Public Key Cryptography

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Outline

1. Background
   - Basic Concepts in Complexity Theory
   - Some Number Theoretic & Algebraic Algorithms
   - Some Computational Hard Problems and their Application
2. Cryptography
3. Public Key Cryptography
   - Diffie Hellman Key Exchange Protocol
   - Public Key Cryptosystems
   - RSA & ElGamal
   - Elliptic Curve Cryptography
   - Other Public Key Cryptosystems: Post Quantum
4. Elliptic Curve Cryptography
5. Open Source Libraries

Outline

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Representation of Numbers

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- Each number can be represented in the form of different basis.
- Any number n between b^{k-1} and b^k is a k-digit number to the base b.
- Number of digits = $[\log_b n] + 1$ (basis b)
- Number of bits (size of number) $[\log_2 n] + 1$

Complexity Theory: I

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Definition (Running Time)

The number of basic (primitives) operations (steps) to execute an algorithm (computational complexity). *Running time of an algorithm is depend on the size of the input.*

Definition (Size of an Input)

In bits, in digits, in bytes, in words etc.....

Definition (Space Complexity)

It measures the amount of temporary storage used when performing a computational task.

Definition (Big- O)

Complexity Theory: II

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$f(n) = O(g(n))$ if there exists a positive constant c and a positive integer $n_\circ$ such that $0 \leqslant f(n) \leqslant cg(n)$ for all $n \geqslant n_\circ$. Note: g is simpler function than f and it does not increase much faster than f.

Some Examples of Big-O

- Let $f(n) = 2n^3 + 3n^2 + 4n + 5$ & $g(n) = n^3$. Then $f = O(g)$,for take $n_0 = 5, c = 3$.
- Let $f(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_0$ with $a_k > 0$, $f(n) = O(n^k)$.
- $x^n = O(e^n)$ for any positive power n

Complexity Theory: III

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Note: The notation Big-O is used to represents an upper bound of the computational complexity of an algorithm in the *worst-case scenario*.

Definition (Small-o)

$f(n) = o(g(n))$ if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \to 0.$$

Note: $g(n)$ is upper bound of $f(n)$ i.e $f(n) << g(n)$.

Some Examples of small-o

- For any positive integer a, $a^n = o((n!))$
- $n! = o(n^n)$

Remark 1: : Other notations : Ω, ω, Θ.

Orders of common functions[2]

$O(1)$	Constant
$O(\log \log n)$	Double-Logarithmic
$O(\log n)$	Logarithmic
$O(\log^c n), c > 1$	Poly-Logarithmic
$O(n^c), 0 < c < 1$	Fractional
$O(n)$	Linear
$O(n^c), c > 1$	Polynomial
$L_n(\alpha, c)$[1]	Sub-exponential
$O(c^n)$	Exponential
$O(n!)$	Factorial

[1] $O(exp(c + O(1))(n)^{\alpha}(\log n)^{1-\alpha})$

[2] http://en.wikipedia.org/wiki/Big_O_notation

Complexity of an Algorithm I

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- Complexity of an algorithm is said to be in polynomial time if its complexity is $O(n^c)$, where n is the bit length of the input, & $c > 1$.
- Algorithms with complexity of form $c^{f(n)}$, where $c > 1$ & f is a polynomial in n are called exponential time algorithm.
- A sub-exponential algorithm is one for which the time complexity is in between of polynomial and exponential $(L_n(\alpha, c))$

Complexity of an Algorithm II

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Definition

A decision problem is said to be in class P if it can be solved in polynomial time.

Example

Instance: $n \in Z^+$
Question: Is n prime?
Answer: Yes, $[O(\log n)^6$ using AKS algo]

Definition

A decision problem is said to be in class NP if 'yes' answer can be verified in polynomial time.

Complexity of an Algorithm III

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Definition

A decision problem is said to be in class *co-NP* if 'no' answer can be verified in polynomial time.

Example

Instance: $n \in Z^+$
Question: Is n composite?

Definition

L_1 and L_2 be two decision problems. L_1 is said to poly-time reduce to L_2, written $L_1 \leqslant_p L_2$, if there is an algorithm that solves L_1 using only polynomial calls to an algorithm for solving L_2 as a subroutine. This means a polynomial time algorithm for L_2 implies a polynomial time algorithm for L_1.

Complexity of an Algorithm IV

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Example

$QRP \leqslant_P IFP$

Definition

A decision problem L is said to be NP-complete if

- $L \in NP$
- $L_1 \leqslant_P L$ for every $L_1 \in NP$.

Example

Subset Sum Problem is **NP** complete problem: given a set of positive integers $\{a_1, a_2, \cdots a_n\}$ and a positive integer s, determine whether or not there is a subset of the a_i that sum to s.

Complexity of an Algorithm V

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Definition

A decision problem is said to be *NP*-hard if any *NP*-complete problem polynomially reduces to it.

Examples

- Computational version of subset sum problem is **NP**-hard problem.

Important Topics of Number Theory

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- The Euclidean Algorithm
- Modular Arithmetic
- Arithmetic functions and their Properties
- Solution of Linear and Quadratic Congruences, CRT
- Primality testing and Factorization

The Euclidean Algorithm

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The Euclidean Algorithm finds the greatest common divisor of two integers a and$b$.
For example, if we want to find $gcd(287, 91)$, we divide 287 by 91:

$$287 = 91 * 3 + 14 .$$

We have if $a|b$ and $a|c$, then $a|(b + c)$.

$$gcd(287, 91) = gcd(91, 14)$$

$$gcd(91, 14) = gcd(14, 7)$$

$$gcd(14, 7) = 7$$

Therefore, $gcd(287, 91) = 7$.

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- **Theorem** The $gcd(a, b)$ is the least positive value of $ax + by$, where x and y range over all integers.
- **Theorem** An integer solution (x, y) of equation $ax + by = c$ exists if and only if c is divisible by $gcd(a, b)$.

- $gcd(a, b, c) = gcd(gcd(a, b), c)$

Number Theoretic Algorithms[3]

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Complexity of Basic Operations in $\mathbb{Z}_n$

Operations		Complexity
Modular Addition	$(a+b) \mod n$	$O(\log n)$
Modular Subtraction	$(a-b) \mod n$	$O(\log n)$
Modular Multiplication	$(a.b) \mod n$	$O((\log n)^2)$
Modular Inversion	$a^{-1} \mod n$	$O((\log n)^2)$
Modular Exponentiation	$a^k \mod n, k < n$	$O((\log n)^3)$

[3] Chapter 2, [2]

Solution of the Congruence

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Consider $f(x) \equiv 0 \pmod{m}$, $f(x)$ being a polynomial with integer coefficients. Any $n \in \mathbb{Z}$ is called a solution of the congruence if $f(n) \equiv 0 \pmod{m}$.
The solution is not unique as for any k, such that $n \equiv k \pmod{m}$,

$$\begin{aligned} & f(k) \equiv f(n) \pmod{m} \\ \Longrightarrow\ & f(k) \equiv 0 \pmod{m} \end{aligned}$$

Thus every congruence having one solution has infinitely many solutions.
The congruence $ax \equiv b \pmod{m}$ is called a linear congruence.

Euler Fermat Theorem

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We define Euler's totient function $\phi(m)$ as

$$\phi(m) = \begin{cases} 1 \text{ when } m = 1, \\ \text{number of positive integers that are } \leq m \text{ and co-prime to } m \end{cases}$$

Euler Fermat Theorem For $(a, m) = 1$, we have

$$a^{\phi(m)} \equiv 1 \mod m.$$

Fermat Little Theorem

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Fermat Little Theorem is a corollary to Euler-Fermat Theorem.
Let p be a prime number. If $(a, p) = 1$ then from Euler Fermat theorem, we have $a^{\phi(p)} \equiv 1 \pmod{p}$ but $\phi(p) = p - 1$, therefore $a^{p-1} \equiv 1$.
Multiplying both side by a, we get $a^p \equiv a \pmod{p}$.

Otherwise, if $(a, p) \neq 1$, then $p|a$ and $a \equiv 0 \pmod{p}$.
$\implies a^p \equiv 0 \pmod{p}$
$\implies a^p \equiv a \pmod{p}$
Which is Fermat Little Theorem.

The Chinese Remainder Theorem (CRT)

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Let $m_1, m_2, \cdots, m_r$ be pairwise mutually prime positive integers. For arbitrary integers $b_1, b_2, \cdots, b_r$, the system of linear congruences

$$\begin{aligned} x &\equiv b_1 \pmod{m_1} \\ x &\equiv b_2 \pmod{m_2} \\ &\vdots \\ x &\equiv b_r \pmod{m_r} \end{aligned}$$

has a unique solution $x = \sum_{k=1}^{r} b_k M_k M_{k'}$ modulo m,
where $M = m_1 m_2 \cdots m_r$ and $M_k = M/m_k$, $M_k M_{k'} \equiv 1 \pmod{m_k}$.

CRT Problem

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To find the least value of x which satisfy,

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 2 \pmod 7$$

The calculations are given in the following table.

b_k	m_k	M_k	$M_k(modm_k)$	$M_{k'}$	$b_k M_k M_{k'}$
2	3	35	2	2	140
3	5	21	1	1	63
2	7	15	1	1	30
				sum	233

The least number of things are obtained by reducing 233 (mod 105) i.e. 23.

Quadratic Residues

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Consider the quadratic congruences of the type

$$x^2 - n \equiv 0 \pmod{p} \qquad \ldots(1)$$

where p is a prime. We see that the congruence can have at most 2 solutions.

If x is a solution, then $(-x)$ is also a solution. Thus, (1) has either two solutions or none. When (1) has solutions, we say that n is a quadratic residue modulo p and we write nRp, otherwise if (1) has no solution then we say that n is a quadratic non-residue modulo p and we write $n\bar{R}p$.

The Legendre's symbol $\left(\frac{n}{p}\right)$ is defined as

$$\left(\frac{n}{p}\right) = \begin{cases} 0 \text{ if } p|n, \\ 1 \text{ if } nRp, \\ -1 \text{ if } n\bar{R}p. \end{cases}$$

Jacobi Symbols

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If P is an odd positive integer with prime factorization

$$P = \prod_{i=1}^{r} p_i^{a_i}$$

The Jacobi symbol $\left(\frac{n}{P}\right)$ is defined as,

$$\left(\frac{n}{P}\right) = \prod_{i=1}^{r} \left(\frac{n}{p_i}\right)^{a_i}, \quad if\ P > 1$$

and

$$\left(\frac{n}{1}\right) = 1$$

Primitive Roots

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Let a and m be relatively prime and $m > 1$. Then a is called a primitive root modulo m if

$$a^k \not\equiv 1 \; mod \; m$$

whenever $k < \phi(m)$.

Theorem:
There are exactly $\phi(p-1)$ primitive roots modulo p.

Computing Euler's function

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- If p is a prime, then for integers $k \geq 1$

$$\phi(p^k) = p^{k-1}(p-1)$$

- If m and n are co-prime, then

$$\phi(mn) = \phi(m).\phi(n)$$

- Therefore, the value of ϕ(n) for any positive integer n can be computed by writing prime factorization of n, for $n = p_1^{a_1}.p_2^{a_2}.....p_t^{a_t}$

$$\phi(n) = (p_1-1).p_1^{a_1-1}.(p_2-1).p_2^{a_2-1}....(p_t-1).p_t^{a_t-1}$$

And so,

$$\phi(n) = n.\prod_{p|n}(1-\frac{1}{p})$$

Some Primality Testing Algorithms [4]

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Algorithms	Type	Complexity
Jacobi Some test	True Primality Test	$O(k(\log n)^{\log\log\log n})$
Elliptic Curve Primality Proving test (ECPP)	-do-	$O(\log n)^{6+\epsilon}$
Agrawal, Kayal, and Saxena Test	-do (deterministic)-	$O((\log n)^{10.5}$
Miller-Rabin test	Probabilist Primality Test	$O((t\log n)^3)$, (t modular exp)
Solovay-Strassen Test	Probabilist Primality Test	$O((t\log n)^3)$, (t modular exp)
Fermat's test	Probabilist Primality Test	$O((t\log n)^3)$, (t modular exp)

[4] Chapter 4, [2]

Fermat's Primality Test

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Fermat's Little theorem for p prime and an integer a such that $p \nmid a$ says that $a^{p-1} \equiv 1 \; mod \; p$.

If $a^{p-1} \not\equiv 1 \; mod \; p$ for some a then p is composite. Thus, if $a^{p-1} \equiv 1 \; mod \; p$ for one or more values of a, then we say that p is probably a prime.

Example: Fermat pseudo-prime: $n = 341$**,**

$$2^{340} \equiv 1 \mod 341 (11 \times 31)$$

Fermat's Primality Test: Algorithm

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Input: n - an integer to test for primality, $n > 3$, k - a parameter that determines the number of times to test for primality.

Output: Composite if n is composite.
Otherwise, probably prime.

Repeat k times.

- Pick 'a' randomly in the range $[2, n-2]$
- If $a^{p-1} \not\equiv 1 \; mod \; p$, then return composite.
- If composite is never returned: return probable prime.

Solovay-Strassen Test

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Concept:

- Euler proved that for an odd prime number p and any integer a, $a^{p-1/2} \equiv \left(\frac{a}{p}\right) \mod p$, where $\left(\frac{a}{p}\right)$ is Legendre symbol.
- Given an odd number n, we can contemplate weather or not the congruence $a^{n-1/2} \equiv \left(\frac{a}{n}\right) \mod n$, holds for various values of the base a. If n is prime, then this congruence holds for all values of a.
- So, if we pick a value of a at random and test the congruence then as soon as we find an a which does not fit the congruence we know that n is not prime.
- Example of Euler Pseudo-prime is $91 = (7 \times 13)$ to the base 9: $9^{45} \equiv (\frac{9}{91}) \equiv 1$

Solovay-Strassen Test: Continued –

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Pick a random integer $a < n$, and do the followings:

- If $gcd(a, n) > 1$, then return composite.
- If $\left(\frac{a}{n}\right)$ is not equal to $a^{n-1/2}$, then return composite.
- Else, return probable (pseudo) prime.

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Exercises

- Complexity for computation of Jacobi /Legendre Symbol.
- Complexity for finding square root of Quadratic Residue Modulo p.
- Complexity for getting solution the system of linear congruences using Chinese Remainder Theorem.

Important Topics of Finite Fields

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- Irreducible and Primitive Polynomials, and methods for testing them.
- Construction of Finite Fields and Basis
- Arithmetic over Finite Fields
- Factorization of polynomial over finite fields
- Combinatorial Problems over Finite Fields

Field

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Definition

A field $(\mathbb{F}, +, \cdot)$ *consists of a set* $\mathbb{F}$ *along with 2 binary operations* $+$ *&* $\cdot$ *on* $\mathbb{F}$ *satisfying the following conditions*

i. $(\mathbb{F}, +)$ *is a commutative group,*

ii. $(\mathbb{F} \setminus \{0\}, \cdot)$ *is also a commutative group and*

iii. *The operation* $\cdot$ *is distributive over* $+$.

Finite Fields

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- A **finite field** is a field $\mathbb{F}$ which contains a finite number of elements.
- If $\mathbb{F}$ is a finite field, then $\mathbb{F}$ contains p^m elements for some prime p and integer $m \geq 1$.
- For every prime power order p^m, there is a ! finite field of order p^m. This field is denoted by $\mathbb{F}_{p^m}$, or sometimes by $GF(p^m)$.
- For $m = 1$, $\mathbb{F}_p$ or $GF(p)$ is a field. If p is a prime then $\mathbb{Z}_p$ is a field.

$$\mathbb{F}_p \cong GF(p) \cong \mathbb{Z}_p.$$

Construction of Finite Field of Order p^m

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- First select an irreducible polynomial $f(x) \in \mathbb{Z}_p[x]$ of degree m.
- The ideal $< f(x) >$ is e a maximal ideal.
- Then $Z_p[x]/ < f(x) >$ is a finite field of order p^m.
- For each $m \geq 1$, $\exists$ a monic irreducible polynomial of degree m over $\mathbb{Z}_p$.
- Hence, every finite field has a polynomial basis representation.

Theorem

The number of monic irreducible polynomials in $\mathbb{F}_q[x]$ of degree n is given by

$$\frac{1}{n}\sum_{d|n}\mu(d)q^{n/d},$$

where μ is Möbius function.

Finite Fields $GF(2^3)$

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- Consider an irreducible polynomial $x^3 + x + 1$ over $GF(2)$
- $GF(2)[x]/ < x^3 + x + 1 >=$
  $\{0, 1, x, x+1, x^2, x^2+1, x^2+x, x^2+x+1\}$
- One to one correspondence between $GF(2^3)$ and $\mathbb{Z}_8$.
  Similarly, $GF(2^3)$ maps all of the polynomials over $GF(2)$ to the 8 polynomials shown above.

Finite Field Basis I

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☞ $GF(p^m)$ is a vector space over $GF(P)$ of dimension m.
☞ Any set of m Linearly Independent elements can be used as a basis.

1. Polynomial Basis: If α is roots of generating polynomial $f(x) = 0$ with $GF(P^m)^\star =< \alpha >$ then $\{0, \alpha^0, \alpha^1, \cdots \alpha^{n-1}\}$ will be basis of $GF(p^m)$.
2. Normal Basis: : Basis of the form $\{\alpha, \alpha^p, \cdots \alpha^{p^{m-1}}\}$ is called normal basis, where α is a normal element.

Finite Field Basis II

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Polynomial Basis			Normal Basis			$\alpha^7 = 1$ Exp.
1	α	α^2	α^3	α^6	α^5	
0	0	0	0	0	0	0
1	0	0	1	1	1	$1 = \alpha^0$
0	1	0	0	1	1	α
0	0	1	1	0	1	α^2
1	1	0	1	0	0	α^3
0	1	1	1	1	0	α^4
1	1	1	0	0	1	α^5
1	0	1	0	1	0	α^6

Figure: Representation of $GF(2^3), f(x) = x^3 + x + 1$

Complexity of Arithmetic Operations in $\mathbb{F}_{p^m}$[5]

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Complexity wrt Polynomial Basis Representation

Operations		Complexity in terms of $\mathbb{Z}_p$-operations
Addition	$(g(x)+h(x)) \mod f(x)$	$O(m)$
Subtraction	$(g(x)+h(x)) \mod f(x)$	$O(m)$
Multiplication	$(g(x).h(x)) \mod f(x)$	$O(m^2)$
Inversion	$g(x)^{-1} \mod f(x)$	$O(m^2)$
Exponentiation	$g(x)^k \mod f(x), k < p^m$	$O((\log p)m^3)$

* $f(x)$ is irreducible polynomial of degree m.

Exercises

- Running time complexity of Extended Euclidean Algorithm in $\mathbb{Z}_p[x]$

[5] Chapter 2, [2]

Hard Problems and One-way Function I

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PKC is based on

- Mathematical problems believed to be *hard* to solve.
  Hard means computationally infeasible
  Hard=2^{128} or more operations : ECC-256,
- Trapdoor one-way function

Hard Problems and One-way Function II

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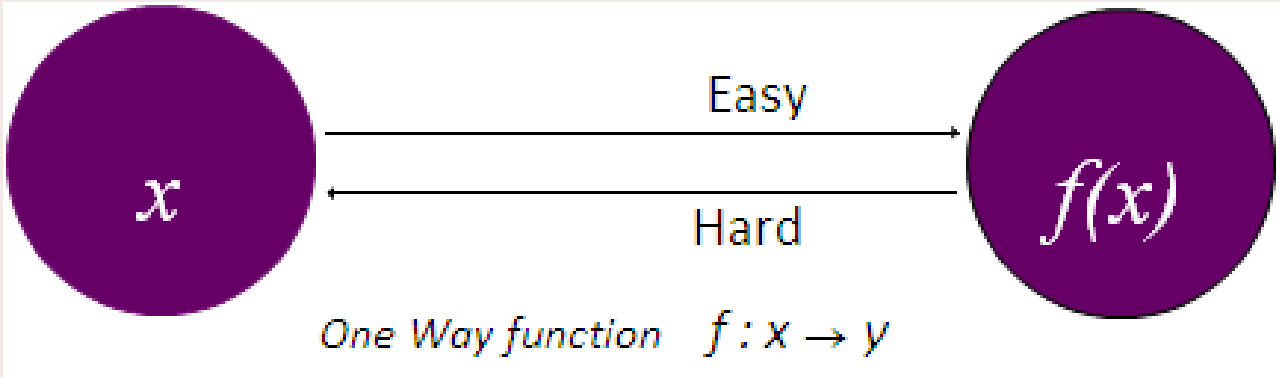
Definition (One-way Function)



Easy: There exists a polynomial-time algorithm that gives $y = f(x)$.

Hard: For given any $y = f(x)$ it is computationally infeasible to find x.

Hard Problems and One-way Function III

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Definition (trapdoor one-way function)

A trapdoor one-way function is a one-way function with the additional property that given some extra information (called the trapdoor information) it becomes feasible to find for any given $y \in Im(f)$, an $x \in X$ such that $f(x) = y$.

Some Computationally Hard Problems I

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- Integer factorization (RSA)
- Subset sum problem (Knapsack)
- Quadratic Residuosity Problem (Rabin)
- DLP in finite fields (El Gamal)
- DLP in elliptic curve over finite fields (ECC) (ElGamal, ECIES, ECDSA)
- CDHP, DDHP, GDHP, GDLP
- Conjugacy Search Problem
- Group action and Semi-group action problem
- BDHP (Joux Protocol)
- Factorization with Discrete Logarithm Problem (Over Group Ring)
- Discrete logarithm with conjugacy Search Problem

Some Computationally Hard Problems II

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- Solving system of multivariate polynomial equations over finite fields and isomorphism of polynomials (HFE, Sflash, Quartz)
- Decoding Problem: GBD/GD (Code based Cryptography: McEliece's Crypto-system)
- Lattice basis reduction (NTRU, FALCON)
- Hash Based PKC (Gravity-SPHINCS), PICNIC, SPHINCS+
- Computational Supersingular Isogeny Problem

Integer factorization (RSA)

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N = Large Composite Integer. IFP: Find factor of N.

Sub-exponential Algorithms

Methods	Running Time
Continued Fraction Method	$T(CFRAC) = O\big((L_N(1/2, \sqrt{2})\big)$
Lenstra's Elliptic Curve Method	$T(ECM) = O\big(L_p(1/2, \sqrt{2}).(\log N)^2)\big)$
Multiple Polynomial Quadratic Field Sieve Method	$T(MPQS) = O\big(L_N(1/2, 3/2\sqrt{2})\big)$
General Number Field Sieve Method	$T(GNFS) = O\big(L_N(1/3, \sqrt[3]{64/9})\big)$
Special Number Field Sieve Method	$T(SNFS) = O\big(L_N(1/3, \sqrt[3]{32/9})\big)$

Integer factorization (RSA)

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Other Integer Factorization Algorithms (Exponential)

Methods	Running Time
Lehman's Method	$O(N^{\frac{1}{3}+\epsilon})$
Shanks' Square Form Factorization Number	$O(N^{\frac{1}{4}})$
Shanks' Class Group Method	$O(N^{\frac{1}{5}+\epsilon})$

DLP in finite fields (El Gamal)

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Definition (Discrete Logarithm Problem (DLP))

- Given $y \in G$, The DLP in G is to find the unique $x \in Z_N$ such that $y = g^x$. Such an x is called the discrete log of y with respect to base g.

DLP in finite fields (El Gamal)

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Algorithm for Solving DLP

Methods	Running Time / Space
Shanks' Baby-Step Giant-Step Algorithm	$O(\sqrt{N}\log N)$ (group Operations) / $O(\sqrt{N})$
Silver-Pohlig-Hellman, $N = \prod_{i=1}^{i=k} p_i^{\alpha_1}$	$O(\sum_{i=1}^{i=k} \alpha_i(\log N + \sqrt{p_i}))$ (group Operations) /Comparable
Pollard's Rho	$O(\sqrt{N})$ (group Operations) /Negligible

DLP in finite fields (El Gamal)

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Algorithm for Solving DLP

Adleman's Index Calculus Algorithm for $\mathbb{F}_p^\star$	$O\left(L_p(1/2,c)\right)$
Coppersmith et al Algo (for $\mathbb{F}_p^\star$)	$O\left(L_p(1/2,c)\right)$ (c is smaller than Adleman's Algo)
Index Calculus Algorithm for $GF(q)$: $q=p^m$	$O\left(L_q(1/2,c)\right)$
Coppersmith Index Calculus Algorithm for $GF(2^m)$	$O\left(L_{2^m}(1/3,c)\right)$
General Number Field Sieve Algorithm (for $GF(q)$)	$O\left(L_q(1/3,c)\right)$, $c=(64/9)^{1/3}$

Outline

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Cryptography for Secure Communication

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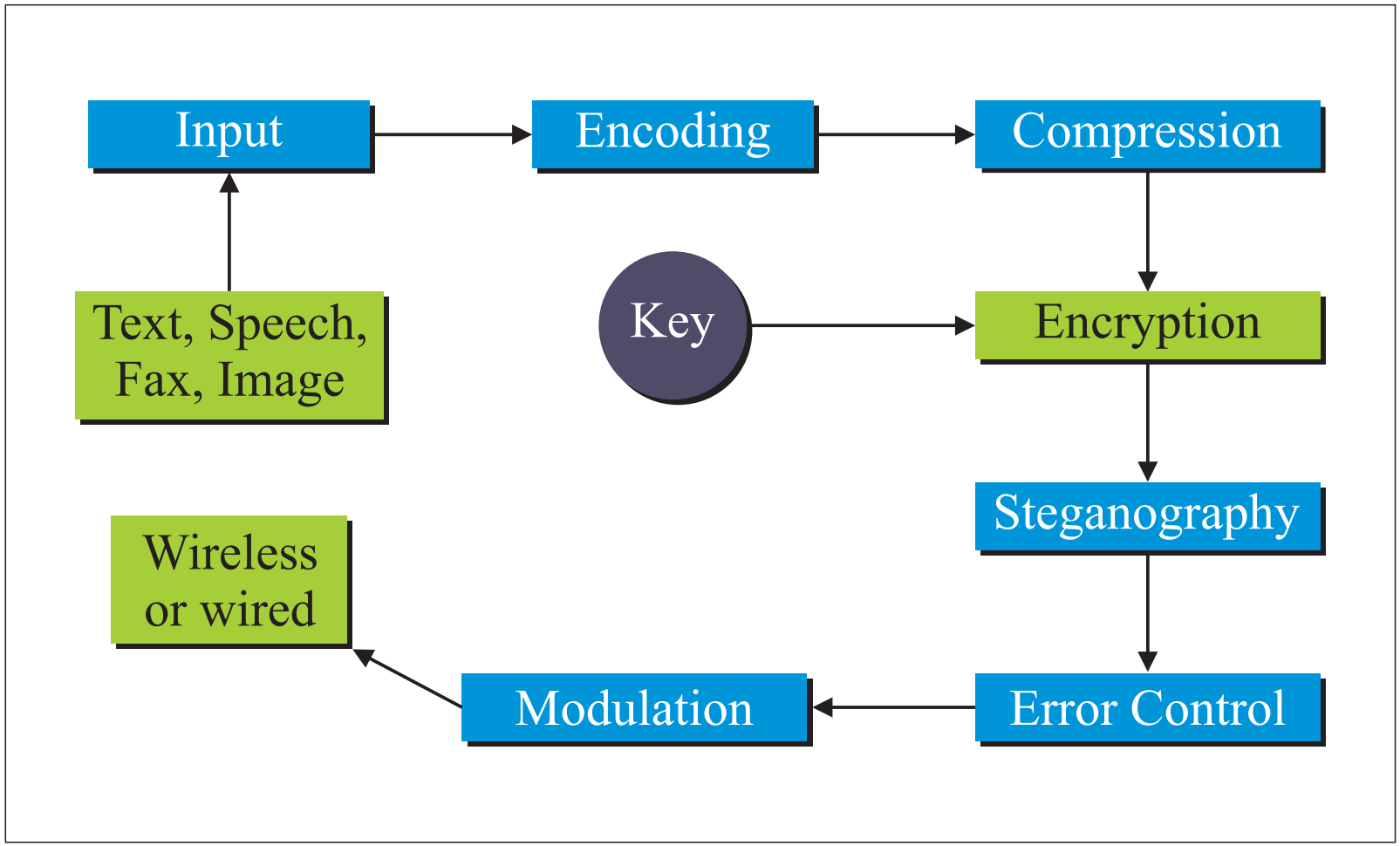

Cryptography for Secure Communication

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Input to Source

- Text
- Speech
- Picture/Movie
- Fax

Example

- Text $\Rightarrow$ ASCII
- Speech $\Rightarrow$ PCM
- Picture/Movie $\Rightarrow$ JPEG, MPEG
- FAX $\Rightarrow$ Modified Huffman Coding

Cryptology

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- **Cryptography** Design of algorithms, systems, protocols which are used to protect information against specific threats. PRIVACY, AUTHENTICATION, INTEGRITY & NON-REPUDIATION.
- **Cryptanalysis** Evolving mathematical methods to check if the specified cryptographic design achieves the desired security goal. It is a science which deals with how to defeat of achieving '*Security Goals*'.
- Cryptology = Cryptography + Cryptanalysis

Cryptology I

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- **Symmetric Key Crypto :** $\rightarrow$ Alice and Bob both agree on common key.
- **Public Key Crypto :** $\rightarrow$ PKC use a pair of keys $\longrightarrow$ public key, private key. Computing the private key form public key has to be intractable.
- **Hash Function:** $\rightarrow$ function that maps a bit string of arbitrary length into an output of fixed number of bits, called message digest or hash value.
- **Digital Signature:** A digital signature is a data string which associates a message (in digital form) with some originating entity.
- **Non-repudiation:** An entity should not be allowed to deny valid signatures made by him.

Cryptology II

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- **Data Origin Authentication /Message-Authentication:** techniques provide to receiver which receives a message assurance of the identity of the party which originated the message.
- **Entity Authentication/Identification:** Alice proves her identity to Bob. Alice demonstrates to Bob her knowledge of a secret piece of information.

Outline

Introduction

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- PKC developed by Diffie-Hellman and Merkle in the mid 1970s.
- In PKC, each user has pair of keys: Private Key and Public Key.
- Every one has access to the public key but private information can be accessed by only the owner.
- PKC depends on computationally hard problems that prevent inverting the public map.
- Computing the private key form public key has to be intractable.

Public-Key Cryptosystems

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Public Key Cryptosystem CS is a five-tuple

$$CS = (\mathcal{P}, \mathcal{C}, \mathcal{K}, \mathcal{E}, \mathcal{D})$$

- $\mathcal{P}$: Plaintext Space, $\mathcal{C}$: Ciphertext Space
- $\mathcal{K}$: Key Space
- $\mathcal{E}$: Encryption Rule, $\mathcal{D}$: Decryption Rule
- $E_e : \mathcal{P} \rightarrow \mathcal{C}, E_e(M) = C, D_d : \mathcal{P} \rightarrow \mathcal{C}, D_d(C) = M$ $(e,d) \in \mathcal{K}, E \in \mathcal{E}, D \in \mathcal{D}$.
- It is ***computationally infeasible*** to compute d from e.
- $D_d(E_e(M)) = M$.
- E and D operations should be efficiently computable.

Advantages and Disadvantages of PKC

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- **Key Security:** Only private key needs to be kept secret.
- **Longevity:** Key pairs may be used without change in most cases over long period of time.
- **Key Management:** In a large network fewer private keys will be required.
- **Key-exchange:** No key exchange is required
- **Digital Signature:** The greatest advantage of PKC.
- **Performance:** It is slow, in general.
- **Dependency:** Role of CA, require PKI.
- **System Security:** Depends on well-defined computational hard problems.

Diffie Hellman Key Exchange Protocol

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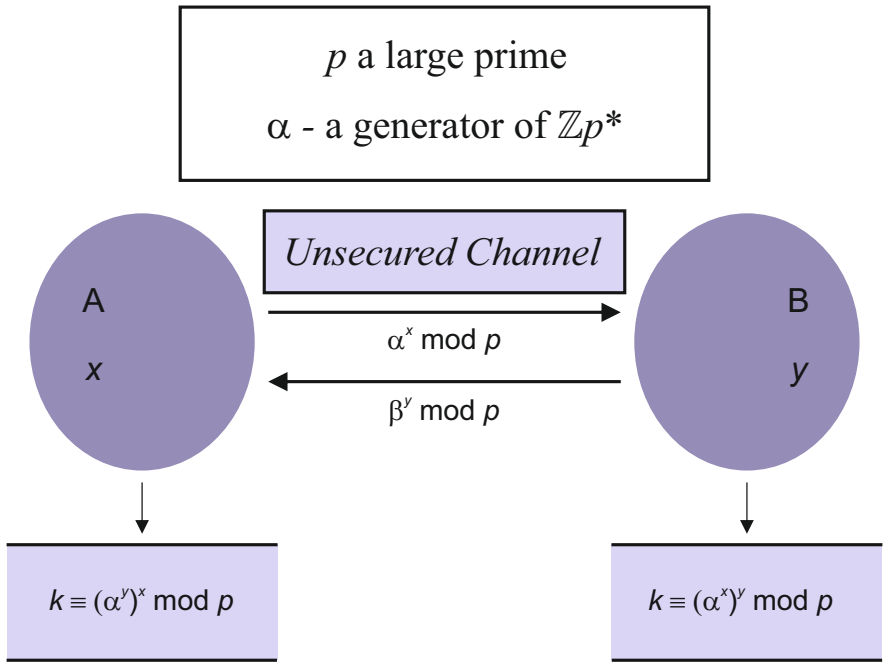
Diffie Hellman Key Exchange Protocol I



- k is the shared secret key

Diffie Hellman Key Exchange Protocol

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- Knowing α, α^x and α^y(but neither x nor y) it is hard to find α^{xy}.
- This is as secure as discrete log is intractable.
- **Idea of this protocol:** The enciphering key can be made public since it is computationally infeasible to obtain the deciphering key from enciphering key.
- This protocol is the ***door-opener*** to Public Key Cryptography.

☞Security: CDHP

Public Key Encryption Schemes

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- Setup
- Key Generation Algorithm
- Encryption
- Decryption

RSA Cryptosystem

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Key Generation

- The first task is to select n. n is normally very large (approx 308 digits) which is a product of two large primes p and q.
- Next a large integer e is chosen such that e is relatively prime to $\phi(n)$. e is usually picked as a prime larger than both $(p-1)$ and $(q-1)$.
- Next d is selected in such a way that: $e.d \equiv 1 \mod \phi(n)$.
- n and e are made public.
- p, q and d are kept private.

Encryption and Decryption

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Encryption

- A obtains B's public key
- Message m is an integer in the interval $[1, n-1]$.
- Compute $c \equiv m^e \mod n$.
- Send the cipher text c to B.

Decryption

- To recover plaintext m from c, B uses his private key d to recover $m \equiv c^d \mod n$.

☞Security: IFP

Example

- Suppose A wants to send the following message to B

$$powera$$

- Let B chooses his $n = 1943 = 29.67$. Then $\phi(n) = 1848$.
- Suppose he picks $e = 701$, then $d = 29$. $\because 26^2 < n < 26^3$, therefore the block size of the plaintext $= 2$.
- $m_1 = po = 15.26 + 14 = 404, m_2 = we = 22.26 + 4 = 576, m_3 = ra = 17.26 + 0 = 42$.
- $c_1 = 404^{701} \equiv 1419(\mod 1943)$. Similarly, A can calculate $c_2 = 344 \& c_3 = 210$.
- Now $c_1 = 1419 = 2.26^2 + 2.26 + 15 = ccp$, $c_2 = 344 = 13.26 + 6 = ng$ & $c_3 = 210 = 8.26 + 2 = ic$ Therefore the cipher text is

$$ccpngic$$

RSA Signature Scheme

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- Setup
- Key Generation Algorithm
- Signature Generation
- Signature Verification

RSA Signature Scheme

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- Setup: common parameters: security level, hash $(h : \{0,1\}^* \longrightarrow Z_n)$.
- Key Generation Algorithm : Public Key of Signer (n, e), private key d.
- Signature Generation: $s = h(m)^d \mod n$.
- Signature Verification: $\tilde{m} = s^e \mod n$, verify that $\tilde{m} = h(m)$, if not reject the signature.

Discrete Log: ElGamal

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Key Generation

- First choose a large prime p such that the DLP is infeasible in $(\mathbb{Z}_p^*, .)$.
- Select a primitive element α of $(\mathbb{Z}_p^*, .)$.
- Pick a random integer $a(1 < a < p-1)$ and compute $\beta \equiv \alpha^a (mod p)$.
- Public Para $= (p, \alpha)$, Public Key$= \beta$ and Private key $= a$.

Encryption and Decryption

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Encryption

- A obtains B's public information (p, α, β).
- He represents the message m as an integer in $\mathbb{Z}_p^*$.
- He chooses a random integer k in $[2, p-2]$.
- Compute $c = (c_1, c_2) \in \mathbb{Z}_p^* \times \mathbb{Z}_p^*$, where $c_1 \equiv \alpha^k(\mod p) \& c_2 \equiv m\beta^k(\mod p)$.

Decryption Using B's private key, he decrypts the ciphertext $c = (c_1, c_2)$ by calculating $m \equiv c_2(c_1^a)^{-1}(\mod p)$.

☞Security: DLP

Example of ElGamal

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- Let $p = 2579$ and $\alpha = 2$, α is a primitive element $\bmod\ 2579$.
- Let $a = 765$ then $\beta \equiv 2^{765}(\ \bmod\ 2579) \equiv 949(\ \bmod\ 2579)$.
- Therefore the public key of B $= (2579, 2, 949)$ and private key is 765.

$$m = 1299$$

- Now suppose A wants to send the message $m = 1299$ to B and A picks the random integer $k = 853$.
- Then A computes $c_1 \equiv 2^{853}\ \bmod\ 2579 \equiv 435\ \bmod\ 2579$ and $c_2 \equiv 1299.949^{853}\ \bmod\ 2579 \equiv 2396(\ \bmod\ 2579)$. Therefore, the ciphertext of A is

$$C = (435, 2396).$$

Digital Signature Algorithm (DSA)

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☞Adopted by NIST in 1994

- Setup: common parameters: security level, hash $(h : \{0,1\}^* \longrightarrow Z_q)$.
- Key Generation Algorithm:
  1. Select primes p, q with $q/(p-1)$
  2. select $g \neq 1 \in \mathbb{Z}_p^\star$ such that $g^q = 1 \mod p$
  3. Select $x \in \mathbb{Z}_q$ and compute $y = g^x \mod p$
  4. public key (p, q, g, y), secret key x

Digital Signature Algorithm (DSA)

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- Signature Generation
  1. Choose $k \in \mathbb{Z}_q^\star$,
  2. Compute $r = (g^k \mod p) \mod q$
  3. $s = k^{-1}(H(m) + xr) \mod q$
  4. Output the signature $\sigma = (s, r)$
- Signature Verification:
  1. Check if $r, s \in \{1, \ldots, q-1\}$, otherwise reject signature
  2. set $w = s^{-1} \mod q$, $u_1 = H(m)w \mod q$, $u_2 = rw \mod q$
  3. Compute $v = (g^{u_1} y^{u_2} \mod p) \mod q$
  4. Accept sign if $v = r \mod q$, and reject otherwise.

☞Security: DLP

RSA Factoring Challanges[7]

- Factorization of Giver Number
- Started by RSA Laboratories: March 1991, Ended: 2009
- RSA-768 factored in Dec 2009
- A lot of research have been carried out to factor RSA number beyond 768-bit. [6]

RSA-768=	123018668453011775513049495838496272077285356959533479219732245215172640050726365751874520219978646938995647494277406384592519255732630345373154826850791702612214291346167042921431160222124047927473779408066535141959745985690214341
	p = 33478071698956898786044169848212690817704794983713768568912431388982883793878002287614711652531743087737814467999489 $\times q$ = 36746043666799590428244633799627952632279158164343087642676032283815739666511279233373417143396810270092798736308917

[6] Cryptology ePrint Archive: http://eprint.iacr.org/2010/006

[7] https://en.wikipedia.org/wiki/RSA_Factoring_Challenge

RSA Factoring Record: Feb 2020

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RSA-250 has 250 decimal digits (829 bits), and was factored in February 2020 by Fabrice Boudot, Pierrick Gaudry, Aurore Guillevic, Nadia Heninger, Emmanuel Thome, and Paul Zimmermann

RSA-250 (829)=	2140324650240744961264423072839333563008614715144755017797754920881418023447140136643345519095804679610992851872470914587687396261921557363047454770520805119056493106687691590019759405693457452230589325976697471681738069364894699871578494975937497937
	p = 6413528947707158027879019017057738908482501474294344720811685963202453234463023862359875266834770873766192558569463979885336 7 $\times q$ = 3337202759497815655622601060535511422794076034476755466678452098702384172921003708025744867329688187756571898625803693206271 1

Post Quantum Cryptography

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- If quantum computers are available then Shor's algorithm runs in polynomail time to solve
  - Integer factorization problem
  - DLP in finite fields & DLP on elliptic curves
  - DLP in general class groups
- The following PKC will be dead: RSA, DLP /ECDLP based Cryptosystems
- Cryptography that will be resistant to attack by quantum computer is known as Post Quantum Cryptography

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Definition (Lattices)

A lattices is defined as the set of all integer combinations of n linearly independent vectors $\mathbf{b_1}, \cdots, \mathbf{b_n}$:

$$\mathfrak{L}(\mathbf{b_1}, \cdots, \mathbf{b_n}) = \left\{ \sum_{i=1}^{i=n} x_i \mathbf{b_i} : x_i \in \mathbb{Z} \text{ for } 1 \leq i \leq n \right\}.$$

The set of vectors $\mathbf{b_1}, \cdots, \mathbf{b_n}$ is called a *basis* for the lattice.
$\mathbf{B} = [\mathbf{b_1}, \cdots, \mathbf{b_n}]$

Lattice basis reduction (NTRU)

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Lattice Problems

- Shortest Vector Problem (SVP): Given a lattice basis $\mathbf{B}$, find the shortest non zero vector in $\mathfrak{L}(\mathbf{B})$.
- Closest Vector Problem (CVP): Given a lattice basis $\mathbf{B}$ and a target vector $\mathbf{t}$ (not necessarily in the lattice), find the lattice point $\mathbf{v}$ closest to $\mathbf{t}$.

Fact: SVP and CVP are NP-hard

Solving System of Multivariate Polynomial Equations Over Finite Field

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Definition ($\mathcal{MQ}$ Problem)

Solve the system $p_1(\mathbf{x}) = p_2(\mathbf{x}) = \cdots = p_m(\mathbf{x})$, where each p_i is a quadratic in $\mathbf{x}$. All coefficients and variables are in $\mathbb{F}_q$, the field with q elements.

- Multivariate Public Key Cryptography is based on hardness of $\mathcal{MQ}$.
- $\mathcal{MQ}$ is an NP-Hard Problem
- HFE, Sflash, Quartz

Decoding Problem: GBD/GD

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Definition (Goppa Bounded Decoding (GBD) Problem)

Instance: An $r \times n$ binary matrix H and word $s \in \mathbb{F}_2^r$.
Question: Is there a word e in $\mathbb{F}_2^n$ of weight $\leq r/\log_2 n$ such that $He^T = s$?

Definition (Goppa Code Distinguishing (GD) Problem)

Instance: An $r \times n$ binary matrix H.
Question: Does H belong to $\mathcal{G}_{n,t}$ for some t. Here $\mathcal{G}_{n,t}$ denotes the set of all parity check matrices of t-error correcting binary Goppa codes of length n.

Decoding Problem: GBD/GD

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- GBD and GD problem are NP-hard. In fact these problems are exponential.
- Security of Code based Cryptography are based on hardness of GBD/GD problem.
- Examples: McEliece's Cryptosystem and Niederreiter.
- Cryptanalytic Attacks: not susceptible to all known attacks
- Key Size: Very Large
- Encryption/Decryption Speed: Reasonable

Outline

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Introduction to Elliptic Curve I

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- An elliptic curve **E** over a field **K** is a non-singular cubic curve in two variables, $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \mathbf{0}$ together with an extra point *the point at infinity*.
- The field **K** is usually taken to be the complex, real, rational, algebraic extensions of rational or a **finite field**.
- Elliptic curves groups for cryptographic applications are examined with the underlying finite fields of characteristic p(where $p > 3$ is a prime) i.e F_{p^m} and fields of characteristic 2 i.e. F_{2^m}.
- The "standard elliptic curve"has the form:

$$y^2 = x^3 + ax + b$$

for some fixed a and b.

Introduction to Elliptic Curve II

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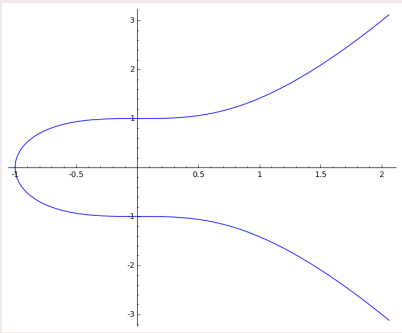
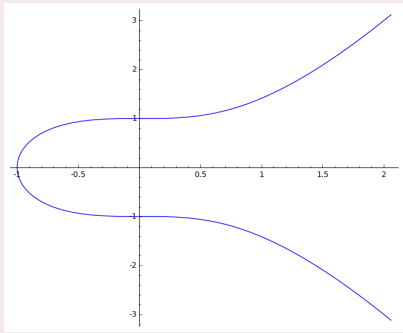
Examples for Characteristic 0:

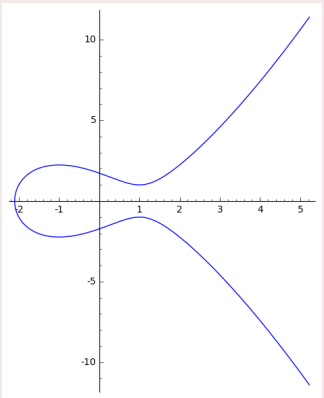
$y^2 = x^3 - 1$



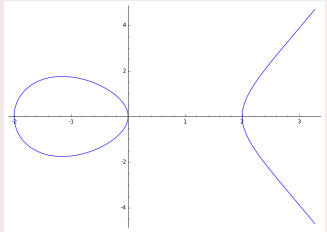
$y^2 = x^3 + 1$



$y^2 = x^3 - 3x + 3$



$y^2 = x^3 - 4x$



Elliptic Curve

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Generalized Form: An elliptic curve E over a field $\overline{K}$ is defined by a Weiestrass equation: $E : y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$, $a_i \in \overline{K}$ together with a point $\mathcal{O}$ called point at *infinity*.

- E/K- if $a_1, a_2, a_3, a_4, a_6 \in K$
- $E(K)$ the set of K-*rational points* of E, with the point $\mathcal{O}$.

$$E(K) = \{(x,y) \in K^2 : y^2+a_1xy+a_3y = x^3+a_2x^2+a_4x+a_6\} \cup \{\mathcal{O}\}. \quad (1)$$

- we define some constants:

$$\left.\begin{aligned} b_2 = a_1^2 + 4a_2,\ b_4 = a_1a_3 + 2a_4,\ b_6 = a_3^2 + 4a_6, \\ b_8 = a_1^2a_6 + 4a_2a_6 - a_1a_2a_4 - a_2a_3^2 - a_4^2, \\ c_4 = b_2^2 - 24b_4, c_6 = -b_2^3 + 36b_2b_4 - 216b_6. \end{aligned}\right\}$$

  discriminant $\Delta = -b_2^2b_8 - 8b_4^3 - 27b_6^2 + 9b_2b_4b_6$.

- $(K) \neq 2, 3, \Delta = (c_4^3 - c_6^2)/1728$.
- The curve $f(x,y) = 0$ is non singular iff at least one of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \neq 0$. Equivalent condition is Discriminant $\Delta \neq 0$.

- Let E be an elliptic curve given by a Weierstrass equation. Then $(E, +)$ is an abelian group with identity element $\mathcal{O}$ and $E(K)$ is a subgroup of E.

- $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on a curve. the explicit formula for $P + Q = (x_3, y_3)$ can be computed easily. If the curve is defined in equation (1), then

$$\begin{aligned} x_3 &= \lambda^2 + a_1\lambda - a_2 - x_1 - x_2 \\ y_3 &= -(\lambda + a_1)x_3 - \beta - a_3 \end{aligned} \tag{2}$$

where $\beta = y_1 - \lambda x_1$ and

$$\lambda = \begin{cases} \frac{y_2 - y_1}{x_2 - x_1} & \text{if } P \neq Q, \\ \frac{3x_1^2 + 2a_2x_1 + a_4 - a_1y_1}{2y_1 + a_1x_1 + a_3}, & \text{if } P = Q. \end{cases}$$

- $-P = (x_1, -y_1 - a_1x_1 - a_3)$

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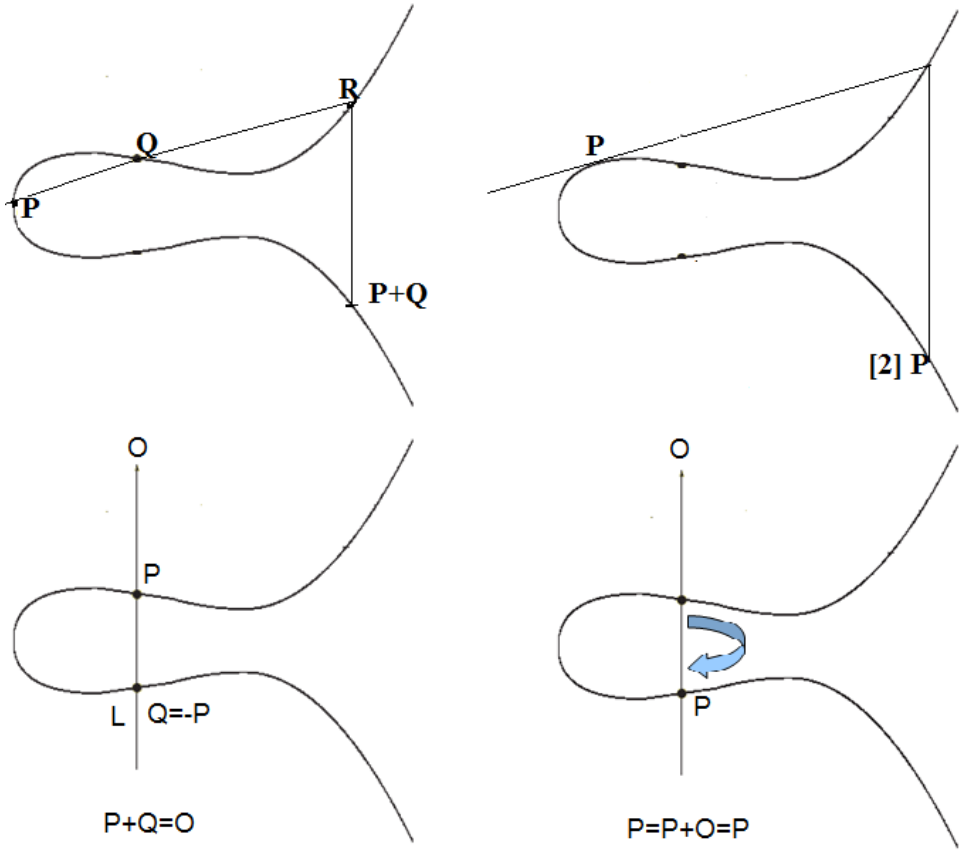

Complexity Analysis

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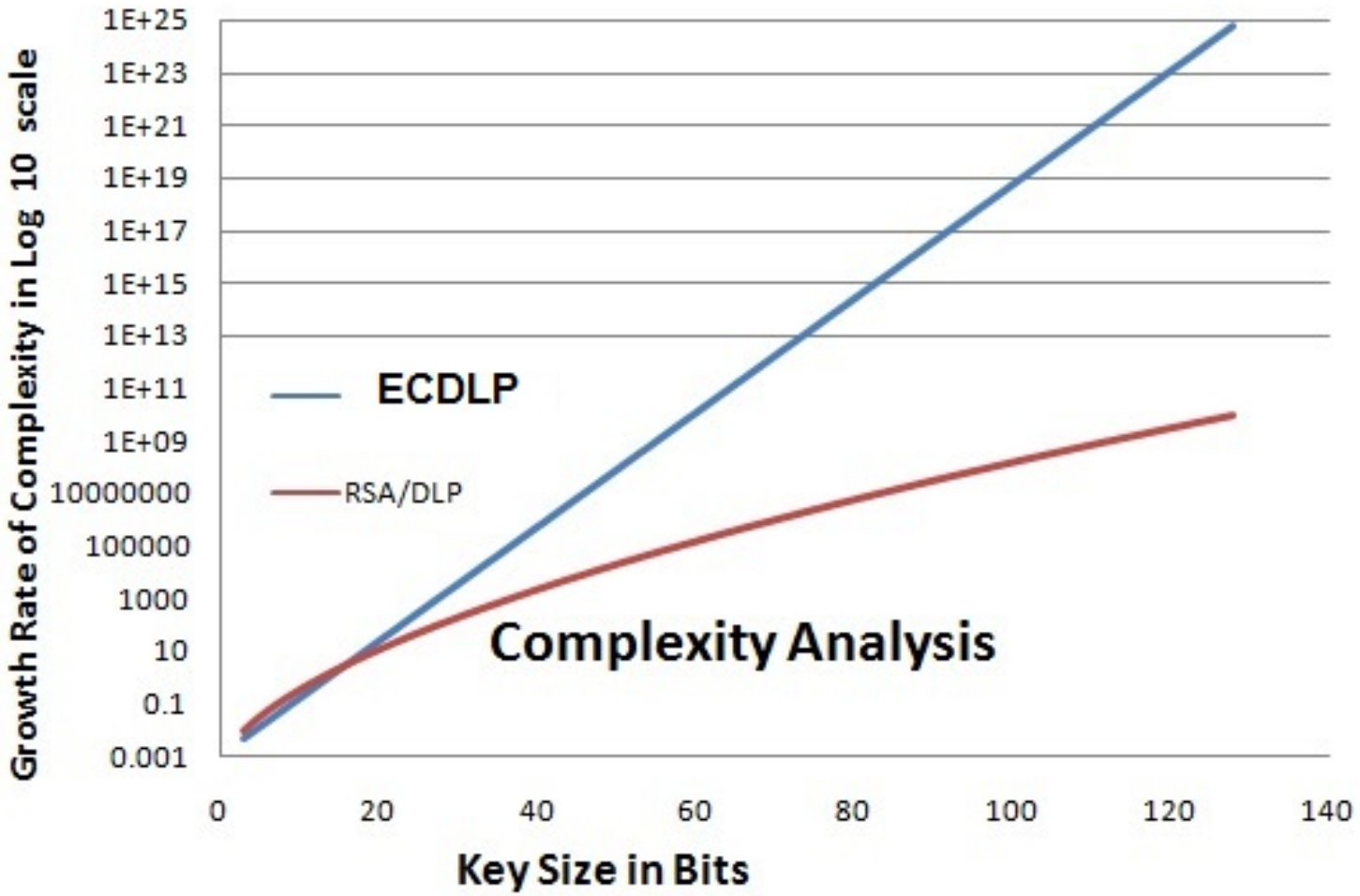

Elliptic Curve based Cryptographic Schemes

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Koblitz (1987) and Miller (1985) first recommended the use of elliptic-curve groups (over finite fields) in cryptosystems. Some cryptographic schemes [6]:

1. Signature Scheme
   - ECDSA
   - EC-KCDSA
2. Public Key Encryption
   - ElGamal Elliptic Curve Encryption.
   - Elliptic Curve Integrated Encryption (ECIES)(Bellare and Rogaway, Variant of ElGamal).
   - Provably Secure Encryption Curve Scheme(PSEC)-Fujisaki and Okamoto.
3. Key Agreement
   - Diffie-Hellman.
   - Station-to-station (Diffie, Van Oorschot, Wiener).
   - ECMQV (Menezes, Qu, Vanstone).

ElGamal Elliptic Curve Cryptosystem

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ElGamal Cryptosystem Consists of:

- Setup: **Domain Parameters** $D = (F_p, E, P, n)$
- Key Generation Algorithm: Key Pair (d, Q), where $Q = d.P$.
- Encryption Algorithm: Ciphertext
- Decryption Algorithm: Plaintext

Encryption

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- **Input:** Elliptic curve domain parameters (F_p, E, P, n), public key $Q = d.P$, plaintext m.
- **Output:** Ciphertext (C_1, C_2)
  1. Represent the message m as a point M in $E(F_p)$.
  2. Select random $k \in [2, n-1]$.
  3. Compute $C_1 = k.P$(scalar multiplication).
  4. Compute $C_2 = M + k.Q$.
  5. Return (C_1, C_2).

Decryption

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- **Input:** Elliptic curve domain parameters (F_p, E, P, n), private key d, ciphertext (C_1, C_2).
- **Output:** Plaintext m
  1. Compute $M = C_2 - d.C_1$.
  2. Return (m).

Key Size Comparison:

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Key sizes (in bits) recommended by the National Institute of Standards and Technology (NIST) to protect keys used in (DES) and (AES) together with the key sizes for RSA, Diffie-Hellman and elliptic curves that are needed to provide equivalent security. [8]

Symmetric Key Size	**RSA and Diffie-Hellman Key Size**	**Elliptic Curve Key Size**
80	1024	160
112	2048	224
128	3072	256
192	7680	384
256	15360	512

[8] https://www.nsa.gov/business/programs/elliptic_curve.shtm

ECC:Advantage

- Shorter Key Length translates to
  1. Faster implementation
  2. Less power consumption
  3. Less silicon area
- Can be implemented in 8/16/32-bits microprocessor with reasonable amount of security.
- Can be implemented in Smart Card, PDA etc..

Implementation of ECC

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Note: Basic necessity for implementation of ECC: a good finite filed library

- Choice of Finite Filed
- Choice of Basis
- Choice of coordinate Systems
- Choice of methods for scaler multiplication

Certicom ECC CChallanges[9]

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- The Challenge is to compute the ECC private keys from the given list of ECC public keys and associated system parameters
- Certicom Proposed two levels of challenges for elliptic curves over prime fields ($\mathbb{F}_p$) and elliptic over extension of finite fields ($\mathbb{F}_{2^n}$).

Level I	level II
109-bit challenge Solved EECp-109 Solved in 2002 EECC2-109 Solved in 2004	163-bit challenge
131-bit challenge	191-bit challenge
	239-bit challenge
	359-bit challenge

- World records: 113-bit Koblitz curve using FPGA-cluster

[9] Certicom Research

Identity Based Cryptography I

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- Conventional Public-key cryptography is dependent on a public-key infrastructure(PKI).
- PKI addresses authentication of public key problem
- A PKI often works with a party trusted by all users, called Certification Authority (CA)
- CA has to generate and sign certificates containing public key of users along with identity.
- PKI- based cryptography is both time-consuming and error-prone.
- In 1984, Shamir introduced the concept of identity-based cryptography.
- It uses user identity attributes, such as email addresses/ phone numbers company address instead of digital certificates

Identity Based Cryptography II

- More precisely, the public key of a user is derived directly from publicly available information.
- Practical realization of identity-based public key cryptography (ID-PKC) came prom pairings.
- In 2001, Boneh and Franklin proposed the first identity-based encryption scheme, using the bilinearity of pairings.
- Computationally Hard prob: Bilinear Diffie Hellman Problem
- Although it provides some advantages over PKI-based approaches, it is not without its drawbacks (Key escrow).

Outline

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Some Open Source Libraries[10]

- SAGE [System for Algebra and Geometry Experimentation]: http://www.sagemath.org/
- PARI/GP: http://pari.math.u-bordeaux.fr/
- crypto - OpenSSL cryptographic library: https://www.openssl.org/docs/crypto/crypto.html
- GAP http://www.gap-system.org
- NTL: A Library for doing Number Theory http://www.shoup.net/ntl/
- MIRACL http://indigo.ie/?mscott/
- GNU MP http://www.swox.com/gmp/

[10] Appendix B, [6]

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