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Neuro-Fuzzy Expert Systems: Relevance, Features and Methodologies

SUSHMITA MITRA AND SANKAR K PAL, FIETE
Machine Intelligence Unit, Indian Statistical Institute, 203 B T Road, Calcutta 700 035, India.

Some of the inherent problems of expert system design, and the relevance of fuzzy logic, neural networks and neuro-fuzzy computing in circumventing them are explained. Various existing models of neuro-fuzzy expert systems are described. A comparative study with the traditional and connectionist versions is given in tabular form. The way of generating linguistic rules is explained with an example. The work on recent approaches using knowledge-based networks in this regard has also been highlighted.

Indexing terms : Neuro-fuzzy computing, Expert systems, Knowledge-based network, Linguistic rule generation

ARTIFICIAL neural networks (ANNs) [1-4] can be formally defined as massively parallel interconnections of processing elements that interact with objects of the real world in a manner similar to biological systems. All information is stored distributed among the various connection weights. The networks can be trained by examples (as is often required in real life) and sometimes they generalize well for unknown test cases.

Fuzzy logic is based on the theory of fuzzy sets and, unlike classical logic, it aims at modeling the imprecise (or inexact) modes of reasoning and thought processes (with linguistic variables) that play an essential role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision. This ability depends, in turn, on our ability to infer an approximate answer to a question based on a store of knowledge that is inexact, incomplete, or not totally reliable.

We see that fuzzy set theoretic models [5, 6] try to mimic human reasoning and the capability of handling uncertainty, whereas the neural network models attempt to emulate the architecture and information representation schemes of the human brain. Integration of the merits of fuzzy set theory and neural network theory therefore promises to provide, to a great extent, more intelligent systems (in terms of parallelism, fault tolerance, adaptivity and uncertainty management) to handle real life recognition / decision making problems. For the last five to seven years, there have been several attempts [7-10] by researchers over the world in making a fusion of the merits of these theories under the heading 'neuro-fuzzy computing' for improving the performance in decision making systems.

As the knowledge base of an expert system is a repository of human knowledge and since some of these may be imprecise in nature, often this may result in a collection of rules and facts which for the most part are neither totally certain nor totally consistent. The expert system is also likely to be required to infer from premises that are imprecise, incomplete or not totally reliable. The uncertainty of information in the knowledge base of the question-answering system thus induces some uncertainty in the validity of its conclusions [11]. Hence a basic problem in the design of exeprt systems is the analysis of the transmitted uncertainty from the premises to the conclusion and the association of a certainty factor [12]. Fuzzy expert systems [12, 13], incorporating the concept of fuzzy sets at various stages, help to a reasonable extent in the management of uncertainty in such situations.

Neural networks are also used in designing expert systems. Such models are called connectionist expert systems [14], and they use the set of connection weights of a trained neural net for encoding the knowledge base for the problem under consideration. The use of ANN helps in (*a*) incorporating parallelism, and (*b*) tackling optimisation problems in the knowledge base space. These models are usually suitable in data-rich environment and seem to be capable of overcoming the problem of the knowledge acquisition bottleneck of traditional expert systems. They help in minimizing human interaction and associated inherent bias during the phase of knowledge base formation (which is time-consuming in the case of traditional models) and also reduce the possibility of generating contradictory rules. Powerful learning techniques exist for generating connectionist networks from training samples. This enables us to automate the construction of knowledge bases for classification-type expert systems. When the connection weights of a trained fuzzy neural net are used as the knowledge base, we call the model a neuro-fuzzy expert system. This enables one to accommodate the merits of neuro-fuzzy computing in expert system design.

The block diagram of the basic modules of an expert system, fuzzy expert system, fuzzy neural net, connectionist expert system, neuro-fuzzy expert system and knowledge-based connectionist expert system are provided in Fig 1. As stated above, a fuzzy neural net constitutes the knowledge base of a neuro-fuzzy expert system. (Note that this excludes other possible integrations, such as bringing the concept of ANN into the framework of fuzzy expert system). While the rules are collected by knowledge engineers for designing the knowledge base of a traditional expert system (or fuzzy expert system), the connectionist models use the trained link weights of the neural net/fuzzy neural net to automatically generate the rules, either

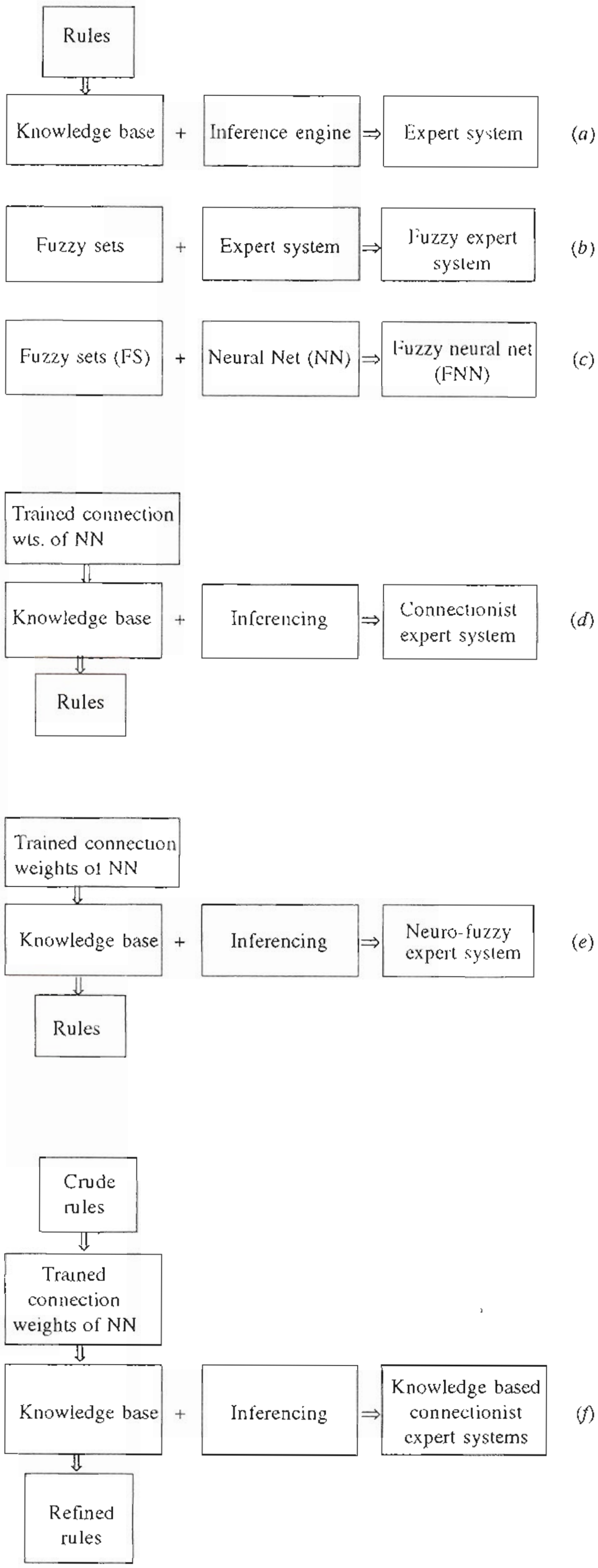

Fig 1 Block diagram of the basic modules of various expert systems

for later use in a traditional version or for providing justification in the case of an inferred decision. This automates and also speeds up the knowledge acquisition process. The use of fuzzy neural nets helps in the handling of uncertainty at various levels (*eg*, input, output, learning and neuronal) and generates fuzzy rules capable of more realistically representing real-life situations. The knowledge-based connectionist expert systems, on the other hand, initially encode crude domain knowledge among the connection weights of the neural net, thereby speeding up the training phase and generating better performance. Refined rules are later extracted from the less redundant trained network.

In this article we provide a review on existing models of neuro-fuzzy expert systems. A survey on connectionist expert systems is also included in this context, for the convenience of the reader. A comparative study is provided in tabular form. Firstly we discuss the general problems of expert system design, and the relevance of fuzzy sets, connectionist models and neuro-fuzzy computing in this line. The utility of knowledge-based networks, which combine some initial knowledge about the problem domain in the network structure and then refine this knowledge through neural learning, is also described later.

EXPERT SYSTEMS : SOME PROBLEMS

The major components of an expert system [15] are the knowledge base, inference engine and user interface. The knowledge base contains the expert-level information necessary to solve problems in a specific domain. This information is generally represented in the form of a set of rules, although frames [16], semantic nets [17] and belief networks [18] are also in vogue. We shall consider rule-based systems in this discussion. Knowledge bases, being domain-specific, are non-transferable. The inference engine interacts both with the knowledge base and a working memory (that records facts about the current problem and is updated with the availability of new information). Pattern matching occurs between the rules in the knowledge base and the facts in the working memory to select the relevant rules applicable. Note that when no matching occurs, no rule is selected, whereas when multiple rules apply, conflict resolution strategies are used to select the most specific one. The same inference engine can be used with different knowledge bases.

We provide here a mathematical formulation of expert systems followed by a discussion on fuzzy logic and its role in the management of uncertainties, the relevance of connectionist models and the need for neuro-fuzzy computing.

Let us consider finding a decision consisting of a sequence of decision elements (or hypotheses) optimizing some criteria in an environment characterized by available information. Let **D** be a candidate decision consisting of n decision elements d_i, where each decision element d_i belongs to a finite, discrete set D^i.

$$\mathbf{D} = (d_1, d_2, \ldots, d_n)\ d_i \in D^i.$$

As a link between the decision and the available information, a number N of measurements (or observations) are available

$$m_i : D \rightarrow \mathcal{M} : D \rightarrow m_i(D) \qquad i = 1, \dots, N$$

where $\mathcal{M}$ is the measurement space. Heuristic functions are used for rating the different candidate decisions according to these measurements. These ratings describe how well (or how likely) a decision (and its associated measurement) fits in with the environment

$$h_i : \mathcal{M} \rightarrow \mathcal{R} : m_i \rightarrow h_i(m_i) \qquad i = 1, \dots, N, \tag{1}$$

where $\mathcal{R}$ is the space of the possible rating values (mostly a subset of the real numbers). Each heuristic can be considered as a piece of knowledge, usually coming from an expert, and is used for partially assessing the quality of the decision. Heuristics are combined to form a global rating r, which is a measure of the quality of the decision

$$r = O\,[h_1(m_1(\mathrm{D})), h_2(m_2(\mathrm{D})), \dots, h_N(m_N(\mathrm{D}))], \tag{2}$$

where O is the combination operator across all heuristics.

The role of fuzzy logic

Fuzzy sets

A fuzzy set A in a space of points $R = \{r\}$ is a class of events with a continuum of grades of membership and is characterized by a membership function $\mu_A(r)$ which associates with each element in R a real number in the interval [0,1] with the value of $\mu_A(r)$ at r representing the grade of membership of r in A. Formally, a fuzzy set A with its finite number of supports $r_1, r_2, \dots, r_t$ is defined as a collection of ordered pairs

$$A = \{(\mu_A(r_i), r_i), \quad i = 1, 2, \dots, t\}$$

where the support of A is an ordinary subset of R and is defined as

$$S(A) = \{r \mid r \in R \text{ and } \mu_A(r) > 0\}$$

Here μ_i, the grade of membership of r_i in A, denotes the degree to which an event r_i may be a member of A or belong to A. Note that $\mu_i = 1$ indicates the strict containment of the event r_i in A. If, on the other hand, r_i does not belong to A then $\mu_i = 0$.

Fuzzy logic is based on the theory of fuzzy sets and, unlike classical logic, it aims at modeling the imprecise (or inexact) modes of reasoning and thought processes (with linguistic variables) that play an essential role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision. This ability depends, in turn, on our ability to infer an approximate answer to a question based on a store of knowledge that is inexact, incomplete, or not totally reliable. In fuzzy logic everything, including truth, is a matter of degree [12]. Zadeh has developed a theory of approximate reasoning based on fuzzy set theory. By approximate reasoning we refer to a type of reasoning that is neither very exact nor very inexact. This theory aims at modeling the human reasoning and thinking process with linguistic variables [19] in order to handle both soft and hard data, as well as various types of uncertainties. Many aspects of the underlying concept have been incorporated in designing decision-making systems [20].

Because fuzzy sets are a generalization of the classical set theory, the embedding of conventional models into a larger setting endows fuzzy models with greater flexibility to capture various aspects of incompleteness or imperfection (*ie*, deficiencies) in whatever information and data are available about a real process. Assignment of membership functions of a fuzzy subset is subjective in nature, and reflects the context in which the problem is viewed. It cannot be assigned arbitrarily. In many cases, it is convenient to express the membership function of a fuzzy subset in terms of standard S and π functions [21].

Let X be a variable which takes values in a universe of discourse U, with the generic element of U denoted by u and $X = u$ signifying that X is assigned the value u, $u \in U$. Let F be a fuzzy subset of U which is characterized by a membership function μ_F. Then F is a fuzzy restriction on X if F acts as an elastic constraint on the values that may be assigned to X such that

$$X = u : \mu_F(u)$$

where $\mu_F(u)$ is interpreted as the degree to which the constraint represented by F is satisfied when u is assigned to X. Let $R(X)$ denote a fuzzy restriction associated with X. Then we write $R(X) = F$. Consider a proposition of the form $p = X$ is F, where X is the name of an object, a variable or a proposition, and F is the name of a fuzzy subset of U. The translation of such a proposition may be expressed as

$$R(A(X)) = F$$

where $A(X)$ is an implied attribute of X which takes values in U and the proposition has the effect of assigning F to the fuzzy restriction on the values of $A(X)$. Let p be the proposition "Z is young", in which young is a fuzzy subset of $U = [0,100]$ characterized by the membership function

$$\mu_{young}(u) = 1 - S(u; 20, 30, 40) \tag{3}$$

where u is the numerical age, and the S-function is defined by

$$S(u; \alpha, \beta, \gamma) = \begin{cases} 0 & \text{for } u \le \alpha \\ 2\left(\frac{u-\alpha}{\gamma-\alpha}\right)^2 & \text{for } \alpha \le u \le \beta \\ 1 - 2\left(\frac{u-\gamma}{\gamma-\alpha}\right)^2 & \text{for } \beta \le u \le \gamma \\ 1 & \text{for } u \le \gamma \end{cases} \tag{4}$$

in which the parameter $\beta = \dfrac{\alpha+\gamma}{2}$ is the crossover point where $S(\beta; \alpha, \beta, \gamma) = 0.5$. In this case, the implied attribute

$A(X)$ is *Age* (Z) and the translation of "Z is young" assumes the form

$$Z \text{ is young} \rightarrow R\ (Age(Z)) = young$$

Some other function could also be used in this regard.

Management of uncertainties

The knowledge base of an expert system contains human knowledge, most of which is imprecise and qualitative. To describe situations where the boundary between competing hypotheses is vaguely defined, human experts use terms such as *very likely, likely, more or less likely, low, medium, high, etc.* Encoding this sort of expertise by probabilities results in the loss of information about this vagueness or imprecision. Using linguistic variables for such terms enables a knowledge engineer to capture the essence of the experts' experience and judgement without attempting to over-quantify intuition. Moreover, facts about the world are rarely known with certainty. Conventional rule-based systems, with two-valued logic usually evade this issue of partial matching.

In conventional statistical designs, the input patterns are quantitatively exact to within the resolution of the sensors used to collect them. However, real processes also may possess imprecise or incomplete input features. In such cases it may become convenient to use linguistic variables and hedges [21] like *low, medium, high, very, more or less, etc* to augment or even replace numerical input feature information. Any input feature value can be described in terms of some combination of membership values in the linguistic property sets *low, medium* and *high*.

The importance of fuzzy logic to the management of uncertainty in expert systems mainly lies in its ability for dealing with fuzzy quantifiers and modifiers. Fuzzy logical systems allow a proposition or conclusion to range over fuzzy subsets (like *very true, more or less true, likely true, etc*) of truth-value sets characterised by their possibility distributions. Fuzzy modifiers like *not, very, more or less, extremely, slightly, much, a little, etc* can also be represented. A fuzzy certainty factor is associated with the conclusion to analyse the transmission and cumulation of uncertainty from the premises to the conclusion. Deduction of conclusions from observations and rules in the knowledge base is made using either truth value restriction or compositional rule of inference. Hence, partial match can occur between the antecedent of a rule and a fact supplied by the user.

In short, fuzzy logic or reasoning [12] provides a natural conceptual framework for knowledge representation and inferencing from knowledge bases that are imprecise, incomplete or not totally reliable. The advantage of using fuzzy reasoning is that it can yield an approximate answer even when probabilistic theories are not applicable, as the latter often require idealized assumptions such as the independence of evidence and the mutual exclusiveness and exhaustiveness of hypotheses.

The range of the space of the measurement values in eqn (1) can now be divided into a number of classes, each characterized by a membership function and a linguistic variable describing how well it fits the hypothesis that the candidate is the solution to the problem. Mathematically,

$$h_i : \mathcal{M} \rightarrow [0, 1]^K : m \rightarrow h_i^K(m) = (\mu_i^1(m), \mu_i^2(m), \dots, \mu_i^K(m)) \tag{5}$$

where K indicates the number of classes. Each linguistic term is a fuzzy set which designates a category partially qualifying a candidate solution in the sense of the considered heuristic (*eg, very likely, likely, not unlikely, etc*). The set of heuristics forms a knowledge base of fuzzy rules whose antecedents are related to the measurements or observations and whose consequent part determines the fuzzy (partial) quality of the decision.

Fuzzy rule-based systems can be incorporated in fuzzy expert systems. Such a system can be expressed by a set of fuzzy inference rules. In each rule, there is a premise and a consequence. The premise is described by a fuzzy proposition and the consequence can be a fuzzy conclusion. A typical fuzzy inference rule for an N-input K-output system can be expressed as

if $\quad x_1$ is A_{i1}, x_2 is A_{i2}, ... , and x_N is A_{iN}

then $\quad y_1$ is B_{i1}, y_2 is B_{i2}, ... , and y_M is B_{iK} ,

where $X = \{x_j, j = 1, 2, \dots, N\} \in \Re^N$ are the inputs to the fuzzy system, $Y = \{y_j, j = 1, 2, \dots, K\} \in \Re^K$ are the outputs, A_{ij} $(j = 1, 2, \dots, N)$ and B_{ij} $(j = 1, 2, \dots, K)$ are fuzzy subsets. Thus a fuzzy rule-based system implements a mapping $\Re^N \rightarrow \Re^K$. Fuzzy inference methods are algorithms that deduce results from the inference rules and the presented inputs. Note that the consequent part of the rules can also be represented by scalars or membership values $\mu_i^j(m)$, $j = 1, 2, \dots, K$, where m refers to the measurement of the input variable x. These rules constitute the knowledge base of the fuzzy expert system.

The various approaches in fuzzy inferencing for expert systems include the approximate analogical reasoning based on similarity measures by Turksen and Zhong [22], the problem-reduction method of Ishizuka *et al* [23], modeling of physicians' decision processes by Esogbue and Elder [24] and inferencing in the framework of inflammatory protein variations by Sanchez and Bartolin [25] (using weighting). Wang and Mendel [26] developed a slightly different method for creating a fuzzy rule base made up of a combination of rules generated from numerical examples and linguistic rules supplied by human experts. The input and output domain spaces are divided into a number of linguistic subspaces. Human intervention is sought to assign degrees to the rules, and conflicts are resolved by selecting those rules yielding the maximum of a computed measure corresponding to each linguistic subspace.

Relevance of connectionist models

The various uncertainty management schemes of traditional expert systems share some common problems. For example, a willing human expert able to accurately quantify expertise is needed. The transfer of the knowledge takes place gradually through many interviews between the expert and the system, and is therefore very time consuming. Usually humans are prone to be easily biased and thus the quality of knowledge extracted from the experts depends greatly on the methods used for assessment. Moreover, large knowledge bases need to be searched quickly and it is also very important to check that this knowledge base remains consistent as more information is accumulated. It would therefore be welcome if knowledge assessment could be automated by freeing it from human intervention, thereby avoiding human bias and subjectivity.

It is worth mentioning that the most difficult, time-consuming and expensive task in building an expert system is constructing and debugging its knowledge base. In practice, the knowledge base construction can be said to be the only real task in building an expert system considering the proliferating presence of expert shells. Several approaches have been explored for easing this knowledge-acquisition bottleneck.

Connectionist expert systems [14] offer an alternative approach both to the knowledge base construction as well as the inferencing phase, providing interaction with the user accompanied by justification(s) of the conclusion(s) reached. Rules are not required to be supplied by humans. Instead, the connection weights of a trained neural network encode among themselves, in a distributed fashion, the information conveyed by the input-output combinations of the training set. The problems faced by traditional expert systems regarding the difficulties in normalizing across different experts' scales, conversion from human expressions to numerical terms, bias of the expert(s), generation of contradictory rules by the experts, *etc*, may be overcome here. The use of the learning technique of neural networks enables the model to extract the information inherent in the data (that is not utilised in the traditional models) and allows dynamical adjustments to changes in the environment. It also enables one to handle a complicated environment for which either no mathematical model exists, or, even if it exists is so strongly nonlinear that a design method does not exist. Besides, the various characteristics of neural nets, *viz*, generalization, tolerance to noise, graceful degradation at the border of the domain of expertise, ability to discover new relations between variables, *etc*, are in-built and hence can be exploited by the connectionist expert systems.

Let us now provide a mathematical formulation of a layered neural network that can be used for constructing a connectionist expert system. A neuron can be depicted as an information processing element which receives an n-dimensional input vector

$$\mathbf{X}(t) = [x_1(t), x_2(t) \ldots , x_n(t)] \in \Re^n \tag{6}$$

and yields a scalar neural output $y(t) \in \Re^1$ at instant t. The input vector, $\mathbf{X}(t) \subset \Re^n$, represents the signals being transmitted from the n-neighboring neurons (including self-feedback signal) and/or the outputs (measurements) from the sensory neurons. Mathematically, the information processing ability of a neuron can be represented as a nonlinear mapping operation

$$\mathbf{X}(t) \in \Re^n \rightarrow y(t) \in \Re^1 \tag{7}$$

A confluence operation $\otimes$ essentially provides a measure of similarity between the neural input vector $\mathbf{X}(t)$ (new information) and the synaptic weight vector $\mathbf{W}(t)$ (accumulated knowledge base). Generally summation and product operations are used in this stage. A nonlinear activation function then performs a nonlinear mapping on the similarity measure through a nonlinear activation function $\psi[\cdot]$. Hence

$$y(t) = \psi[\mathbf{W}(t) \otimes \mathbf{X}(t)] \tag{8}$$

A neural network can be viewed as a collection of such neurons connected to each other according to a specific topology. It, therefore, performs a mapping from the n-dimensional input space to a K-dimensional output space such that

$$\mathbf{X}(t) \in \Re^n \rightarrow \mathbf{Y}(t) \in \Re^K \tag{9}$$

where K refers to the number of output classes in case of a classifier. The error-based learning algorithm is represented as

$$\mathbf{W}(t+1) = \mathbf{W}(t) = \eta \Delta \mathbf{W}(t) \tag{10}$$

for the N_w connection weights of the neural net.

Connectionist expert systems use the connection weights W of the trained neural network (eqn (10)) to form the knowledge base. The magnitudes of these connection weights are used to generate rules in order to justify any decision. The maximum weighted paths from the output layer to the input layer are used in the process [14, 27]. Note that in traditional expert systems, the knowledge base is formulated in terms of rules by interaction with the experts. On the other hand, here the rules may be automatically extracted from the trained connection weights, that form the knowledge base. This procedure will be discussed in more detail later.

Need for neuro-fuzzy computing

Both neural networks and fuzzy systems are trainable dynamic systems that estimate input-output functions. They estimate a function without any mathematical model and learn from experience with sample data. A fuzzy system adaptively infers and modifies its fuzzy associations from representative numerical samples. Neural networks, on the other hand, can blindly generate and refine fuzzy rules from training data [28]. Fuzzy systems and neural networks also differ in how they estimate sampled functions, the kind of samples used and how they rerpesent and store these samples. Fuzzy systems estimate functions with fuzzy set samples (A_i, B_i), while neural systems use numerical point samples (x_i, y_i), where both kinds of samples reside in the input-output product space $X \times Y$. Hence the input-output mapping corresponds to $f: X \rightarrow Y$ in both cases.

The fuzzy theory is considered to be advantageous in the logical field, and in handling higher order processing easily. The higher flexibility is a characteristic feature of neural nets produced by learning, and hence this suits data-driven processing better [29].

For the last few years, researchers all over the world [7-10, 30, 31] have been trying to combine the merits of fuzzy and neural approaches under the heading neuro-fuzzy computing for building more intelligent decision making systems. This enables one to incorporate the generic advantages of artificial neural networks like massive parallelism, robustness and learning in data-rich environments into the expert system model. The modelling of imprecise and qualitative knowledge as well as the transmission of uncertainty are possible through the use of fuzzy logic. Besides this generic advantage, the neuro-fuzzy approach provides some application specific merits in the following way. For example, in the case of classification-type connectionist expert systems one is typically interested in exploiting the capability of neural nets in generating the required (linearly nonseparable) decision regions. The uncertainties involved in the input description and output decision are also taken care of by the concept of fuzzy sets. It is observed that in certain cases a neuro-fuzzy model performs better than either a neural network or a fuzzy system considered individually [32, 33].

Keeping in view eqns (6)-(10) defining a neural net, let us now provide a mathematical formulation of a layered fuzzy neural net that can be used for designing a neuro-fuzzy expert system. A fuzzy neural network can incorporate fuzziness at the input-output level, in the connection weights, in the confluence operation or in the activation function. Let the fuzzy input and output vectors be represented as $\mathbf{X}$ and $\mathbf{Y}$ respectively where these correspond to fuzzy numbers or intervals or the augmented space consisting of linguistic terms. Similarly, the connection weight vector may be represented as $\mathbf{W}$. Arithmetic operations like fuzzy addition and fuzzy multiplication can be used in the new confluence operation $\otimes$. The nonlinear activation function ψ can incorporate fuzzy logic operations like AND, OR and NOT. Hence the resultant mapping from the n-dimensional input space to the K-dimensional output space becomes

$$\mathbf{X}(t) \in \Re^n \rightarrow \mathbf{Y}(t) \in \Re^K \tag{11}$$

where a single fuzzy neuron implements the nonlinear operation

$$y(t) = \psi[\mathbf{W}(t) \otimes \mathbf{X}(t)] \tag{12}$$

The learning algorithm now becomes

$$\mathbf{W}(t+1) = \mathbf{W}(t) + \eta \Delta \mathbf{W}(t) \tag{13}$$

for the N_w connection weights of the fuzzy neural net.

The neuro-fuzzy expert systems use the connection weights W of the fuzzy neural net (eqn (13)) to form the corresponding knowledge base. The connection weights encode the knowledge base of the problem during training by using the training set $\{\mathbf{X}_i, \mathbf{Y}_i\, i = 1, \ldots, N\}$, where the implemented mapping is $\Re^n \rightarrow \Re^K$. Note that the antecendent $\mathbf{X}_i$ and the consequent $\mathbf{Y}_i$ may involve linguistic terms, or fuzzy intervals / numbers, or fuzzy membership values in [0, 1]. Fuzzy rules may be extracted using the connection weights of the network by backtracking along with maximum weighted paths [27].

Before we describe various neuro-fuzzy expert system models, let us provide a brief survey on the characteristics of connectionist (nonfuzzy) expert systems for the convenience of readers. This also includes knowledge-based nets which are recently being used for this purpose.

CONNECTIONIST EXPERT SYSTEMS

Some models

Here we consider a few of the existing layered connectionist expert systems modelled by Gallant [14], Saito and Nakano [34], Lacher *et al* [35] and Poli *et al* [36]. The inputs and outputs consist of crisp variables in all cases. Generally the symptoms are represented by the input nodes while the diseases and possible treatments correspond to the intermediate and/or output nodes [14, 34]. The linear discriminant network of [14] (dealing with sacrophagal problems) is generated from the dependency information regarding the variables, that is provided by the expert in the form of an adjacency matrix. This is then trained by the simple Pocket Algorithm. The absence of the hidden nodes and nonlinearity limits the utility of the system in modeling complex decision surfaces. The multilayer network in [34] is designed for detecting headache. A patient responds to a questionnaire regarding her perceived symptoms and these constitute the input to the network.

Lacher *et al* [35] have designed event-driven, acyclic networks of neural objects called expert networks. The network is built under the commercial shell M1. There are regular nodes and operation nodes (for conjunction and negation). Input weights are hard-wired, while the output weights of a node are adaptive. Antecedents of a disjunction in a rule are simplified to generate a set of individual rules before formulating the initial network architecture. The back-propagation algorithm is modified to work in the event-driven environment, where both forward and backward signals propagate in data-flow fashion. The form of the rules (coarse knowledge) is tuned with the associated certainty factors (fine knowledge) and the resultant network trained for better performance.

A novel approach to designing a modular connectionist expert system, called Hypernet, has been reported by Poli *et al* [36]. The feed forward network consists of a reference-generating module, a drug compatibility module and a therapy-selecting module in order to simulate the physician's reasoning as closely as possible. The user-friendly system provides graphics interface for easy handling as well as verification of decisions. The model is implemented for diagnosing and treating hypertension. The performance is good due to the embedded modularity of the network.

Rule generation is also possible for the models in [14, 34]. In [34] the doctor is supplied with information regarding possible diagnoses based on the output node values. Relation factors, estimating the strength of the relationship between symptom(s) and disease(s), are extracted from the network and used to help doctors. Rules are generated from the changes in levels of input and output units; the connection weights are not involved in the process. These rules are then used to allow the patient to confirm the symptoms initially provided by her to the system, in order to eliminate noise from the answers. The model in [14] incorporates inferencing/forward chaining, confidence estimation, backward chaining and explanation of conclusions by 'If- Then' rules. In order to generate a rule, the attributes with greater inference strength (magnitude of connection weights) are selected and a conjunction of the more significant premises is formed to justify the output concept. Here, the user can also be queried to supplement incomplete input information.

An MLP-based model for the identification of EEG power spectra of rats in depression has been reported recently by Mitra *et al* [37]. The input consists of frequency, represented both as individual values and as non-overlapping bands, normalized in the range [0, 1]. The output refers to the control and depressed states. It has been observed that the role of exercise reverses the effect of stress. Rules have also been generated in terms of linguistic labels *small* and *large* corresponding to the relative values of the features. Note that this is slightly different from the crisp rules, indicating the presence or absence of certain features (symptoms) as in [14 ,34].

Knowledge based networks

Recently, there have been some attempts in improving the performance of expert systems by using knowledge-based networks which use the domain knowledge to determine the initial structure of the network. Such a model has the capability of outperforming a standard MLP as well as other related algorithms including symbolic and numerical ones [38,39]. However, in the absence of knowledge one has to resort to a purely data-driven mode of learning as in the simple connectionist expert models. When the initial knowledge fails to explain many instances, additional hidden units and connections need to be added (often empirically). The initial encoded knowledge may be refined with experience by performing learning in the data environment. The resulting networks generally involve less redundancy in their topology.

Let us first provide a mathematical formulation in line with the modelling in eqns (6)-(10), before proceeding on to the survey. The knowledge-based nets implement a mapping

$$\mathbf{X}'(t) \in \Re^{n} \rightarrow \mathbf{Y}(t) \in \Re^{K} \quad (14)$$

from the n'-dimensional input space to the K-dimensional output, space, where

$$y(t) = \psi' [\mathbf{W}'(t) \otimes \mathbf{X}'(t)] \quad (15)$$

The learning algorithm becomes

$$\mathbf{W}'(t+1) = \mathbf{W}'(t) + \eta \Delta \mathbf{W}'(t) \quad (16)$$

for the N'_w connection weights such that $N'_w \leq N_w$ of eqn (10).

The knowledge-based models discussed here [38-40] involve crisp inputs and outputs. The initial domain knowledge, in the form of rules, is mapped into the multi-layer feedforward network topology using binary link weights to maintain the semantics. Yin and Liang [40] have employed a 'gradually-augmented-node' learning algorithm to incrementally build a dynamic knowledge base capable of both acquiring new knowledge and relearning existing information. The rules are explicitly represented among the condition nodes, rules nodes and action nodes and the algorithm gradually builds the multilayer feedforward network. This connectionist incremental expert model is used as an animal identification system whose network structure is changed dynamically according to the new environment or through human intervention. In Fu's model [38] hidden units and additional connections are introduced appropriately when the network performance stagnates during training using backpropagation. Weight decay, pruning of weights and clustering of hidden units are incorporated to improve the generalization of the network.

Towell and Shavlik [39] have designed a hybrid learning system for problems from molecular biology. Disjunctive rules are rewritten as multiple conjunctive rules while building the network structure. Nodes and links are incorporated, upon instructions from the user, to augment the knowledge-based module. Expansion of the network guided by both the domain theory and training data has been reported by Opitz and Shavlik [41]. Dynamic addition of hidden nodes are made by heuristically searching through the space of possible network topologies, in a manner analogous to the adding of rules and conjuncts to the symbolic rulebase.

A way of using the knowledge of the trained neural model to extract the revised rules for the problem domain is described in [38, 42]. Meaningful rules can be extracted from the knowledge-based network in refined form by employing clustering, averaging, elimination, optimization and simplification [42]. The algorithm considers groups of links as equivalence classes, thereby generating a bound on the number of rules rather than establishing a ceiling on the number of antecedents. Note that this approach differs from that in [34], where a breadth-first search is employed to exhaustively find those input settings that cause the weighted sum to exceed the bias at a node.

NEURO-FUZZY EXPERT SYSTEMS

We now provides a review on neuro-fuzzy models for inferencing and rule generation, with the objective of generat-

TABLE 1 Comparative study of various expert systems

	Expert system	Connectionist expert system	Neuro-Fuzzy expert system	Knowledge-based connectionist/Neuro-Fuzzy expert system
Knowledge base	Knowledge acquisition and representation in the form of rules, frames, semantic nets or belief networks	Connection weights of trained neural net that were initialised with small random values	Connection weights of trained fuzzy neural net that were initialised with small random values	Connection weights of trained nonfuzzy/fuzzy neural net that were initialised with crude domain knowledge in rule form with binary link weights [38-40], (63-66], apriori class information and distribution of pattern points [68]
Knowledge refinement	Addition of new knowledge (say, as new rules)	Empirical addition of hidden nodes/ links	Empirical addition of hidden nodes/ links	Network optimization using growing and pruning of nodes/links, based on training data and additional knowledge [38-41, 68]
Inferencing	Matching facts with the existing knowledge base	Presentation of crisp input, forward pass and generation of crisp output	Presentation of fuzzy input, forward pass and generation of fuzzy output	Presentation of input, forward pass and generation of output
Rule generation		Crisp rules obtained during backward pass using changes in levels of input and output units [34], magnitude of connection weights [14, 37]	Fuzzy rules obtained during backward pass using node activations and link weights [27, 54-59]	Rules obtained during backward pass [38, 42, 64]; Negative rules also possible [68]

ing expert systems. A comparative analysis of the basic features of these models with those of the traditional and connectionist (non-fuzzy) versions is provided in Table 1.

Ways of integration

The state-of-the-art for the various techniques of combining neural networks and fuzzy sets involves synthesis at various levels. We categorize the different fusion methodologies, made so far, as follows [43].

1. incorporating fuzziness into the neural network framework : fuzzifying the input data, assigning fuzzy labels to the training samples, possibly fuzzifying the learning procedure and obtaining neural network outputs in terms of fuzzy sets [44, 45];
2. designing neural networks guided by fuzzy logic formalism : designing neural networks to implement fuzzy logic and fuzzy decision making, and to realize membership functions representing fuzzy sets [46, 47];
3. changing the basic characteristics of the neurons : neurons are designed to perform various operations used in fuzzy set theory (like fuzzy union, intersection, aggregation represented by AND, OR and hybrid operators) instead of the standard multiplication and addition operations [48, 49];
4. making the individual neurons fuzzy : the input and output of the neurons are fuzzy sets and the activity of the networks involving the fuzzy neurons is also a fuzzy process [50]; and
5. using measures of fuzziness as the error or instability of a network : the fuzziness / uncertainty measures of a fuzzy set are used to model the error or instability or energy function of the neural network based system [51].

As the existing neuro-fuzzy expert systems fall under categories 1 and 3 only, we shall not be concerned with the remaining groups (dealing mainly with classification or control problems) in this discussion.

Various methodologies

Neuro-fuzzy expert systems use the connection weights of trained fuzzy neural nets for encoding the knowledge base, thereby enabling one to incorporate the advantages of fuzzy set theory into the connectionist expert system model. Besides the generic advantages of neural networks and fuzzy systems, like parallelism, robustness, adaptivity and handling of uncertainty, one can incorporate their application specific merits in this paradigm. For example, the capability of neural nets in generating linearly non-separable decision regions can be exploited. Moreover, the modelling of uncertainty in the input description and output decision can be tackled by the concept of fuzzy sets. As an illustration of the characteristics of neuro-fuzzy expert systems, the models by Hayashi [52], Hudson *et al* [53], Sanchez [45], Mitra and Pal [27] and Romaniuk and Hall [54] are described here. Note that while the last model falls under category 3 of the fusion methodologies, the remaining models pertain to category 1.

Yoshida *et al* [55] have defuzzified real-life fuzzy data, using the Level Set representation, to produce the crisp inputs $\{+1, -1, 0\}$ required by the distributed single-layer perceptron-based model trained with the Pocket-algorithm for diagnosing hepatobiliary disorders. All contradictory training data are excluded, as these cannot be tackled by the model. In Hayashi's extension [52], the input layer consists of both fuzzy and crisp cell groups while the output is modeled only by fuzzy cell groups. The crisp cell groups are represented by m cells taking on two values in $\{(+1, +1, \ldots, +1), (-1, -1, \ldots, -1)\}$. Fuzzy cell groups, on the other hand, use binary m-dimensional vectors, each taking on values in $\{+1, -1\}$. Linguistic relative importance terms like *very important* and *moderately important* are allowed in each proposition; linguistic truth values like *completely true*, *true*, *possibly true*, *unknown*, *possibly false*, *false* and *completely false* are also assigned by the domain experts depending on the output values. Multiple correct pattern classes, using different linguistic truth values, is possible.

Hudson *et al* [53] use input nodes that simply represent the data values for signs, symptoms and test results (may be continuous or discrete). The interactive nodes account for the interactions which may occur between these parameters. A feed-forward neural network model is used for detecting carcinoma of the lung. Information is extracted directly from the accumulated data and then combined with a rule-based expert system incorporating approximate reasoning techniques. The learning method is an adaptation of the potential function approach to pattern recognition and is used to determine the weighting factors as well as the relative strengths of rules for two-class problems.

Sanchez [45] has associated two types of connection weights, *viz*, primary linguistic weights and secondary numerical weights to generate the knowledge base for a biomedical application (inflammatory protein variations) using a feedforward network. Triangular membership functions like *negative large*, *negative medium*, *negative small*, *approximately zero*, *positive small*, *positive medium* and *positive large*; or, *decreased*, *normal* and *increased* account for the linguistic weights while the quantitative weights lie in the range [0, 1]. The linguistic weights are tuned according to the information provided from the input-output examples while the numeric weights and the network topology are determined by solving fuzzy relation equations.

A cell recruitment learning algorithm, capable of forgetting previously learned facts by learning new information, has been employed by Romaniuk and Hall [54] to build a fuzzy connectionist expert system for determining the creditworthiness of credit applicants. The network consists of positive and negative collector cells along with unknown and intermediate cells and can handle fuzzy or uncertain data. Fuzzy functions like maximum, minimum and negation are applied at the neuronal levels depending upon the corresponding bias values. This incremental learning algorithm can be used either in conjunction with an existing knowledge base or alone.

Extraction of fuzzy 'If-Then' production rules is possible in [54-56], using a top-down traversal involving analysis of the node activations, their bias and the associated link weights. Rhee and Krishnapuram [56] have reported a method for rule generation from minimal approximate fuzzy aggregation networks. They estimate the linguistic labels and the corresponding triangular membership functions for the input features from the training data. Hybrid operators with compensatory behaviour, whose parameters can be learned during gradient-descent to estimate the type of aggregation, are employed at the neuronal level. Pruning of redundant features and/or hidden nodes helps in generating appropriate rules in terms of AND-OR operators that are represented by these hybrid functions.

Mitra and Pal [27] have reported the use of a fuzzy MLP for classification and rule generation. The input is represented in terms of π-functions corresponding to the linguistic properties *low*, *medium* and *high*. Handling of inputs in numeric, linguistic and set forms is possible. The output is in terms of fuzzy class membership values and enables efficient handling of overlapping pattern classes. The antecedent parts of rules are generated by backtracking along the maximum-weighted connection paths of the trained network. The consequent part is determined from a certainty measure which expresses the confidence (belief) of an output decision. The node excitations corresponding to a test pattern determines the appropriate 'If-Then' parts of a rule generated to justify an inferred decision. Note that this investigation provides a basic module for designing a classification type connectionist expert system. The rules thus obtained can also constitute the knowledge base of a traditional expert system in the same application domain. Here (unlike the other models) both the antecedent and consequent parts of these rules are provided in linguistic (or natural) form. Linguistic hedges/modifiers like *very*, *more* or *less* and *not* can be represented as antecedent clauses.

Consider the simple 3-layered network given in Fig 2 demonstrating a simple rule generation instance regarding class 1 [27]. A sample set of connection weights w^h_{ji}, input activation y_i^o and the corresponding linguistic labels are depicted in the figure. The solid and dotted-dashed paths (that have been selected) terminate at input neurons i_s and i_n respectively. The dashed lines indicate the paths not selected, using the w_{ji}^h and y_i^h values during backtracking. We select only those maximum

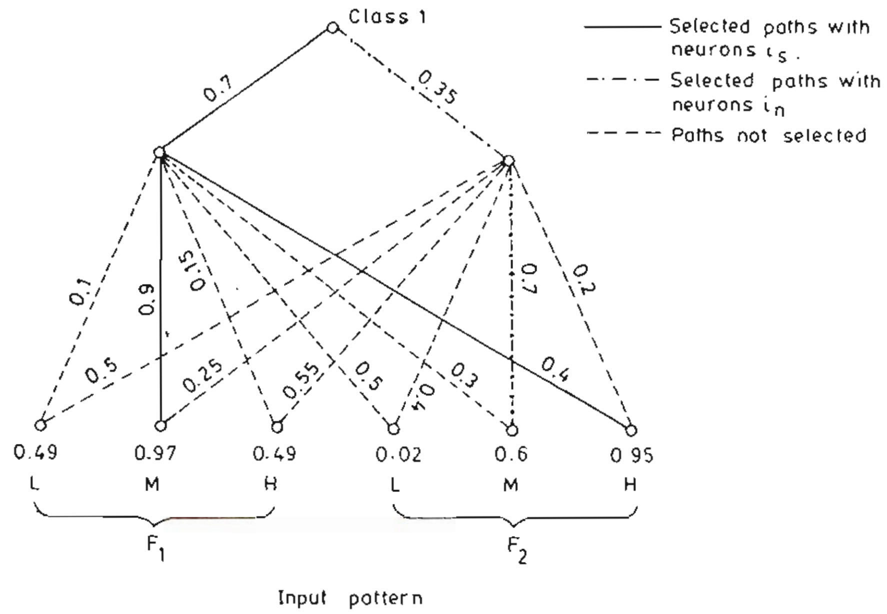

Fig 2 Example to demonstrate rule generation scheme by back-tracking

weighted paths from the output to the input layer, such that all neurons lying along them have $y_i^h > 0.5$. Let the certainty measure for the output neuron under consideration be 0.7. This corresponds to the label *likely* in the consequent part. Then the rule generated by the model in this case to justify its conclusion regarding class 1 would be

If F_1 is *very medium* AND F_2 is *high*

then *likely* class 1.

We generate clauses for an 'If-Then' rule until the net pathweights $wet_i{}^o$ satisfy the relation

$$\sum_{i_s} wet_{i_s}{}^o > 2 \sum_{i_n} wet_{i_n}{}^o$$

Here the net path weights are found to be 2.7 (= 1.6 + 1.1) and 1.05 for the i_s and i_n neurons respectively, such that 2.7 > 2 * 1.05. The modifier *very* (corresponding to F_1) is obtained by selecting the one having the *minimum* distance from the input vector. Similarly, in the case of F_2, modifiers are required using this minimum distance criterion.

The user can be queried in case of unknown or non available input features. Handling of missing or incomplete inputs is also possible. Applications have been made for vowel recognition and detection of *Kala-azar* (a tropical disease). This has been extended in [57] to design a neuro-fuzzy expert system for diagnosing hepatobiliary disorders. Here the linguistic labels at the input can be automatically tuned from the training data.

Another interesting application has also been reported [58] using the unsupervised, self-organizing Kohonen's net. This approach is completely different from the fuzzy Kohonen's net, in unsupervised mode, as reported in [46, 47]. The network has been modified to incorporate linguistic π-functions and contextual class information at the input, thereby enabling it to function under partial supervision. Unlike the other methods (involving layered feed forward nets under full supervision), this fuzzy version of the Kohonen's net has been effectively used for classification, querying and rule generation. Note that the three models [27, 57, 58] fall under category 1 of the fusion methodology.

A fourth model, using logical AND-OR functions (in terms of product-probabilistic sum and max-min) at the neuronal level, has been reported [59]. This is grouped under category 3. It has been observed that more meaningful rules (in terms of AND-OR clauses) can be generated here in case of simpler problems, although the classification performance is better in case of the more generalized sigmoidal function of [27, 57].

It is worth mentioning that all these models incorporate overlapping linguistic lables, represented by π-functions, at the input. This is different from the approach of Keller *et al* [60] where trapezoidal possibility distributions, sampled at discrete points, are used to represent fuzzy linguistic terms and modifiers. The concept of class membership helps the models to tackle overlapping and fuzzy pattern classes. This approach is an extension to the work of Keller and Hunt [44] for multiclass problems using multilayer networks. Another approach for fuzzification at input and output has been reported by Ishibuchi *et al* [61] using interval vectors. Although this is different, it will not be elaborated here as it does not cover the domain of connectionist expert system design or rule generation. Moreover, the conventional triangular membership functions used in control problems are also slightly different from the π-functions. It is to be noted that the triangular functions can be used in place of the more general continuous π-functions if desired.

Fuzzy knowledge based networks

Some attempts on using neuro-fuzzy approaches to the design of knowledge-based systems have also been recently reported. A brief survey on this field is provided here based on the studies of Masuoka *et al* [62], Kosko [63], Machado and Rocha [64], Pedrycz and Rocha [65] and Hirota and Pedrycz [66]. The first two approaches fall under category 1 of the fusion methodologies while the rest can be grouped under category 3. Analogous to the idea of eqns (11) - (13), the mapping from the n'-dimensional input space to the K-dimensional output space can be represented here as

$$\mathbf{X}'(t) \in \Re^{n'} \rightarrow \mathbf{Y}(t) \in \Re^{K} \tag{17}$$

where

$$y(t) = \psi\,[\mathbf{W}'(t) \otimes \mathbf{X}'(t)] \tag{18}$$

The learning algorithm becomes

$$\mathbf{W}'(t+1) = \mathbf{W}'(t) + \eta \Delta \mathbf{W}'(t) \tag{19}$$

for the N'_w connection weights such that $N'_w \le N_w$ of eqn (13). These trained connection weights form the knowledge base for the neuro-fuzzy knowledge-based expert system.

Knowledge extracted from experts in the form of membership functions and fuzzy rules (in AND-OR form) is used to build and preweight the neural net structure which is then tuned using training data. The model by Masuoka *et al* [62] consists of the input variable membership net, the rule net, and the output variable net. Fuzzy signed digraph with feedback, termed fuzzy cognitive map, has been used by Kosko [63] to represent knowledge. Additive combination of augmented connection matrices are employed to include the views of a number of experts for generating the knowledge network.

Machado and Rocha [64] have used a connectionist knowledge base involving fuzzy numbers at the input layer, fuzzy AND at the hidden layers and fuzzy OR at the output layer. The hidden layers chunk input evidences into clusters of information for representing regular patterns of the environment. The output layer computes the degree of possibility of each hypothesis. The initial network architecture is generated using knowledge graphs elicited from experts by the application of the knowledge acquisition technique of [67]. The experts express their knowledge about each hypothesis of the problem domain by selecting an appropriate set of evidences and building an acyclic weighted AND-OR graph to describe how these must be combined to support decision making.

Pedrycz and Rocha [65] have used basic aggregation neurons (AND-OR) and referential processing units (matching, dominance and inclusion neurons) to design knowledge-based networks. The inhibitory and excitatory characteristics are captured by embodying direct and complemented input signals and fully supervised learning is employed. Another related approach by Hirota and Pedrycz [66] has incorporated the use of fuzzy clustering for developing the geometric constructs leading to the design of knowledge-based networks.

Most of these models are mainly concerned with the encoding of initial knowledge by a fuzzy neural network followed by refinement during training. Extraction of fuzzy rules in this framework has been attempted in [62, 64]. Inference, inquiry and explanation are possible during consultation in [64]. Mitra *et al* [68] have recently designed a neuro-fuzzy knowledge based system for classification and rule generation. This approach falls under category 1 of the fusion methodologies. Here crude initial domain knowledge is encoded among the connection weights using the apriori class information (and their complements) and the distribution of pattern points in the feature space. An accurate estimation of the links connecting the output and hidden layers (in terms of the preceding layer link weights and node activations) is provided. The input, output and learning scheme are similar to that in [27]. Node growing and link pruning are incorporated to generate the optimal network architecture. Inferencing, querying and rule generation are demonstrated (as in [27]) for recognizing vowels and diagnosing hepatobiliary disorders. Negative rules, indicative of cases where a pattern does not belong to a class, can also be generated. This is specially suitable in the ambiguous cases where positive rules (dealing with the belongingness of a pattern to a particular class) cannot be obtained. The performance of the knowledge-based net is seen to be superior as compared to the models incorporating no initial knowledge.

CONCLUSIONS AND DISCUSSION

A review on neuro-fuzzy expert systems, along with their relevance, characteristics and merits, has been provided. Neuro-fuzzy models have been found to incorporate both the generic and application-specific merits of neural networks as well as fuzzy systems. This has resulted in the generation of more intelligent decision making systems. We have also included a brief survey on connectionist expert systems (without incorporating fuzzy sets) for the convenience of the readers. The use of knowledge-based networks has been discussed as one of the latest entrants in this field. A comparative study of the various methodologies has been provided in tabular form.

One of the major problems in connectionist/neuro-fuzzy expert system design is the choice of the optimal network structure. This has an important bearing on any performance evaluation. Moreover, the models are generally very much data dependant and the appropriate network size also depends on the available training data.

Various methodologies developed for selecting the optimal network structure include growing and pruning of nodes/links, employing genetic algorithms and embedding initial knowledge in the network topology. The latter approach has been investigated to some extent in the knowledge-based networks. Genetic algorithms have been used for determining the optimal set of connection weights of an MLP [69]. However, the use of rought sets [70] in this regard remains an interesting proposition. Exploration of these techniques should constitute an important area for future research in expert system design.

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AUTHORS

Nikhil R Pal obtained BSc with honors in physics and Master of Business Management from the University of Calcutta in 1979 and 1982 respectively. He obtained MTech (Comp Sc) and PhD (Comp Sc) from the Indian Statistical Institute in 1984 and 1991 respectively. Currently he is an Associate Professor in Machine Intelligence Unit of Indian Statistical Institute, Calcutta. From Sept 1991 to Feb 1993 and July 1994 to Dec 1994 he was with the Computer Science Department of the University of West Florida. He was a guest faculty of the University of Calcutta also. His research interest includes image processing, pattern recognition, fuzzy sets theory, measures of uncertainty, neural networks, genetic algorithms, and fuzzy logic controllers. He is an associate editor of the International Journal of Approximate Reasoning and IEEE Transactions on Fuzzy Systems.

* * *

Sitabhra Sinha obtained his BSc and MSc degrees in Physics from the University of Calcutta in 1992 and 1994, respectively. He is currently pursuing a PhD degree at the Machine Intelligence Unit of the Indian Statistical Institute, Calcutta.

His research interests include artificial neural networks, brain process modelling, nonlinear dynamics & cooperative phenomena in network systems, and cellular automata.

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Jayadeva received his BTech and PhD degrees in Electrical Engineering from the Indian Institute of Technology, Delhi in 1988 and 1993, respectively. He is currently an Assistant Professor in the same departemnt. His research interests include Artificial and Biological Neural Networks, their applications and Implementation; Parallel Algorithms; VLSI CAD; VLSI Design; and Optimization. He is the author/co-author of over 13 publications in refereed journals and conferences, as well as a chapter in a book on Optimality in Biological and Artificial Networks, to be published by MIT Press. He is a recipient of the Young Scientist Award from the International Union of Radio Science (1996), the Young Scientist Medal of the Indian National Science Academy (1995), the IEEE Aerospace, Electronic Systems and Communication Chapter India award in the area of Aerospace and Electronic Systems (1991), and the Rajiv Bambavale Cash Prize for the best BTech project in Electrical Engineering (1988).

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Nicolaos B Karayiannis received the Diploma degree in Electrical Engineering from the National Technical University of Athens in 1983, and the MASc and PhD degrees in Electrical Engineering from the University of Toronto in 1987 and 1991, respectively.

He is currently an Assistant Professor in the department of Electrical Engineering, University of Houston. He has published more than 60 papers, including 19 in technical journals, and is the co-author of the book Artificial Neural Networks: Learning Algorithms, Performance Evaluation, and Applications (Kluwer, 1993). His current research interests include supervised and unsupervised learning, applications of artificial neural networks in image processing and analysis, applications of fuzzy logic in neural network training, and fuzzy learning vector quantization.

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Gopathy Purushothaman received the BE degree from the University of Madras (1989), the MTech degree from the Indian Institute of Technology, Kanpur (1990), and the MS degree from the University of Houston (1994). He is presently a PhD candidate in Electrical and Computer Engineering at the University of Houston.

He received the Urvish Medh Memorial Award from the University of Houston in 1995, the Theoretical Development Award at the Artificial Neural Networks in Engineering Conference (Missouri, USA) in 1994, the Jawaharlal Nehru Memorial Award from the Government of India and the University of Madras Gold Medal, in 1989.

His research interests are computational neuroscience, psychophysics and artificial neural networks.

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Laxmidhar Behera was born on January 1967. He received the bachelor's degree and master's degree in Electrical Engineering from Regional Engineering College, Rourkela, in 1988 and 1990 respectively. He carried out his doctoral research at IIT, Delhi, from 1991 to 1995.

At present he is working as Assistant Professor at Birla Institute of Technology and Science, Pilani. He is now actively associated with the research work carried out at Centre for Robotics, BITS, Pilani. Previously he worked as lecturer at REC, Rourkela, from 1990 to 1991. His areas of interests are intelligent robotics, neural networks, adaptive fuzzy control and evolutionary computation.

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