Summer Internship in Cryptology 2020

R C Bose Centre for Cryptology and Security, Indian Statistical Institute

Solve the following problems and submit the solutions in the appropriate fields of the online Application Form to apply for the Summer Internship program in Cryptology (2020) organized by the R C Bose Centre for Cryptology and Security, Indian Statistical Institute, Kolkata. You may refer to any academic resource for information regarding these problems, but sharing the problems on online discussion forums or blogs will lead to disqualification of your application.

Please keep a soft copy (scanned image, doc, or pdf file) of your detailed solution/ calculation readily available. If selected for the second round you may be asked to submit it. Your detailed solution/ calculation should contain the essential steps to solve the problem also it should contain your name and the name of your institute. It is your responsibility to maintain sufficient clarity, legibility, and authenticity of your solution.
Please carefully verify your answers before submitting the application form. We will not entertain your requests for modification of your answers/ resubmission of your application under any circumstances.

The online application form is available at `www.isical.ac.in/~rcbose/internship`.

Problem 1: Count Triangles

Three persons are said to form a TRIANGLE if all of them know each other or none of them know each other. Six persons are chosen from a population. What is the minimum number of TRIANGLEs that can be found among these six persons? The minimum is taken over all possible population and all possible choice of six persons.

Problem 2: Find the Number

Consider the following ten sets of natural numbers:

$P_1 = \{n \in \mathbb{N} : \ n \text{ is a perfect square}\}$.

$P_1' = \{n \in \mathbb{N} : 7|n\}$.

$P_2 = \{n \in \mathbb{N} : n+1 \text{ is a prime}\}$.

$P_2' = \{n \in \mathbb{N} : n \text{ has two digits}\}$.

$P_3 = \{n \in \mathbb{N} : n+1 \text{ is a perfect square}\}$.

$P_3' = \{n \in \mathbb{N} : n \text{ has more than two digits}\}$.

$P_4 = \{n \in \mathbb{N} : n \text{ has 8 as a digit}\}$.

$P_4' = \{n \in \mathbb{N} : n \text{ has 3 as a digit}\}$

$P_5 = \{n \in \mathbb{N} : 11|n\}$.

$P_5' = \{n \in \mathbb{N} : \text{the sum of the digits of } n \text{ is a prime}\}$.

Let

$$P = \bigcap_{i=1}^{5} \left(P_i \triangle P_i'\right),$$

where $A \triangle B$ denotes the symmetric difference between the sets A and B. Find the smallest element in P.

Problem 3: Crack the Cipher

Renowned cryptanalyst Prof. Shanku was presented with an intercept:

ILKAI LKAIF VSTDS PSVLI UTFLK
02 10 8 7 000 0

Along with the above short ciphertext the intelligence agencies also provided the following background information on it:

- It is confirmed that the sender and the receiver has neither met each other nor communicated by other means for a long time. Thus it is very unlikely that they have a shared key with them which they are using for communication.
- Both the sender and the receiver has immense knowledge about old communication/encryption technologies. Sleuths confirm that among other things the sender was recently consulting books on Morse codes in the local library.

Help Prof. Shanku cryptanalyze the intercept.