International Journal of Systems Science, 1996, volume 27, number 12, pages 1179–1193

Review

Neuro-fuzzy computing for image processing and pattern recognition

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The relevance of integration of the merits of fuzzy set theory and neural network models for designing an efficient decision making system is explained. The feasibility of such systems and different ways of integration, so far made, in the context of image processing and pattern recognition are described. Scope for further research and development is outlined. An extensive bibliography is also provided.

1. Introduction

Pattern recognition (Duda and Hart 1973, Tou and Gonzalez 1974) and machine learning form a major area of research and development that encompasses the processing of pictorial and other non-numerical information obtained from the interaction between science, technology and society. The second motivation for this spurt of activity in this field is the need for the people to communicate with computing machines in their natural mode of communication. The third and most important motivation is that scientists are also concerned with the idea of designing and making automata that can carry out certain tasks as we human beings do. The most salient outcome of these motivations is the concept of future generation computing systems.

Machine recognition of patterns can be viewed as a twofold task, consisting of learning the invariant and common properties of a set of samples characterizing a class, and of deciding that a new sample is a possible member of the class by noting that it has properties common to those of the set of samples. Therefore, the task of pattern recognition by a computer can be described as a transformation from the measurement space M to the feature space F and finally to the decision space D, i.e. $\boxed{M} \rightarrow \boxed{F} \rightarrow \boxed{D}$.

When the input pattern is a grey tone image, the measurement space involves some processing tasks such as enhancement, filtering, noise reduction, segmentation, contour extraction and skeleton extraction, in order to extract salient features from the image pattern. This is what is basically known as image processing (Gonzalez and Wintz 1987, Rosenfeld and Kak 1982). The ultimate aim is to make its understanding, recognition and interpretation from the processed information available from the image pattern. Such a complete image recognition/interpretation system is called a vision (Marr 1982, Ballard and Brown 1982) system which may be viewed as consisting of three levels namely, low level, mid level and high level.

In a pattern recognition or vision system, uncertainties can arise at any phase of the aforementioned tasks resulting from incomplete or imprecise input information, ambiguity or vagueness in input images, ill-defined and/or overlapping boundaries among the classes or regions, and indefiniteness in defining/extracting features and relations among them. Any decision taken at a particular level will have an impact on all higher level activities. It is therefore required for a recognition system to have sufficient provision for representing the uncertainties involved at every stage, so that the ultimate output (results) of the system can be associated with the least uncertainty (and not be affected or biased very much by the earlier or lower level decisions).

The utility of fuzzy set theory (Zadeh 1965, Kaufmann 1980, Dubois and Prade 1980, Bezdek 1981, Kandel 1982, 1986, Kaufmann and Gupta 1985, Pal and Majumder 1986) in handling uncertainty (Bezdek and Pal 1992, Klir and Folger 1988), arising from deficiencies of information

Received 3 July 1995. Revised 15 April 1996. Accepted 18 April 1996.

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available from a situation (as mentioned before) in pattern recognition problems, has adequately been addressed in the literature (Kaufmann 1980, Dubois and Prade 1980, Bezdek 1981, Kandel 1982, 1986, Kaufmann and Gupta 1985, Pal and Majumder 1986, Bezdek and Pal 1992, Klir and Folger 1988, Zadeh *et al.* 1975, Pedrycz 1990. Yager and Zadeh 1992). This theory provides an approximate, yet effective and more flexible means of describing the behaviour of systems which are too complex or too ill-defined to admit precise mathematical analysis by classical methods and tools. Since the theory of fuzzy sets is a generalization of classical set theory, it has greater flexibility to capture faithfully the various aspects of incompleteness or imperfection (i.e. deficiencies) in information of a situation. This theory is also reputed to mimic the human reasoning process for decision making.

Again, for any pattern recognition, image analysis or vision system, one desires to achieve robustness of the system with respect to random noise and failure of components, and to obtain output in real time. Moreover, a system can be made artificially intelligent if it is able to emulate some aspects of human information processing system. Neural network (NN) (Rumelhart *et al.* 1986, Kohonen 1989, Hopfield 1984, Gronberg 1988, Fukushima 1980, Pao 1987, Wassermann 1990, Kosko 1992, Chua and Yang 1988, Ghosh *et al.* 1994, Gelenbe 1991) based approaches are attempts to achieve these goals. A neural network can formally be defined as: *a massively parallel interconnected network of simple (usually adaptive) processing elements which is intended to interact with the objects of the real world in the same way as biological systems do.* The architecture of the network depends on the goal one is trying to achieve. The massive connectivity among the neurons usually makes the system fault tolerant (with respect to noise and component failure) while the parallel processing capability enables the system to produce output in real time. Moreover, most of the image analysis operations are co-operative in nature and the tasks of recognition mostly need formulation of complex decision regions. Neural network models have the capability of achieving these properties. All these characteristics, therefore, suggest that image processing and pattern recognition problems can be considered as prospective candidates for neural network implementation.

Thus we see that fuzzy set theoretic models try to mimic human reasoning and uncertainty handling capabilities, whereas neural network models attempt to emulate the architecture and information representation schemes of the human brain. Integration of the merits of these two technologies therefore promises to provide, to a great extent, more intelligent systems (in terms of parallelism, fault tolerance, adaptivity and uncertainty management) to handle real-life recognition problems. A large number of researchers are now concentrating on exploiting these modern concepts to solve complex problems in various fields under a new branch, called *neuro-fuzzy* computing. The present article deals with the utility and feasibility of such an integrated system. The rest of the article is organized as follows. In § 2, the relevance of fuzzy set theoretic methods for pattern recognition/image analysis is described. The relevance of neural network based techniques in this context is described in § 3. In § 4, various ways of fusing fuzzy set theory and neural network models are provided. Suggestions for further research are given in § 5. Concluding remarks can be found in § 6.

2. Relevance of fuzzy set theory in pattern recognition

Fuzzy sets were introduced in 1965 by Zadeh as a new way to represent vagueness in everyday life. They are generalizations of conventional (crisp) set theory. Conventional sets contain objects that satisfy precise properties required for membership. Fuzzy sets, on the other hand, contain objects that satisfy imprecisely defined properties to varying degrees. A fuzzy set A of the universe X is defined as a collection of ordered pairs

$$A = \{(\mu_A(x), x), \forall x \in X\}, \tag{1}$$

where $\mu_A(x)$ $(0 \leq \mu_A(x) \leq 1)$ gives the degree of belonging of the element x to the set A or the degree of possession of an imprecise property represented by A. Since the theory of fuzzy sets is a generalization of classical set theory, it has greater flexibility to capture faithfully the various aspects of incompleteness or imperfection in information of a situation. The flexibility of fuzzy set theory is associated with the elasticity property of the concept of its membership function. The grade of membership is a measure of the compatibility of an object with the concept represented by a fuzzy set. The higher the value of membership, the lesser will be the amount (or extent) to which the concept represented by a set needs to be stretched to fit an object. Different aspects of fuzzy set theory including membership functions, basic operations and uncertainty measures can be found in the work of Kaufmann (1980), Dubois and Prade (1980), Bezdek (1981), Kandel (1982, 1986), Kaufmann and Gupta (1985), Pal and Majumder (1986), Bezdek and Pal (1982) and Klir and Folger (1986).

In this section we will explain some of the uncertainties which one often encounters while designing a pattern recognition system, and the relevance of fuzzy set theory in handling them.

Let us consider, first of all, the case of processing and recognition of a grey-tone image pattern. A grey tone image possesses ambiguity within each pixel because of the possible multi-valued levels of brightness. This pattern uncertainty is due to inherent vagueness rather than randomness. If the grey levels are scaled to lie in the range [0, 1], we can regard the grey level of a pixel as its degree of belonging (membership) in the set of high-valued ('bright') pixels; thus a grey tone image can be viewed as a fuzzy set. Regions, features, primitives, properties, and relations among them that are not crisply defined can similarly be regarded as fuzzy subsets (Prewitt 1970, Rosenfeld 1984, Pal 1992, Krishnapuram and Keller 1992, Pal and Ghosh 1992 a). Basic principles and operations of image processing and recognition in the light of fuzzy set theory are available in Pal and Majumder (1986). Uncertainty in an image pattern may be explained in terms of greyness ambiguity or spatial (geometrical) ambiguity or both. Greyness ambiguity means 'indefiniteness' in deciding whether a pixel is white or black. Spatial ambiguity refers to 'indefiniteness' in the shape and geometry of a region within the image. For example, greyness ambiguity measures are reflected by the index of fuzziness (Kaufmann 1980) and entropy (De Luca and Termini 1972, Xie and Bedrosian 1984, Kosko 1986, Pal and Pal 1989), whereas spatial ambiguity measures are represented by fuzzy geometrical properties (Rosenfeld 1984, 1992, Pal and Ghosh 1990, 1992 a, Pal and Rosenfeld 1988, 1991, Ghosh 1992, Dubois and Jaulent 1987).

Conventional approaches to image analysis and recognition (Gonzalez and Wintz 1987, Rosenfeld and Kak 1982, Marr 1982) consist of segmenting the image into meaningful regions, extracting their edges and skeletons, computing various features (e.g. area, perimeter, centroid etc) and primitives (e.g. line, corner, curve etc) of, and relationships among, the regions, and finally, developing decision rules and grammars for describing, interpreting and/or classifying the image and its subregions. In a conventional system each of these operations involves crisp decisions (i.e. yes or no, black or white, 0 or 1) to make regions, features, primitives, properties, relations and interpretations crisp.

Since the regions in an image are not always crisply defined, uncertainty can arise within every phase of the aforesaid tasks. Any decision made at a particular level will have an impact on all higher level activities. An image recognition system should have sufficient provision for representing and manipulating the uncertainties involved at every processing stage; i.e. in defining image regions, features and relations among them, so that the system retains as much of the 'information content' of the data as possible. If this is done, the ultimate output (result) of the system will possess minimal uncertainty (and unlike conventional systems, it may not be biased or affected as much by lower level decision components).

For example, consider the problem of object extraction from a scene. Now, the question is 'how can one define exactly the target or object region in a scene when its boundary is ill-defined?' Any hard thresholding made for the extraction of the object will propagate the associated uncertainty to subsequent stages (e.g. thinning, skeleton extraction, primitive selection, etc) and this might, in turn, affect feature analysis and recognition. A similar case arises with the task of skeletonization and contour detection of a region. Thus, it is convenient, natural and appropriate to avoid committing ourselves to a specific (hard) decision (e.g. segmentation, edge detection and skeletonization), by allowing the segments or skeletons or contours to be fuzzy subsets of the image, the subsets being characterized by the possibility (degree) to which each pixel belongs to them. Prewitt (1970) first suggested that the results of image segmentation should be fuzzy subsets, rather than ordinary subsets. Similarly, while describing relations among different components and features or classifying the sub-regions it is necessary to make the decision-making algorithms flexible by providing soft decisions.

Some of the areas of image analysis where the theory of fuzzy sets has been adequately applied are mentioned here. Computation of fuzzy geometric properties is described by Rosenfeld (1984, 1992), Pal and Ghosh (1990, 1992 a), Ghosh (1992), Dubois and Jaulent (1987) and Rosenfeld and Klette (1985). Fuzzy segmentation is discussed by (Pal and Rosenfeld 1989, Pal *et al.* 1983, Keller and Carpenter 1990, Murthy and Pal 1990, Pal and Ghosh 1992 b, Huntsberger *et al.* 1985, Trivedi and Bezdek 1980, Cannon *et al.* 1986, Hall *et al.* 1992). The problem of evaluation of image quality has also been dealt with using fuzzy set theory (Pal and Majumder 1986, Pal *et al.* 1983, Xie 1990, Tanaka and Sugano 1991, Kundu and Pal 1990). Other image operations, like thinning and edge detection, have been developed by Pal and Majumder (1986), Pal and Rosenfeld (1991), Pal (1990), Goetcherian (1980). Pal and King (1983). There have been several attempts to extract fuzzy primitives (or features) from fuzzy edges and segmented outputs of the image regions for shape analysis, matching, and recognition. Readers may consult Lee (1975), Pal and Wang (1992).

Let us now consider the case of a decision-theoretic approach to pattern classification. With the conventional probabilistic and deterministic classsifiers (Duda and Hart 1973, Tou and Gonzalez 1974), the features characterizing the input patterns are considered to be quantitative (numeric) in nature. The patterns having imprecise or incomplete information are usually ignored or discarded from their designing and testing processes (Duda and Hart 1973, Tou and Gonzalez, 1974, Fukunaga 1972, Devijver and Kittler 1982). The impreciseness (or ambiguity) may arise from various causes. For example, instrumental error or noise corruption in the experiment may lead to only partial or partially reliable information being available on a feature measurement F, e.g. F is about 500 (say) or F is between 400 and 500 (say). Again, in some cases the expense incurred in extracting the exact value of a feature may be high, or it may be difficult to decide on the actual salient features to be extracted. Sometimes, it may become convenient to use linguistic variables and hedges, e.g. small, medium, high, very, more or less etc, in order to describe the feature information (e.g. F is very small). In such cases, it is not appropriate to give exact representation to uncertain feature data. Rather, it is reasonable to represent uncertain feature information by fuzzy subsets.

Again, uncertainty in classification or clustering (Anderberg 1973) of patterns may arise from the overlapping nature of the various classes. This overlapping may result from fuzziness or randomness. Moreover, the concept of clustering, in practice, is a fuzzy notion (because the technique is unsupervised and we do not have any information on the class structures or labelled samples). In the conventional technique, it is usually assumed that a pattern may belong to only one class, which is not necessarily true in real-life applications. A pattern can and should be allowed to have degrees of membership in more than one class. It is, therefore, necessary to convey this information while classifying a pattern or clustering a data set.

Similarly, consider the problem of determining the boundary or shape of a class from its training set. Classical approaches attempt to estimate the exact shape for the class by determining the boundary which contains or passes through some or all of the sample points; which in a practical case may not be the right one. It may be necessary to extend the boundaries to some extent to cover the possible portions uncovered by the sample points. The extended portions should have lower possibility to be in the class than the portions explicitly highlighted by the sample points. The size of the extended regions should also decrease with an increase in the number of sample points. This leads one to determine a fuzzy shape and boundary of a pattern class.

From the aforementioned examples, we see that the concept of fuzzy sets can be used at the feature level in representing input data as an array of membership values denoting the degree of possession of certain properties, in representing linguistically phrased input features for their processing, in weakening the strong commitments for extracting ill-defined image regions, properties, primitives, and relations among them, and at the classification level, for representing class membership of objects, and for providing an estimate (or a representation) of missing information in terms of membership values.

The capability of fuzzy set theory in pattern recognition problems has been reported adequately. For example, feature extraction is dealt with by Bezdek and Castelaz (1977), Pal and Chakraborty (1986). Classification of patterns including linguistic representation of inputs (Nath and Lee 1983, Nath *et al.* 1985, Mandal *et al.* 1992, Pal and Mandal 1992) can be found in Pal and Majumder (1986), Kandel (1986), Roubens (1978), Keller *et al.* (1985), Bellman *et al.* (1966), Bezdek *et al.* (1986), Devi and Sarma (1986). Clustering techniques and their validations are discussed by Bezdek (1981), Ruspini (1969), Gitman and Levine (1970), Dunn (1973), Windham (1981, 1983), Bacher (1978), Backer and Jain (1981), Roubens (1982), Xie and Beni (1991), Dave (1990). A review showing the development of this area has been provided recently by Bezdek and Pal (1992).

3. Relevance of neural network approaches

Neural network (NN) models (Rumelhart *et al.* 1986, Kohonen 1989, Hopfield 1984, Grossberg 1988, Fukushima 1980, Pao 1987, Wassermann 1990, Kosko 1992, Chua and Yang 1988, Ghosh *et al.* 1994, Gelenbe 1991) try to emulate the biological neural network/nervous system with electronic circuitry. NN models have been studied for many years with the hope of achieving human-like performance (artificially), particularly in the field of pattern recognition, by capturing the key ingredients responsible for the remarkable capabilities of the human nervous system. Note that these models are extreme simplifications of the actual human nervous system.

NNs are designated by the network topology, connection strength between pairs of neurons (called weights), node characteristics and the status updating rules. Node characteristics mainly specify the primitive types of operations it can perform, like summing the weighted inputs coming to it and then amplifying it or doing some fuzzy aggregation operations. The updating rules may be for weights and/or states of the processing elements (neurons). Normally, an objective function is defined which represents the complete status of the network and the set of minima of it corresponds to the set of stable states of the network. Since there are interactions among the neurons the collective computational property inherently reduces the computational task and makes the system fault tolerant. Thus, NN models are also suitable for tasks where collective decision making is required. Hardware implementations of neural networks have been attempted by authors in a Special Issue on Neural Net Hardware Design (IEEE 1992a) and by Ramacher and Ruckert (1991), Mead (1989), Robinson *et al.* (1992)and Harrer *et al.* (1992).

Neural network based systems are usually reputed to enjoy the following major characteristics:

(i) adaptivity—adjusting the connection strengths to new data/information;
(ii) speed—due to massively parallel architecture;
(iii) robustness—to missing, confusing, ill-defined/noisy data;

(iv) ruggedness—to failure of components;

(v) optimality—as regards error rates in performance.

For any pattern recognition system, one desires to achieve the above-mentioned characteristics. Moreover, there exists some direct analogy between the working principles of many pattern recognition tasks and neural network models. For example, image processing and analysis in the spatial domain mainly employ simple arithmetic operations at each pixel site in parallel. These operations usually involve the information of the neighbouring pixels (cooperative processing) in order to reduce local ambiguity and to attain global consistency. An objective measure is required (representing the overall status of the system), the optimum of which represents the desired goal. The system thus involves collective decisions. On the other hand, we notice that neural network models are also based on parallel and distributed working principles (all neurons work in parallel and independently). The operations performed at each processor site are also simpler and independent of the others. The overall status of a neural network can also be measured.

Let us consider, in particular, the case of pixel classification. A pixel is normally classified into different classes depending on its grey value, positional information and contextual information (collected from the neighbours). Pixels at different sites can be classified independently. The mathematical operations needed for this task are also simple. A neural network architecture in which a single neuron is assigned to a pixel and is connected to its neighbours can therefore be applied for this task. The neurons operate in parallel and are independent of each other. The local interconnections provide the contextual information (which can be adaptive or dynamic also) for classification.

Again, the task of recognition in a real-life problem involves searching a complex decision space. This becomes more complicated particularly when there is no *a priori* information on class distribution. Neural network based systems use adaptive learning procedures, learn from examples and attempt to find a useful relation between input and output, however complex it may be, for decision-making problems. Neural networks are also reputed to model complex nonlinear boundaries and to discover important underlying regularities in the task domain. These characteristics demand that methods are needed for constructing and refining neural network models for various recognition tasks. For example, consider the case of supervised classification. Here each pattern is characterized by a number of features. Different features usually have different amounts of weight in characterizing the classes. A collective decision, taking into account all the features, is made for assignment of class labels to an input. A multi-layer perceptron in which the input layer has neurons equal to the number of features and the output layer has neurons equal to the number of classes, can therefore be used to tackle this classification problem. Here the importance of different features will automatically be encoded in the connecting links during training. The nonlinear decision boundaries are modelled and class labels are assigned by taking collective decisions.

Major areas in which neural networks have been applied in order to exploit the computational power, and to make robust decisions are as follows.

(i) Pattern recognition (Lippmann 1989, Pal and Mitra 1992, Burr 1988, Lee *et al.* 1990, Simpson 1992, 1993, Gorman and Sejnowski 1988, Hirari and Tsukui 1990, Kanaoka *et al.* 1992, Giles *et al.* 1988, Khotanzad and Lu 1990, Khotanzad *et al.* 1993, Mitra and Pal 1994 a, Newton *et al.* 1992, Bruke 1991, Basak *et al.* 1993 a, Guyon 1991, Fukushima 1992, Martin and Pittman 1991, Knerr *et al.* 1992).

(ii) Image preprocessing (Cottrel *et al.* 1987, Cottrel and Munro 1988, Luttrell 1989, Chen *et al.* 1991, Manjunath *et al.* 1990, Silverman and Noetzel 1990, Silverman 1991, Ghosh *et al.* 1991, 1992, 1993 Ghosh and Pal 1992, Shah 1990, Cortes and Hertz 1989, Blanz and Gish 1991, Yu and Tsai 1992, Babaguchi *et al.* 1991, Jang 1991a Widrow and Winter 1988, Basak *et al.* 1994, Bedini and Tonazzini 1990).

(iii) Scene analysis (Newton *et al.* 1990, Nasrabadi and Li 1991, Jamison and Schalkoff 1988, Nasrabadi and Choo 1992, Sawaragi *et al.* 1992).

(iv) Text processing (Burr 1988, Sejnowski and Rosenberg 1987).

(v) Expert system design/rule generation (Gallant 1989, Sanchez 1990, Jang 1991b, Narazaki and Ralescu 1992, Shastri 1988, Mitra and Pal 1994c,d).

(vi) Optimization problems (Hopfield and Tank 1985, 1986, Basak *et al.* 1993 b, Bruck and Sanz 1988).

(vii) Controller design (Jang 1992, Lee 1991, Nguyen and Widrow 1990, Gupta *et al.* 1989, Yager 1992, Hayashi *et al.* 1992, Berenji 1992, Berenji and Khedkar 1992).

(viii) Natural language processing (Rocha *et al.* 1992, McClelland 1985).

(ix) Approximate reasoning (Takagi *et al.* 1992, Romaniuk and Hall 1992).

(x) Speech recognition (Lippmann 1989, Weibel *et al.* 1989, Franzani 1987, Kohonen 1988).

4. Integration of the theories of fuzzy sets and neural networks

As mentioned before, fuzzy set theory provides an approximate but effective and flexible way of representing, manipulating and utilizing vaguely-defined data and information, and of describing the behaviours of systems that are too complex or too ill-defined to admit precise mathematical analysis by classical methods and tools. Successful use of fuzzy logic to create many commercial products has been made recently in Japan. This, in turn, has increased interest among engineers, researchers and company executives to understand and explore this technology further. Although the approach tries to model the human thought process in a decision-making system, it has no relation to the architecture of the human neural information processing system, nor does it take into consideration the information storage technique of human beings, and sometimes it is computationally intensive.

Human intelligence and discriminating power are mainly attributed to the massively connected network, of biological neurons in the human brain. We mentioned earlier that attempts have recently been made to emulate electronically the architecture and information representation scheme of human neural networks under the name *artificial neural network* models. The collective computational abilities of the densely interconnected nodes or processors may provide a material technique, at least to a great extent, for solving highly complex real-life problems in a manner similar to a human being.

It, therefore, appears that integration of the merits of these two technologies can provide more intelligent systems (in terms of parallelism, fault tolerance, adaptivity and uncertainty management) to handle real-life recognition problems. These promises have motivated (during the last 5–7 years) a large number of researchers to exploit these modern concepts for solving real-world problems, leading to the development of a new paradigm called *neuro-fuzzy computing*. Besides the generic advantages of parallelism, fault-tolerance and uncertainty handling, the neuro-fuzzy paradigm sometimes provides some application specific advantages. This will be apparent later. The fusion or integration (Bezdek and Pal 1992, Bezdek 1992, Pal and Ghosh 1994, Ghosh and Pal 1993), made so far, can be categorized under the following headings.

4.1. *Incorporating fuzziness into neural network frameworks*

This approach includes fuzzifying the input data, assigning fuzzy labels to training samples, and obtaining outputs of neural networks in terms of fuzzy sets (Fig. 1).

Here, the integration can be viewed as incorporating the concept of fuzziness into a neural network framework for building fuzzy neural network classifiers. For example,

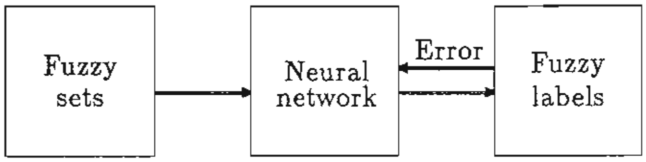

Figure 1. Neural network implementing fuzzy classifier.

the output of the neurons in the output layer during both training and testing phases can be a fuzzy label vector, the input could be some fuzzy properties and the learning procedure can also be fuzzified. In this case. the network itself is functioning as a fuzzy classifier. Keller and Hunt (1985) first suggested a way of incorporating the concept of fuzzy sets into perceptrons (single-layer) for pattern recognition. They described a method for fuzzifying the labelled target data for training the perceptron. Instead of giving hard labels to the training samples, membership functions denoting their degrees of belonging to the classes were used as labels. Instead of using the weight updating as

$$W \leftarrow W + cX_k \tag{2}$$

(c is a constant and X_k is the input data) they used

$$W \leftarrow W + |u_{1k} - u_{2k}|^m cX_k, \tag{3}$$

where m is a constant and u_{ik} denotes the degree of belonging of X_k to the ith class. Assignment of membership functions to the label vectors also provided a good stopping criterion for linearly non-separable classes (where the classical perceptron usually oscillates). Note that the benefits that we realize from such an integration are mostly generic in nature.

The concept of fuzzy sets has recently been introduced by Pal and Mitra (1992) and Mitra and Pal (1996) in various stages of multi-layer perceptrons and Kohonen's model for designing both supervised and unsupervised fuzzy classifiers for uncertainty analysis and recognition of patterns. The self-organizing network developed for fuzzy partitioning of patterns takes membership values corresponding to linguistic properties (e.g. small, medium and high) along with some contextual class information as input. An index of disorder based on mean square distance between input and weight vectors has been defined in order to provide a quantitative measure for the ordering of the output space. The method based on the multi-layer perceptron, on the other hand, involves assignment of appropriate weights to the back-propagated errors depending on the membership values at the corresponding outputs. Its input can also be in terms of linguistic properties. Incorporation of fuzziness also makes the system less oscillatory in addition to providing superior performance for overlapping classes. The system is found to be robust with respect to fuzzification of input properties. These modified versions also provided better performance (application-specific advantage) for certain non-convex decision regions (Pal and Mitra 1994) as compared with the classical methods and conventional connectionist approaches as they incorporate more local information of the feature space by decomposing it into 3^N (N being the dimension of feature space) sub-regions through the properties small, medium and high. A similar concept of fuzzy labels has also been utilized by Hall (1991) for learning in a network.

A method has been suggested by Kammerer (1992) to incorporate known uncertainty of the data in the computational processes of neural networks. A measure of certainty is used on each input element in order to modulate the element's contribution to the whole input activity. Some improvements on the classification accuracy have been demonstrated on optical character recognition problems. The technique basically shows an effect of fuzzifying input on the classification accuracy.

Traditional rule-based expert systems encode the knowledge base in the form of *if-then* rules; while connectionist expert systems (Gallant 1988) use the set of connection weights of the trained neural net model for this purpose. Traditional expert systems have some drawbacks in eliciting knowledge from experts, and in learning the rules. Some advantages of neural networks (like training by examples, dynamic adjustment to the changes in the environment, ability to generalize, tolerance to noise, and ability to discover new relations between variables) may be incorporated in expert systems to remove these drawbacks. Sanchez (1990) has developed a fuzzy version of a classificatory connectionist expert system. Here, the knowledge base is generated from a set of training examples and is stored as connection strengths. He has associated two types of connection weights, e.g. primary linguistic weights and secondary numerical weights to generate the knowledge base for a biomedical application (inflammatory protein variations) using a feedforward network. Triangular membership functions like negative large, negative medium, negative small, approximately zero, positive small, positive medium and positive large; or decreased, normal and increased account for the linguistic weights while the quantitative weights lie in the range [0,1]. The linguistic weights are tuned according to the information provided by the input–output examples while the numeric weights and the network topology are determined by solving fuzzy relational equations.

An application of a fuzzy multi-layer perceptron and Kohonen's model for designing classificatory connectionist expert systems is described by Pal and Mitra (1992), Mitra and Pal (1994 a,b). These models can handle impreciseness in the input representation and provide output decisions as class labels with certainty factors. The input can be in quantitative, linguistic or set form. These models have been found to be useful in querying, inferencing and generating if-then rules for medical diagnosis with incomplete symptoms. An attempt has also been made by Gorzalczay and McLeish (1992) to build an expert system which can handle both numerical and linguistic medical data.

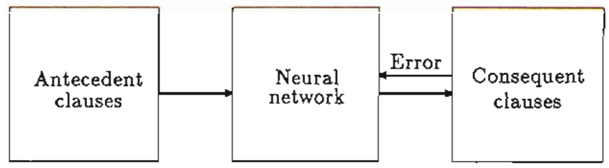

Figure 2. Neural network implementing fuzzy logic.

4.2. *Designing neural networks guided by fuzzy logic formalism*

The second fusion methodology includes designing neural networks to implement fuzzy logic and fuzzy decision making, and to realize membership functions representing fuzzy sets (Fig. 2).

Here neural networks are used for a variety of computational tasks within the framework of a pre-existing fuzzy model (i.e. implementation of fuzzy logic formalism using neural networks). The use of multi-layer feed-forward neural networks for implementing fuzzy logic (if–then rules) for decision making systems is made by Keller *et al.* (Keller and Tahani 1991, Keller *et al.* 1992, Keller and Krishnapuram 1992). It has been shown that the networks designed for implementing fuzzy rules can learn and extrapolate complex relationships between antecedents and consequent clauses for rules containing single, conjunctive and disjunctive antecedent clauses. For rules having con-junctive clauses, the architecture has a fixed number of neurons in the input layer for each antecedent clause. The neurons corresponding to an antecedent clause are again connected to a particular set of neurons in the hidden layer. The neurons in the output layer, on the other hand, are connected to all the neurons in the hidden layer. For implementing rules with disjunctive antecedent clauses, one more hidden layer was necessary. Keller *et al.* (1992) attempted to embed *a priori* knowledge of each rule directly into the weights of the network, whereas others applied the standard back-propagation learning algorithm for learning weights. The generalizing ability of neural networks has been exploited here in formulating robust decision rules. This is useful in image analysis problems. An attempt is also made in this line by Takagi *et al.* (1992) to design structured neural networks to perform if-then fuzzy inference rules.

The use of neural networks for realizing fuzzy membership functions for recognition problems has been modelled by Ishibuchi and Tanaka (1990), Takagi and Hayashi (1991). A method has been suggested by Ishibuchi and Tanaka (1990) to identify real-valued and interval-valued membership functions from a set of given input–output data using a feed-forward layered neural network and back-propagation of error. Suggestions are also given by Yamakawa and Furukawa (1992) to design membership functions of fuzzy neurons (to be discussed in § 4.4.4).

Based on similar concepts, a great deal of effort has been given for designing neural network driven optimal decision rules for fuzzy controllers (Gupta *et al.* 1989, Yager 1992, Hayashi *et al.* 1992, Berenji 1992, Berenji and Khedkar 1992). For example, a system for implementing fuzzy logic controllers using a neural network was designed by Yager (1992) where the linguistic values associated with the fuzzy control rules are realized by separate neural network blocks. Emphasis is also given to adjust membership functions of the linguistic labels used in control rules. This avoids, to an extent, the subjective selection of membership functions in fuzzy control systems. An attempt was made by Nauck and Kruse (1992) to adapt membership functions in a linguistic-variable-based fuzzy control environment by using neural network principles.

Huntsberger and Ajjimarangsee (1990) have modified Kohonen's network by adding one more layer for generating a fuzzy self-organizing feature map. Fuzziness is also incorporated in the learning process by replacing the learning rate, usually found in Kohonen-type update rules for the weight vectors, with fuzzy membership of the nodes in each class. They have also shown that the results produced by this fuzzy version of Kohonen's algorithm are similar to those obtained by fuzzy *c*-means algorithms (Bezdek 1981). Parallel implementations of this technique are also suggested. Further modification on the rate of learning is done by Bezdek *et al.* (1992) and a relationship between the fuzzy version of Kohonen's algorithm and the fuzzy *c*-means algorithm has been established.

Another neural network architecture which can be used for fuzzy clustering and classification was suggested by Newton *et al.* (1992). The system uses a control structure similar to that found in the adaptive resonance theory of Carpenter and Grossberg (Grossberg 1998), and employs a learning strategy, similar to that of a fuzzy *c*-means algorithm, to update the centroid position of the clusters. Functionally the architecture is similar to the leader clustering algorithms. The algorithm has also been used (Mitra and Pemaraju 1992) to cluster the simulation data of a tethered satellite system to estimate the range of delta voltage necessary to maintain the desired length rate of tether.

A supervised neural network classifier that utilizes min-max hyperboxes as fuzzy sets (which are aggregated into fuzzy set classes) was introduced by Simpson (1991). The network has a three-layered architecture having input, hidden and output layers. Each hidden-layer neuron represents a hyperbox fuzzy set having two types of connections from the input layer representing the min and max points of the inputs. Learning is a single-pass procedure. The model is suitable for finding reasonable decision boundaries in overlapping classes and for learning highly nonlinear relations. Similar concepts were used for clustering by Simpson (1993).

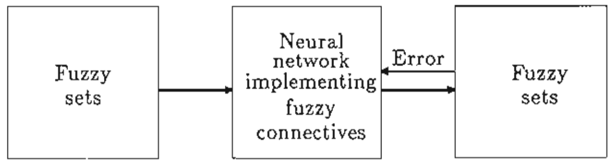

Figure 3. Neural network implementing fuzzy connectives.

In NASA's Johnson Space Center, a study was made (Lea *et al.* 1992) for observing tether oscillations present during retrieval of a tethered satellite with a space time neural network (Villarreal and Shelton 1992). The study shows that the problem requires high momentum and very low learning rate for the network.

4.3. *Changing the basic characteristics of the neurons*

Here, the neurons are designed to perform various operations used in fuzzy set theory (like fuzzy union, intersection, aggregation) instead of doing the standard multiplication and addition operations (Fig. 3).

Krishnapuram and Lee (1992 a,b) used fuzzy set connectives in multi-layer network structures suitable for pattern recognition and other decision making systems. Various union, intersection, generalized mean and multiplicative hybrid operators (which are used in the fuzzy set literature to aggregate imprecise information in order to arrive at a decision in uncertain environments) are implemented by layered networks. The hybrid (compensatory) model used was the γ-model of Zimmerman and Zysno (1983) where the output is expressed as

$$y = \left(\prod_i x_i^{\delta_i}\right)^{1-\gamma}\left(1 - \prod_i (1 - x_i)^{\delta_i}\right)^{\gamma}, \tag{4}$$

where $\sum_{i=1}^{m} \delta_i = 1$ and $0 \leq \gamma \leq 1$. $x_i \in [0, 1]$ are the inputs. δ_i is the weight associated with x_i and γ controls the degree of compensation. The hybrid opterator can behave as a union, intersection or mean operator with different sets of parameters, which can be learned through a training procedure. An iterative algorithm to determine the type of aggregation function and its parameters at each node in the network is also provided; thereby making the network more flexible. The approach provides a tool for modelling and managing uncertainty in the process of combination of evidence from complementary and supplementary knowledge sources. The technique also provides a mechanism for selecting powerful features and discarding irrelevant features via the detection of redundancy. The training procedure of the multiplicative γ-model is slow. To achieve faster convergence the additive γ-model is studied, under the above framework, by Keller and Chen (1992) as an alternative connective in such networks.

Gupta (1992) suggested the use of generalized AND (which can be expressed using the notion of triangular norms) and OR (represented by triangular co-norm) operations for fuzzy signals (signals bounded by the graded membership function over the unit interval [0, 1]) instead of multiplication and summation operations as used in standard neural networks. Thus for fuzzy inputs, $\boldsymbol{x}(t) \in [0, 1]^n$ and synaptic strengths $\boldsymbol{w}(t) \in [0\ 1]^n$ the weighted synaptic signal $\boldsymbol{z}(t) \in [0, 1]^n$ is defined as

$$z_i(t) = w_i(t) \text{ AND } x_i(t), \quad i = 1, 2, \ldots, n \tag{5}$$

and the aggregated input to a neuron is

$$u_i(t) = \underset{i}{\text{OR}}\ z_i(t). \tag{6}$$

The nonlinear mapping with threshold $w_0 \in [0, 1]$ is then defined as

$$v_i(t) = [u_i(t) \text{ OR } w_0(t)]^{\alpha}, \tag{7}$$

where α is a positive quantity. For $0 < \alpha \leq 1$ the operation corresponds to the *dilation* operation of a fuzzy set and for $\alpha > 1$ it corresponds to the *concentration* operation (Pal and Majumder 1986).

Recently, a fuzzy neural network using logical operations namely max–min and product–probabilistic sum has been developed by Mitra and Pal (1994a) for both classification and rule generation with linguistic properties as input. For the purpose of rule generation and inferencing, the user could be queried for more essential feature information in the case of partial input. The model is likely to be suitable for data-rich environments. The use of logical neurons helps in generating rules in more appropriate forms and makes its hardware realization easier. The system is robust with respect to input fuzzification.

Pedrycz (1991) tried to introduce fuzziness in neural networks in a different way. He pointed out some analogies between structures involving composite operators and a certain class of neural networks. Links are established between neural network architectures and relational systems in terms of fuzzy relational equations. The proposed architecture is based exclusively on set theoretic operations. The individual neurons perform logical operations (like max, min) which are mainly used in set theory instead of arithmetic operations. The problem of learning of connection strengths or weights was also studied and relevant learning rules were proposed. A performance index, called the equality index, is also introduced to keep track of these logical operations. He has also suggested (Pedrycz 1992) the design of neural networks to implement logic operations (like AND, OR, MATCH) used in fuzzy set theory to detect regions of patterns (using reference neurons) and combining them to yield a final classification decision.

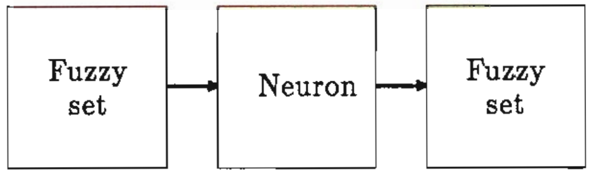

Figure 4. Block diagram of a fuzzy neuron.

4.4. *Making the individual neurons fuzzy*

Another method of integration is to make the individual neurons fuzzy (Lee and Lee 1975, Yamakawa and Tomada 1989). Here, input to a neuron is a fuzzy set and the output also is a fuzzy set (Fig. 4). The activity of networks involving fuzzy neurons is a fuzzy process.

The idea was originally introduced by Lee and Lee (1975). Some of the concepts of fuzzy set theory are employed to define a fuzzy neuron, which is a generalization of the classical neuron. The activity of a fuzzy neuron is a fuzzy process. The input to such a neuron is a fuzzy set and the outputs are equal to some positive numbers, μ_j $(0 < \mu_j \leq 1)$, if it is firing and zero if it is quiet. μ_j denotes the degree to which the jth output is fired. Unlike conventional neurons, such a neuron has multiple outputs (set). The utility of neural networks with such fuzzy neurons has been demonstrated for synthesizing fuzzy automata. This concept, although introduced long ago, has not been explored much as compared to others.

4.5. *Measures of fuzziness as error or instability of a network*

Integration of the concept of fuzzy sets and neural network has also been made by using the fuzziness/uncertainty measures of a fuzzy set to model the error or instability or energy function of a neural network based system. An attempt has been made in this context by Ghosh *et al.* (1993) to incorporate various fuzziness measures in a multi-layer network to make it able to perform (unsupervised) self-organizing tasks in image processing, in general, and object extraction in particular (Fig. 5). The network architecture is basically a feedforward one with a feedback path. In each layer every neuron corresponds to an image pixel. Each neuron is connected to the corresponding neuron in the previous layer and its neighbours. The status of neurons in the output layer is described as the membership value to a fuzzy set representing object regions. A fuzziness measure (e.g. index of fuzziness and entropy, Pal and Majumder 1986) of this set is used to quantify system error (instability of the network) and it is back-propagated to correct weights.

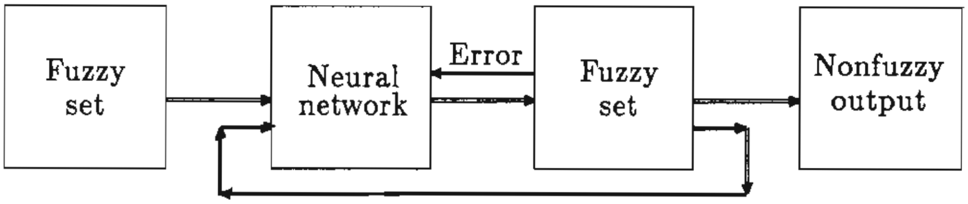

Figure 5. Layered network implementing self-organization.

After the weights have been adjusted the output of the neurons in the output layer is fed back to the corresponding neurons in the input layer. The second pass is then continued with this as input. The iteration (updating of weights) is continued until the network stabilizes, i.e. the error value (measure of fuzziness) becomes negligible. This integration makes it possible for a layered network (which is usually used as a supervised classifier) to act as an unsupervised one, in addition to providing a robust and noise-insensitive segmentation algorithm. Here the neuro-fuzzy integration also provides an application specific advantage. As such, we cannot always generalize this concept to use an MLP as an unsupervised classifier. In Kios and Liu (1992), an approach is also mentioned to design an optimal network architecture by optimization of the fuzziness of a set.

Note that the aforesaid attempts of integration for neuro-fuzzy computing are mainly made in the field of pattern recognition and to some extent in fuzzy logic control. Literature on neuro-fuzzy image processing is not adequate at this moment. For further references on this approach one can refer to Bezdek and Pal (1992), Kosko (1992), IEEE (1992, c, d, e), Archer and Wang (1991), Carpenter *et al.* (1991), Special Issue (1992), Cohen and Hudson (1992), Pedrycz and Card (1992), Lin and Lee (1992) and Werbos (1992 a,b).

5. Scope for further work

The fusion of fuzzy logic and neural network theories has been tried out at various levels (namely input, neuron characteristic, output) mainly in pattern recognition problems. The literature is relatively poor on such attempts for performing image processing tasks. Besides that, most of the attempts are only preliminary. Their usefulness and validity should be studied rigorously by choosing problems from different aspects of image processing and pattern recognition. The complexity of the algorithms needs to be analysed. Recent studies are trying to establish links between some of the existing fuzzy set-theoretic algorithms and neural network-based approaches (Bezdek 1992, Huntsberger and Ajjimarangsee 1990, Bezdek *et al.* 1992, Pedrycz and Card 1992).

The incorporation of fuzzy set concepts in the design of layered networks has been found to increase their capability. Such incorporation in various stages of self-organizing or auto-associative network-based algorithms may be tried out in order to handle ill-defined input data more efficiently. Further investigations along these lines may include: effect of fuzzification of input data and output labels on learning, use of fuzzy geometrical properties as input features for training a network, and defining fuzzy memberships in noisy environments. Consideration of fuzzy geometric properties as input would enable the system to handle directly raw images without doing pre-processing for analysis and recognition. Besides this, fuzzy set theoretic algorithms may also be altered so that they can be implemented on a particular type of neural-network architecture.

Preliminary attempts have been made to design neural networks governed by a fuzzy logic formalism based on if-then rules. Their application to processing and analysis of ill-defined images needs to be investigated, in addition to designing appropriate membership function and optimal networks for implementing a given set of rules. Again, other standard operators like sharpening, complementing, bounded sum, probabilistic sum, bounded difference etc, which are useful for pattern recognition need neural network implementations. Similarly, the utility of fuzzy geometrical properties as objective functions (or energy functions) of neural network-based systems for image processing and recognition constitute another possibility for further investigation.

6. Conclusions

The relevance of fuzzy set theoretic approaches to certain tasks of pattern recognition and image analysis is described. Characteristics of neural networks and their importance for designing pattern recognition systems are mentioned. Utility, feasibility and different ways of fusion of these two technologies, so far made, for designing more intelligent systems have been described. It is observed that besides the generic advantages, like parallelism, fault tolerance, robustness and modelling vagueness, the neuro-fuzzy approach provides some application specific merits (Pal and Mitra 1992, Mitra and Pal 1994 b, c, Ghosh *et al.* 1993). From the discussions it appears that this new area of neuro-fuzzy computing will continue to be in the frontline of research for the next ten years, if not more.

Acknowledgments

A part of this work was done when Professor S. K. Pal held Jawaharlal Nehru Fellowship and Dr A. Ghosh held a Research Associateship of the Council of Scientific and Industrial Research, India. Valuable discussion with Dr N. R. Pal is gratefully acknowledged.

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