms. Such a system requires many inputs from a variety of independent sources, e.g., ranging

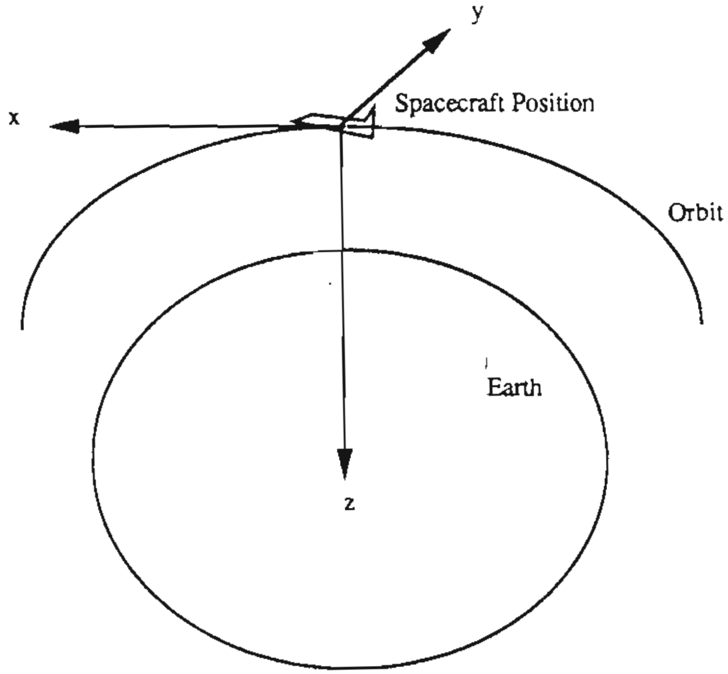


Fig. 2. The local vertical local horizontal (LVLH) coordinate frame.

and visual sensors, navigation systems, object recognition systems, human inputs from ground-based or space-based stations, on-board planning systems, diagnostic systems that report on the health of various systems, including individual sensors, and redundancy management systems. Some specific problems are tracking of moving objects with sensors such as cameras, radar, lasers, or star trackers. In the event of multiple objects in the vicinity of the desired target vehicle, it must be possible to recognize the proper one, and for final approach to the vehicle it will be necessary to recognize objects on the target vehicle such as docking ports or grapple fixtures.

The next important task is trajectory control, especially the control of relative position with respect to the target vehicle. This must be performed during the rendezvous. In some segments, control has to be very precise, whereas in other segments the accuracy requirements may be relaxed to some extent. Trajectory control requires a continuous knowledge of current state, which is typically derived from several sensor measurements. It also requires the information regarding a desired state typically provided by the guidance systems. It should be noted that the information required for trajectory control is continuously changing with time and is highly dependent on the accuracy of sensor measurements.

Similarly, attitude control is required throughout the mission. A robust attitude control enhances trajectory control because the execution of desired delta-V is much more accurate. Poor attitude control can definitely result in a mission failure. It should be noted that rotational control has to be very precise during the final approach and docking segments, because the coupling between the rotational changes and the relative distances is significantly high. Again note that the knowledge regarding current as well as the desired attitude is required, and this information changes with time.

Both of the above tasks require processing of sensor data and its synthesis. All measurements must be accurately interpreted, and action must be taken accordingly. Because several sensors are used, proper data fusion must be performed and each measurement must be used in its proper context. Otherwise, the probability of mission failure increases very significantly. This data fusion task necessarily includes the monitoring task that must be continuously performed, and any deviations from the planned trajectory must be reported immediately.

Once the chaser spacecraft gets close to the satellite, its approach to the docking port must be carefully maintained with tight control of its translational as well as rotational state. The controller must be very precise and must have a fine tuning capability. At the end of the approach task, it must initiate docking and rigidizing procedures, which will use a completely different set of sensors. There must be some provision for a recovery procedure in case of a docking failure. When the crew performs these functions, they interpret the measurements according to their training and take action according to the procedures developed in a mission simulator. These procedures typically include steps in case of a docking failure. The autonomous vehicle must have the same capability for mission success.

The vehicle must prepare for return to base with or without the payload. These preparations could be very lengthy or very short depending on what procedure the crew decides to use and how their sequence of actions is organized. In any event, thinking like the crew will definitely help to solve the problem of increasing autonomy in rendezvous operations.

Camera Tracking System

Let us now discuss, in particular, the development of a camera tracking control system in orbital operation. Advanced sensor systems with intelligence and distributed nature are required for activities like proximity

operations and traffic control around the Space Station Freedom (SSF). There will be several sensors of different types for providing various measurements simultaneously as input to such a system. The system uses the object position in a scene as input and controls the gimbal drives to keep it in the Field Of View (FOV) of the camera, as shown in Figure 3.

Tracking of an object means aligning the pointing axis of a camera along the object's line of sight. The monitoring camera is typically mounted on the pan and tilt gimble drives which are capable of rotating the pointing axis within a certain range. The task of the tracking controller is to command these gimble drives so that the pointing axis of the camera is along the line of sight vector which is estimated from the measurements.

For the fuzzy logic–based tracking controller, the inputs are range and line of sight vector, and the outputs are the commanded pan and tilt rates. The line of sight vector is input in terms of pixel position in the camera FOV. When an image is received, it is processed to determine the location of the object in the camera frame which has the vertical, horizontal and pointing vectors as three axes. The centroid of the image is computed and is used as the current location of the object in the viewing plane. This plane is a Cartesian coordinate plane having vertical and horizontal axes. The size of the viewing plane is 170×170 pixel with origin at the upper left corner (Figure 3). The range of the object is received from the laser range finder as a measurement. These three parameter values constitute the input vector to the controller.

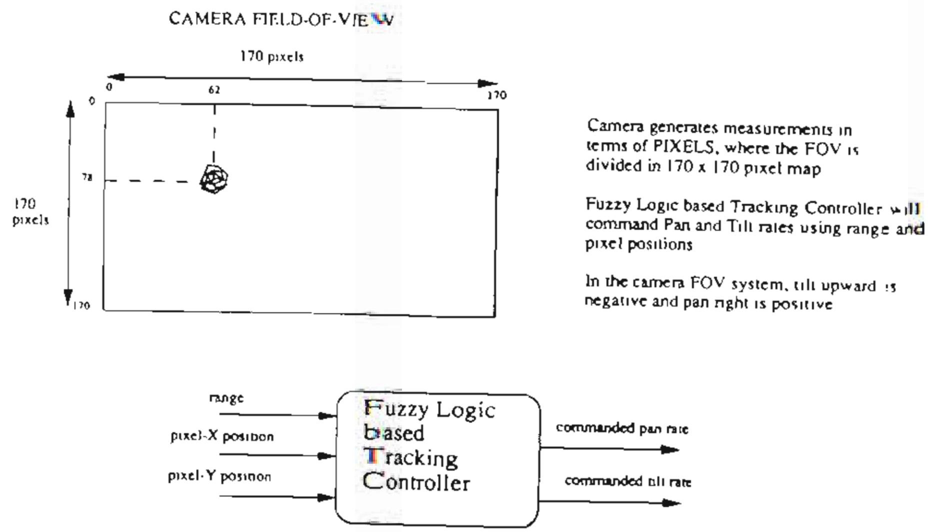


Fig. 3. Concept of a camera tracking system.

Application of Camera Tracking System for Traffic Management

In the future, the SSF may have many vehicles approaching and departing the facility simultaneously. The crew onboard SSF will then have to perform the traffic management function very actively for safety reasons. The camera tracking system can be used effectively during these operations and can help the crew to efficiently manage the traffic around SSF. During assembly and other extravehicular operations, tracking and monitoring of other objects around SSF is required for mission success.

The capabilities of the tracking controller can be further increased to perform other functions such as approaching toward the object, grapple, object identification, traffic management, and caution and warning to crew. Fast-moving objects can be identified easily via prediction of position and thus collision avoidance can also be achieved. Because the system can work as a stand-alone system at the command level and will interrupt the operations flow only if necessary, it can become a node in a distributed sensor system.

Reinforcement Learning for External Environment during Docking and Repair Operations

A space shuttle crew initiates the proximity operation procedures and docking maneuvers, when the orbiter is within 1,000 feet of the payload. It is expected that the payload will remain in a stable attitude and in nearly the same orbit during this entire period of time. Typically, the crew performs an approach known as the v-bar approach, keeping manual control of the orbiter. Docking maneuvers with the payload are also performed manually. The manual procedures and algorithms used during these tasks by the crew are developed using the real-time shuttle mission simulator facility on the ground.

During proximity operations, if the algorithms require some adjustment, the so-called fine tuning, it is performed real-time, even if it was not learned in the real-time simulation. Real-time adjustments are achieved based upon the current situation (e.g., the satellite is not in a stable attitude or its orbit is constantly changing) and goal achievements. Thus, the crew constantly learns and updates these procedures and algorithms as their experience base builds up.

2.2 CONTROL OF MARS ROVER FOR SAMPLE COLLECTION

The space exploration initiative of the U.S. includes plans for unmanned missions by NASA to Mars to investigate the terrain and to

process soil samples in advance of a manned mission [12]. Surface exploration of Mars involves an environment where uncharted obstacles are likely to interfere with maneuvering of a rover vehicle. In addition, since the worst case round trip communications time between Earth and Mars will require 20 minutes, Earth-based tele-robotic control of the Mars rover will be extremely difficult and time consuming, and could seriously endanger the success of the mission. Hence, autonomy is needed for operations of vehicles.

It is anticipated that specific target locations will be designated for sample gathering. In maneuvering autonomously from one position to another, the rover will need to avoid a variety of obstacles (e.g., boulders or troughs) that cannot be identified prior to the mission and that can block the shortest path to the target. Therefore, the rover must interpret the obstacle size and its distance on the basis of imprecise sensor measurements. Once the unforeseen obstacles that can cause a hazard to the vehicle are determined, one can plan a trajectory to avoid them, so that the physical integrity of the rover is maintained while minimizing the time and distance required to reach a target position.

3. AMBIGUITY, UNCERTAINTY, AND FUZZY LOGIC/MATHEMATICS

In the previous section, we explained the various space station problems where autonomous operations are necessary in order to have a successful mission by making these tasks efficient. As we know, real life problems are rarely free from uncertainty, which usually emerges from the deficiencies of information available from a situation. The deficiencies may result from incomplete, imprecise, not fully reliable, vague or contradictory information depending on the problem. Let us now describe the various uncertainties and ambiguities involved in the aforesaid autonomous space research operation and how the concept of fuzzy sets and approximate reasoning can provide a helpful tool for handling them in order to develop an efficient expert system.

Because the techniques of processing and recognition of image patterns interact with and support a large percentage of space control problems (e.g., docking, proximity operation, camera tracking and collision avoidance), the types of uncertainties and ambiguities involved in those problems may be categorized, broadly, in two groups, namely, uncertainty in processing and interpreting of an image pattern, and uncertainty in the subsequent control operations. In other words, the uncertainty may arise from ambiguity in image data (due to noise and/or fuzziness, say) and in its processing, ambiguity in exact meaning of the terms used by the crew (i.e., the terms represented in the knowledge domain) and ambiguity in rules itself (e.g., doubtful connection between the antecedent and consequent in an inference rule).

3.1. *IMAGE AMBIGUITY AND UNCERTAINTY IN ITS PROCESSING/ANALYSIS*

A gray tone image possesses some ambiguity within the pixels due to the possible multivalued levels of brightness. This pattern indeterminacy is due to inherent vagueness rather than randomness. The conventional approach to image analysis and recognition consists of segmenting (hard partitioning) the image space into meaningful regions, extracting its different features (e.g., edges, skeletons, centroid of an object), computing the various properties of and relationships among the regions, and interpreting and/or classifying the image. Because the regions in an image are not always crisply defined, uncertainty can arise at every phase of the aforesaid tasks. Any decision taken at a particular level will have an impact on all higher level activities. Therefore, a recognition system (or vision system) should have sufficient provision for representing the uncertainties involved at every stage, i.e., in defining image regions, its features, and relations among them, and in their matching, so that it retains as much as possible the information content of the original input image for making a decision at the highest level. The ultimate output (result) of the system will then be associated with least uncertainty (and unlike conventional systems it will not be biased or affected very much by the lower level decisions).

For example, consider an operation control rule in the camera tracking problem:

If the object (target) is *far-left*, then rotate the camera to the left side.

Now, the question is "How can someone define exactly the target or object region or its contours in a scene when its boundary is ill-defined?" Any hard thresholding made for its extraction will propagate the associated uncertainty to the following stages, and this might affect its feature analysis and recognition. Similar is the case with docking operation for servicing, and Mars rover control problem where the autonomous operations involve the task of recognizing an object (either as target or as obstacle) from a gray image and/or determining its depth (and distance) from the contours of two stereo images.

From the aforesaid discussion, it becomes therefore convenient, natural and appropriate to avoid committing ourselves to a specific (hard) decision

(e.g., segmentation/thresholding, edge detection and skeletonization) by allowing the segments or skeletons or contours to be fuzzy subsets of the image; the subsets being characterized by the possibility (degree) of a pixel belonging to them.

Similarly, for describing and interpreting ill-defined structural information in a pattern, it is natural to define primitives and relations among them using labels of fuzzy sets. For example, primitives may be defined in terms of arcs with varying grades of membership from 0 to 1, and production rules of a grammar may be fuzzified to account for the fuzziness in physical relation among the primitives; thereby increasing the generative power of a grammar.

The incertitude in an image pattern may be explained in terms of grayness ambiguity or spatial (geometrical) ambiguity or both. Grayness ambiguity means "indefiniteness" in deciding a pixel as white or black. Spatial ambiguity refers to "indefiniteness" in shape and geometry (e.g., in defining centroid, sharp edge, perfect focusing, etc.) of a region. There is another kind of uncertainty which may arise from the subjective judgement of an operator in defining the grades of membership of the object regions. This has been explained in Section 4 in terms of uncertainty in membership function.

3.2. *UNCERTAINTY IN CONTROL OPERATIONS*

Let us now discuss the nature of uncertainty on control operations. For this, we consider some typical rules which a human operator (crewman) follow during the rendezvous vehicle control [6].

If the rendezvous vehicle's orientation with respect to a desired pointing vector to the target vehicle is *close* to the required orientation, then no action is necessary.
If the orientation *significantly* deviates from the required, then take *appropriate* action to correct the problem.
If the measurement residual is *small* with respect to the expected value as determined from pre-mission studies and the residual change is *small* with respect to expected propagation error and noise in the sensor for *several* consecutive measurements, then allow the Kalman filter to process data.

Similarly, consider the rule mentioned in the previous section for camera tracking problem:

If the object (target) is *far-left*, then rotate the camera to the left side.

Note that the aforesaid control statements (rules) contain inexact (fuzzy) variables or terms such as *close, significantly, appropriate, small, far-left, several*. During actual operations, these terms and the rules are interpreted and executed by the crew based on their experience (expertise). Therefore, it is required to model the crewman's reasoning and commonsense thought process in handling uncertainties in decision-making tasks while designing an autonomous unmanned system.

3.3. *SOME KEY FEATURES OF FUZZY LOGIC/FUZZY MODELS*

Logic, according to Webster's dictionary, is the science of the normative formal principles of reasoning. In this sense, fuzzy logic (based on the theory of fuzzy sets) is concerned with the formal principles of approximate reasoning, with precise (classical) reasoning viewed as a limiting case. Unlike classical logic, it aims at modeling the imprecise (or inexact) modes of reasoning and thought process (with linguistic variable) that play an essential role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision. This ability depends, in turn, on our ability to infer an approximate answer to a question based on a store of knowledge that is inexact, incomplete, or not totally reliable [16].

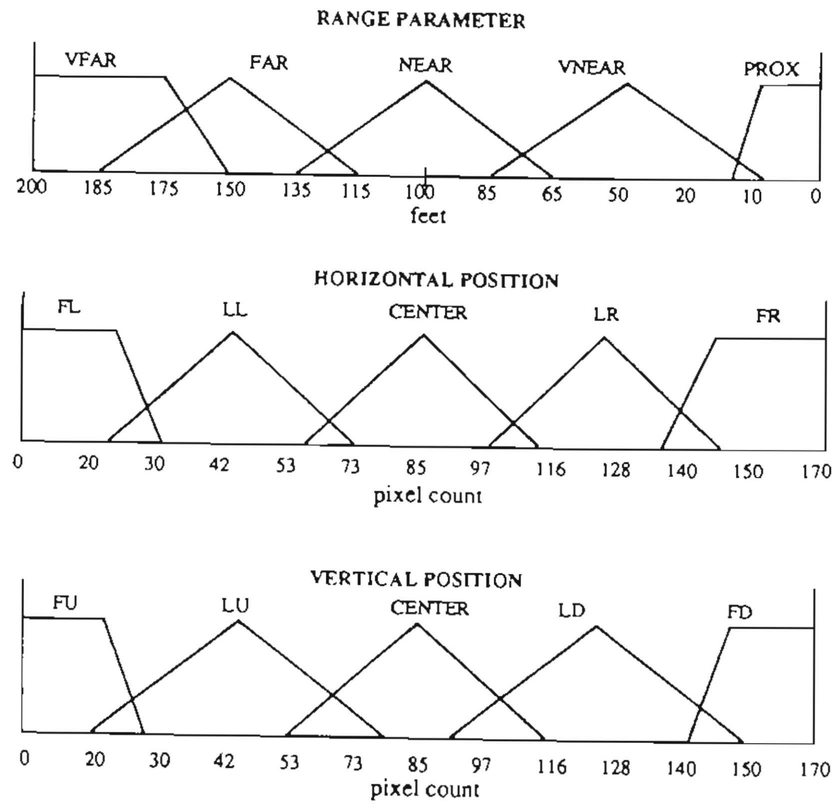
For example, consider the rules described in Section 3.2 for orbital operations. The reason why classical logical systems cannot cope with rules of this type is as follows. First, they do not provide a system for representing the meaning of propositions expressed in a natural language when the meaning is imprecise. Second, in those cases in which the meaning can be represented symbolically in a meaning representation language (e.g., a semantic network or a conceptual-dependency graph), there is no mechanism for inference.

Fuzzy logic addresses these problems in the following ways. First, the meaning of a lexically imprecise proposition is represented as an elastic constraint on a variable. Second, the answer to a query is deduced through a propagation of elastic constraints [16].

The role of fuzzy logic in space autonomy research problems has been adequately addressed in a series of papers [5–12] where the human capability of common sense reasoning in decision-making tasks was modeled and such models were integrated with expert systems and engineering control system technology to create a system that performs comparably to a manned system. A few of the applications along with their merits are briefly outlined below for the convenience of readers.

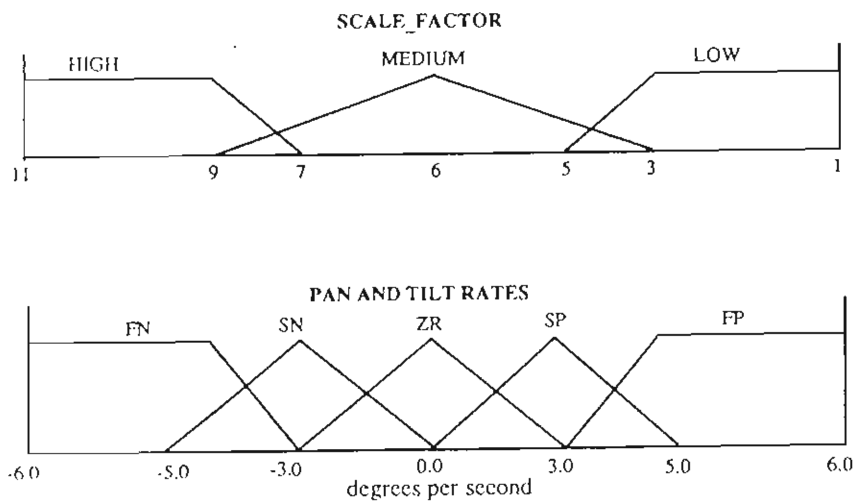
Let us consider, first of all, the problem of camera tracking of an object. Membership functions for the range, horizontal and vertical positions are shown in Figure 4 and the membership functions for the scale factor, pan and tilt rates are shown in Figure 5. The scale factor parameter is used as an intermediate step and provides the desired flexibility of changing the responsiveness of the fuzzy controller for pan and tilt rates.

The desired image location is the center [i.e., at location (85,85)] of the viewing plane. If the current location is close to the center, then rotation of the pointing axis is not required. If the location is to the left of center then a left rotation is necessary. Similarly, if the image is down from the horizontal line then a downward rotation is required. These rotations are determined using the position and range measurements and the rule base



KEY : VFAR - Very Far, VNEAR - Very Near, PROX - Proxomity zone
FL - Far Left, LL - Little Left, LR - Little Right, FR - Far Right
FU - Far Up, LU - Little Up, LD - Little Down, FD - Far Down

Fig. 4. Membership functions for input parameters for camera tracking system.



KEY : FN - Fast Negative, SN - Slow Negative, ZR - Zero,
FP - Fast Positive, SP - Slow Positive

Fig. 5. Membership functions for scale factor and output parameters for camera tracking system

as shown in Table 1. First the range (distance from the target) measurement is fuzzified and the value of the scale factor is determined based on the scale factor rules. Necessary defuzzification processing is performed to compute the crisp value of the scale factor. Then, the scale factor and the position measurements are provided to the next set of rules to determine the rate at which the gimble drives should be rotated. For example, if the distance (range) membership function is *very far* or *far* then the scale factor is *low*. If the scale factor is *low* and the horizontal position membership function is *far-left* then the pan rate membership function is *fast-negative* (Table 1).

There are 30 rules that determine both pan and tilt rates. Again, the necessary defuzzification processing is performed to compute the crisp values of the pan and tilt rates which can be sent to the gimble drives as command values.

The camera is rotated based on these commands within the limits of its gimble rates and angles. New line of sight measurements in the camera FOV are obtained for the next cycle and the processing is repeated. The cycle time is based on the processing time required for the following

TABLE 1
Rule Base for the Tracking Task

	Distance Membership Functions				
	VFAR	FAR	NEAR	VNEAR	PROX
Scale_Factor	LOW	LOW	MED	HIGH	HIGH

		Horizontal Position Membership Functions				
		FL	LL	CENTER	LR	F
Scale_Factor	LOW	FN	SN	ZR	SP	FP
	MED	SN	SN	ZR	SP	SP
	HIGH	SN	ZR	ZR	ZR	SP
		Pan_Rate Membership Functions				

		Vertical Position Membership Functions				
		FD	LD	CENTER	LU	FU
Scale_Factor	LOW	FP	SP	ZR	SN	FN
	MED	SP	SP	ZR	SN	SN
	HIGH	SP	ZR	ZR	ZR	SN
		Tilt_Rate Membership Functions				

Note: Negative Tilt_rate means the pointing axis going upward in FOV.

functions: (i) determining object pixel positions, (ii) obtaining a range measurement, (iii) rotating the gimble drives at a desired rate, and (iv) the requirements to track the object within a certain performance envelope. Typical cycle time ranges between 0.1 to 1.0 second.

This camera tracking system will involve low power sensors as compared to an active sensor system, e.g., Radar in the Ku band range, or LADAR using laser frequency. Typically, an active sensor radiates a power pulse towards a target and receives back a reflected pulse. Based on the power transmitted, power received and the time between these pulses, parameters like range and range rate are calculated. On the other hand, the camera tracking system does not need to radiate power. Since there is already a shortage of power, an important consumable, onboard the SSF, availability of low power sensors is very important for continuous operations. The SSF can afford to keep this type of a sensor working around the clock without having much impact on the power management or other computational load on the main computers.

Similarly, in the case of docking and repair operation using reinforcement learning, it has been shown that the fuzzy logic controller can perform the same activities autonomously using sensor measurements as inputs. Fuzzy membership functions and the associated rule base [6, 7] have been developed utilizing the same procedures used by the crew during mission operations. A fuzzy reinforcement learning method [17] has been developed at Ames Research Center (ARC) using the inverted pendulum. The fuzzy controller can be combined with the reinforcement learning technique to give it a capability to learn real-time and improve its performance. With this capability, the fuzzy controller can adapt to a new environment and adjust its membership functions and/or rules to appropriately perform the tasks, given enough training instances.

Let us now consider the case of the Mars rover for sample collection. A fuzzy logic approach to trajectory control has been developed [10] which allows the rover to avoid these hazards during the sample collection process. The fuzzy trajectory controller receives the goal or target point from the planner and uses X and Y position errors as well as orientation (Yaw) error in the control system frame and commands the rover in terms of steering angle and velocity. The fuzzy rule-base containing 112 rules for the controller, has been designed to drive the rover towards the X-axis of the control error frame. As the rover approaches this axis, the rover is commanded to the correct orientation error and then slowly drives towards the target point.

The X and Y position error variables were modeled as a shouldered membership set of five piecewise linear functions with a universe of discourse ranging from -100 to 100 meters. The orientation or yaw error variable was modeled as an unshouldered membership set of seven functions with a universe of discourse ranging from $-180°$ to $180°$. The steering variable was modeled as an unshouldered membership set of five functions with a universe of discourse ranging from $-30°$ to $30°$. Finally, the velocity variable was modeled as an unshouldered membership set of seven functions with a universe of discourse ranging from -5 to 5 meters/second.

Preliminary results have shown that the trajectory controller can reach the target position and attitude within 0.0005 meters on the x-error axis, 0.25 meters on the y-error axis, and 0.45 degrees yaw error. It is believed that these accuracies can be reduced by altering the membership function sets for the inputs and outputs. Further testing will facilitate the tailoring of the membership functions to the fuzzy rule set.

While the application of the theory of fuzzy logic and approximate reasoning in control theory was in the process of development, there was another important branch, called fuzzy image processing and pattern recognition, that grew up in parallel based on the realization that many of the basic concepts in pattern analysis, e.g., the concept of an edge or a corner, do not lend themselves to precise definition. Or, a pattern may belong to more than one class; the degree of belonging to the class being characterized by a membership function.

If the gray levels in an image are scaled to lie in the range [0, 1], we can regard the gray level of a pixel as its degree of membership in the set of high-valued ("bright") pixels. Thus, a gray scale image can be regarded as a fuzzy set, and any "object," skeleton," or "contour" may be viewed as its fuzzy subsets. Basic principles and operations of image processing and recognition in the light of fuzzy set theory are available in [18].

As mentioned before, it is natural and also appropriate to avoid committing ourselves to a specific hard decision (for segmentation/thresholding and skeletonization) when the regions in an image are ill-defined or fuzzy, and similarly to define primitives and relation among them using labels of fuzzy sets for describing and interpreting ill-defined structural information in a pattern (when the pattern indeterminacy is due to inherent vagueness rather than randomness). There had been a great deal of development of theory and uncertainty measures in this context [19–25]. These consist of generalization of many standard geometric properties of and relations among regions in an image, and of the entropy definition by extending them to the fuzzy case.

Such an extension, called fuzzy geometry [19, 21, 22], includes the topological concept of connectedness, adjacency and surroundedness, convexity, area, perimeter, compactness, height, width, length, breadth, index of area coverage, major axis, minor axis, diameter, extent, elongatedness, adjacency and degree of adjacency. These measures have been found to reflect the spatial (geometrical) ambiguity of an image. Similarly, the grayness ambiguity can be measured by index of fuzziness, fuzzy correlation, higher order entropy and hybrid entropy of an image [23–25]. These parameters can be optimized with respect to a particular membership function (S-type or π-type) in order to provide soft decision for image description and analysis, and to obtain a desired fuzzy output, with its binary (crisp) version viewed as its limiting case.

Another generalization, called fuzzy medial axis transformation (FMAT), has recently been made by Pal and Rosenfeld [26]. FMAT of an image provides its gray skeleton which can be considered as its core (prototype) version for the purpose of image matching. This can also be used for image representation and reconstruction.

Fuzzy logic and approximate reasoning has also been found to be successful in formulating the tasks of knowledge acquisition and recognition [27, 28] where imprecise (e.g., missing feature, linguistic feature, set theoretic feature) input data/statement has been quantified heuristically to assign a numerical value for its further processing. Since we are mainly interested here in addressing the various ambiguities and their management from the pattern recognition point of view, these tools along with their performance will be explained in detail in Sections 5 and 6.

4. UNCERTAINTY AND FLEXIBILITY IN MEMBERSHIP FUNCTION

From the aforesaid discussion, it is seen that the theory of fuzzy sets has promise in handling various uncertainties and ambiguities to a reasonable extent in autonomous space research applications. Since this theory is a generalization of the classical set theory, it has greater flexibility to capture faithfully the various aspects of incompleteness or imperfection (i.e., deficiencies) in information of a situation. The flexibility of fuzzy set theory is associated with the elasticity property of the concept of its membership function. The grade of membership is a measure of the compatibility of an object with the concept represented by a fuzzy set. The higher the value of membership, the lesser will be the amount (or extent) to which the concept represented by a set needs to be stretched to fit an object.

Because the grade of membership is both subjective and dependent on context, some difficulty of adjudging the membership value still remains. In other words, the problem is how to assess the membership of an element to a set. This is an issue where opinions vary, giving rise to uncertainties. Two operators, namely "Bound Functions" [14] and "Spectral Fuzzy Sets" [13], have recently been defined to analyze the flexibility and uncertainty in membership function evaluation. These are explained below.

4.1. *BOUND FUNCTIONS*

Consider, for example, a fuzzy set "tall." This is represented by an S-type function which is a nondecreasing function of height. Now, the question is, can any such nondecreasing function be taken to represent the above fuzzy set? Intuitively, the answer is "no." Bounds for such an S-type membership function μ have recently been reported by Murthy and Pal [14] based on the properties of fuzzy correlation [29]. The correlation

measure between two membership functions μ_1 and μ_2 relates the variation in their functional values. The main properties on which the correlation was formulated are as follows:

P_1: If for higher values of μ_1, μ_2 takes higher values and for lower values of μ_1, μ_2 also takes lower values, then $C(\mu_1, \mu_2) > 0$.
P_2: If $\mu_1 \uparrow$ and $\mu_2 \uparrow$, then $C(\mu_1, \mu_2) > 0$.
P_3: If $\mu_1 \uparrow$ and $\mu_2 \downarrow$, then $C(\mu_1, \mu_2) < 0$.

[$\uparrow$ denotes increases and $\downarrow$ denotes decreases.]

These properties have further been explained in Section 5.1. It is to be mentioned that P_2 and P_3 should not be considered in isolation of P_1. Had this been the case, one can cite several examples when $\mu_1 \uparrow$ and $\mu_2 \uparrow$ but $C(\mu_1, \mu_2) < 0$ and $\mu_1 \uparrow$ and $\mu_2 \downarrow$ but $C(\mu_1, \mu_2) > 0$. Subsequently, the types of membership functions which should preferably be avoided in representing fuzzy sets are categorized with the help of correlation. Bound functions h_1 and h_2 are accordingly derived [11] in order to restrict the variation in the μ function. They are

$$h_1(x) = 0, \quad 0 \leq x \leq \epsilon$$
$$= x - \epsilon, \quad \epsilon \leq x \leq 1 \tag{1}$$

$$h_2(x) = x + \epsilon, \quad 0 \leq x \leq 1 - \epsilon$$
$$= 1, \quad 1 - \epsilon \leq x \leq 1 \tag{2}$$

where $\epsilon = 0.25$. The bounds for membership function μ are such that

$$h_1(x) < \mu(x) \leq h_2(x) \quad \text{for} \quad x \in [0, 1].$$

For x belonging to any arbitrary interval, the bound functions will be changed proportionately. For $h_1 \leq \mu \leq h_2$, $C(h_1, h_2) \geq 0$, $C(h_1, \mu) \geq 0$ and $C(h_2, \mu) \geq 0$. The function μ lying in between h_1 and h_2 does not have most of its variation concentrated (i) in a very small interval, (ii) towards one of the end points of the interval under consideration, and (iii) towards both the end points of the interval under consideration. In other words, the membership function μ of a fuzzy set should not have, in any interval of the domain, an abrupt change from nonmembership to membership or vice versa, because this can make the representation of a fuzzy set crisp. Figure 6 shows such bound functions. It is to be noted that Zadeh's standard S function [18, 30] satisfies these bounds.

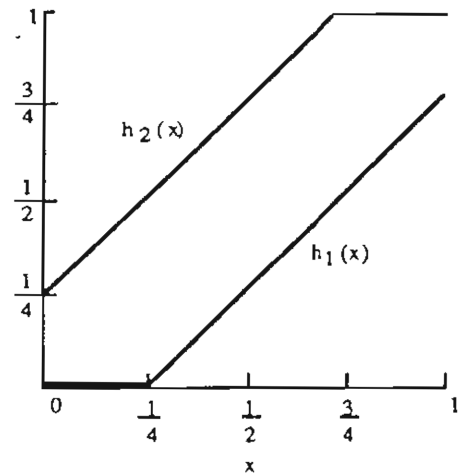


Fig. 6. Bounds for S-type membership functions.

The significance of the bound functions in selecting an S-type function μ for image segmentation problem has been reported in detail in [31]. It has been shown that for detecting a minimum in the valley region of a histogram, the window length w of the function $\mu:[0,w]\rightarrow[0,1]$ should be less than the distance between two peaks around that valley region. The ability to make the fuzzy set theoretic approach flexible and robust will be demonstrated further in Section 5.3.

4.2. *SPECTRAL FUZZY SETS*

The concept of spectral fuzzy sets is used where, instead of a single unique membership function, a set of functions reflecting various opinions on membership elements is available so that each membership grade is attached to one of these functions. By giving due respect to all the opinions available for further processing, it reduces the difficulty (ambiguity) in selecting a single function. A spectral fuzzy subset F having n supports is characterized by a set or a band (spectrum) of r membership functions (reflecting r opinions) and may be represented as

$$F=\bigcup_j\left[\bigcup_i \mu_F^i(x_j)/x_j\right],\qquad x_j\in\psi \tag{3}$$

$$i=1,2,\ldots,r;\ j=1,2,\ldots,n.$$

r, the number of membership functions, may be called the cardinality of the opinion set. $\mu_F^i(x_j)$ denotes the degree of belonging of x_j to the the set F according to ith membership function. The various properties and operations related to it are available in [13].

The incertitude or ambiguity associated with this set is of twofold, namely, ambiguity in assessing a membership value to an element ($d1$) and ambiguity in deciding whether an element can be considered to be a member of the set or not ($d2$). Obviously, $d2$ is related to a fuzzy set and its functional nature is the same as H^1 (Equation 7 with $r=1$). On the other hand, $d1$ reflects the amount of disparity (disagreement) within opinions because of the spectral nature. Regarding $d1$ it has been observed that human beings do not find it very difficult to assign memberships to elements which either have very low or very high possibility of belonging to that set. In other words, the difference in opinions is low for those elements (supports) whose degree of inclusion or possibilities of belonging to a set is subjectively very low or very high. The variation in opinion, on the other hand, is high for those supports whose degree of belongingness is fairly medium. For example, consider a spectral fuzzy set labelled "tall men" over the range 5 to 7 ft. The difference in opinions (or difficulty in assessing a membership) as expressed would be high around 5 ft 5 inch as opposed to those around 5 or 7 ft. Similarly, if someone is asked to bring a glass of water several times with the same glass, the variation in the amount of water will be less compared to the case when half a glass of water is asked for. Similar observations may be made when the task is performed by several people. It therefore appears that the variation in membership function assignment is high for the elements having fairly medium belongingness. Based on this concept, "Spectral Index" θ is defined as [13]

$$\theta(F)=d_1(F)=\frac{1}{n}\sum_j\theta_j,\qquad j=1,2,\ldots,n \tag{4}$$

$$\theta_j=\frac{S}{r_{C_2}}\left[\sum_{l=1}^{r-1}\sum_{i=l+1}^{r}|\mu_F^l(x_j)-\mu_F^i(x_j)|\right]$$

$$1\geqslant\theta_j\geqslant 0 \tag{5}$$

where

$$S=1\Big/\frac{r}{2(r-1)},\qquad \text{if } r \text{ is even} \tag{6a}$$

$$=1\Big/\frac{(r+1)}{2r},\qquad \text{if } r \text{ is odd} \tag{6b}$$

ν provides, in a global sense, a quantitative measure of the average differences (or disagreement) of opinion, in assigning a membership value to a supporting element.

The (dis)similarity between the concept of spectral fuzzy sets and those of the other tools such as probabilistic fuzzy set, interval-valued fuzzy set, fuzzy set of type 2, or ultrafuzzy set [32–36] (which have also considered the difficulty in settling a definite degree of fuzziness or ambiguity) has been explained in [13]. Fuzzy set of type 2 is characterized by a fuzzy membership function the grade of which is a fuzzy set in the unit interval [0, 1] rather than a point in [0, 1]. A fuzzy set of type n is also characterized by n step recursively defined ambiguity. An ultrafuzzy set is a type 2 fuzzy set which expresses membership as fuzzy numbers e.g., "approximately 0.6," "close to 0.8," etc. In probabilistic set theory, a concept of probabilistic measure space is introduced through a parameter space which plays an important role in dealing with ambiguity. The spectral fuzzy sets, on the other hand, handles the aforesaid difficulty in settling a membership value or a degree of fuzziness by incorporating a collection of individual opinions on it. In other words, it is characterized with a set of membership functions (where each membership grade is attached to *one* of the membership functions) rather than with set-valued membership grades (where the source of membership grades is forgotten). The extent of opinion (spectrum) may be restricted by the bound functions explained before. Variance of probabilistic set is to some extent analogous to the measure $d1$ of the spectral fuzzy set.

The concept has been found to be significantly useful [13] in segmentation of ill-defined regions where the selection of a particular threshold becomes questionable as far as its certainty is concerned. In other words, questions may arise like, "where is the boundary" or "what is the certainty that a level l, say is a boundary between object and background." The opinions on these queries may vary from individual to individual because of the differences in opinion in assigning membership values to the various levels. In handling this uncertainty, the algorithm gives due respect to various opinions on membership of gray levels for object region, minimizes the image ambiguity $d(=d1+d2)$ over the resulting band of membership functions and then takes a soft decision by providing a set of thresholds (instead of a single) along with their certainty values. A hard (crisp) decision obviously corresponds to one with maximum d value, i.e., the level at which opinions differ most.

It is to be noted that the results that would have been obtained by considering the membership functions individually must not necessarily be the same as those obtained here, because the latter one involves a collective decision. Furthermore, the problems of edge detection and skeleton extraction (where incertitude arises from ill-defined regions and various opinions on membership values), and any expert system type application (where differences in experts' opinions lead to an uncertainty) may also be similarly handled within this framework.

5. IMAGE AMBIGUITY AND UNCERTAINTY MEASURES

Let us explain in this section the various image information measures (arising from both fuzziness and randomness) and tools, and their relevance for the management of uncertainty in different operations for its processing/analysis. These are classified mainly in two groups, namely, grayness ambiguity/uncertainty and spatial ambiguity/uncertainty.

5.1. *GRAYNESS AMBIGUITY MEASURES*

The definitions of some of the measures which are formulated recently to represent grayness ambiguity in an image X with dimension $M \times N$ and levels L (based on individual pixel as well as a collection of pixels) are listed below.

rth Order Fuzzy Entropy

$$H^r(X) = (-1/k) \sum_i \{\mu(s_i^r) \log\{\mu(s_i^r)\} + \{1-\mu(s_i^r)\} \log\{1-\mu(s_i^r)\}\}$$

$$i = 1, 2, \ldots, k \quad (7)$$

s_i^r denotes the ith combination (sequence) of r pixels in X. k is the number of such sequences. $\mu(s_i^r)$ denotes the degree to which the combination s_i^r, as a whole, possesses some image property μ.

Hybrid Entropy

$$H_{hy}(X) = -P_w \log E_w - P_b \log E_b \quad (8)$$

$$E_w = (1/MN) \sum_m \sum_n \mu_{mn} \exp(1-\mu_{mn})$$

with

$$E_b = (1/MN) \sum_m \sum_n (1 - \mu_{mn}) \exp(\mu_{mn})$$

$$m = 1, 2, \ldots M; \quad n = 1, 2, \ldots N.$$

μ_{mn} denotes the degree of "whiteness" of the (m,n)th pixel. P_w and P_b denote probability of occurrences of white ($\mu_{mn} = 1$) and black ($\mu_{mn} = 0$) pixels respectively. E_w and E_b denote the average likeliness (possibility) of interpreting a pixel as white and black respectively.

Correlation

$$C(\mu_1, \mu_2) = 1 - 4\left[\sum_m \sum_n \{\mu_{1mn} - \mu_{2mn}\}^2\right] \Big/ (X_1 + X_2) \tag{9}$$

$$C(\mu_1, \mu_2) = 1 \qquad \text{if } X_1 + X_2 = 0$$

with

$$X_1 = \sum_m \sum_n \{2\mu_{1mn} - 1\}^2 \tag{10a}$$

$$X_2 = \sum_m \sum_n \{2\mu_{2mn} - 1\}^2 \tag{10b}$$

$$m = 1, 2, \ldots M; \quad n = 1, 2, \ldots N$$

μ_{1mn} and μ_{2mn} denote the degree of possessing the properties μ_1 and μ_2 respectively by the (m,n)th pixel. $C(\mu_1, \mu_2)$ denotes the correlation between two such properties μ_1 and μ_2 (defined over the same domain).

These expressions (Equations 7-10) are the versions extended to the two dimensional image plane from those defined [23, 29] for a fuzzy set. $H^r(X)$ gives a measure of the average amount of difficulty in taking a decision whether any subset of pixels of size r possesses an image property or not. Note that, no probabilistic concept is needed to define it. If $r = 1$, $H^r(X)$ reduces to (nonnormalized) entropy as defined by De Luca and Termini [37]. $H_{hy}(X)$, on the other hand, represents an amount of difficulty in deciding whether a pixel possesses a certain property μ_{mn} or not by making a prevision on its probability of occurrence. (It is assumed here that the fuzziness occurs because of the transformation of the complete white (1) and black pixels (0) through a degradation process; thereby modifying their values to lie in the intervals [0, 0.5] and [0.5, 1] respectively). Therefore, if μ_{mn} denotes the fuzzy set "object region," then the amount of ambiguity in deciding x_{mn} a member of object region is conveyed by the term hybrid entropy depending on its probability of occurrence. In the absence of fuzziness (i.e., with exact defuzzification of the gray pixels to their respective black or white version), H_{hy} reduces to the two state classical entropy of Shannon, the states being black and white. Since a fuzzy set is a generalized version of an ordinary set, the entropy of a fuzzy set deserves to be a generalized version of classical entropy by taking into account not only the fuzziness of the set but also the underlying probability structure. In that respect, H_{hy} can be regarded as a generalized entropy such that classical entropy becomes its special case when fuzziness is properly removed.

Note that the Equations (7) and (8) are defined using the concept of logarithmic gain function. Similar expressions using exponential gain function, i.e., defining the entropy of an n-state system as [38, 39]

$$H = \sum_i p_i e^{1 - p_i}, \qquad i = 1, 2, \ldots, n \tag{11}$$

are available in [23].

All these terms, which give an idea of "indefiniteness" or fuzziness of an image may be regarded as the measures of average intrinsic information which is received when one has to make a decision (as in pattern analysis) in order to classify the ensembles of patterns described by a fuzzy set.

$H^r(X)$ has the following properties:

Pr 1: H^r attains a maximum if $\mu_i = 0.5$ for all i.
Pr 2: H^r attains a minimum if $\mu_i = 0$ or 1 for all i.
Pr 3: $H^r \geq H^{*r}$, where H^{*r} is the rth order entropy of a sharpened version of the fuzzy set (or an image).
Pr 4: H^r is, in general, not equal to $\bar{H}^r$, where $\bar{H}^r$ is the rth order entropy of the complement set.
Pr 5: $H^r \leq H^{r+1}$ when all $\mu_i \in [0.5, 1]$.
$H^r \geq H^{r+1}$ when all $\mu_i \in [0, 0.5]$.

The 'sharpened' or 'intensified' version of X is such that

$$\mu_{x^*}(x_{mn}) \geq \mu_x(x_{mn}) \quad \text{if } \mu_x(x_{mn}) \geq 0.5$$

and

$$\mu_{x^*}(x_{mn}) \leq \mu_x(x_{mn}) \quad \text{if } \mu_x(x_{mn}) \leq 0.5 \tag{12}$$

When $r = 1$, the property Pr 4 is valid only with the "equal" sign. The property Pr 5 (which does not arise for $r = 1$) implies that H^r is a monotonically nonincreasing function of r for $\mu_i \in [0, 0.5]$ and a monotonically nondecreasing function of r for $\mu_i \in [0.5, 1]$ (when the "min" operator has been used to get the group membership value).

When all μ_i values are the same, $H^1(X) = H^2(X) = \cdots = H^r(X)$. This is because of the fact that the difficulty in taking a decision regarding possession of a property on an individual is the same as that of a group selected therefrom. The value of H^r would, of course, be dependent on the μ_i values.

Again, the higher the similarity among singletons (supports) the quicker is the convergence to the limiting value of H^r. Based on this observation, an index of similarity of supports of a fuzzy set may be defined as $S = H^1/H^2$ (when $H^2 = 0$, H^1 is also zero and S is taken as 1). Obviously, when $\mu_i \in [0.5, 1]$ and the *min* operator is used to assign the degree of possession of the property by a collection of supports, S will lie in $[0, 1]$ as $H^r < H^{r-1}$. Similarly, when $\mu_i \in [0, 0.5]$, S may be defined as H^2/H^1 so that S lies in $[0, 1]$. The higher the value of S the more alike (similar) are the supports of the fuzzy set with respect to the fuzzy property μ. This index of similarity can therefore be regarded as a measure of the degree to which the members of a fuzzy set are alike.

Therefore, the value of first order fuzzy entropy (H^1) can only indicate whether the fuzziness in a set is low or high. In addition to this, the value of H^r, $r > 1$ also enables one to infer whether the fuzzy set contains similar supports (or elements) or not. The similarity index thus defined can be successfully used for measuring interclass and intraclass ambiguity (i.e., class homogeneity and contrast) in pattern recognition and image processing problems.

$H^1(X)$ is regarded as a measure of the average amount of information (about the grey levels of pixels) which has been lost by transforming the classical pattern (two-tone) into a fuzzy (gray) pattern X. Further details on this measure with respect to image processing problems are available in [18, 40, 41]. It is to be noted that $H^1(X)$ reduces to zero whenever μ_{mn} is made 0 or 1 for all (m, n), no matter whether the resulting defuzzification (or transforming process) is correct or not. In the following discussion, it will be clear how H_{hy} takes care of this situation.

Let us now discuss some of the properties of $H_{hy}(X)$. In the absence of fuzziness when MNP_b pixels become completely black ($\mu_{mn} = 0$) and MNP_w pixels become completely white ($\mu_{mn} = 1$), then $E_w = P_w$, $E_b = P_b$ and H_{hy} boils down to the two state classical entropy

$$H_c = -P_w \log P_w - P_b \log P_b, \qquad (13)$$

the states being black and white. Thus, H_{hy} reduces to H_c only when a proper defuzzification process is applied to detect (restore) the pixels. $|H_{hy} - H_c|$ can therefore be treated as an objective function for enhancement and noise reduction. The lower the difference, the lesser is the fuzziness associated with the individual symbol and the higher will be the accuracy in classifying them as their original value (white or black). (This property is lacking with the $H^1(X)$ measure which always reduces to zero irrespective of the defuzzification process.) In other words, $|H_{hy} - H_c|$ represents an amount of information that was lost by transforming a two-tone image to a gray tone.

For a given P_w and P_b ($P_w + P_b = 1, 0 \leq P_w, P_b \leq 1$), of all possible defuzzifications, the proper defuzzification of the image is the one for which H_{hy} is minimum.

$$\text{If } \mu_{mn} = 0.5 \text{ for all } (m, n) \text{ then } E_w = E_b$$

$$\text{and } H_{hy} = -\log(0.5 \exp 0.5) \qquad (14)$$

i.e., H_{hy} takes a constant value and becomes independent of P_w and P_b. This is logical in the sense that the machine is unable to make a decision on the pixels since all μ_{mn} values are 0.5.

Let us now consider the measure correlation $C(\mu_1, \mu_2)$ of Equation (9). This has the following properties.

(a) If for higher values of $\mu_1(x)$, $\mu_2(x)$ takes higher values and the converse is also true then $C(\mu_1, \mu_2)$ must be very high.
(b) If with increase of x, both μ_1 and μ_2 increase then $C(\mu_1, \mu_2) > 0$.
(c) If with increase of x, μ_1 increases and μ_2 decreases or vice versa then $C(\mu_1, \mu_2) < 0$.
(d) $C(\mu_1, \mu_1) = 1$
(e) $C(\mu_1, \mu_1) \geq C(\mu_1, \mu_2)$
(f) $C(\mu_1, 1 - \mu_1) = -1$
(g) $C(\mu_1, \mu_2) = C(\mu_2, \mu_1)$
(h) $-1 \leq C(\mu_1, \mu_2) \leq 1$
(i) $C(\mu_1, \mu_2) = -C(1 - \mu_1, \mu_2)$
(j) $C(\mu_1, \mu_2) = C(1 - \mu_1, 1 - \mu_2)$

Correlation of an image indicates the characteristics of relative variation between its two properties μ_1 and μ_2. Based on these characteristics, bound functions are defined as shown in Section 4.1. If one of these properties is considered to be the nearest crisp (two tone) property of the other (say, $\mu_1 = 1$ if $\mu_2 > 0.5$ and $\mu_1 = 0$ if $\mu_2 \leq 0.5$) then $C(\mu_1, \mu_2)$ lies in

[0, 1]. In other words, if μ_2 denotes a bright image plane of an image X having cross-over point at s, say, and is dependent only on gray level, then μ_1 represents its closest two tone version thresholded at s. Therefore, by varying s of the μ_2 plane, an optimum version of μ_2 (i.e., optimum fuzzy segmented version of the image) can be obtained for which correlation is maximum. Various segmentation algorithms based on transitional correlation and within class correlation have been derived [42] using the cooccurrence matrix.

5.2. *SPATIAL AMBIGUITY MEASURES BASED ON FUZZY GEOMETRY OF IMAGE*

Many of the basic geometric properties of and relationships among regions have been generalized to fuzzy subsets. Some of these geometrical properties of a fuzzy digital image subset (characterized by piece-wise constant membership function $\mu_X(x_{mn})$ or simply μ) as defined by Rosenfeld [19], and Pal and Ghosh [21, 22] are listed below with illustrations. These may be viewed as providing measures of ambiguity in geometry (spatial domain) of an image.

Compactness

$$\text{comp}(\mu) = \frac{a(\mu)}{(p(\mu))^2} \tag{15}$$

where

$$a(\mu) = \sum \mu \tag{16}$$

$$p(\mu) = \sum_{i,j,k} |\mu(i) - \mu(j)||A(i,j,k)| \tag{17}$$

$a(\mu)$ denotes area of μ. $p(\mu)$, the perimeter of μ, is just the weighted sum of the lengths of the arcs $A(i,j,k)$ along which the region $\mu(i)$ and $\mu(j)$ meet, weighted by the absolute difference of these values. Physically, compactness means the fraction of maximum area (that can be encircled by the perimeter) actually occupied by the object [19]. In the nonfuzzy case, the value of compactness is maximum for a circle and is equal to $\pi/4$. In the case of the fuzzy disc, where the membership value is only dependent on its distance from the center, this compactness value is $\geqslant \pi/4$. Of all possible fuzzy discs compactness is therefore minimum for its crisp version.

EXAMPLE 1. Let μ be of the form

0.2	0.4	0.3
0.2	0.7	0.6
0.6	0.5	0.6

Then area $a(\mu) = 4.1$, perimeter $p(\mu) = 2.3$ and $\text{comp}(\mu) = 0.775$.

Height and Width

$$h(\mu) = \sum_n \max_m \mu_{mn} \tag{18}$$

and

$$w(\mu) = \sum_m \max_n \mu_{mn} \tag{19}$$

So height/width of a digital picture is the sum of the maximum membership values of each row/column [19]. For the fuzzy subset μ of Example 1, height is $h(\mu) = 0.4 + 0.7 + 0.6 = 1.7$ and width is $w(\mu) = 0.6 + 0.7 + 0.6 = 1.9$.

Length and Breadth

$$l(\mu) = \max_m \left(\sum_n \mu_{mn}\right) \tag{20}$$

and

$$b(\mu) = \max_n \left(\sum_m \mu_{mn}\right) \tag{21}$$

The length/breadth of an image fuzzy subset gives its longest expansion in the column/row direction [21]. If μ is crisp, $\mu_{mn} = 0$ or 1; then length/breadth is the maximum number of pixels in a column/row. Comparing Equations (20) and (21) with (18) and (19) we notice that the length/breadth takes the summation of the entries in a column/row first and then maximizes over different column/rows, whereas the height/width maximizes first the entries in a column/row and then sums over different columns/rows. For the fuzzy subset μ in example 1, $l(\mu) = 0.4 + 0.7 + 0.5 = 1.6$ and breadth is $b(\mu) = 0.6 + 0.5 + 0.6 = 1.7$.

Index of Area Coverage

$$\mathrm{IOAC}(\mu) = \frac{a(\mu)}{l(\mu)b(\mu)} \tag{22}$$

In the nonfuzzy case, the IOAC has value of 1 for a rectangle (placed along the axes of measurement) [21]. For a circle this value is $\pi r^2/(2r*2r) = \pi/4$. IOAC of a fuzzy image represents the fraction (which may be improper also) of the maximum area (that can be covered by the length and breadth of the image) actually covered by the image.

For the fuzzy subset μ of Example 1, the maximum area that can be covered by its length and breadth is $1.6 \times 1.7 = 2.72$ whereas, the actual area is 4.1, so the IOAC $- 4.1/2.72 = 1.51$. Note the difference between IOAC(μ) and comp(μ). Again, note the following relationships

$$l(X)/b(X) \leq 1 \tag{23a}$$

$$b(X)/w(X) \leq 1 \tag{23b}$$

When equality holds for (23a) or (23b), the object is either vertically or horizontally oriented.

Major Axis: Find the length of the object. Now rotate the axes through an angle θ, θ varying between 0° and 90°. The angle for which length is maximum is said to be the angle of inclination of the object (with the vertical). The corresponding axis along which the length is maximum is said to be the major axis. The length along the major axis denotes the expansion of the object [22].

Minor Axis: The axis perpendicular to major axis, for which breadth is maximum, is defined as the minor axis of the object [22].

Center of Gravity: The center of gravity (C.G) of an object can be defined in various ways [22]. Two such definitions are given here.

(a) C.G of an object can be defined as the point of intersection of the major and the minor axes.

(b) Take any pixel as the center. Take a neighborhood of radius r. Find the energy (area) of the circle. Now shift the center of the circle over all the pixels of the object. The center for which the energy is maximum is defined as the C.G. If there is any tie then increase the radius and obtain the C.G.

For the fuzzy subset μ of Example 1, length is $l(\mu) = 1.6$ and breadth is $b(\mu) = 1.7$. Now, if we rotate the object by 45° then its length is $l(\mu) = 0.6 + 0.7 + 0.6 = 1.9$.

Hence the object is inclined at an angle of 45° with vertical axis. So by major axis of this image we mean the axis inclined at an angle of 45° with the vertical. Similarly the minor axis of this object is inclined at an angle of 45° with horizontal. Trivially the C.G of this object is through the pixel having membership 0.7.

Density

$$d(\mu) = \sum_{i=1}^{N} \mu(i)/N \tag{24}$$

N denotes the number of supports of μ (i.e., summation is taken over pixels for which μ is nonzero). The maximum value of density is 1, and this value occurs only for a nonfuzzy case. Density can be used for finding the C.G of an image. If we break the image into different regions then the region having the maximum density may be regarded as containing the C.G.

Degree of Adjacency: The degree to which two regions S and T of an image are adjacent is defined as [22]:

$$a(S,T) = \sum_{p \in BP(S)} \frac{1}{1 + |\mu(p) - r(q)|} * \frac{1}{1 + d(p)} \tag{25}$$

Here $d(p)$ is the shortest distance between p and q, q is a border pixel (BP) of T and p is a border pixel of S. The other symbols have the same meaning as in the previous discussion.

The degree of adjacency of two regions is maximum ($= 1$) only when they are physically adjacent i.e., $d(p) = 0$ and their membership values are also equal i.e., $\mu(p) = r(q)$. If two regions are physically adjacent then their degree of adjacency is determined only by the difference of their membership values. Similarly, if the membership values of two regions are equal their degree of adjacency is determined by their physical distance only. The readers may note the difference between Equation (25) and the adjacency definition given by Rosenfeld [19].

EXAMPLE 2. Let us consider a scene having two regions (Figures 7A–7C). The gray levels of the regions in Figures 7A and 7B are identical but the regions in Figure 7B closer to each other compared to that in Figure 7A. So the degree of adjacency of the regions in Figure 7B should be more than that in Figure 7A. In Figures 7B and 7C the relative positions of the regions are the same, but the difference of gray levels of the border pixels in Figure 7B is more than that in Figure 7C. So from intuition, we would expect the degree of adjacency of the regions in Figure 7C to be more than that in Figure 7B.

The $a(S,T)$ values computed with Equation (20) are 0.062, 0.143 and 0.189 for Figures 7A, 7B, and 7C, respectively. So, the results agree well with our intuition.

5.3. *SOME EXAMPLES ON FUZZY IMAGE PROCESSING OPERATIONS*

In this section, we describe some algorithms to show how the aforesaid information measures and geometrical properties (which reflect grayness ambiguity and spatial ambiguity in an image) can be incorporated in handling uncertainties in various operations, e.g., gray level thresholding, enhancement, contour detection, and skeletonization by avoiding hard decisions and providing output in both fuzzy and nonfuzzy (as a special case) versions. It is to be noted that these low level operations (particularly image segmentation and object extraction) play a major role in an image recognition system. As mentioned before (in Section 3.1), any error made in this process might propagate to feature extraction and classification.

Threshold Selection (Fuzzy Segmentation)

Given an L level image X of dimension $M \times N$ with minimum and maximum gray values ℓ_{min}, and ℓ_{max} respectively, the algorithm for its fuzzy segmentation into object and background may be described as follows:

Step 1. Construct the membership plane using Zaden's S function [18, 30] as

$$\mu_{mn} = \mu(\ell) = S(\ell; a, b, c) \tag{26}$$

(called bright image plane if the object regions possess higher gray values) or

$$\mu_{mn} = \mu(\ell) = 1 - S(\ell; a, b, c) \tag{27}$$


```
                9GKPSTVPNGDC78
                9GJKSTVTVJCA88
                96HJOVUMTRPHA8
                9AJNSUVMKGA978
                9GHKMSTVRPHCA8

AS49HKOTTSNKGC8
AFGKLPSUTPMJGF8
AFSHSTVUJHCA998
ADEGKOVSSJDA988
AAHKRTMJFA89876
ABJOSTTVUSKFGA8
AAGHKUVRMHFC968
```

(a)

```
   9GKPSTVPNGDC78
   9GJKSTVTVJCA88
   96HJOVUMTRPHA8
   9AJNSUVMKGA978
   9GHKMSTVRPHCA8

AS49HKOTTSNKGC8
AFGKLPSUTPMJGF8
AFSHSTVUJHCA998
ADEGKOVSSJDA988
AAHKRTMJFA89876
ABJOSTTVUSKFGA8
AAGHKUVRMHFC968
```

(b)

```
   9GKPSTVPNGDC78
   9GJKSTVTVJCA88
   96HJOVUMTRPHA8
   9AJNSUVMKGA978
   9HKPTSTMKFDCA8

AS49HKOTTSNKGC8
AFGKLPSUTPMJGF8
AFSHSTVUJHCA998
ADEGKOVSSJDA988
AAHKRTMJFA89876
ABJOSTTVUSKFGA8
AAGHKUVRMHFC968
```

(c)

Fig. 7. (a) A scene with two regions. (b) Scene same as in A, but the relative positions of the regions are different. (c) Scene same as in B, but the gray levels of the border pixels of one region are different.

(called dark image plane if the object regions possess lower gray values) with cross-over point b and a band width $\Delta b = b - a = c - b$.

Step 2. Compute the parameter $I(X)$ where $I(X)$ represents either grayness ambiguity or spatial ambiguity (as designated by H', correlation, compactness, IOAC and adjacency, say) or both (i.e., product of grayness and spatial ambiguities).

Step 3. Vary b between ℓ_{min} and ℓ_{max} and select those b for which $I(X)$ has local minima or maxima depending on $I(X)$. (Maxima correspond for the correlation measure only.) Among the local minima/maxima, let the global one have cross-over point at s.

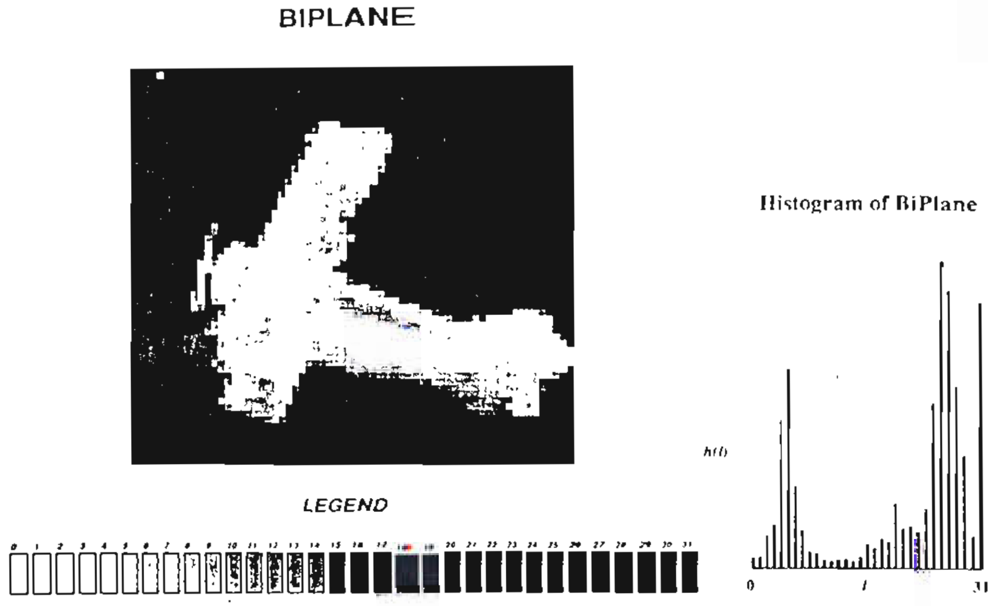
The level s, therefore, denotes the cross over point of the fuzzy image plane μ_{mn}, which has minimum grayness and/or geometrical ambiguity. The μ_{mn} plane then can be viewed as a fuzzy segmented version of the image X. For the purpose of nonfuzzy segmentation, we can take s as the threshold (or boundary) for classifying or segmenting an image into object and background. The faster methods of computation of the fuzzy parameters are explained in [22].

Note that we considered Zadeh's standard S function in the above algorithm to represent the fuzzy set "bright image plane," and $w = 2\Delta b$ is the length of the window (such that $[0, w] \to [0, 1]$) which was shifted over the entire dynamic range. As w decreases, the $\mu(x_{mn})$ plane tends to have more intensified contrast around the cross-over point, thus resulting in a decrease of ambiguity in X. As a result, the possibility of detecting some undesirable thresholds (spurious minima) increases because of the smaller value of Δb. On the other hand, an increase in w results in a higher value of fuzziness and thus leads towards the possibility of losing some of the weak minima.

The criteria regarding the selection of membership functions and the length of window (i.e., w) have been reported recently by Murthy and Pal [31] assuming continuous functions for both histogram and membership function. For a fuzzy set "bright image plane," the membership function $\mu: [0, w] \to [0, 1]$ should be such that

(i) μ is continuous, $\mu(0) = 0$, $\mu(w) = 1$
(ii) μ is monotonically non-decreasing, and
(iii) $\mu(x) = 1 - \mu(w - x)$ for all $x \in [0, w]$.

Furthermore, μ should satisfy the bound criteria derived based on the correlation (Section 4.1). For example, consider a 32 level image of a biplane (Figure 8) of dimension 64×64 along with its histogram. It has been shown that the resulting thresholds are always satisfactory (lying approximately in the range 10–15 [31]) when any monotonically nondecreasing functions (e.g., different forms (orders) of symmetric functions



(a) (b)

Fig. 8. (a) Biplane image. (b) Histogram of biplane image.

and/or asymmetric functions as expressed by Equations 29 and 30) lying within the bound functions $h1$ and $h2$ (Figure 6) are used. It has also been shown that for detecting a minimum in the valley region of a histogram, the window length w of the μ function should be less than the distance between two peaks around that valley region. This proves the flexibility of the approach.

If, instead of a single membership function, we have a set of monotonically nondecreasing functions to represent a collection of various opinions on the bright membership plane $\mu_x(x_{mn})$ and we wish to give due respect to all of these opinions, then we can use the concept of spectral fuzzy sets (Section 4.2) to minimize the parameter "spectral index" (Equation 4) in addition to one of those represented by $I(X)$ to manage this uncertainty. Consequently, we will make a soft decision by providing a set of thresholds associated with their respective certainty values.

Let the various opinions on membership function representing the bright image plane be denoted as

$$\mu_x(x_{mn}) = \mu_x(\ell)$$
$$= \{S(\ell; a, b, c), g_1(\ell), g_2(\ell)\} \qquad \ell = 0, 1, 2, \ldots, L-1 \quad (28)$$

where

$$g_1(\ell) = ((\ell - a)/(c - a))^{\beta} \tag{29}$$

and

$$g_2(\ell) = 1 - (1 - (\ell - a)/(c - a))^{\gamma} \tag{30}$$

ℓ denotes the gray level value of x_{mn}. β and γ are positive constants and they control the height and width (i.e., spectrum) of μ_X plane. Then the sets of thresholds $\{s\}$ obtained along with their certainty values $d(s)$ are shown in Table 2 for three different sets of membership functions containing six, seven and eight opinions. The results correspond to $\Delta b = 4$. Note that the $d(s)$ values shown are not normalized. The set of thresholds obtained here for different sets of β and γ is same, but with different orders of certainty values depending on difference in opinions. The thresholds, as seen from the histogram, are intuitively appealing. If one is interested in having a nonfuzzy single threshold, he can select the one with highest $d(s)$ value because it has the most disagreement in opinion as far as its belonging to either the object or background is concerned.

Let us now describe another way of extracting an object by minimizing higher order entropy (Equation 7) of both object and background regions using an inverse π function as shown by the solid line in the Figure 9. Unlike the previous algorithm, the membership function does not need any parameter selection to control the output.

Suppose s is the assumed threshold so that the gray level ranges $[1, s]$ and $[s+1, L]$ denote, respectively, the object and background of the image X. The inverse π-type function to obtain μ_{mn} values of X is generated by

TABLE 2

Soft Thresholds for Biplane Extraction

$\beta = 1.0, 2.0, 3.0$; $\gamma = 1.5, 2.0$		$\beta = 1.0, 2.0, 3.0, 3.5$; $\gamma = 1.5, 2.0$		$\beta = 1.0, 2.0, 3.0, 3.5, 4.0$; $\gamma = 1.5, 2.0$	
$\{s\}$	$d(s)$	$\{s\}$	$d(s)$	$\{s\}$	$d(s)$
11	1.73	11	1.77	11	1.79
12	1.98	12	2.07	12	2.13
13	1.66	13	1.85	13	2.01
14	1.42	14	1.79	14	2.10

taking union of $S(x;(s-(L-s)), s, L)$ and $1 - S(x;1, s, (s+s-1))$, where S denotes the standard S function defined by Zadeh. The resulting function as shown by the solid line, makes μ lie in $[0.5, 1]$. Because the ambiguity (difficulty) in deciding a level as a member of the object or the background is maximum for the boundary level s, it has been assigned a membership value of 0.5 (i.e., cross-over point). Ambiguity decreases (i.e., degree of belongingness to either object or background increases) as the gray value moves away from s on either side. The μ_{mn} thus obtained denotes the degree of belongingness of a pixel x_{mn} to either object or background. Since s is not necessarily the mid point of the entire gray scale, the membership function (solid line of Figure 9) may not be a symmetric one. It is further to be noted that one may use any linear or nonlinear equation (instead of Zadeh's standard S function) to represent the membership function in Figure 9.

Therefore, the task of object extraction is to

Step 1. Compute the rth order fuzzy entropy of the object H_O^r and the background H_B^r considering only the spatially adjacent sequences of pixels present within the object and background respectively. Use the 'min' operator to get the membership value of a sequence of pixels.

Step 2. Compute the total rth order fuzzy entropy of the partitioned image as

$$H_s^r = H_O^r + H_B^r.$$

Step 3. Minimize H_s^r with respect to s to get the threshold for object background classification.

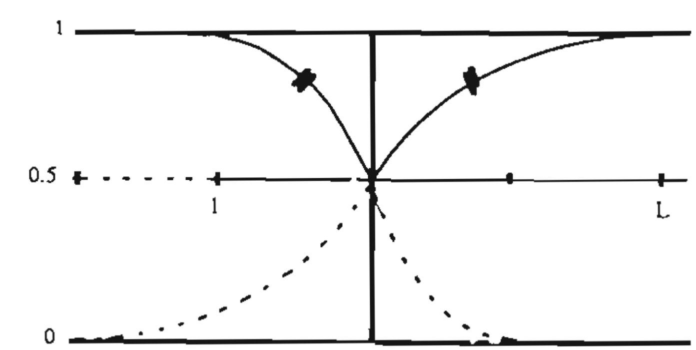


Fig. 9. Inverse π function (solid line) for computing object and background entropy.

Referring back to the Section 5.1, we have seen that H^2 reflects the homogeneity among the supports in a set, in a better way than H^1 does. The higher the value of r, the stronger is the validity of this fact. Thus, considering the problem of object-background classification, H^r seems to be more sensitive (as r increases) to the selection of the appropriate threshold; i.e., the improper selection of the threshold is more strongly reflected by H^r than H^{r-1}. For example, the thresholds obtained by the H^2 measure has more validity than those by H^1 (which only takes into account the histogram information). Similar arguments hold good for even higher order ($r > 2$) entropy.

The methods of object extraction (or segmentation) we described above are all based on gray level thresholding. Another way of doing this task is by pixel classification. The details on this technique using fuzzy c-means, fuzzy isodata, fuzzy dynamic clustering, and fuzzy relaxation are available in [18, 24, 43–49].

Contour Detection

Edge detection is also an image segmentation technique where the contours/boundaries of various regions are extracted based on the detection of discontinuity in grayness. The key factors of this approach are as follows: (i) most of the information of an image lies on the boundaries between different regions where there is a more or less abrupt change in gray levels, and (ii) the human visual systems seem to make use of edge detection, but not of thresholding.

To formulate a fuzzy edge detection algorithm, let us describe an edginess measure based on H^1 (Equation 7) which denotes an amount of difficulty in deciding whether a pixel can be called an edge or not. Let $N^3_{x,y}$ be a 3×3 neighborhood of a pixel at (x, y) such that

$$N^3_{x,y} = \{(x,y),(x-1,y),(x+1,y),(x,y-1),(x,y+1),(x-1,y-1),$$

$$(x-1,y+1),(x+1,y-1),(x+1,y+1)\} \quad (31)$$

The edge-entropy, $H^E_{x,y}$ of the pixel (x, y), giving a measure of edginess at (x, y) may be computed as follows. For every pixel (x, y), compute the average, maximum and minimum values of gray levels over $N^3_{x,y}$. Let us denote the average, maximum and minimum values by Avg, Max, Min

respectively. Now define the following parameters.

$$D = \max\{\text{Max} - \text{Avg}, \text{Avg} - \text{Min}\} \quad (32)$$

$$B = \text{Avg} \quad (33)$$

$$A = B - D \quad (34)$$

$$C = B + D \quad (35)$$

A π-type membership function (Figure 10) is then used to compute μ_{xy} for all $(x, y) \in N^3_{x,y}$, such that $\mu(A) = \mu(C) = 0.5$ and $\mu(B) = 1$. It is to be noted that $\mu_{xy} \geq 0.5$. Such a $\mu_{x,y}$, therefore, gives the degree to which a gray level is close to the average value computed over $N^3_{x,y}$. In other words, it represents a fuzzy set "pixel intensity close to its average value," averaged over $N^3_{x,y}$. When all pixel values over $N^3_{x,y}$ are either equal or close to each other (i.e., they are within the same region), such a transformation will make all $\mu_{xy} = 1$ or close to 1. In other words, if there is no edge, pixel values will be close to each other and the μ values will be close to one (1); thus resulting in a low value of H^1. On the other hand, if there is an edge (dissimilarity in gray values over $N^3_{x,y}$), then the μ values will be more away from unity; thus resulting in a high value of H^1. Therefore, the entropy H^1 over $N^3_{x,y}$ can be viewed as a measure of edginess ($H^E_{x,y}$) at the point (x, y). The higher the value of $H^E_{x,y}$, the stronger is the edge intensity and the easier is its detection. Such an entropy plane will represent the fuzzy edge detected version of the image. As mentioned before, there are several ways in which one can define a π-type function as shown in Figure 10.

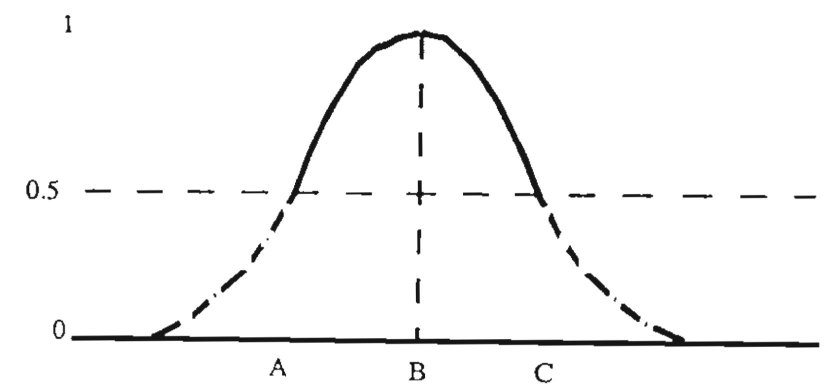


Fig. 10. π Function for computing edge entropy.

The proposed entropic measure is less sensitive to noise because of the use of a dynamic membership function based on a local neighborhood. The method is also not sensitive to the direction of edges. Other edginess measures and algorithms based on fuzzy set theory are available in [18, 50–52].

Optimal Enhancement Operator Selection

When an image is processed for visual interpretation, it is ultimately up to the viewers to judge its quality for a specific application and how well a particular method works. The process of evaluation of image quality therefore becomes subjective which makes the definition of a well processed image an elusive standard for comparison of algorithm performance. Again, it is customary to have an iterative process with human interaction in order to select an appropriate operator for obtaining such a desired processed output. For example, consider the case of contrast enhancement using a nonlinear functional mapping. Not every kind of nonlinear function will produce a desired (meaningful) enhanced version. The questions that automatically arise are as follows. Given an arbitrary image which type of nonlinear functional form will be best suited without prior knowledge on image statistics (e.g., in remote applications like space autonomous operations where frequent human interaction is not possible) for highlighting its object? Knowing the enhancement function how can one quantify the enhancement quality for obtaining the optimal one? Regarding the first question, even if we are given the image statistics, it is possible only to estimate approximately the function required for enhancement and the selection of the exact functional form still needs human interaction in an iterative process. The second question, on the other hand, needs individual judgment, which makes the optimum decision subjective.

The method of optimization of the fuzzy geometrical properties and entropy has been found recently [53] to be successful in providing quantitative indices in order to avoid such human iterative interaction in selecting an appropriate nonlinear function and to make the task of subjective evaluation objective.

Fuzzy Skeleton Extraction and FMAT

Let us now explain two methods for extracting the fuzzy skeleton of an object from a gray tone image without getting involved into its (questionable) hard thresholding. The first one is based on minimization of the parameter IOAC (Equation 22) or compactness (Equation 15) with respect to α-cuts over a fuzzy "core line" (or skeleton) plane. The membership value of a pixel to the core line plane depends on its property of possessing maximum intensity, and property of occupying vertically and horizontally middle positions from the ϵ-edges (pixels beyond which the membership value in the fuzzy segmented image becomes less than or equal to ϵ, $\epsilon > 0$) of the object [54]. If a nonfuzzy (or crisp) single pixel width skeleton is deserved, it can be obtained by a contour tracing algorithm [55] which takes into account the direction of contour. Note that the original image cannot be reconstructed, like the other conventional techniques of gray skeleton extraction [48, 56–58], from the fuzzy skeleton obtained here.

The second method is based on fuzzy medial axis transformation (FMAT) as defined by Pal and Rosenfeld [26] using the concept of fuzzy disks. A fuzzy disk with center P is a fuzzy set in which membership depends only on the distance from P. For any fuzzy set f, there is a maximal fuzzy disk $g_P^f \leqslant f$ centered at every point P, and f is the *sup* of the g_P^f's. (Moreover, if f is fuzzy convex, so is every g_P^f, but not conversely.) We call a set S_f of points f-sufficient if every $g_P^f \leqslant g_Q^f$ for some set of Q in S_f; evidently f is then the *sup* of the g_Q^f's. In particular, in a digital image, the set of Q's at which g^f is a (nonstrict) local maximum is f-sufficient. This set is called the *fuzzy medial axis* of f, and the set of g_Q^f's is called the *fuzzy medial axis transformation* (FMAT) of f. These definitions evidently reduce to the standard one if f is a crisp set.

For a gray tone image X (denoting the nonnormalized fuzzy "bright image" plane), the FMAT algorithm computes, first of all, various fuzzy disks centered at the pixels and then retains a few (as small as possible) of them, as designated by g_Q's, so that their union can represent the entire image X. That is, the pixel value at any point t can be obtained from such union operation, as t has membership value equal to its own gray value (i.e., equal to its nonnormalized membership value to the bright image plane) to one of those retained disks.

For example, consider a 5×5 image X as shown in Figure 11. The lower left pixel of intensity 4 has coordinate (1, 1). Fuzzy disks (upright square of odd side length) for all the border pixels have values {5, 0} except the one at position (1, 1) for which $g_P = \{4, 0\}$. The pixels having intensity 6 have disk values of {6, 5, 0} except the one at (2, 2) for which it is {6, 4, 0}. The center pixel has $g_P = \{7, 6, 4\}$. In these sets of disk values, the first entry denotes the nonnormalized membership value of the pixel itself to that disk (i.e., membership value of the disk at $r = 0$). The consecutive entries denote similarly the memberships at $r = 1, 2, \ldots$. The pixels constituting the fuzzy medial axis are marked bold. (Note that if we had the pixel

5	5	5	5	5
5	6	6	6	5
5	6	7	6	5
5	6	6	6	5
4	5	5	5	5

Fig. 11. A 5 × 5 digital image. Pixels belonging to fuzzy medial axis (FMA) are marked bold. Pixels belonging to reduced fuzzy medial axis (RFMA) are underlined.

intensity 4 of X replaced by 5, the FMAT would have been reduced to only one disk with $g_{(3,3)} = \{7,6,5\}$).

In order to restore the deleted pixels, simply put all the disk values of FMA pixels at those locations back. In case a location has more than one such values, select the largest one.

It is to be noted that the above representation is redundant i.e., some more disks can further be deleted without affecting the reconstruction. The redundancy in pixels (fuzzy disks) from the fuzzy medial axis output can be reduced by considering the criterion $g_p^l(t) \leq \sup g_{Ql}^l(t) = 1,2,\ldots$ instead of $g_p^l(t) \leq g_Q^l(t)$. In other words, eliminate many other g_p^l's for which there exists a set of g_{Ql}^l's whose *sup* is greater than or equal to g_p^l. For example, the point at location (3,4) in Figure 11 can be removed because it is contained in the union of the fuzzy disks around (3,3) and (2,4) (or (4,4)) i.e., $g_{(3,4)} \leq \sup\{g_{(3,3)}, g_{(2,4)}\}$ (or $\leq \sup\{g_{(3,3)}, g_{(4,4)}\}$) for all pixels in X. Similar is the case with the pixel at location (4,3) which can also be removed. The pixels representing the final reduced MA are underlined in Figure 11. Let RFMAT denote the FMAT after reducing its redundancy.

To demonstrate its applicability on a real image we consider Figure 12A as input. Figure 12B denotes its RFMAT output. Therefore, the fuzzy medial axis provides a good skeleton of the darker (higher intensity) pixels in an image apart from its exact representation. It is to be mentioned here that such a representation may not be economical in a practical situation. The details on this feature and the possible approximation in order to make it practically feasible are available in [59].

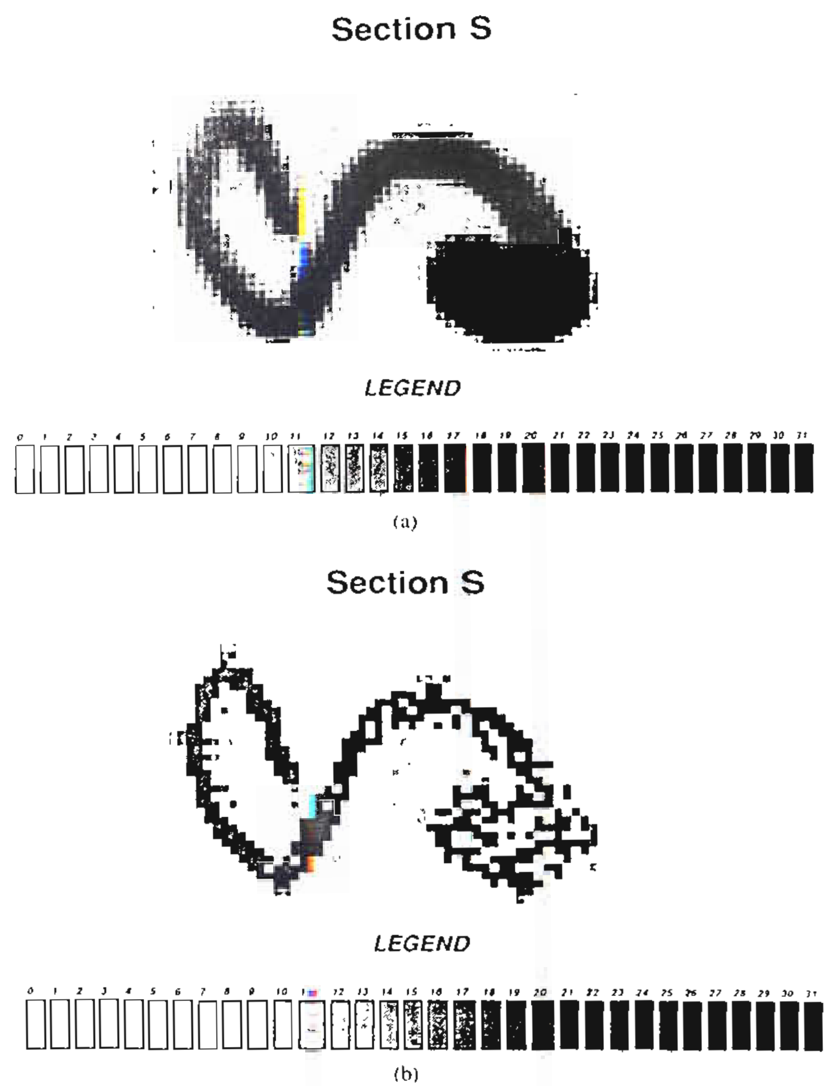


Fig. 12. A: 36 × 60 "S" image. B: RFMAT output of "S" image.

Note that the membership values of the disks contain the information of image statistics. For example, if the image is smooth the disk will not have abrupt change in its values. On the other hand, it will have abrupt change in case the image has salt and pepper noise or edginess. The concept of fuzzy MAT can therefore be used as spatial filtering (both high pass and low pass) of an image by manipulating the disk values to the extent of desire and then putting them back while reconstructing the processed image. Automatic manipulation (modification) of disk values can further be guided by entropy $H^{(1)}$ (as described by Equation 36) of a one dimensional string.

5.4 MEASURES BASED ON CLASSICAL IMAGE ENTROPY AND GRAY LEVEL STATISTICS

Let us now discuss some of the image entropy measures that provide various image information and uncertainty in scene analysis based on the probability of cooccurrences of pixels in an image. These measures do not involve the concept of fuzzy set theory rather, they use the gray level statistics computed from histogram and cooccurrence matrix.

qth Order Image Entropy

$$H^{(q)} = (-1/q) \sum_i p(s_i) \log_2 p(s_i) \quad (36)$$

where $p(s_i)$ is the probability of a sequence s_i of gray levels of length q, and summation is taken over all gray level sequences of length q. $H^{(q)}$ is a monotonic decreasing function of q and

$$\lim_{q \to \infty} H^{(q)} = H, \text{ the entropy of the image.}$$

If $q = 1$ i.e., sequence of length 1, p_i becomes the probability of occurrence of the level i. $H^{(1)}$, which is a function of histogram only, may be called global entropy of the image. For $q = 2$, p_{ij} represents the probability of cooccurrence of the levels i and j, and $H^{(2)}$, called local entropy of order 2, can be obtained from the cooccurrence matrix. Note that $H^{(1)}$ computed from the histogram has less importance than $H^{(2)}$ in describing (and processing) an image, because the histogram remains the same irrespective of the arrangements (permutation) of the points in the image space, whereas the cooccurrence matrix changes [60].

Conditional Image Entropy

$$\begin{aligned} H(c) &= (H(O/B) + H(B/O)) \\ &= -\sum_{x_i \in O} \sum_{y_j \in B} p(x_i/y_j) \log_2 p(x_i/y_j) \\ &\quad - \sum_{y_j \in B} \sum_{x_i \in O} p(y_j/x_i) \log_2 p(y_j/x_i) \end{aligned} \quad (37)$$

where the object O and the background B contain the pixels x_i and y_j respectively. The first term denotes the conditional entropy of the object given the background, i.e., average amount of information that may be obtained from the object given that one has viewed the background [60]. Similarly, the second term represents the information obtained from the background when one has viewed the object. The pixel y_j, in general, can be an mth order neighbor of the pixel x_i, i.e., y_j can be the mth pixel after x_i.

Various segmentation and object extraction algorithms using these measures (for $q = 2$ and $m = 2$) have been developed by Pal and Pal [60]. The criterion used is to maximize either the sum of the second order local entropy of an object and background, or the second order conditional entropy with respect to a partitioning of the image space. In that sense the local entropy of the object and background gives information about the spatial ambiguity by measuring their intraset homogeneity, whereas the conditional entropy conveys information on interset contrast between object and background. (Note that the expressions given above use the concept of logarithmic entropy. Similar expressions using exponential behavior of entropy (Equation 11) are available in [38, 39].)

Positional Image Entropy

The aforesaid entropic measures $H^{(q)}$ and $H(c)$ are able to extract the object irrespective of its position on the scene, and if the object is moved from one place to another keeping the gray level distribution the same these measures do not change. In scene analysis, robotics and computer vision it becomes highly desirable to have the information about the location of the object within a scene. For example, in docking and camera tracking problems of space autonomous research, we are interested in determining the location of a particular shuttle or satellite in the scene. A measure, called positional entropy H^P, has recently been introduced by

Pal and Pal [39] that may be used to find the approximate position of an object constituted by the range of levels $[0, s]$, say, in a scene. Its definition in terms of exponential gain function (Equation 11) is given below:

$$H^p = \sum_{\ell=0}^{s} w_\ell p_\ell e^{1-p_\ell} \tag{38}$$

where

$$w_\ell = \sum_{f_{ij}=\ell} w_{ij} \tag{39}$$

$$w_{ij} = 1/d_{Aij} \qquad \text{when } r > 0 \tag{40a}$$

$$w_{ij} = 1/d_{Dij} \qquad \text{when } r < 0 \tag{40b}$$

$$w_{ij} = 1/d_{Eij} \qquad \text{when } r = 0 \quad \text{and} \quad \sum_{i,j \in \text{obj}} (j - \bar{J}) \neq 0 \tag{40c}$$

$$w_{ij} = 1/d_{Gij} \qquad \text{when } r = 0 \quad \text{and} \quad \sum_{i,j \in \text{obj}} (j - \bar{J}) = 0 \tag{40d}$$

$$r = (1/\sigma_i \sigma_j O_s) \sum_{i,j \in \text{obj}} (i - \bar{I})(j - \bar{J}) \tag{41}$$

$$\sigma_i = \sqrt{\sum (i - \bar{I})^2 / O_s} \qquad \sigma_j = \sqrt{\sum (j - \bar{J})^2 / O_s} \tag{42}$$

$$\bar{I} = M/2 + 0.5 \qquad \bar{J} = N/2 + 0.5 \tag{43}$$

$M \times N$ is the size of the image and O_s is the size of the object. p_ℓ denotes the probability of occurrence of the gray level ℓ. f_{ij} denotes intensity of the (i,j)th pixel. d_{xij} denotes the distance of the (i,j)th pixel from the point x (x can be any of the points A, D, G, and E in Figure 13). r denotes the 'object correlation' such that it can have either a positive, negative or zero value depending on the position of the object in the scene. For example, r is positive if the major portion of the object is along the diagonal AC (Figure 13), r is negative when the object is concentrated along the diagonal BD and r is zero when the object lies symmetrically on axis EF or axis GH.

Equation (38) is seen to take into account the position of the object on both the image plane and on the gray scale. That is, if the same object

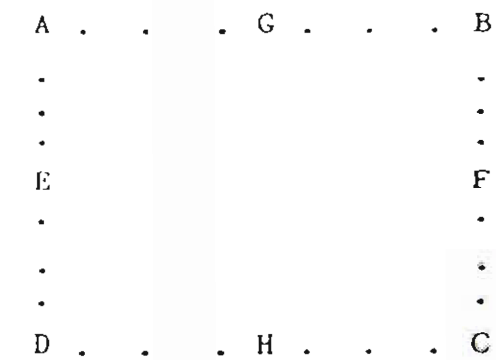


Fig. 13. Different positions for object for correlation measure.

(with same gray level distribution) is placed at different positions on the scene, H^p will change. Similarly, keeping the position of the object fixed, if the gray level distribution is changed, then H^p will also change. The details on its behavior with r are available in [39]. It is to be noted that H^p provides an *approximate* location of an object with reference to some reference points in Figure 13. Further investigation is required to incorporate the fuzzy hedges such as "high," "low," and "very" in identifying a location more specifically.

Sharpness / Contrast Measures

In space station autonomous operations (e.g., docking and camera tracking), we often need to measure the sharpness in focusing an object by a camera. This may be obtained based on the gray level statistics computed from histogram and cooccurrence matrix. A simple measure is to compute the "gray level variance"

$$\sigma^2 = (1/N) \sum_i (\ell_i - \bar{\ell})^2 h_i \tag{44}$$

with

$$\bar{\ell} = (1/N) \sum_i \ell_i h_i \qquad N = \sum_i h_i \tag{45}$$

from the histogram of the image. Here, h_i denotes the number of occurrences of the ith level ℓ_i. $\bar{\ell}$ denotes the mean gray level of the image. N

is the number of pixels in the image. If σ^2 is small the gray levels are all close to the mean.

As mentioned before, statistics computed from the histogram has some limitations. A measure of contrast based on second order statistics (co-occurrence matrix) is to compute the entropy of order 2

$$H^{(2)} = (-1/2) \sum_i \sum_j p_{ij} \log_2 p_{ij} \tag{46}$$

from Equation (36). As expected, $H^{(2)}$ possesses maximum value when p_{ij} (the probability of cooccurrence of the levels i and j) are all equal. Its value decreases when the diagonal concentration increases, i.e., when the contrast in the image decreases.

Another measure of contrast using the cooccurrence matrix and the characteristics of human visual system (HVS) may be defined as

$$C = \sum_i \sum_j \frac{|i-j|}{i+j} p_{ij} \tag{47}$$

According to the logarithmic behavior of HVS, the discrimination ability of a region decreases with an increase in the value of its background intensity. Again, the contrast value should not change if the object and background are interchanged [61]. The factor $|i-j|/(i+j)$ ensures these properties. Obviously, the C value decreases as the diagonal concentration of the cooccurrence matrix increases and the background intensity of a region increases. The use of HVS in automatic selection of valid edge points of an image is described in [62].

Entropy as a Similarity Measure

The entropy $H^{(q)}$ (Equation 36) can also be used as a similarity measure between two images. For a particular value of q, the lower the difference between two images with respect to $H^{(q)}$ the higher is their similarity. Again, the higher the value of q, the more the validity of this concept of similarity. The usefulness of this concept has been demonstrated in [63].

6. UNCERTAINTY IN KNOWLEDGE ACQUISITION AND MATCHING

In the previous sections, we have discussed, in detail, various measures (both fuzzy set theoretic and classical) for ambiguity in an image and their applications in representing and handling the various uncertainties that might arise in some of the important operations in low level vision. The processed output can be obtained in both fuzzy and crisp (as a special case) forms. As mentioned before, these low level operations (particularly image segmentation and object extraction) play major roles in an image recognition system. Any error made in this process might propagate to the higher level tasks, i.e., in feature extraction, description and classification/analysis. Let us now explain, in brief, how the uncertainty in the higher levels of a recognition system can be represented and managed with the notion of fuzzy set theory.

In picture recognition and scene analysis problems, the structural information is very abundant and important, and the recognition process includes not only the capability to assign the input pattern to a pattern class, but also the capacity to describe the characteristics of the pattern that make it ineligible for assignment to another class. In such cases complex patterns are described as hierarchical or tree-like structures of simpler subpatterns and each simpler subpattern is again described in terms of even simpler subpatterns and so on. Evidently, for this approach to be advantageous, the simplest subpatterns, called pattern primitives are to be selected.

Another activity which needs attention in this connection is the subject of shape-analysis that has become an important subject in its own right. Shape analysis is of primal importance in feature/primitive selection and extraction problems. Description of shape can be done in two ways, e.g., in terms of scalar measurements and through structural descriptions. In this connection, it needs to be mentioned that shape description algorithms should be information-preserving in the sense that it is possible to reconstruct the shapes with some reasonable approximation from the descriptors.

As described in Section 5.2, the fuzzy geometrical parameters also provide scalar measurements of shape of a gray image. Having extracted these fuzzy geometrical properties of an image, one can go by the decision theoretic approaches for its recognition. The way uncertainties, impreciseness and incompleteness in an input pattern can be handled heuristically has been reported recently by Pal and Mandal [28]. Their system provides a natural decision which is associated with a confidence factor denoting the degree of certainty of the decision. The concept of determining multiclass (fuzzy) boundary and shape [64] of a pattern class from its sampled points (prototypes) has also been introduced in order to increase the efficiency of the system. In this connection, mention must be made of the development of a generalized guard zone algorithm (GGA) recently reported through a series of papers [65–70] for learning class parameters in presence of mislabelled, noisy or doubtful training samples. The algorithm uses thresholds dynamically to reject those undesirable samples to implement a restricted updating program. The algorithm has been found to be useful in many real life pattern recognition problems by providing much improved performance as compared to the usual nonsupervized recognition systems, particularly when the initial estimates are weak or poor. Its convergence properties and automatic selection of guard zone dimension are mathematically established using the model of Chittineni [71]. For uncertainty analysis in feature selection and extraction problems, the readers may refer to [49, 72–76].

Let us now consider the syntactic approach of description and recognition of an image based on the primitives extracted from the structural information of its shape. The syntactic approach to pattern recognition involves the representation of a pattern by a string of concatenated subpatterns called primitives. These primitives are considered to be the terminal alphabets of a formal grammar whose language is the set of patterns belonging to the same class. The task of recognition therefore involves a parsing of the string.

Because of the ill-defined character of the structural information, the uncertainty may arise both in defining primitives and in relations among them. In order to handle them, the syntactic approach has incorporated the concept of fuzzy sets at two levels. First, the pattern primitives are themselves considered to be labels of fuzzy sets, i.e., such subpatterns as "almost circular arcs," "gentle," "fair," and "sharp" curves are considered. Secondly, the structural relations among the subpatterns may be fuzzy, so that the formal grammar is fuzzified by the weighted production rules and the grade of membership of a string is obtained by min-max composition of the grades of the production used in the derivations.

Pal and King [18, 55] viewed the primitives like line and curve in terms of arcs with varying grades of membership from 0 to 1; 0 representing a straight line and 1 representing a sharp arc. For automatic extraction of primitives from gray-tone edge-detected images they defined membership functions for vertical line, horizontal line and oblique line from the angle of inclination, and the degree of arcness of a line segment from the co-ordinates of its end points. Their effectiveness in recognizing x-ray images, hand and wrist bones, and nuclear patterns of brain neurosecretory cells is demonstrated in [77–79]. Huntsberger et al. [80] has interpreted the shape parameters of triangle, rectangle and quadrangle in terms of membership for "approximate isosceles triangles," "approximate equilateral triangles," and "approximate right triangles" and so on for their classification in a color image.

In order to represent the uncertainty in physical relations among the primitives, the production rules of a formal grammar are fuzzified to account for the fuzziness in relation among the primitives, thereby increasing the generative power of a grammar. Such a grammar is called fuzzy grammar [81, 82]. DePalma and Yau [83] introduced the concept of fractionally fuzzy grammars with a view to improving the effectiveness of a syntactic recognition system.

The incorporation of the element of fuzziness in defining "sharp," "fair," and "gentle" curves in the grammars enables one to work with a much smaller number of primitives. By introducing fuzziness in the physical relations among the primitives, it was also possible to use the same set of production rules and non-terminals at each stage. This is expected to reduce, to some extent, the time required for parsing in the sense that parsing needs to be done only once at each stage, unlike the case of the nonfuzzy approach [77], where each string has to be parsed more than once, in general, at each stage. However, this merit has to be balanced against the fact that the fuzzy grammars are not as simple as the corresponding nonfuzzy grammars.

Recently, the rule based systems have gained popularity in high level vision activities. By modelling the rules and facts in terms of fuzzy sets, it is possible to make inferences using the concept of approximate reasoning. Nafarich and Keller [84] designed such a system for automatic target recognition using about 40 rules. A knowledge based approach using Dempster-Shafer theory of evidence [2] has also been formulated [85] for managing uncertainty in object recognition problem when features fail to be homogeneous. Meaningful pay-offs are defined in this context. The problem is tackled by considering masses with fuzzy focal elements. They also developed an evidential approach to problem solving when a large number of knowledge systems (which might give contradictory or inconsistant information) are available [27]. It is to be mentioned in this connection that the definitions of credibility and plausibility of Dempster-Shafer theory of evidence when the evidences and propositions are both fuzzy in nature are available in [86, 87].

Another way of handling uncertainty in knowledge acquisition based on the theory of rough sets, a concept introduced by Pawlak [3], is reported in [88]. They considered uncertainties arising from the inconsistencies in different actions of different experts for the same object, or from the different actions of the same expert for different objects described by the same values of conditions. The method involves learning from examples. For a set of conditions of the information systems, and a given action of an expert, lower and upper approximations of a classification, generated by the action, have been computed with the help of rough set theory. Based on these approximations, the rules produced from the information stored in a data base are categorized as certain and possible. The certain rules may be propagated separately during the inference process, producing new certain rules. Similarly, the possible rules may be propagated in a parallel way.

It is to be mentioned here that the fuzzy set theory and the rough set theory are independent and offer alternative approaches to deal with uncertainty. However, there is a connection between rough set theory and Dempster-Shafer theory, though they have been developed separately. Dempster-Shafer theory uses belief function as a main tool, whereas the rough set theory makes use of the family of all sets with common lower and upper approximations [4, 88].

7. USE OF NEURAL NETWORKS

Artificial neural networks are signal processing systems that emulate the human brain, i.e., the behavior of biological nervous systems by providing a mathematical model of combination of numerous neurons connected in a network. The theory of fuzzy subsets, on the other hand, can model the human thinking process and behavior, and provides an approximate and yet effective means for describing the characteristics of a system which is too complex or ill-defined to admit of precise mathematical analysis. The fusion of these two new technologies therefore promise enormous intellectual and material gains in the field of computer and system science by incorporating the similarity in their logical operations and learning processes, and combining their individual merits.

A large number of researchers are now concentrating on exploiting this modern concept in the field of pattern recognition and machine vision [89–91]. Use of neural nets for the object extraction problem using self organizing neural nets, and using Gibbs distribution in a modified version of the Hopfield network has recently been made by Ghosh et al. [92–94]. The basic aim of their approach is to emulate the human neural information processing system so that it becomes effective even when the input image information is ill defined and/or incomplete/noisy. Mitra and Pal [95, 96] have introduced the concept of fuzzy sets in designing classifiers (both supervised and unsupervised) for uncertainty analysis and recognition of patterns using Kohonen's model and the multilayer perceptron. A self organizing artificial neural network capable of fuzzy partitioning of patterns has been developed which takes membership values to linguistic properties (e.g., low, medium, and high) along with some contextual class information as the input vector. An index of disorder based on mean square distance between input and weight vectors has been defined in order to provide a quantitative measure for the ordering of the output space. The method based on the multilayer perceptron, on the other hand, involves assignment of appropriate weights to the backpropagated errors depending on the membership values at the corresponding outputs. During training, the learning rate is gradually decreased until the network converges to a minimum error solution. The performance is compared with that of the conventional model and Bayes' classifier. Though the effectiveness of the classifiers is demonstrated on speech (patterns of biological origin and contains enormous fuzziness or ambiguity because of speakers sex, age, mood, dialect, etc.) recognition, the problem of image recognition under uncertainty can easily be dealt with within this framework. For example, the fuzzy geometrical properties (Section 5.2) of a pattern can be used as features for learning the network parameters. The fuzzy segmented version, fuzzy edge detected version or fuzzy skeleton of an image may also be used along with their degrees (values) of ambiguities for the purpose of network training and its recognition.

Use of neural nets has also been made in space control problems using reinforcement learning [9, 17]. Research is going on at the Johnson Space Center to demonstrate their success in tether control operation. A space-time neural network has been reported [97] which incorporates the dimension of time to the backpropagation network algorithm in order to make effective system modelling. This allows the networks to possess an adaptive temporal memory without introducing additional nonlinearities into the learning laws. Its success in predicting sun spots has been demonstrated. Further references on the fusion of neural nets and fuzzy logic regarding development of an intelligent system are available in [98–105]. In the next section we will be explaining the basic concept of genetic algorithms (another new technology) and the possibility of its being useful in space station autonomous research.

8. USE OF GENETIC ALGORITHMS

Genetic algorithms (GAs) are highly parallel, mathematical, adaptive search procedures (i.e., problem-solving methods) based loosely on the processes or mechanics of natural genetics and Darwinian survival of the fittest. They model operations found in nature to form an efficient search that is effective across a broad spectrum of problems. These algorithms apply genetically-inspired operators to populations of potential solutions in an iterative fashion, creating new populations while searching for an optimal (or near-optimal) solution to the problem at hand. Population is a key word here: the fact that many points in the space are searched in parallel sets genetic algorithms apart from other search operators. Another important characteristic of genetic algorithms is that they are very effective when searching (e.g., optimizing) function spaces that are not smooth or continuous—functions that are very difficult (or impossible) to search using calculus-based methods. Genetic algorithms are also blind: that is, they know nothing of the problem being solved other than payoff or penalty information.

The basic iterative model of the GAs is shown in Figure 14 [106, 107]. A new population is created from an existing population by means of *evaluation*, *selection*, and *reproduction*. This process repeats until the population converges on an optimal solution or some other stopping condition is reached.

The initial population consists of a set of individuals (i.e., potential solutions) generated randomly or heuristically. In the classical genetic algorithm, each member is represented by a fixed-length binary string of bits (a *chromosome*) that encodes parameters of the problem. This encoded string can be decoded to give the integer values for these parameters.

Once the initial population has been created, the evaluation phase begins. The genetic algorithms require that members of the population can

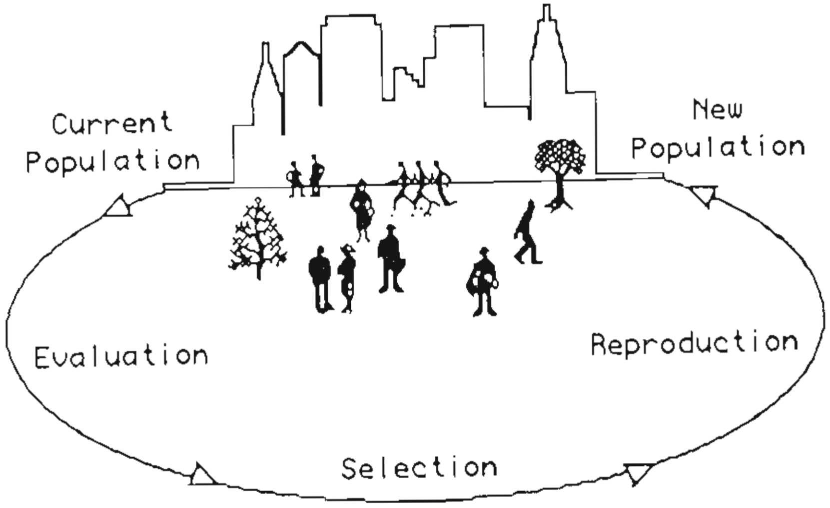


Fig. 14. The iterative genetic algorithm model.

be differentiated according to *goodness* or *fitness*. The members that are more fit are given a higher probability of participating during the selection and reproduction phases. Fitness is measured by decoding a chromosome and using the decoded parameters as input to the objective function. The value returned by the objective function (or some transformation of it) is used as the fitness value.

During the selection phase, the population members are given a target sampling rate that is based on fitness and determines how many times a member will mate during this generation—that is, how many offspring from this individual will be created in the next population. The target sampling rate (usually not a whole number) must be transformed into an integer number of matings for each individual. There are many ways of determining the target sampling rate and the actual number of matings. Suffice it to say that individuals that are more fit are given a reproductive advantage over less fit members.

During the reproduction phase, two members of the mating pool (i.e., members of the population with nonzero mating counts) are chosen randomly and genetic operators are applied to their genetic material to produce two new members for the next population. This process is repeated until the next population is filled. The recombination phase usually involves two operators: cross-over and mutation. During cross-over, the two parents exchange substring information (genetic material) at a random position in the chromosomes to produce two new strings. Cross-over occurs according to a cross-over probability, usually between 0.5 and 1.0. The cross-over operation searches for better *building blocks* within the genetic material that combine to create optimal or near-optimal problem parameters and, therefore, problem solutions, when the string is decoded.

The mutation operation is a secondary genetic algorithm. It is used to maintain diversity in the population—that is, to keep the population from prematurely converging on one solution—and to create genetic material that may not be present in the current population. The mechanics of the mutation operation are simple: for each position in a string created during cross-over, change the value at that position according to a mutation probability. The mutation probability is usually very low—less than 0.05.

Given the proper domain representation, genetic algorithms will evolve toward nearly optimal solutions by building up complicated structures out of small components, which can be described as schemata. A typical expression characterizing this phenomenon, as described by Goldberg [15], is of the form

$$m(H,t+1) \geqslant m(H,t)\frac{f(H)}{\bar{f}}\left(1-p_c\frac{\delta(H)}{L-1}-o(H)p_m\right) \qquad (48)$$

where H stands for a particular schema, $m(H,t)$ is a measure of the number of copies of the schema in the population at time t, f is the fitness function, $f(H)$ is the average fitness of the schema, $\bar{f}$ is the average fitness of the entire population, L is the length of a solution, p_c and p_m are the probabilities of cross-over and mutation, and $\delta(H)$ and $o(H)$ are the defining length and order of the schema. The fractional term indicates that the number of schema in the population will change proportional to the average fitness of solutions containing the schema relative to the average fitness of the population. Of particular interest is the last term of the equation which is a measure of the extent to which a cross-over or mutation is likely to disrupt the schema. Note that the greater the defining length or order of the schema, the greater the chances that it will be altered by the cross-over or mutation, thus driving down the number of schema in the population. Consequently, schemata that are short and low order are probabilistically favored by the genetic algorithm, and any GA which is to work effectively must contain operators that prefer such small, low order building blocks.

GAs differ from many conventional search algorithms in the following ways. They consider many points in the search space simultaneously, not a single point, and therefore have less chance of converging to local optima. They deal directly with strings of characters representing the parameter sets, not the parameters themselves. They use probabilistic rules to guide their searching process instead of deterministic rules.

In handling uncertainty in pattern analysis and control tasks of space autonomous operations, GAs may be helpful in determining the appropriate membership functions and rules, and in providing a reasonably suitable solution as far as extraction of object regions or skeletons is concerned. For this purpose, a suitable fuzzy fitness function needs to be defined depending on the problem. Fuzziness may also be incorporated in the encoding process by introducing a membership function representing the degree of similarity/closeness between the chromosome parameters (strings). For example, consider a scene analysis problem where the relations amongst various segments (or objects) may be defined in terms of fuzzy labels such as close, around, partially behind, occluded, etc. Given a labelling of each of the segments the degrees to which each relationship fits each pair of segments can be measured. These measures can be combined to define an overall fuzzy fitness function. Given this fitness function, the relations amongst objects and the relations amongst classes to which the objects belong, a genetic algorithm searches the space to find the best solution in determining a class to be associated most appropriately to each object. An approach based on genetic algorithm for scene labelling is reported in [108].

In Section 5.3, we have seen that the task of extracting fuzzy medial axis transformation (FMAT) of an image involves enormous computation and it is not guaranteed whether the resulting output provides a compact minimal set for image representation. Searching based on GAs may be helpful in this case. Let us list below a few key features for implementation of GAs in selecting an optimal FMAT.

- The status of each MA pixel may be encoded as 0's (or 1's) representing deletion from (or inclusion in the) the FMAT.
- Since GAs are blind, the pay off or the objective function for measuring the goodness of a set of MA pixels should involve both "minimum number of MA fuzzy disks" and "maximum compactness in MA pixels" as criteria. The second criterion may include the measures like fuzzy compactness (Equation 15) or index of area coverage (Equation 22).
- The goodness of reconstruction can be measured by higher order entropy (Equation 36), which takes into account the consecutive occurrences of the pixels.
- Some trade off is needed between the selection of disk size r and the total number of disk values.
- Extend the one-dimensional genetic operators to two-dimensional image space.

9. DISCUSSION

Various space station problems where research is being conducted at the Johnson Space Center for automating certain tasks are explained. The uncertainties that might arise in analyzing image patterns and the subsequent control operations are discussed. The role of fuzzy logic in representing these uncertainties is explained. Various fuzzy set theoretic and probabilistic tools for measuring information on grayness ambiguity and spatial ambiguity in an image are listed along with their characteristics. Some examples of low-level vision operations (e.g., segmentation, skeleton extraction, and edge detection), whose outputs are responsible for the overall performance of a vision system, are considered in order to demonstrate the effectiveness of these tools in managing uncertainties by providing both soft and hard decisions. Uncertainty in determining a membership function in this regard and the tools for its management are also explained. Apart from representing and managing uncertainties, the tools based on fuzzy set theory and probability theory can also be used for providing quantitative measures in order to avoid the subjective judgment on the quality of processed output and to avoid human intervention in space autonomous operations. Most of the algorithms and tools described here have been developed recently by the author with his colleagues. Some of the illustrations are taken from the existing literature and are put here together in a unified frame. Processing of color image has not been considered here. Some recent results on image information and processing in the notion of fuzzy logic are available in [46, 109, 110].

Uncertainties involved in other parts of a recognition system such as primitive extraction/analysis and syntactic classification, and knowledge acquisition are discussed in brief along with the features of Dempster-Shafer theory and the rough set theory. Some of the recent attempts of the researchers on fusion of fuzzy set theory and neural networks for better handling of uncertainty (in the sense of parallel processing, robustness, and also performance) in pattern analysis problems are mentioned. Finally, the key features of genetic algorithms along with its possibility of being used successfully in this context are explained.

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