Minicrypt Primitives with Algebraic Structure

Sikhar Patranabis (Applied Crypto Group, ETH Zürich)

Based on joint work(s) with:

Navid Alamati (University of Michigan)

Luca De Feo (IBM Zürich)

Hart Montgomery, Arnab Roy (Fujitsu Labs of America)

Disclaimers:

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This is a Sunday afternoon and many of you are tired after a long week

But if you are still here, thank you and let's have some fun….

A Tale of Five Worlds [Impagliazzo’95]

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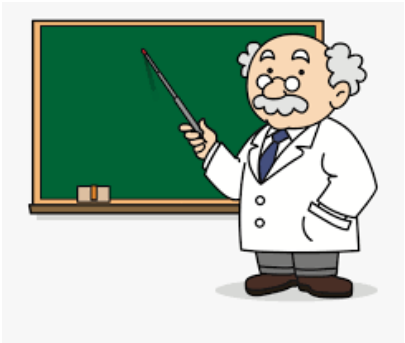

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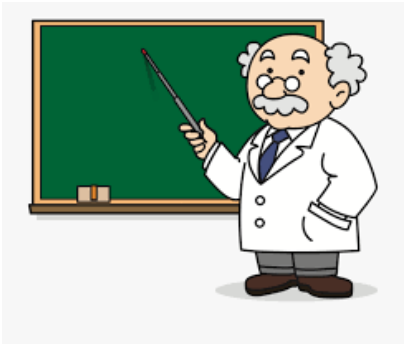

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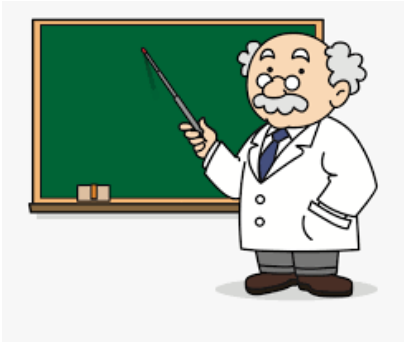

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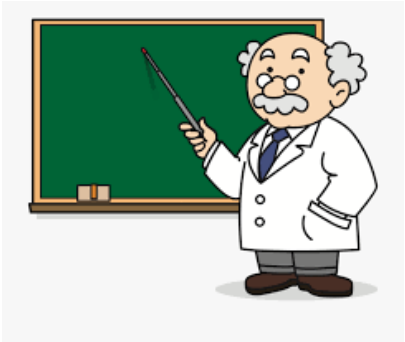

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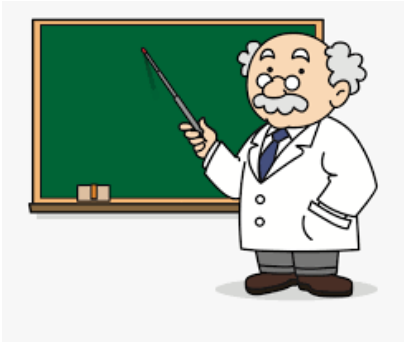

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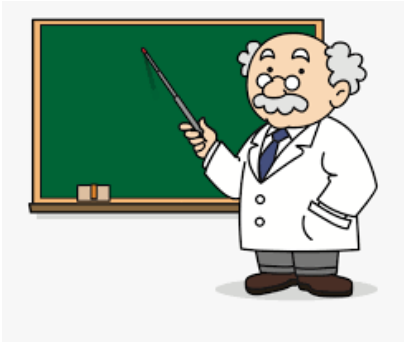

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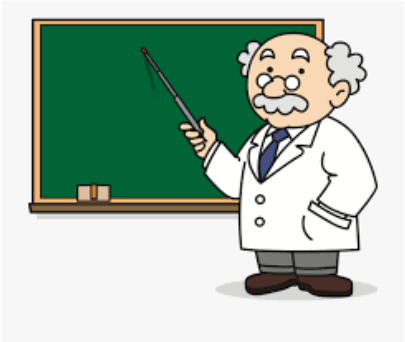

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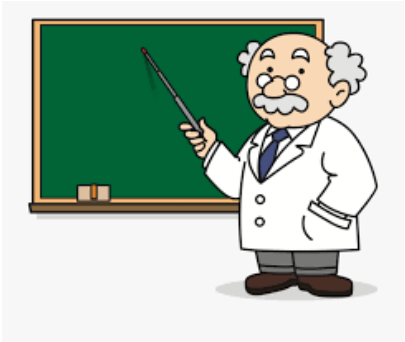

Minicrypt + ? = Cryptomania

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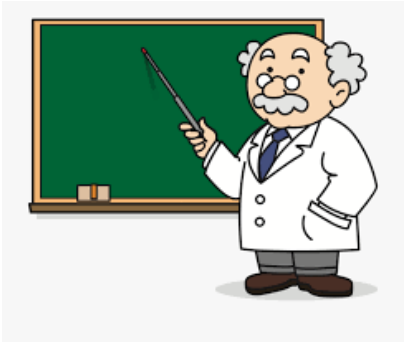

This Talk

Minicrypt + Algebraic Structure = Cryptomania

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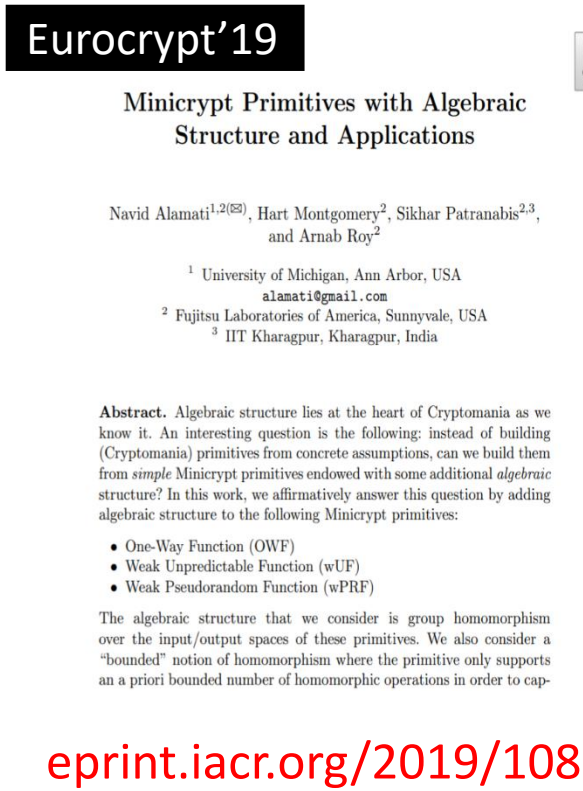
Baby Steps….

Eurocrypt'19



Minicrypt Primitives with Algebraic Structure and Applications

Navid Alamati[1,2(✉)], Hart Montgomery[2], Sikhar Patranabis[2,3], and Arnab Roy[2]

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- One-Way Function (OWF)
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The algebraic structure that we consider is group homomorphism over the input/output spaces of these primitives. We also consider a "bounded" notion of homomorphism where the primitive only supports an a priori bounded number of homomorphic operations in order to cap-

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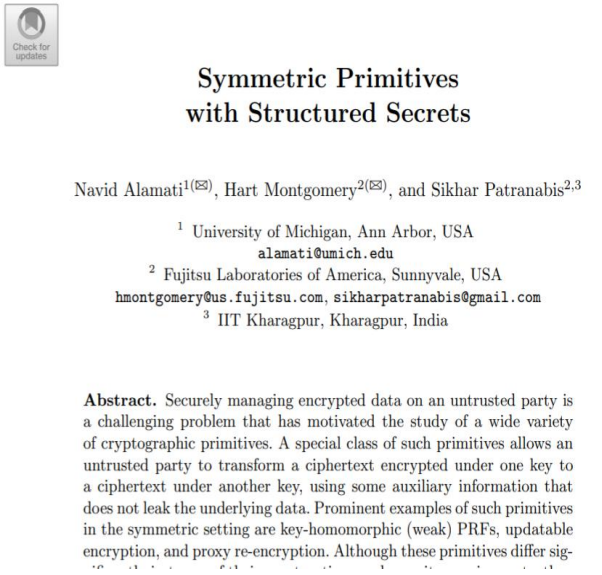

Symmetric Primitives with Structured Secrets

Navid Alamati[1(✉)], Hart Montgomery[2(✉)], and Sikhar Patranabis[2,3]

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Abstract. Securely managing encrypted data on an untrusted party is a challenging problem that has motivated the study of a wide variety of cryptographic primitives. A special class of such primitives allows an untrusted party to transform a ciphertext encrypted under one key to a ciphertext under another key, using some auxiliary information that does not leak the underlying data. Prominent examples of such primitives in the symmetric setting are key-homomorphic (weak) PRFs, updatable encryption, and proxy re-encryption. Although these primitives differ sig-

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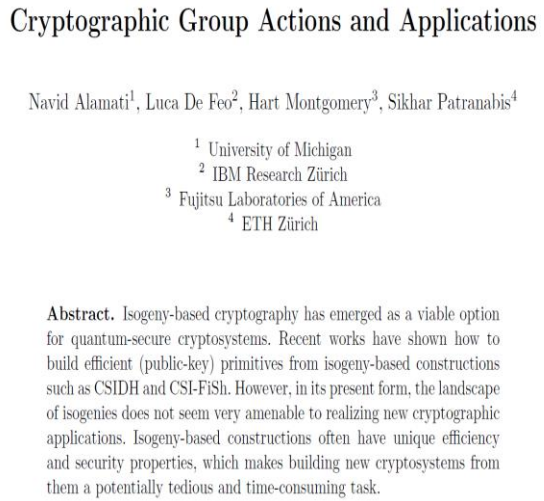
Asiacrypt'20 (Conditionally accepted)



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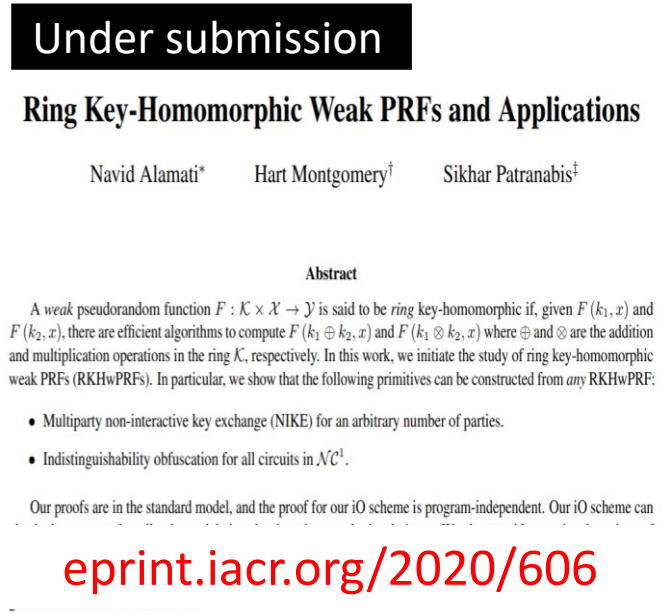

Ring Key-Homomorphic Weak PRFs and Applications

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A *weak* pseudorandom function $F : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$ is said to be *ring* key-homomorphic if, given $F(k_1, x)$ and $F(k_2, x)$, there are efficient algorithms to compute $F(k_1 \oplus k_2, x)$ and $F(k_1 \otimes k_2, x)$ where $\oplus$ and $\otimes$ are the addition and multiplication operations in the ring $\mathcal{K}$, respectively. In this work, we initiate the study of ring key-homomorphic weak PRFs (RKHwPRFs). In particular, we show that the following primitives can be constructed from *any* RKHwPRF:

- Multiparty non-interactive key exchange (NIKE) for an arbitrary number of parties.
- Indistinguishability obfuscation for all circuits in $\mathcal{NC}^1$.

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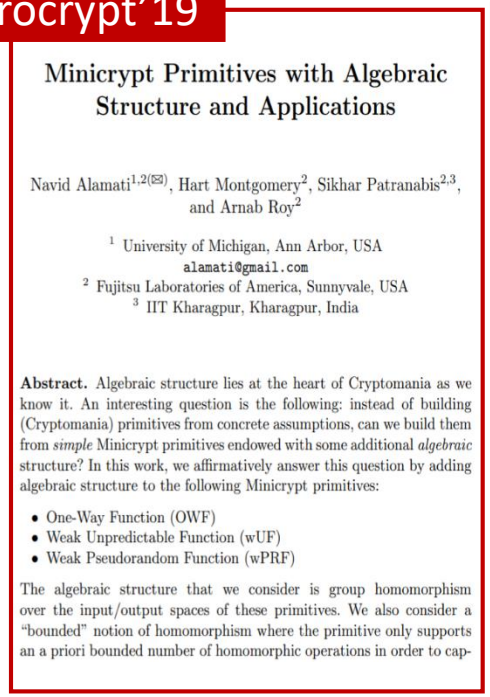

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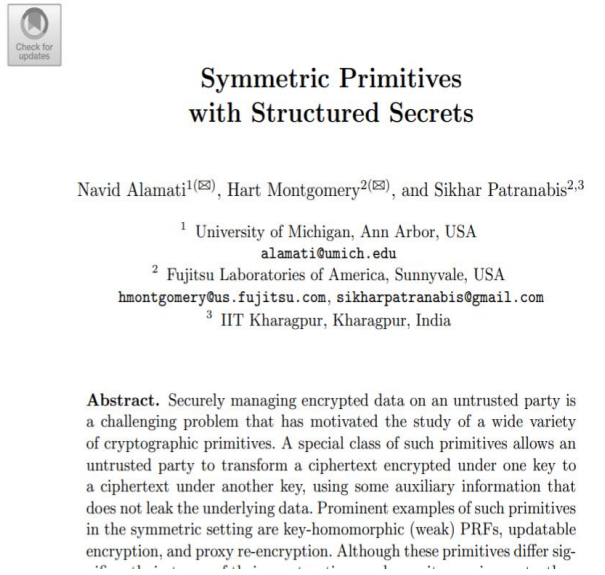

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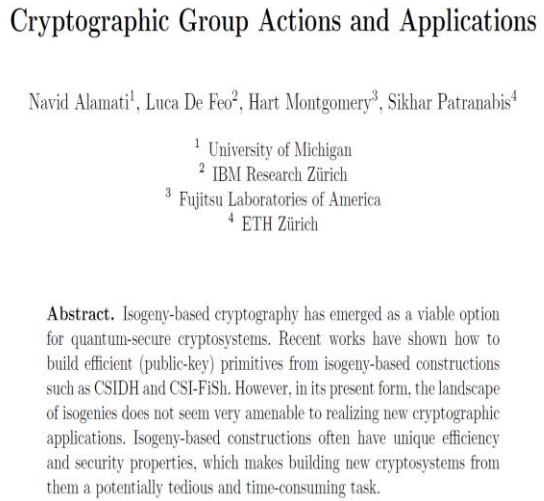

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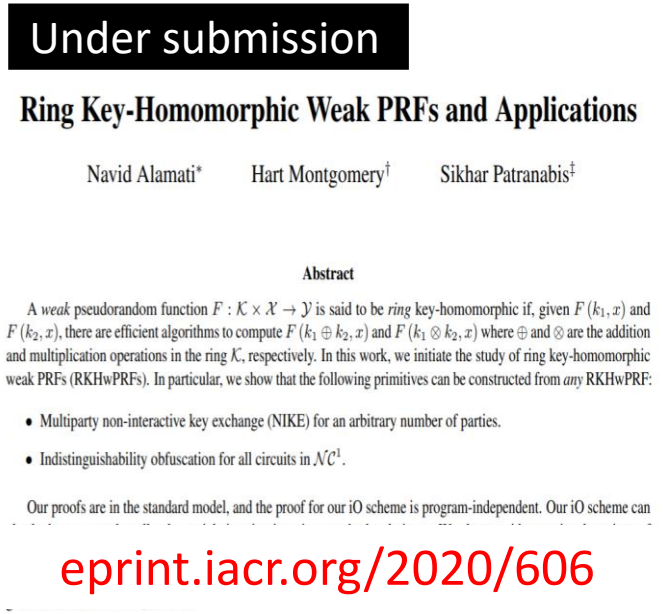

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Building Blocks

- Minicrypt primitives
  - One-Way Functions (OWFs)
  - Weak Unpredictable Functions (wUFs)
  - Weak Pseudorandom Functions (wPRFs)

Building Blocks – This Talk

- Minicrypt primitives + Algebraic structure
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Building Blocks – This Talk

Homomorphism between the input and output spaces

- Minicrypt primitives + Algebraic structure
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Building Blocks – This Talk

Homomorphism between the input and output spaces

- Minicrypt primitives + Algebraic structure
  - Homomorphic One-Way Functions (HOWFs)
  - Input-Homomorphic Weak Unpredictable Functions (IHwUFs)
  - Input-Homomorphic Weak Pseudorandom Functions (IHwPRFs)

Definitions

Homomorphic One-Way Function

A one-way function $f : \mathcal{X} \to \mathcal{Y}$ such that:

- $(\mathcal{X}, \bigoplus)$ and $(\mathcal{Y}, \bigotimes)$ are efficiently samplable groups
- for all $x_1, x_2 \in \mathcal{X}$

$$f\,(x_1 \oplus x_2) = f(x_1) \otimes f(x_2)$$

Definitions

Input-Homomorphic weak Unpredictable Function

A weak unpredictable function $F : \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}$ such that:

- $(\mathcal{X}, \bigoplus)$ and $(\mathcal{Y}, \bigotimes)$ are efficiently samplable groups
- for all $k \in \mathcal{K}$ and for all $x_1, x_2 \in \mathcal{X}$

$$F\,(k, x_1 \bigoplus x_2) = F(k, x_1) \bigotimes F(k, x_2)$$

Definitions

Input-Homomorphic weak PRF

A weak PRF $F : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$ such that:

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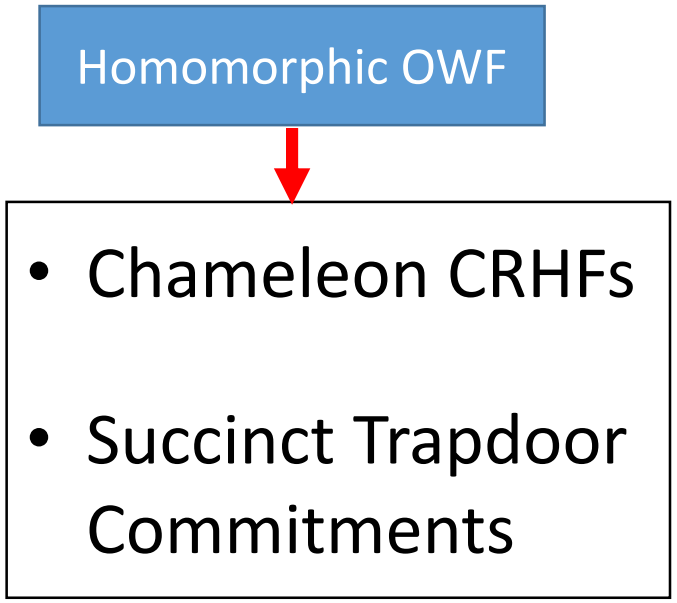
“Bounded” Homomorphism

- Minicrypt primitives + Algebraic Structure
  - (Bounded) Homomorphic One-Way Functions
  - (Bounded) Input-Homomorphic Weak Unpredictable Functions
  - (Bounded) Input-Homomorphic Weak Pseudorandom Functions

Outline

- **Our results**
- An alternative approach to designing primitives
- A cryptoplexity hierarchy in Cryptomania
- Technical overview
- Summary and open questions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• Chameleon CRHFs
• Succinct Trapdoor Commitments

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Input-Homomorphic weak UF

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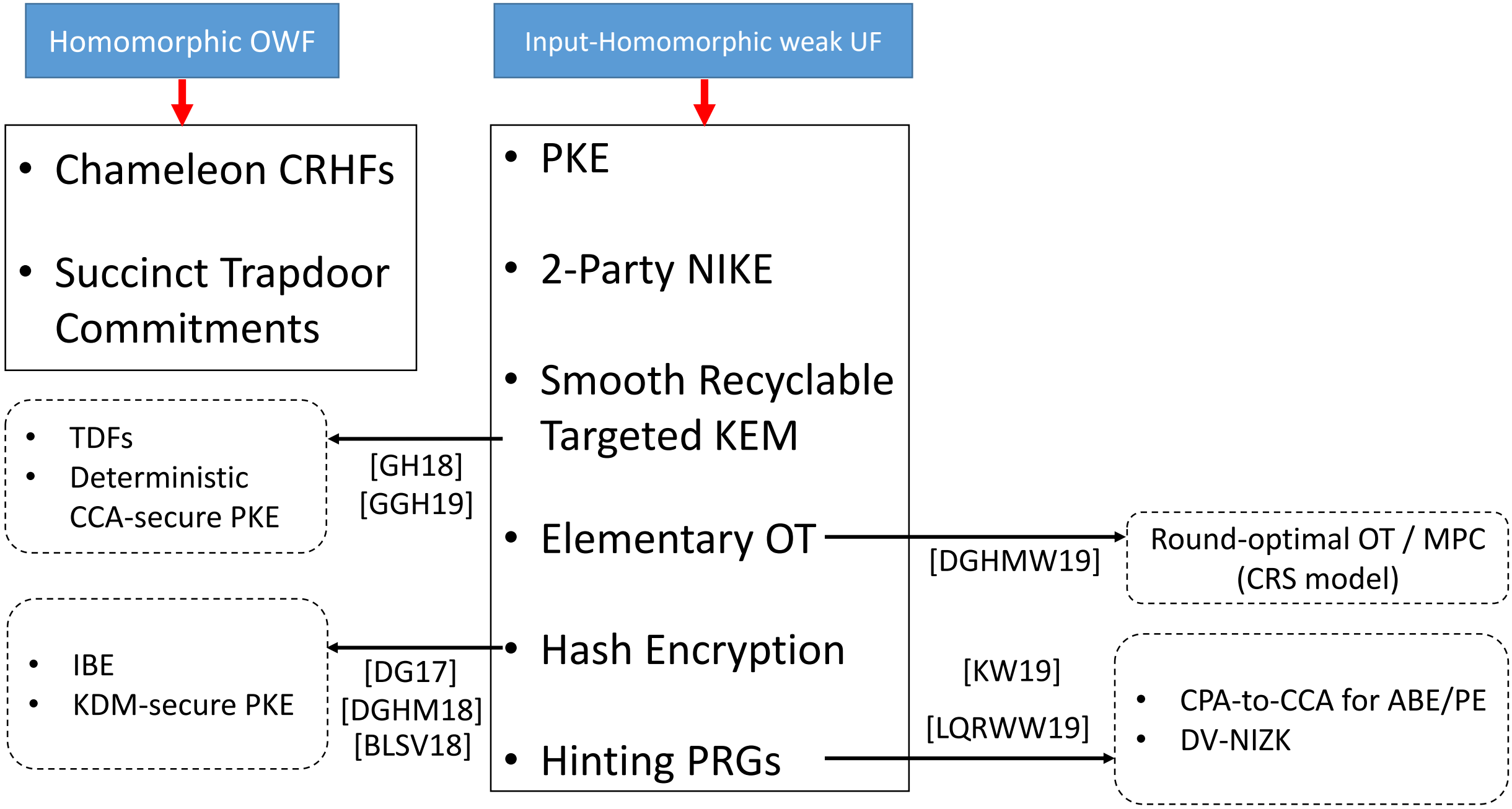
- 2-Party NIKE
- Smooth Recyclable Targeted KEM
- Elementary OT
- Hash Encryption
- Hinting PRGs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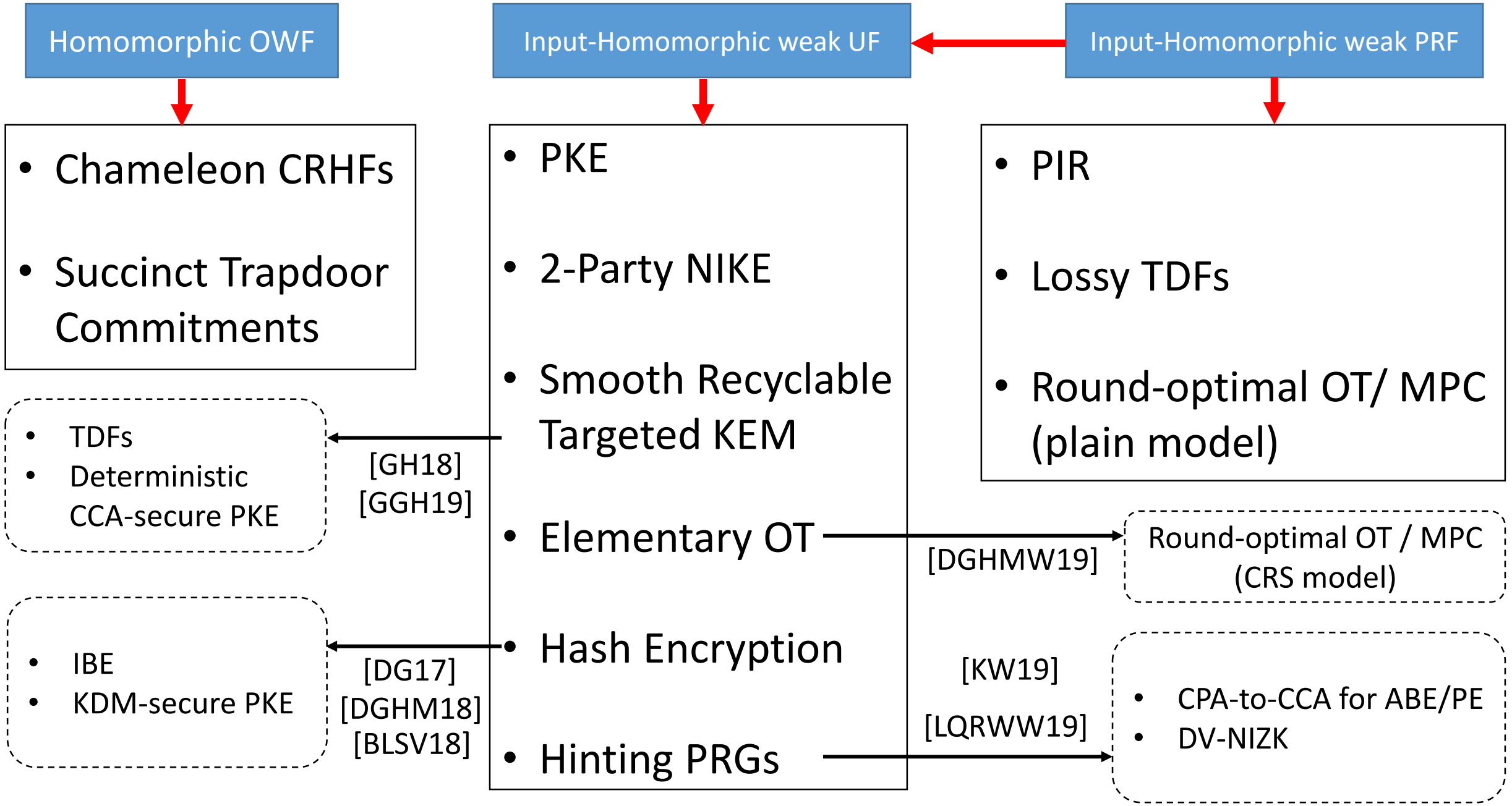
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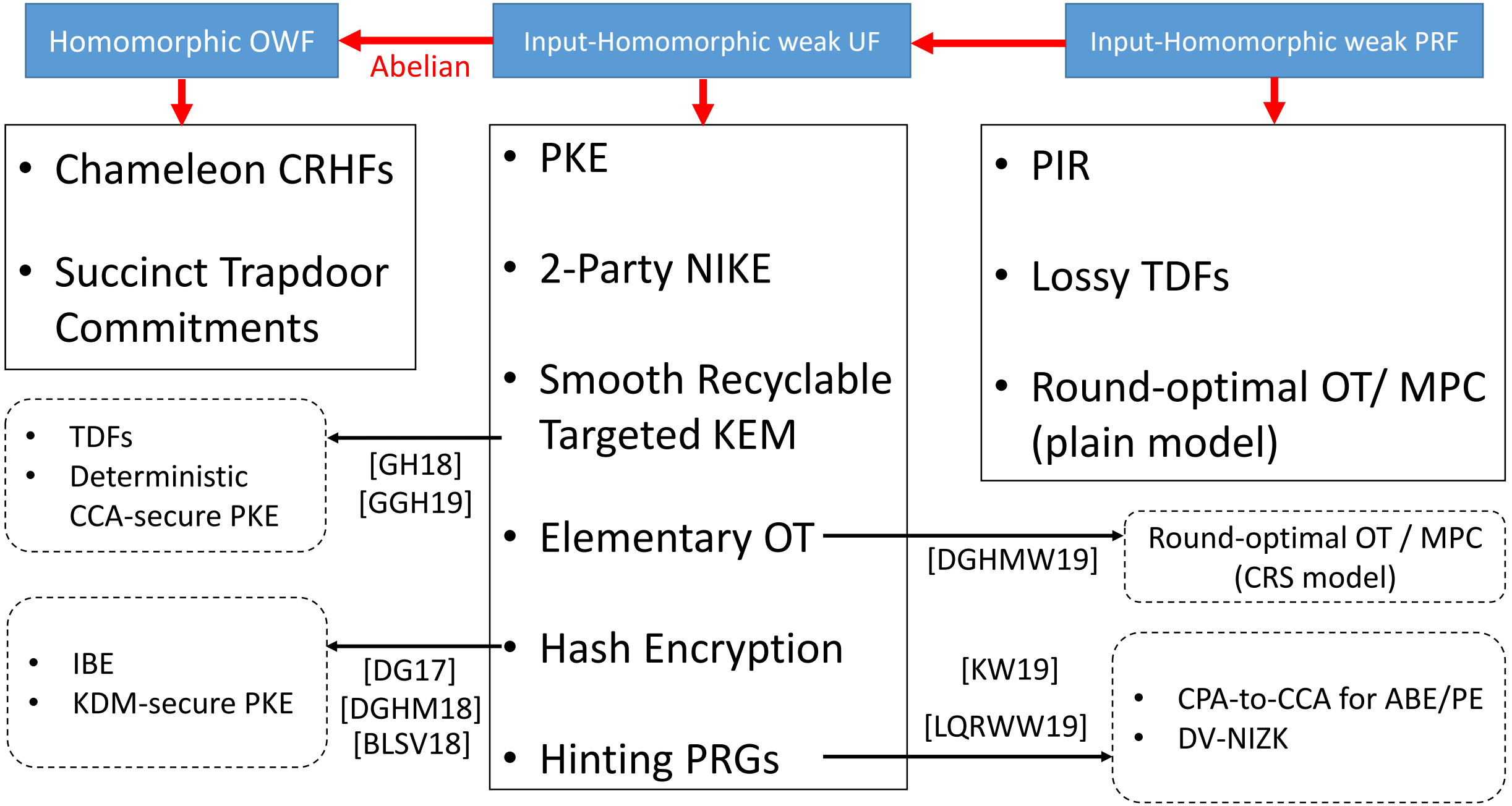
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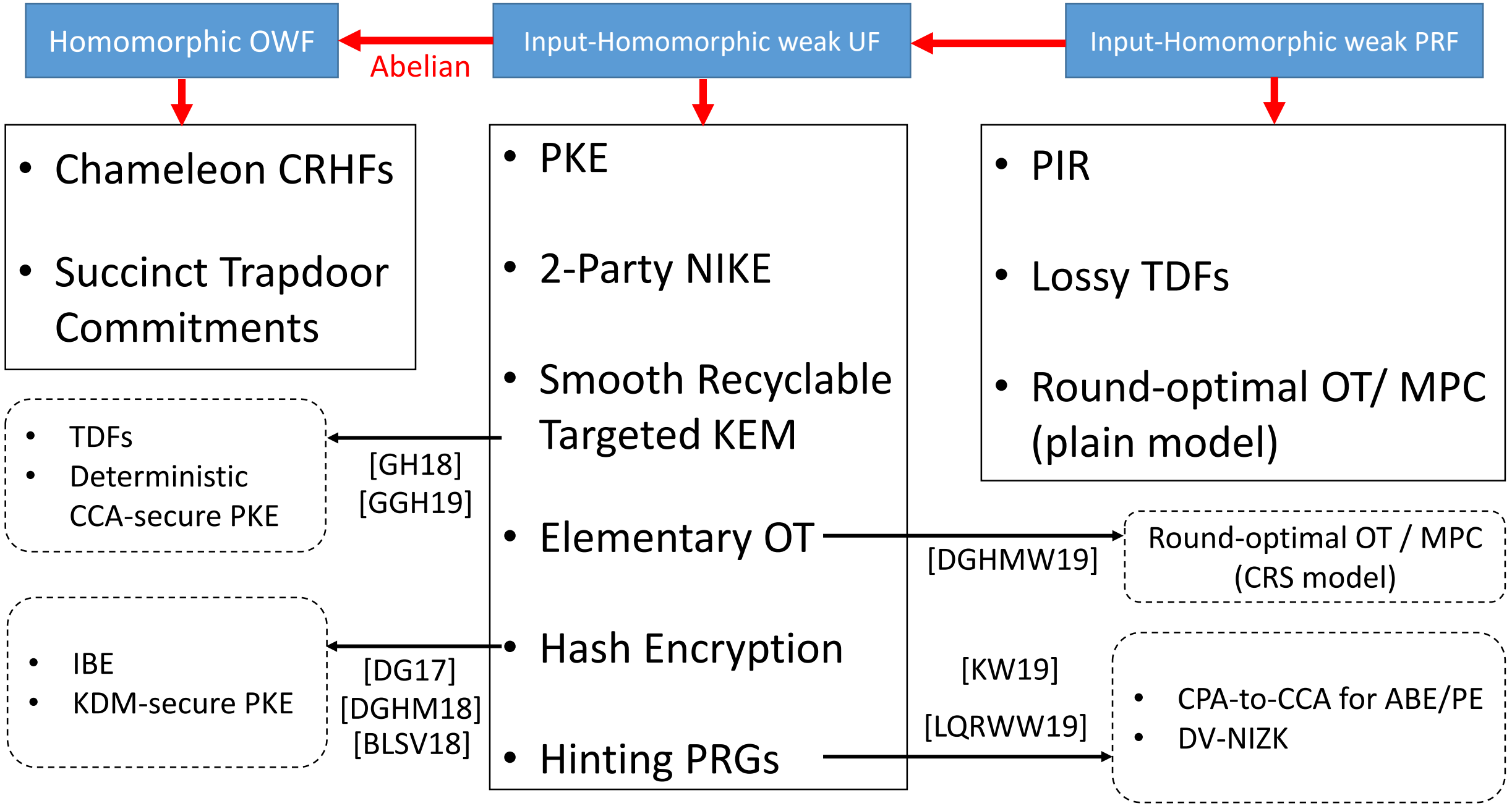
All implications hold for bounded homomorphism



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Assumptions
• Discrete log • Search LWE • Square-root finding
• CDH • RSA
• DDH/MDDH/D-Lin • Decision LWE • QR/DCR • Approx. GCD • Hidden number problem • Finite Field isomorphism

Primitives
• Schnorr-style signatures • Chameleon CRHFs • Succinct commitments
• 2-Party NIKE • CPA-Secure PKE • Trapdoor Functions • IBE • KDM-Secure PKE • DV-NIZK
• PIR • Lossy TDFs • OT and MPC

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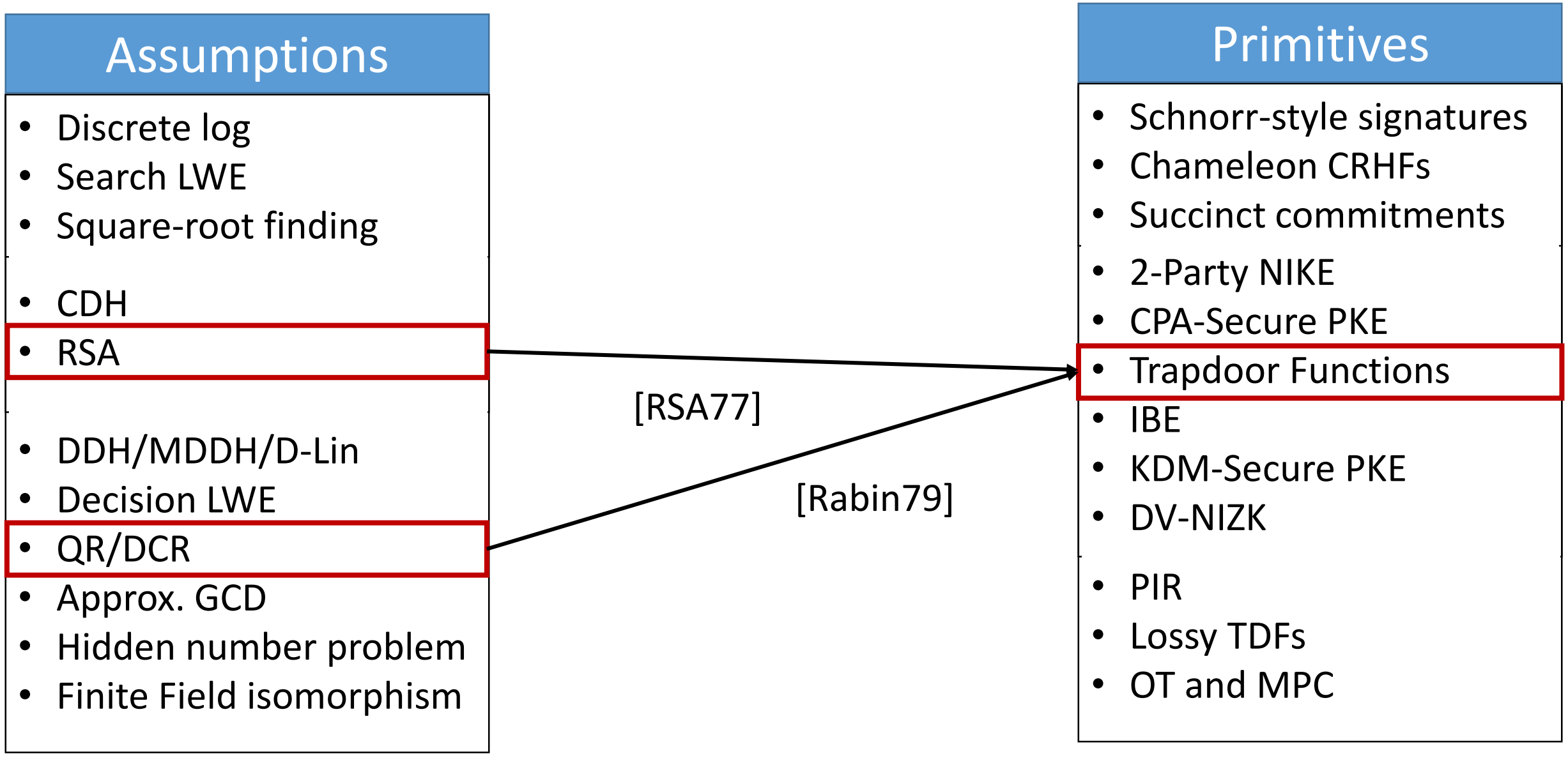
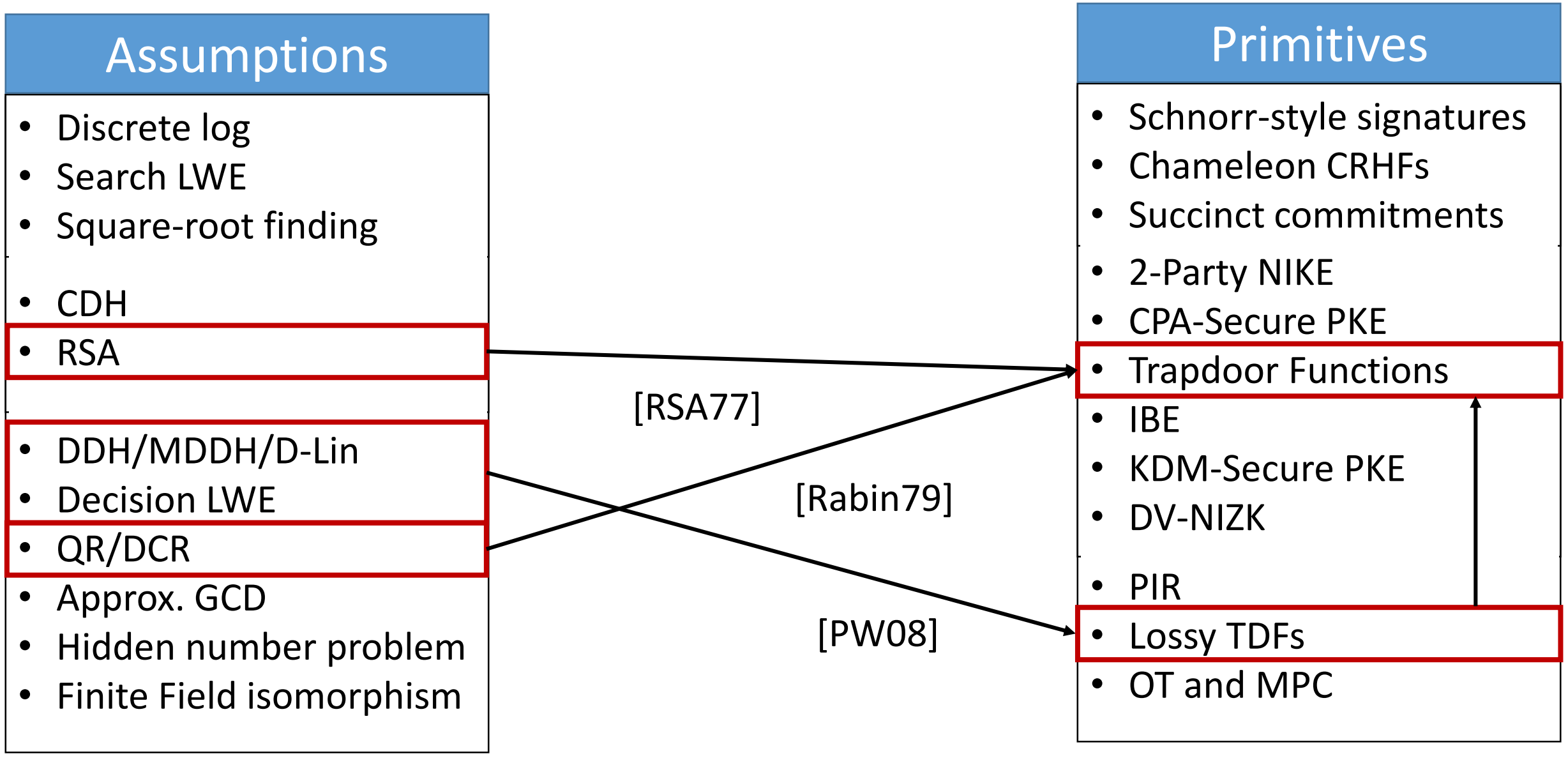
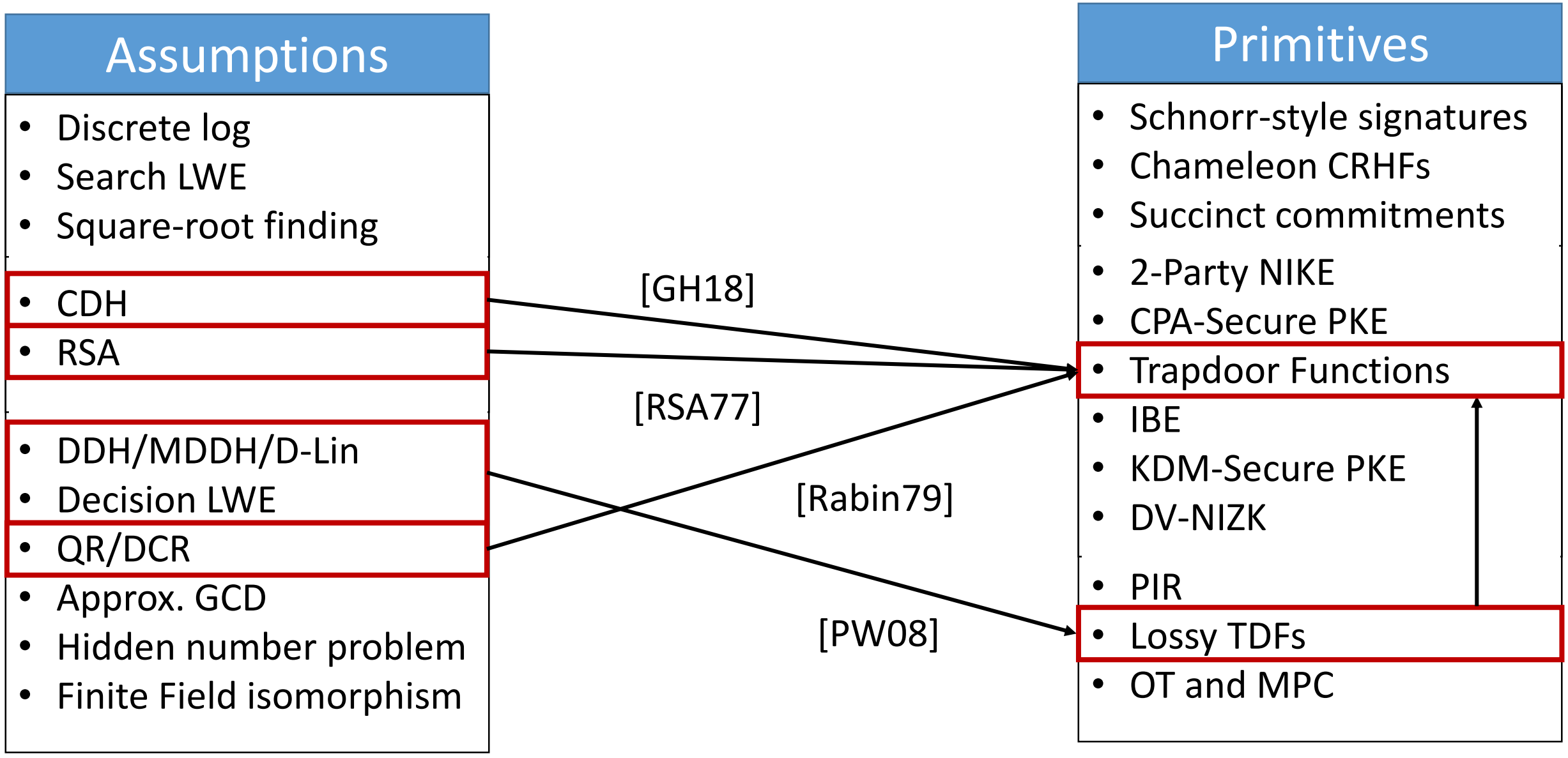
Traditional Approach to Cryptography



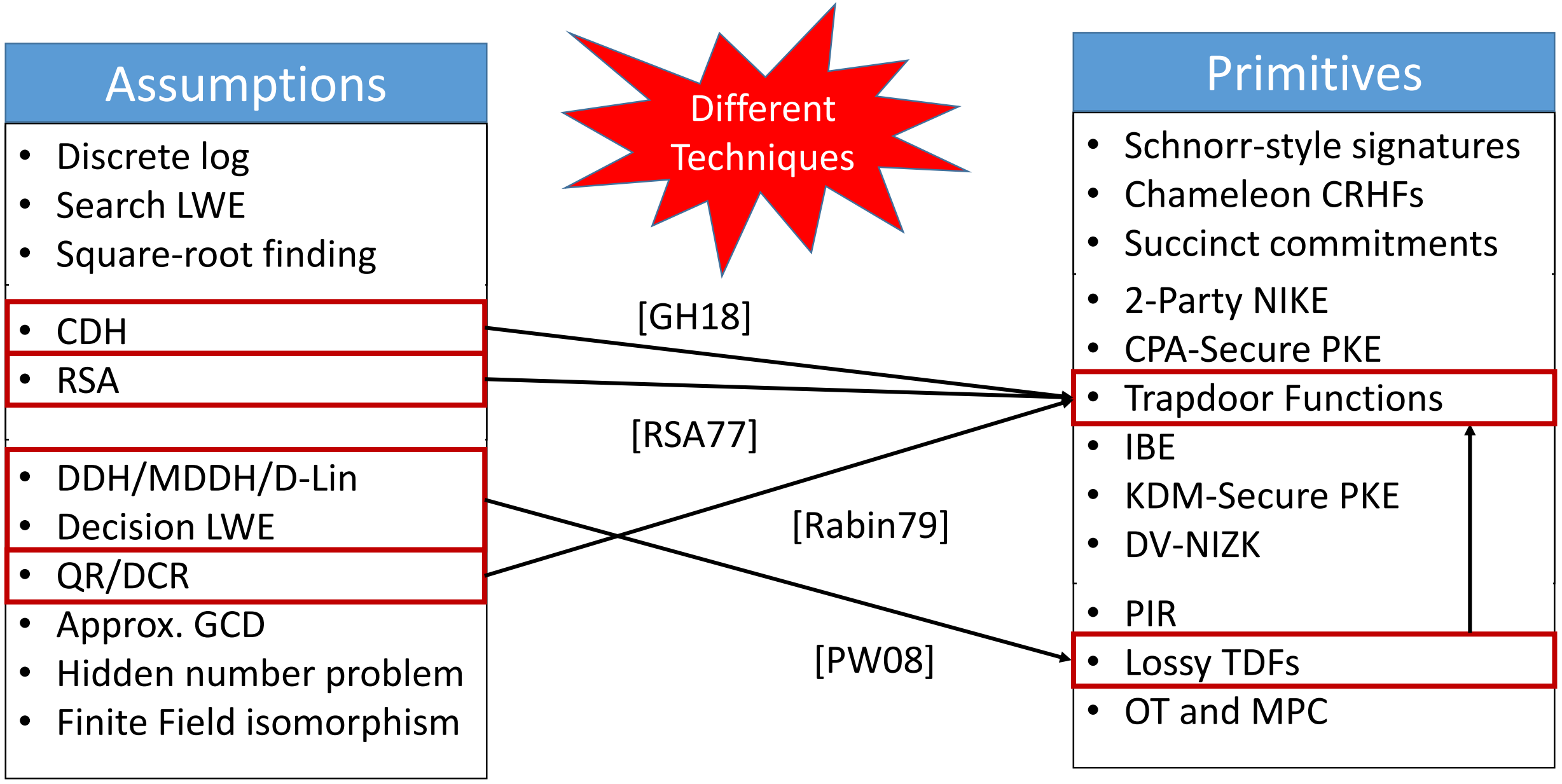
Traditional Approach to Cryptography



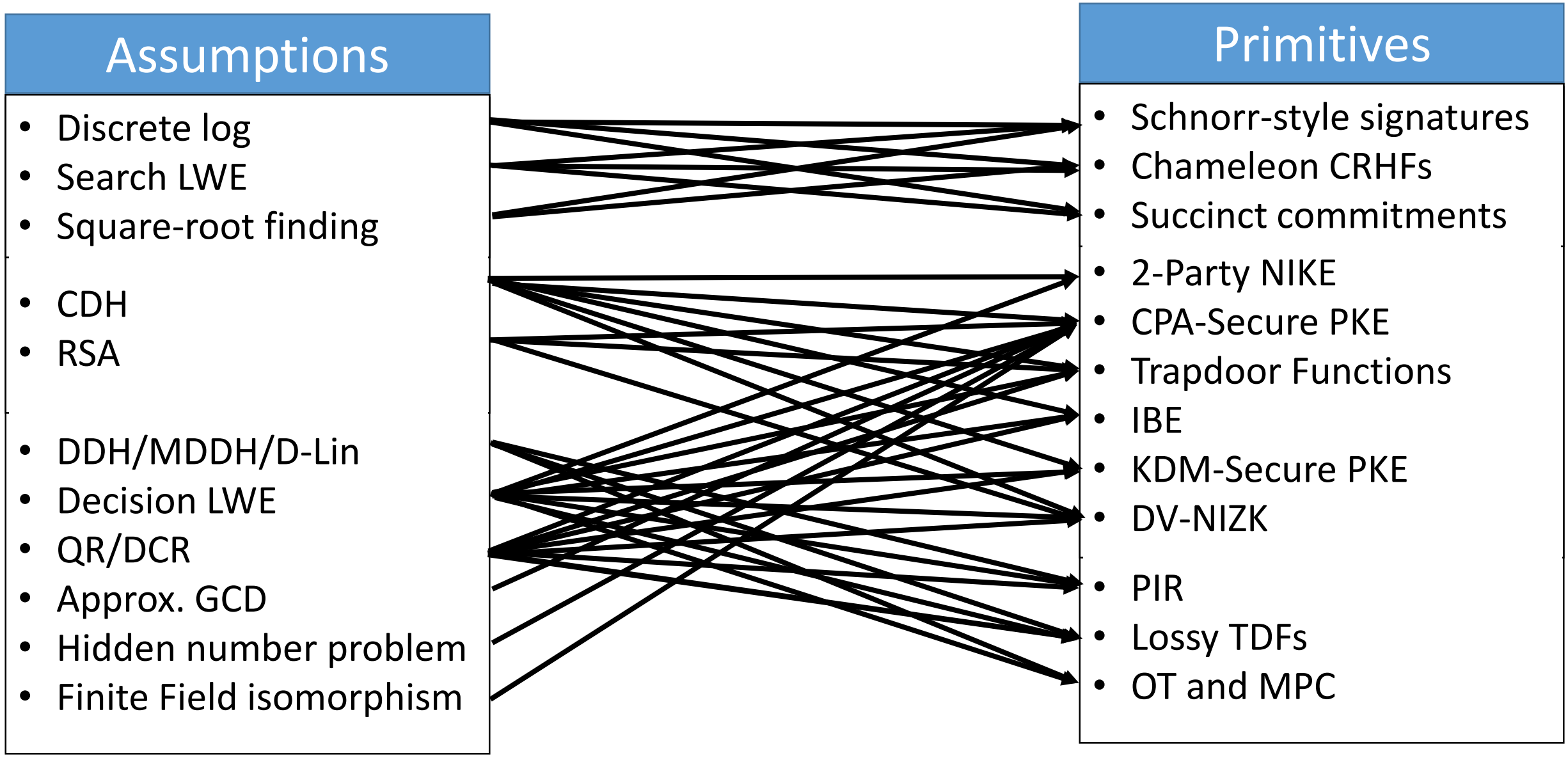
Traditional Approach to Cryptography



Traditional Approach to Cryptography

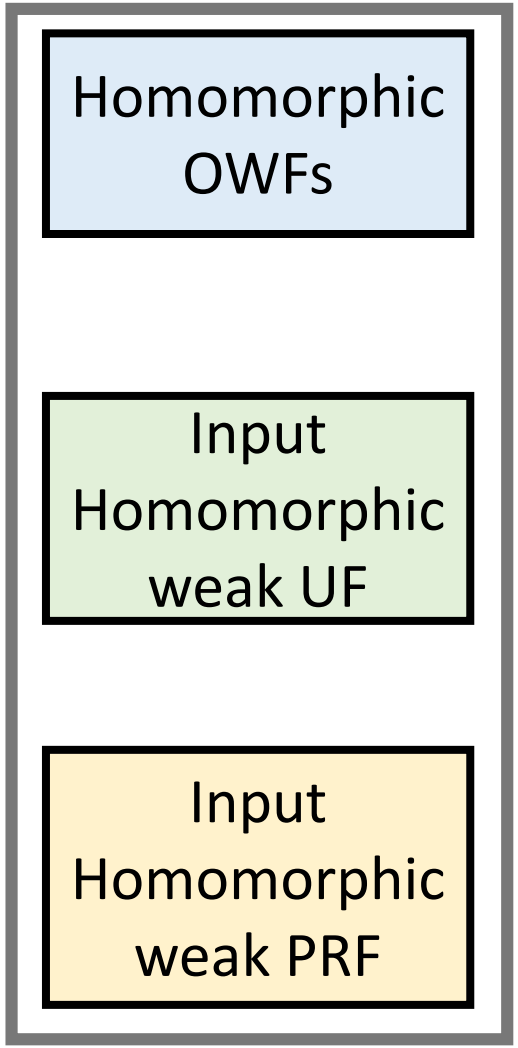


Traditional Approach to Cryptography



A “Narrow-Waist” Approach

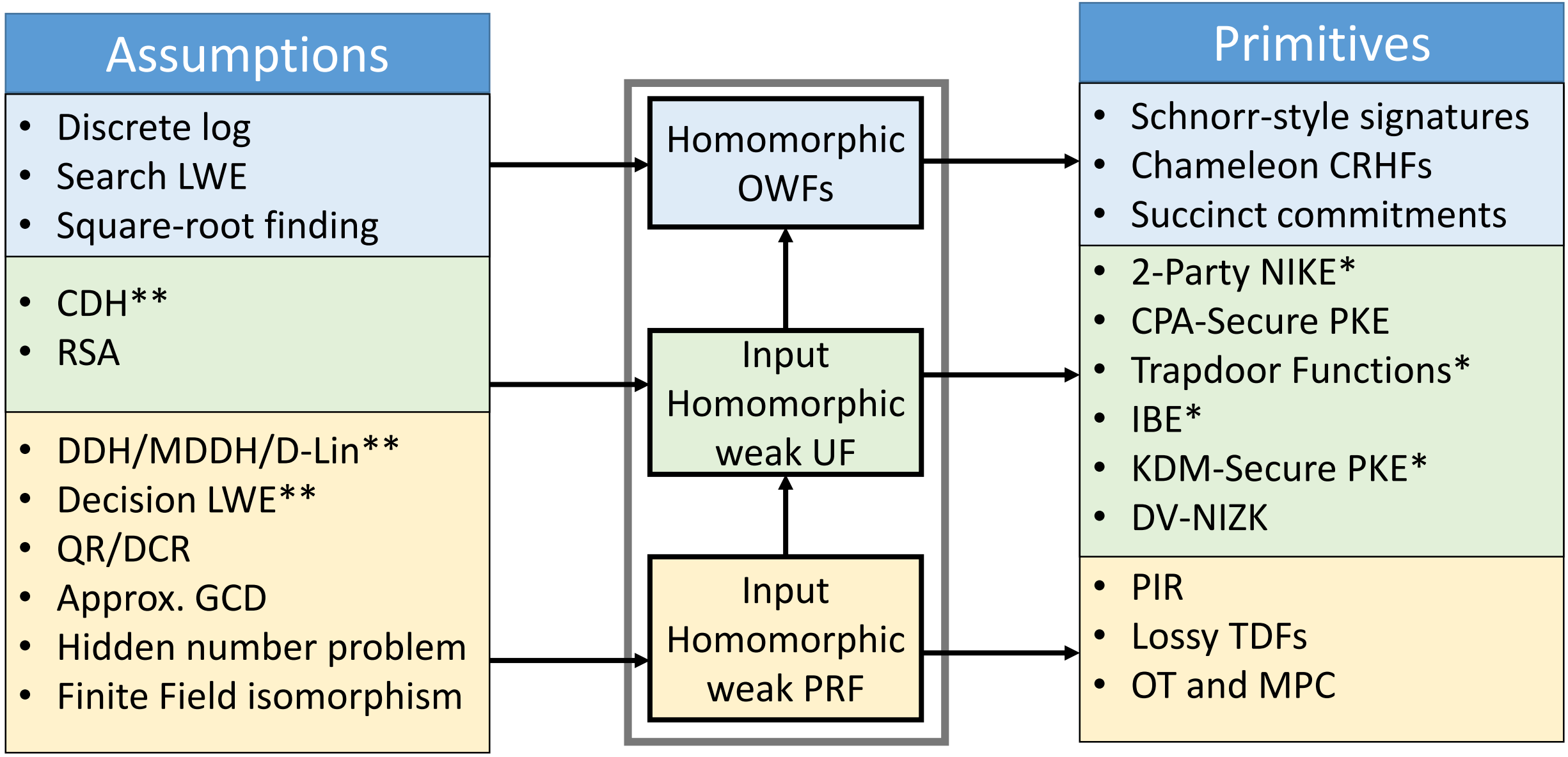
Assumptions
• Discrete log • Search LWE • Square-root finding
• CDH • RSA
• DDH/MDDH/D-Lin • Decision LWE • QR/DCR • Approx. GCD • Hidden number problem • Finite Field isomorphism



Primitives
• Schnorr-style signatures • Chameleon CRHFs • Succinct commitments
• 2-Party NIKE • CPA-Secure PKE • Trapdoor Functions • IBE • KDM-Secure PKE • DV-NIZK
• PIR • Lossy TDFs • OT and MPC

A “Narrow-Waist” Approach

* primitives need some additional structure from ** assumption



A “Narrow-Waist” Approach

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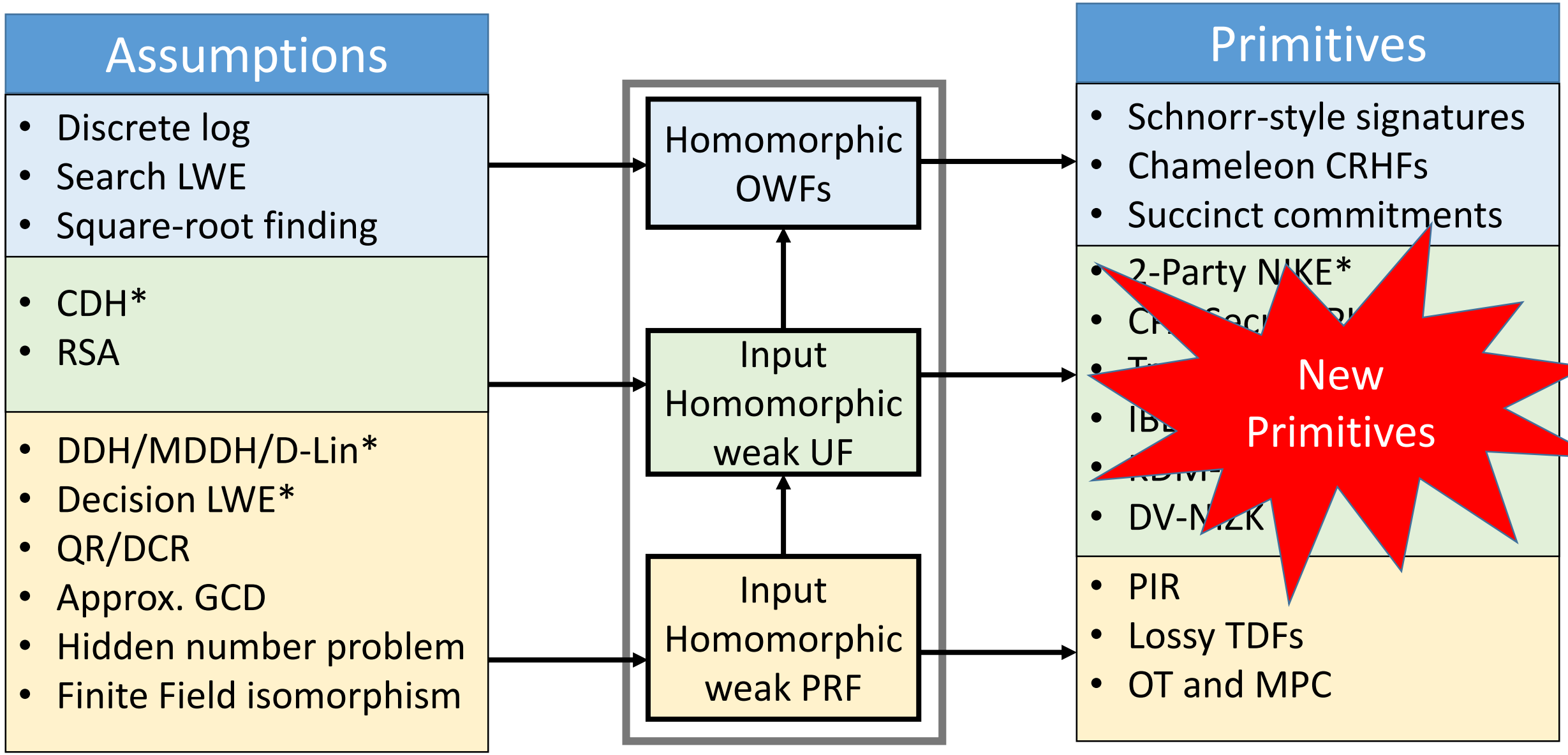
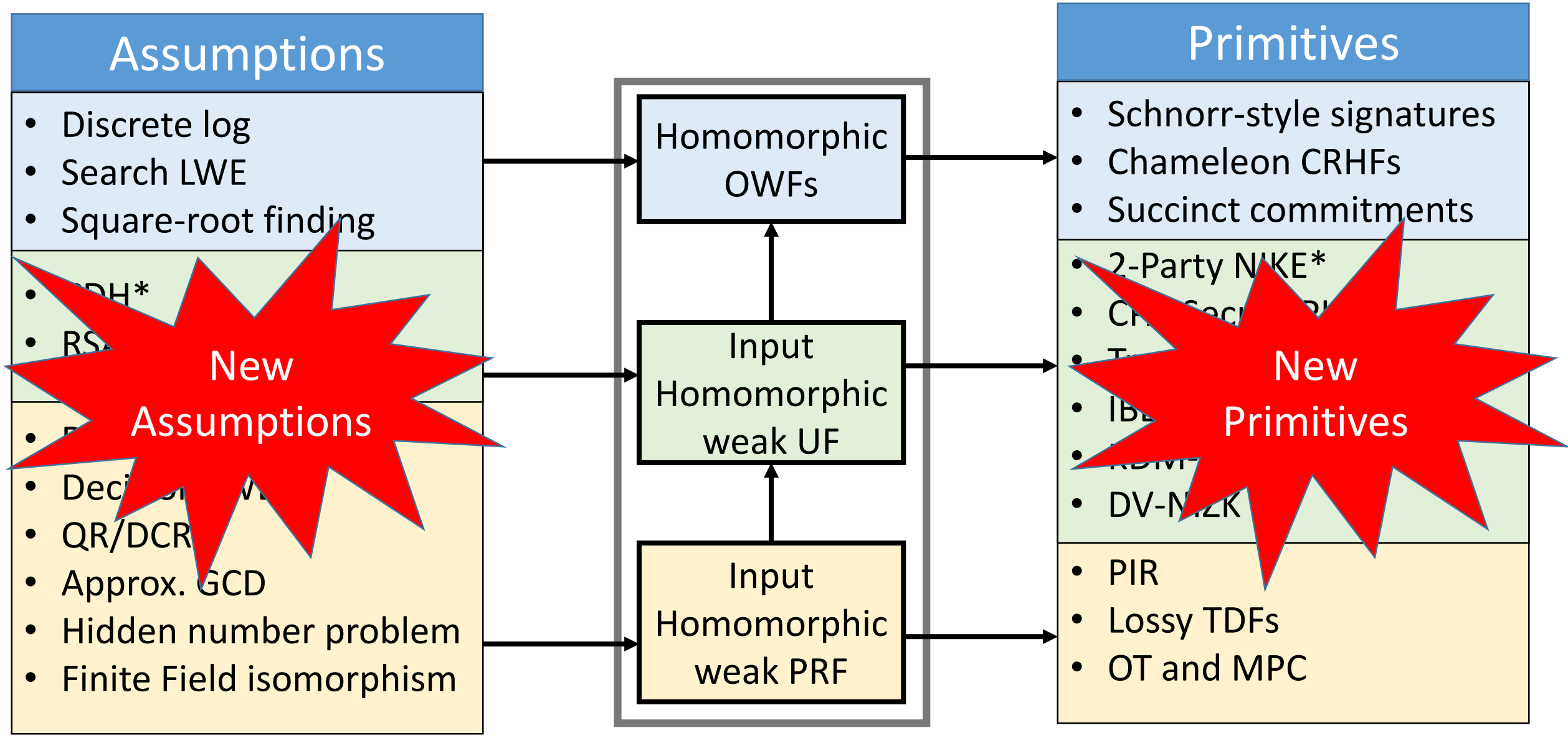


A “Narrow-Waist” Approach

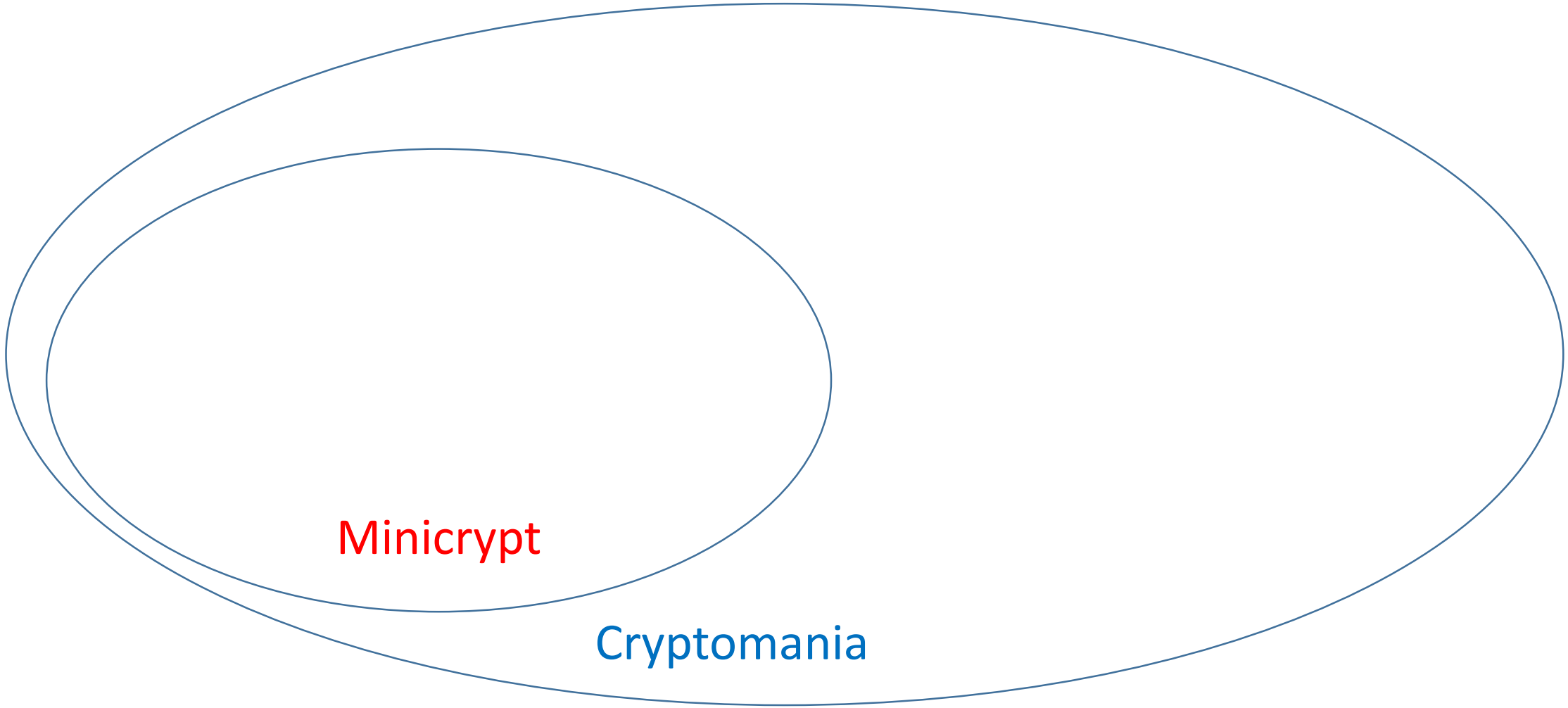
* primitives need some additional structure from ** assumption



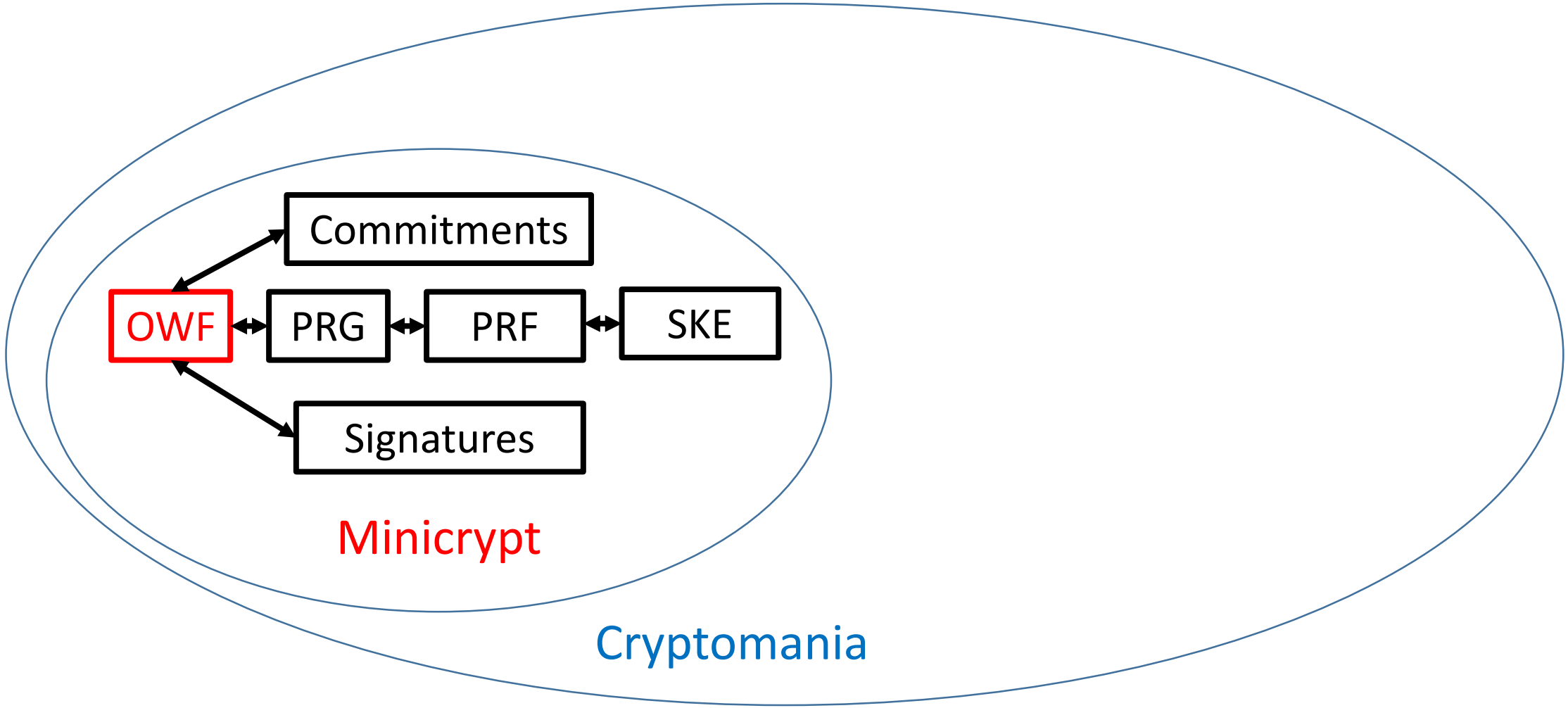
Outline

- Our results
- An alternative approach to designing primitives
- **A cryptoplexity hierarchy in Cryptomania**
- Technical overview
- Summary and open questions

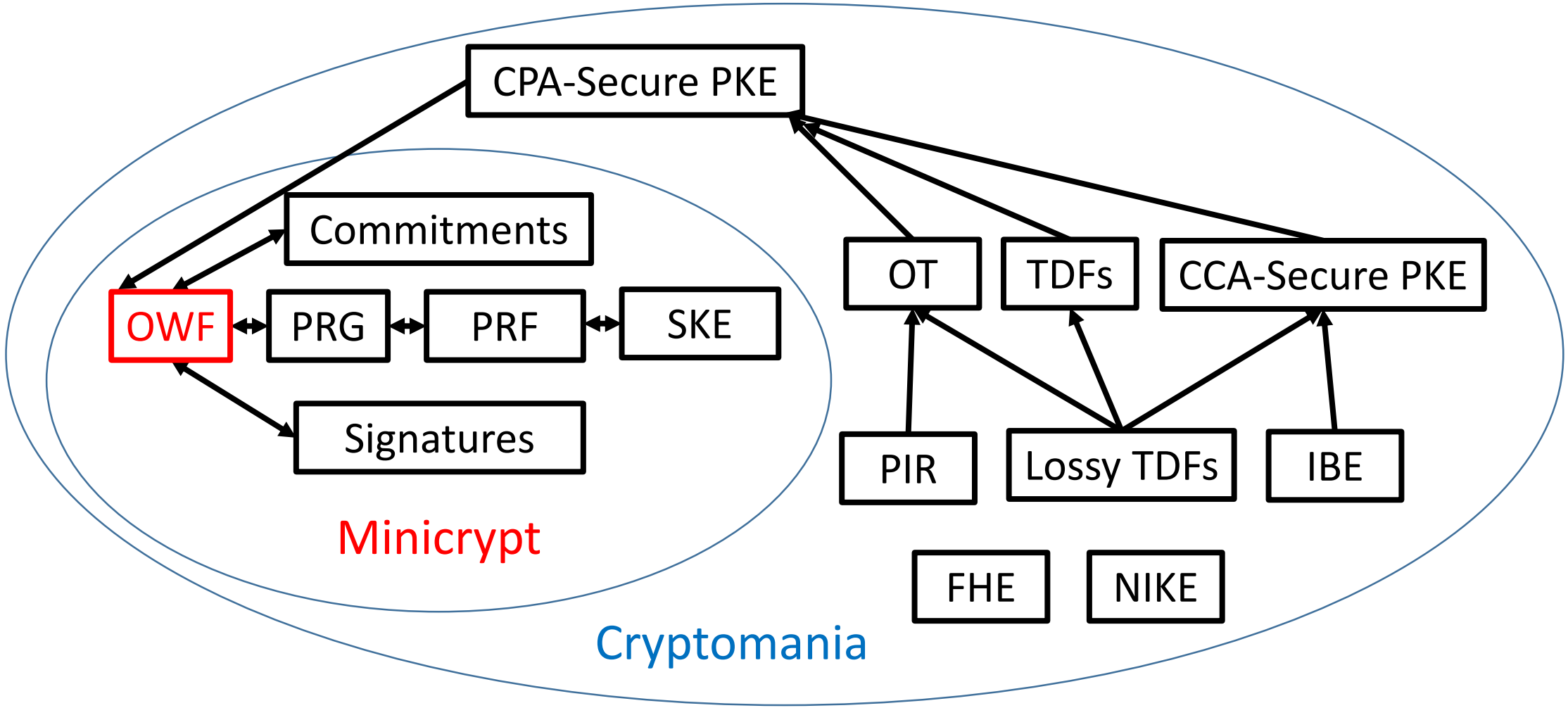
Cryptoplexity Perspective



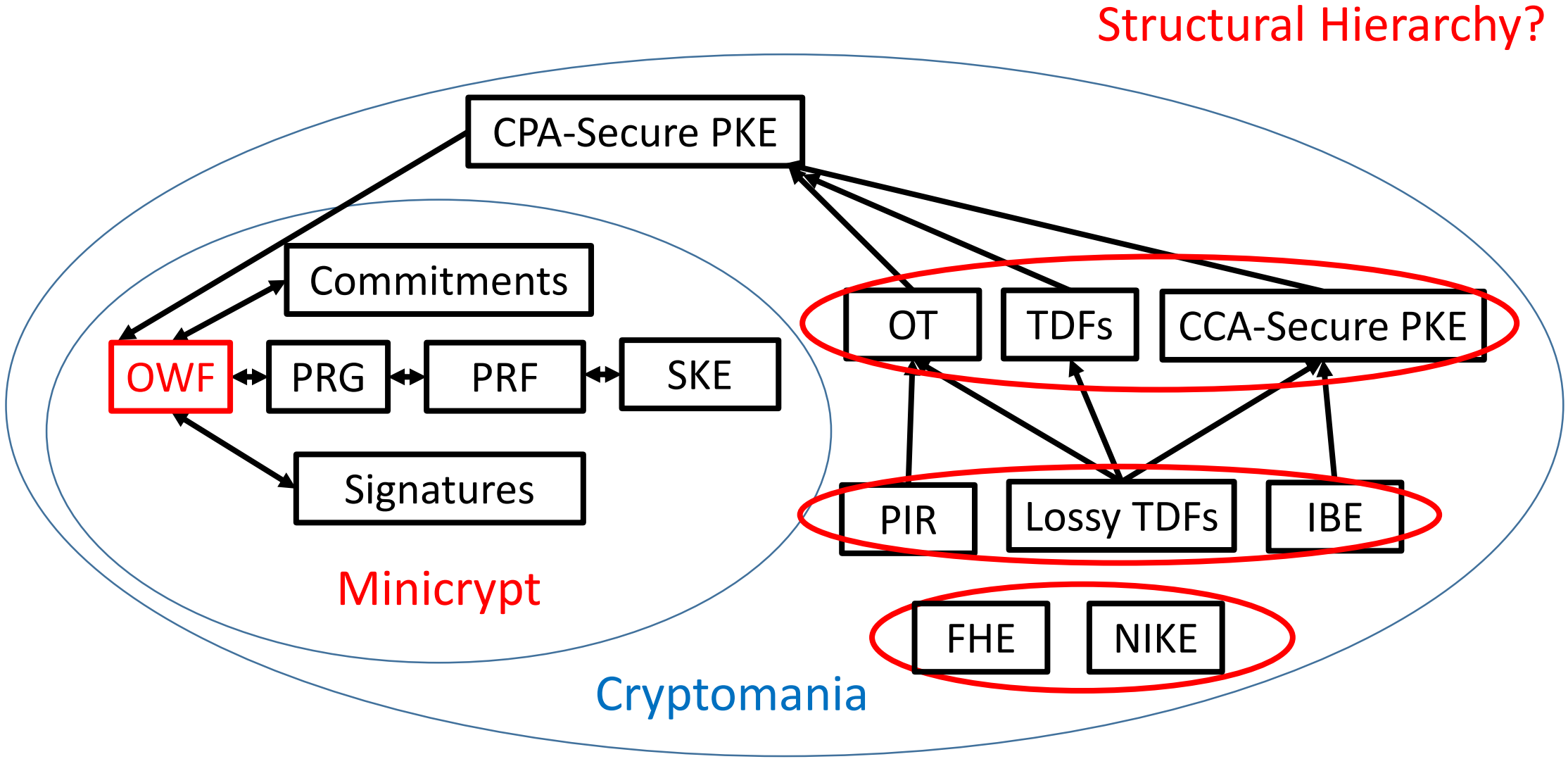
OWFs are “Minicrypt-Complete”



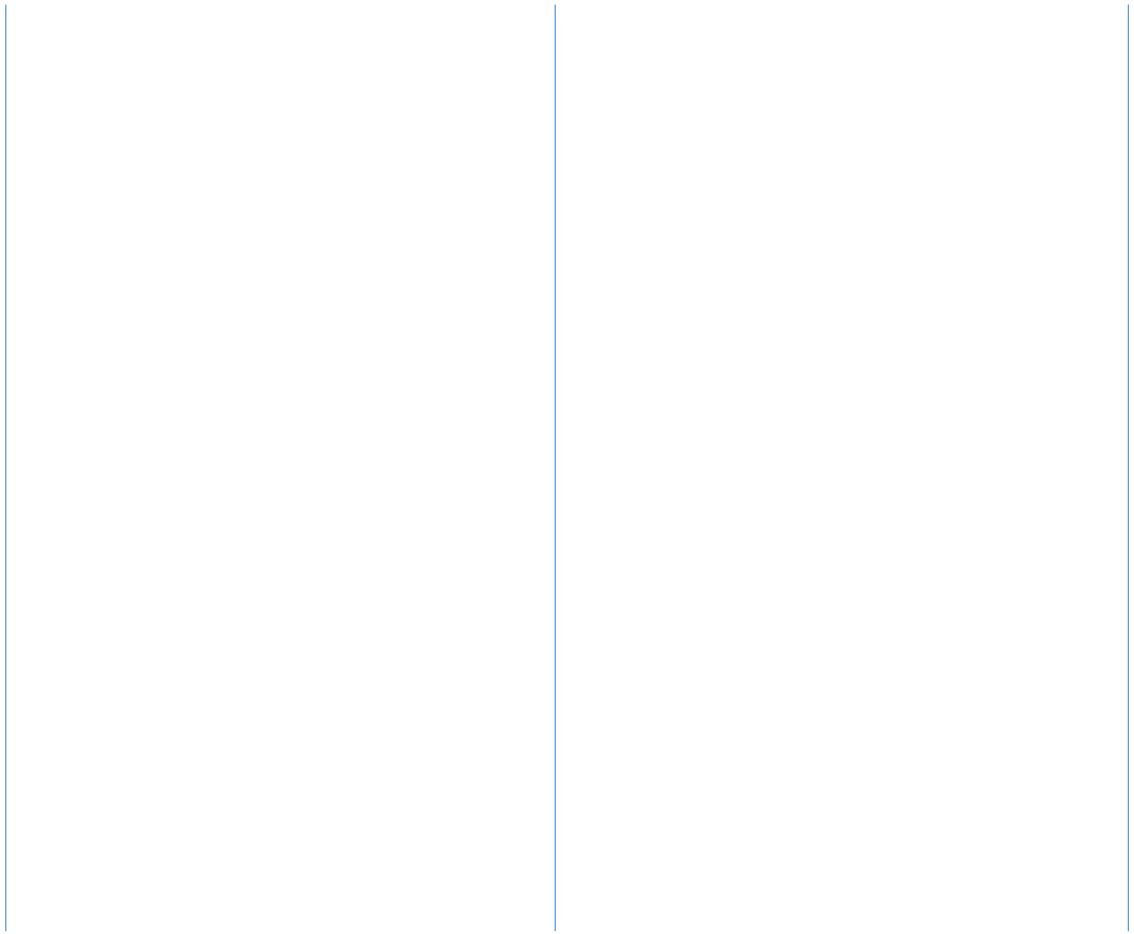
What about Cryptomania?



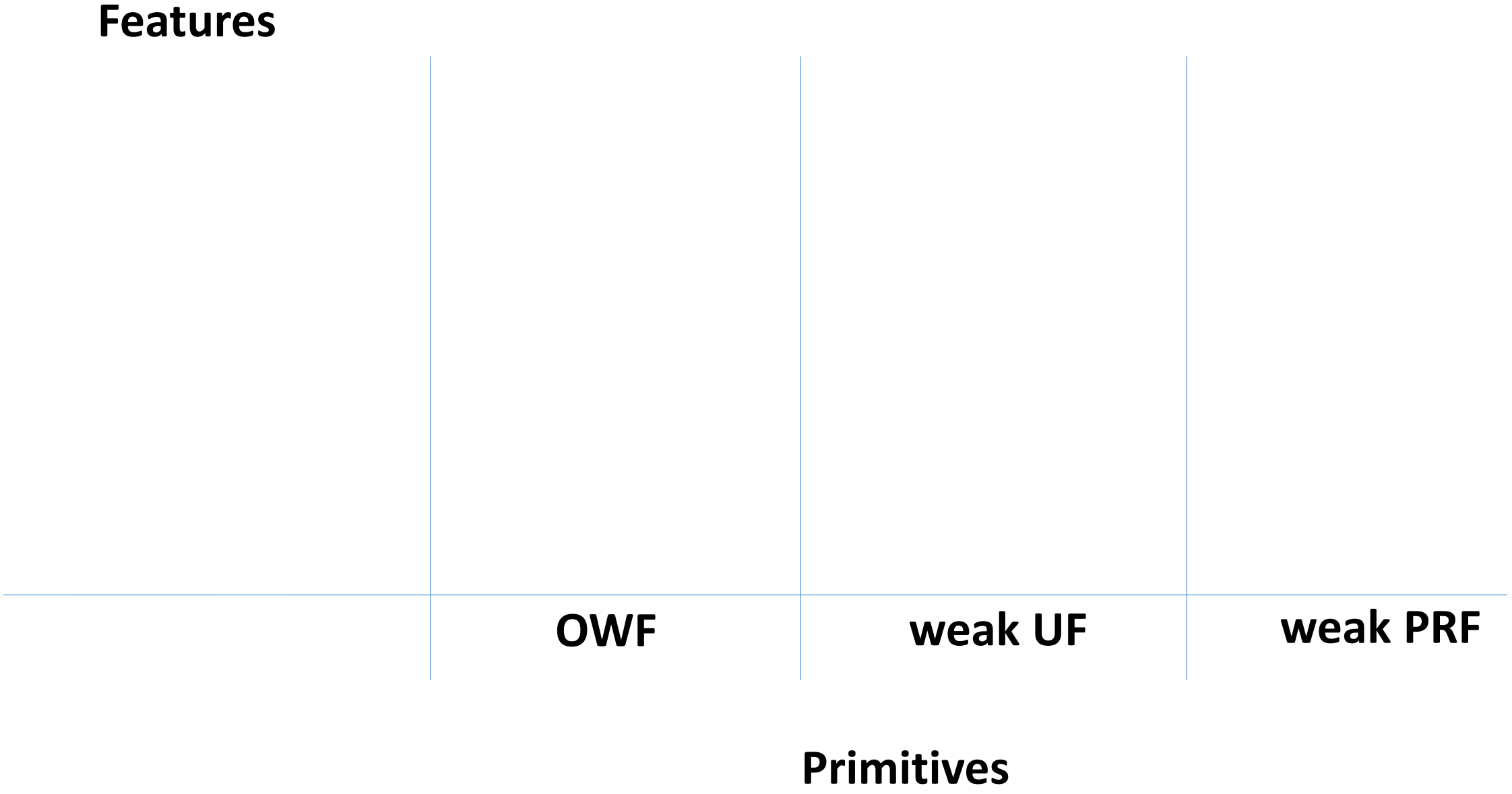
What about Cryptomania?



A Potential Cryptoplexity Hierarchy



A Potential Cryptoplexity Hierarchy



A Potential Cryptoplexity Hierarchy

Features

Dual-computability		2-party NIKE, PKE	
	OWF	**weak UF**	**weak PRF**

Primitives

A Potential Cryptoplexity Hierarchy

Features

Group Input-Homomorphism	Trapdoor CRHF, Succinct Commitments	TDF, IBE, DV-NIZK KDM-Secure PKE	PIR, Lossy TDF, OT, MPC
Dual-computability		2-party NIKE, PKE	
	OWF	**weak UF**	**weak PRF**

Primitives

A Potential Cryptoplexity Hierarchy

Features

Ring Input-Homomorphism			Fully-Homomorphic Encryption
Group Input-Homomorphism	Trapdoor CRHF, Succinct Commitments	TDF, IBE, DV-NIZK KDM-Secure PKE	PIR, Lossy TDF, OT, MPC
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Primitives

A Potential Cryptoplexity Hierarchy

Features

Ring Input-Homomorphism			Fully-Homomorphic Encryption
Group Key-Homomorphism ↓ **Group Input-Homomorphism**	Trapdoor CRHF, Succinct Commitments	TDF, IBE, DV-NIZK KDM-Secure PKE	PIR, Lossy TDF, OT, MPC
Dual-computability		2-party NIKE, PKE	
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Primitives

A Potential Cryptoplexity Hierarchy

Features			
Ring Key-Homomorphism			Multi-party NIKE, Indistinguishability Obfuscation
Ring Input-Homomorphism			Fully-Homomorphic Encryption
Group Key-Homomorphism ↓ **Group Input-Homomorphism**	Trapdoor CRHF, Succinct Commitments	TDF, IBE, DV-NIZK KDM-Secure PKE	PIR, Lossy TDF, OT, MPC
Dual-computability		2-party NIKE, PKE	
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		Primitives	

A Potential Cryptoplexity Hierarchy

Features	Field key/input homomorphism is impossible!		
Ring Key-Homomorphism			Multi-party NIKE, Indistinguishability Obfuscation
Ring Input-Homomorphism			Fully-Homomorphic Encryption
Group Key-Homomorphism ↓ **Group Input-Homomorphism**	Trapdoor CRHF, Succinct Commitments	TDF, IBE, DV-NIZK KDM-Secure PKE	PIR, Lossy TDF, OT, MPC
Dual-computability		2-party NIKE, PKE	
	OWF	**weak UF**	**weak PRF**
		Primitives	

A Potential Cryptoplexity Hierarchy

Field key/input homomorphism is impossible!

More Structure

Features			
Ring Key-Homomorphism			Multi-party NIKE, Indistinguishability Obfuscation
Ring Input-Homomorphism			Fully-Homomorphic Encryption
Group Key-Homomorphism ↓ **Group Input-Homomorphism**	Trapdoor CRHF, Succinct Commitments	TDF, IBE, DV-NIZK KDM-Secure PKE	PIR, Lossy TDF, OT, MPC
Dual-computability		2-party NIKE, PKE	
	OWF	**weak UF**	**weak PRF**

Primitives

Outline

- Our results
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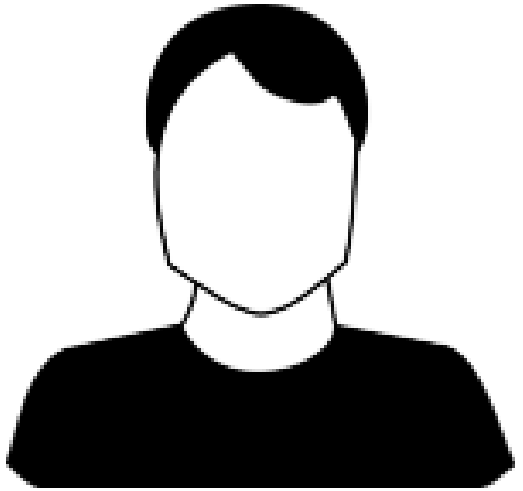
- **Techniques**
- Summary and open questions

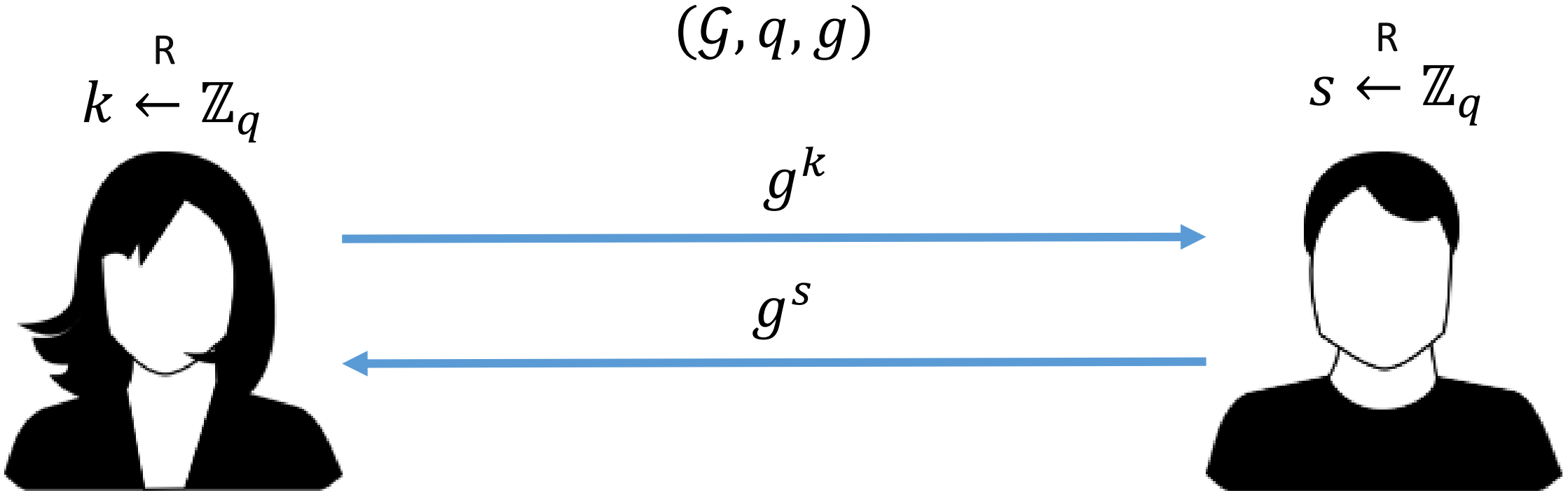
Two-party non-interactive key-exchange (NIKE) from Input-Homomorphic weak UF

Two-party NIKE from CDH (Computational Diffie-Hellman)

$$(\mathcal{G}, q, g)$$



Two-party NIKE from CDH (Computational Diffie-Hellman)



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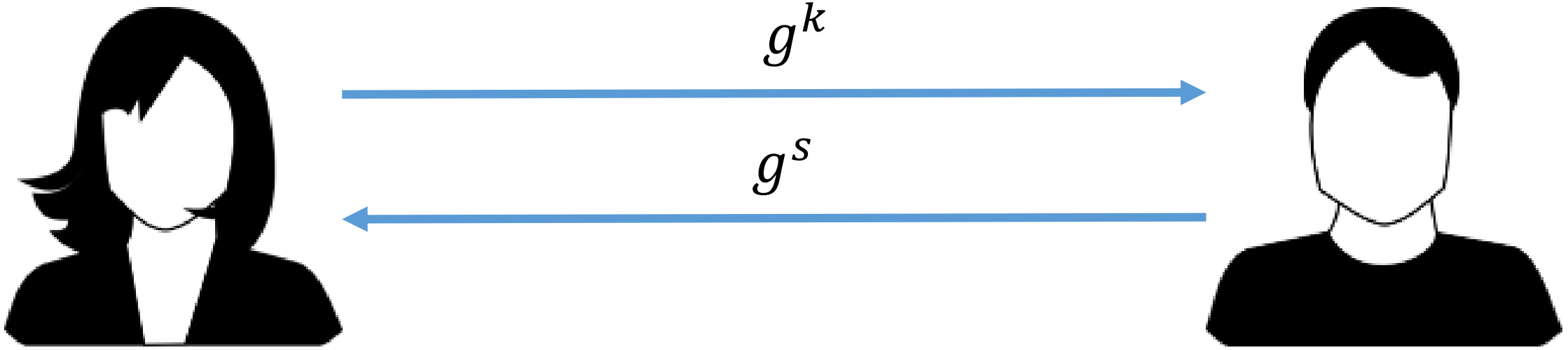


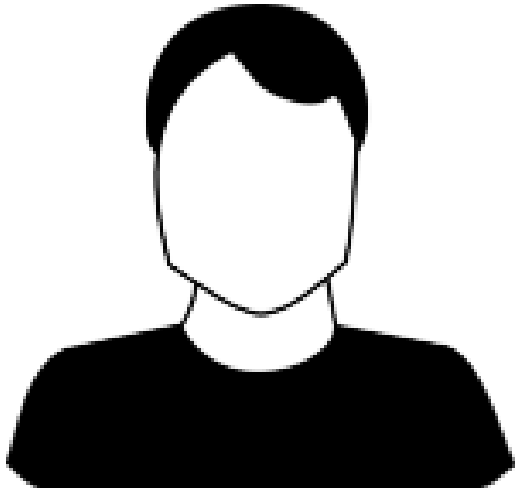
GL (g^{ks})

GL(): Goldreich-Levin Hardcore Predicate [GL89]

GL (g^{ks})

Two-party NIKE from CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right) \qquad n > 3 \log |\mathcal{G}|$$



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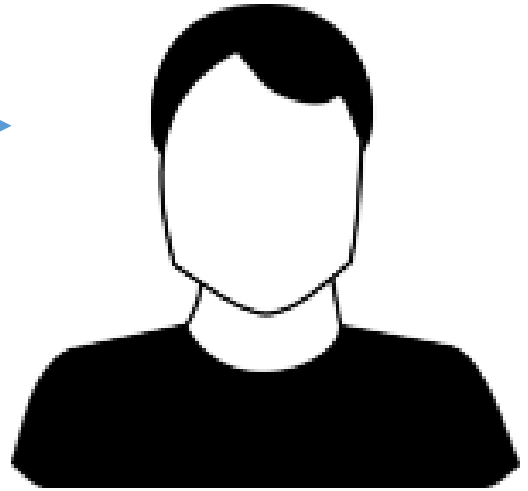
$$k \xleftarrow{R} \mathbb{Z}_q$$

$$\begin{matrix} g_{1,0}^k & \cdots & g_{n,0}^k \\ g_{1,1}^k & \cdots & g_{n,1}^k \end{matrix}$$



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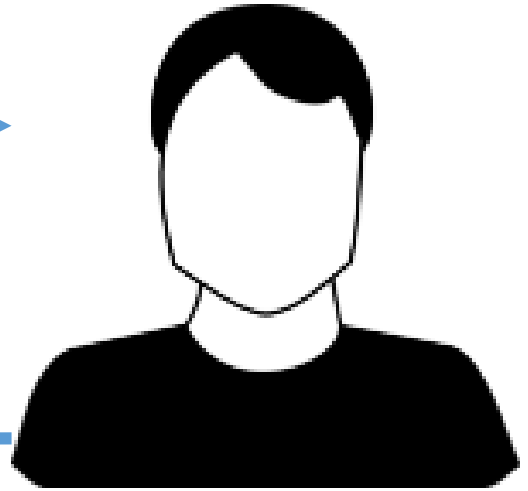
$k \xleftarrow{R} \mathbb{Z}_q$

$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n$

$$\begin{matrix} g_{1,0}^k & \cdots & g_{n,0}^k \\ g_{1,1}^k & \cdots & g_{n,1}^k \end{matrix}$$

$$\prod_{j=1}^{n} g_{j,s_j}$$



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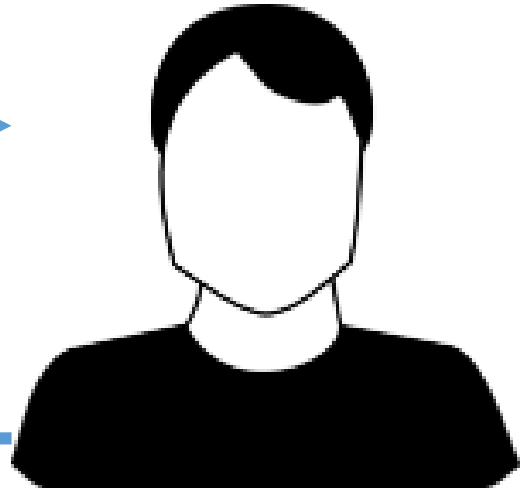

$\prod_{j=1}^{n} g_{j,s_j}^k$

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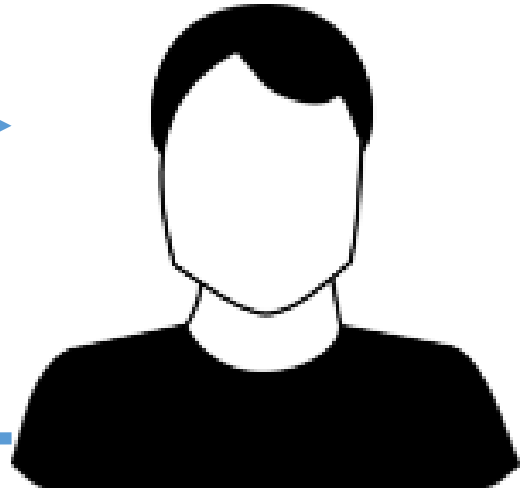

$\mathrm{GL}(\prod_{j=1}^{n} g_{j,s_j}^k)$

$\mathrm{GL}(\prod_{j=1}^{n} g_{j,s_j}^k)$

Two-party NIKE from CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right) \qquad n > 3\log|\mathcal{G}|$$

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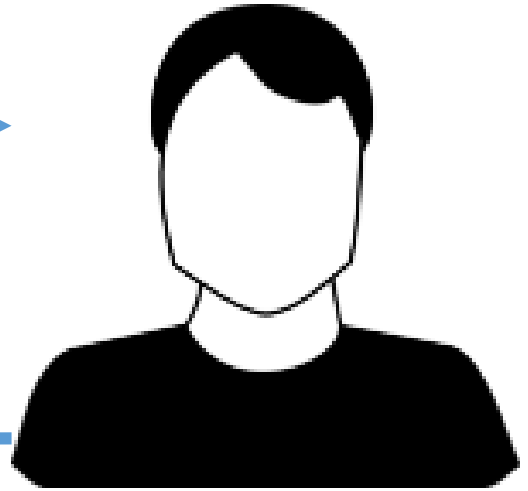

$\mathbf{s}$ is computationally hidden (by DLOG)

$\mathrm{GL}(\prod_{j=1}^{n} g_{j,s_j}^k)$

$\mathrm{GL}(\prod_{j=1}^{n} g_{j,s_j}^k)$

Two-party NIKE from CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right) \qquad n > 3\log|\mathcal{G}|$$

$k \xleftarrow{R} \mathbb{Z}_q$

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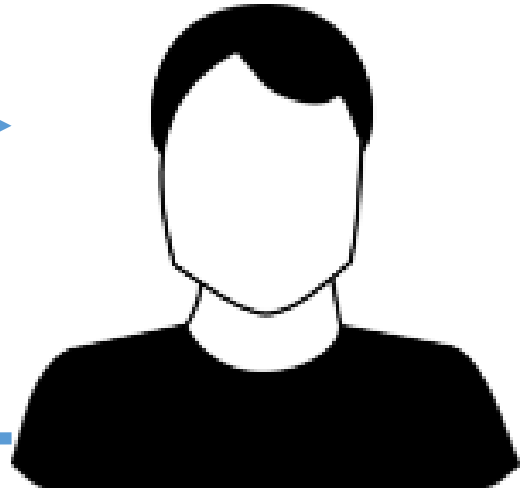
$$\begin{matrix} g_{1,0}^k & \cdots & g_{n,0}^k \\ g_{1,1}^k & \cdots & g_{n,1}^k \end{matrix}$$

By LHL

$$\prod_{j=1}^{n} g_{j,s_j} \approx_s g^*$$



$\mathrm{GL}(\prod_{j=1}^{n} g_{j,s_j}^k) \approx_s \mathrm{GL}(g^{*k})$

$\mathrm{GL}(g^{*k}) \approx_s \mathrm{GL}(\prod_{j=1}^{n} g_{j,s_j}^k)$

Two-party NIKE from CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right) \qquad n > 3\log|\mathcal{G}|$$

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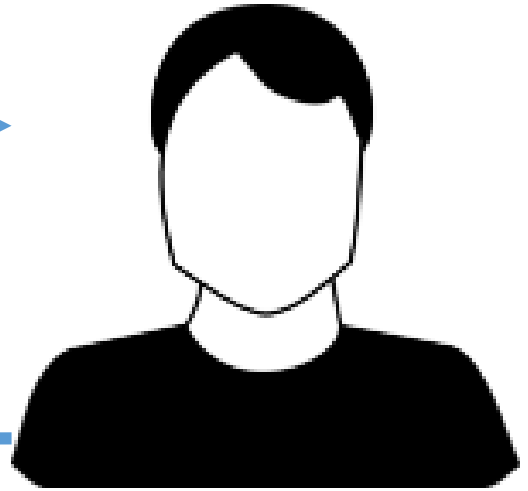
$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n$



By LHL

$$\prod_{j=1}^{n} g_{j,s_j} \approx_s g^*$$

$$\mathrm{GL}\left(\prod_{j=1}^{n} g_{j,s_j}^k\right) \approx_s \boxed{\mathrm{GL}(g^{*k})} \approx_c b' \approx_c \boxed{\mathrm{GL}(g^{*k})} \approx_s \mathrm{GL}\left(\prod_{j=1}^{n} g_{j,s_j}^k\right)$$

By CDH

Translating to Input Homomorphic weak UF

CDH

IHwUF

Translating to Input Homomorphic weak UF

CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{G}|$$

IHwUF

$$\left(F\colon \mathcal{K} \times \mathcal{X} \to \mathcal{Y} \quad \begin{matrix} x_{1,0} & \cdots & x_{n,0} \\ x_{1,1} & \cdots & x_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{X}|$$

Translating to Input Homomorphic weak UF

CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{G}|$$

$$k \xleftarrow{R} \mathbb{Z}_q \qquad \begin{matrix} g_{1,0}^k & \cdots & g_{n,0}^k \\ g_{1,1}^k & \cdots & g_{n,1}^k \end{matrix}$$

IHwUF

$$\left(F\colon \mathcal{K} \times \mathcal{X} \to \mathcal{Y} \qquad \begin{matrix} x_{1,0} & \cdots & x_{n,0} \\ x_{1,1} & \cdots & x_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{X}|$$

$$k \xleftarrow{R} \mathcal{K} \qquad \begin{matrix} F(k, x_{1,0}) & \cdots & F(k, x_{n,0}) \\ F(k, x_{2,0}) & \cdots & F(k, x_{n,1}) \end{matrix}$$

Translating to Input Homomorphic weak UF

CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{G}|$$

$$k \xleftarrow{R} \mathbb{Z}_q \qquad \begin{matrix} g_{1,0}^k & \cdots & g_{n,0}^k \\ g_{1,1}^k & \cdots & g_{n,1}^k \end{matrix}$$

$$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n \qquad \prod_{j=1}^{n} g_{j,s_j}$$

IHwUF

$$\left(F\colon \mathcal{K} \times \mathcal{X} \to \mathcal{Y} \quad \begin{matrix} x_{1,0} & \cdots & x_{n,0} \\ x_{1,1} & \cdots & x_{n,1} \end{matrix}\right)$$

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$$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n \qquad \bigoplus_{j=1}^{n} x_{j,s_j}$$

Translating to Input Homomorphic weak UF

CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{G}|$$

$$k \xleftarrow{R} \mathbb{Z}_q \qquad \begin{matrix} g_{1,0}^k & \cdots & g_{n,0}^k \\ g_{1,1}^k & \cdots & g_{n,1}^k \end{matrix}$$

One-way by DLOG

$$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n \qquad \prod_{j=1}^{n} g_{j,s_j}$$

IHwUF

$$\left(F\colon \mathcal{K} \times \mathcal{X} \to \mathcal{Y} \quad \begin{matrix} x_{1,0} & \cdots & x_{n,0} \\ x_{1,1} & \cdots & x_{n,1} \end{matrix}\right)$$

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$$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n \qquad \bigoplus_{j=1}^{n} x_{j,s_j}$$

Translating to Input Homomorphic weak UF

CDH

$$\left(\mathcal{G}, q, \begin{matrix} g_{1,0} & \cdots & g_{n,0} \\ g_{1,1} & \cdots & g_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{G}|$$

$$k \xleftarrow{R} \mathbb{Z}_q \qquad \begin{matrix} g_{1,0}^k & \cdots & g_{n,0}^k \\ g_{1,1}^k & \cdots & g_{n,1}^k \end{matrix}$$

One-way by DLOG

$$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n \qquad \prod_{j=1}^{n} g_{j,s_j}$$

IHwUF

$$\left(F\colon \mathcal{K} \times \mathcal{X} \to \mathcal{Y}, \begin{matrix} x_{1,0} & \cdots & x_{n,0} \\ x_{1,1} & \cdots & x_{n,1} \end{matrix}\right)$$

$$n > 3 \log |\mathcal{X}|$$

$$k \xleftarrow{R} \mathcal{K} \qquad \begin{matrix} F(k, x_{1,0}) & \cdots & F(k, x_{n,0}) \\ F(k, x_{2,0}) & \cdots & F(k, x_{n,1}) \end{matrix}$$

Is this one-way in general?

$$\mathbf{s} = (s_1, \ldots, s_n) \xleftarrow{R} \{0,1\}^n \qquad \bigoplus_{j=1}^{n} x_{j,s_j}$$

One-Wayness of Subset Sums

Theorem:

If $\mathcal{X}$ is the input space of any input homomorphic weak UF

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Explains the structural similarity underlying these assumptions

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If $\mathcal{X}$ is the input space of any input homomorphic weak UF then the subset sum problem (with appropriate parameters) is average-case hard on $\mathcal{X}$

$$\begin{matrix} x_{1,0} & \dots & x_{n,0} \\ x_{1,1} & \dots & x_{n,1} \end{matrix}, \quad x^* \xleftarrow{R} \mathcal{X}$$

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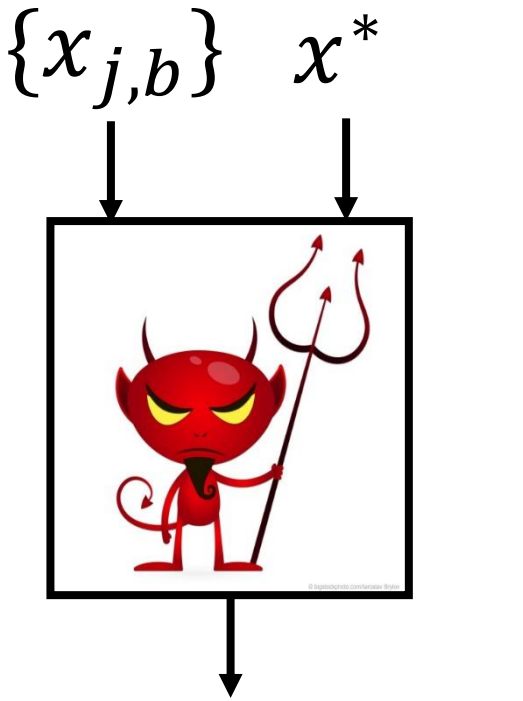

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$\{x_{j,b}\}$ x^*

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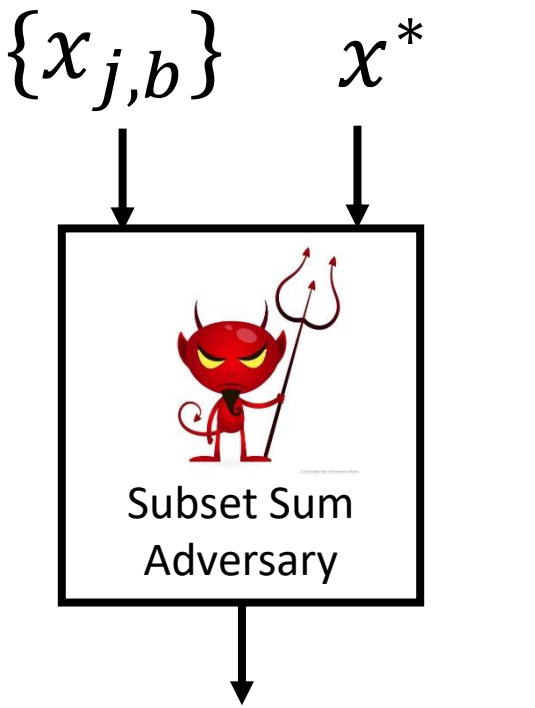

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→ Adversary against IHwUF $F: \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$

IHwUF Adversary

$\{x_{j,b}\}$ x^*

Subset Sum Adversary

$\mathbf{s} = (s_1, \ldots, s_n)$

$\{x_{j,b}, F(k, x_{j,b})\}$

$k \leftarrow \mathcal{K}$

x^*

y^*

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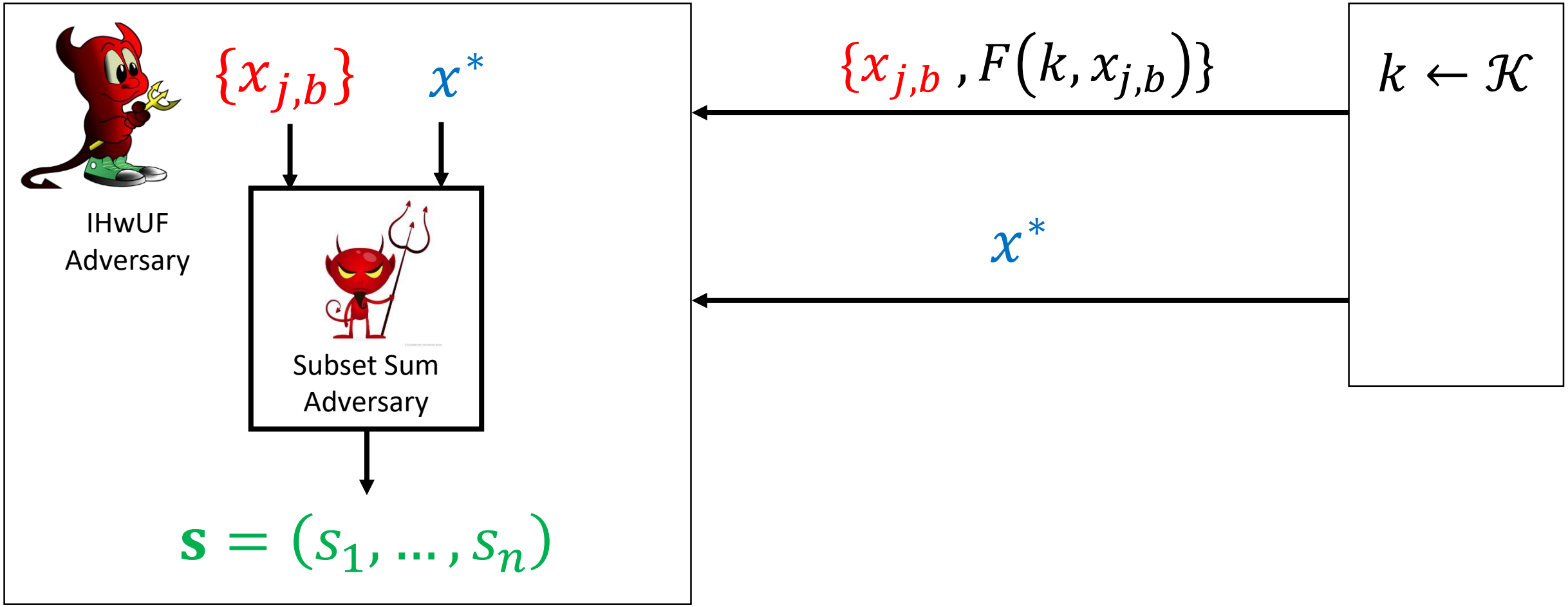
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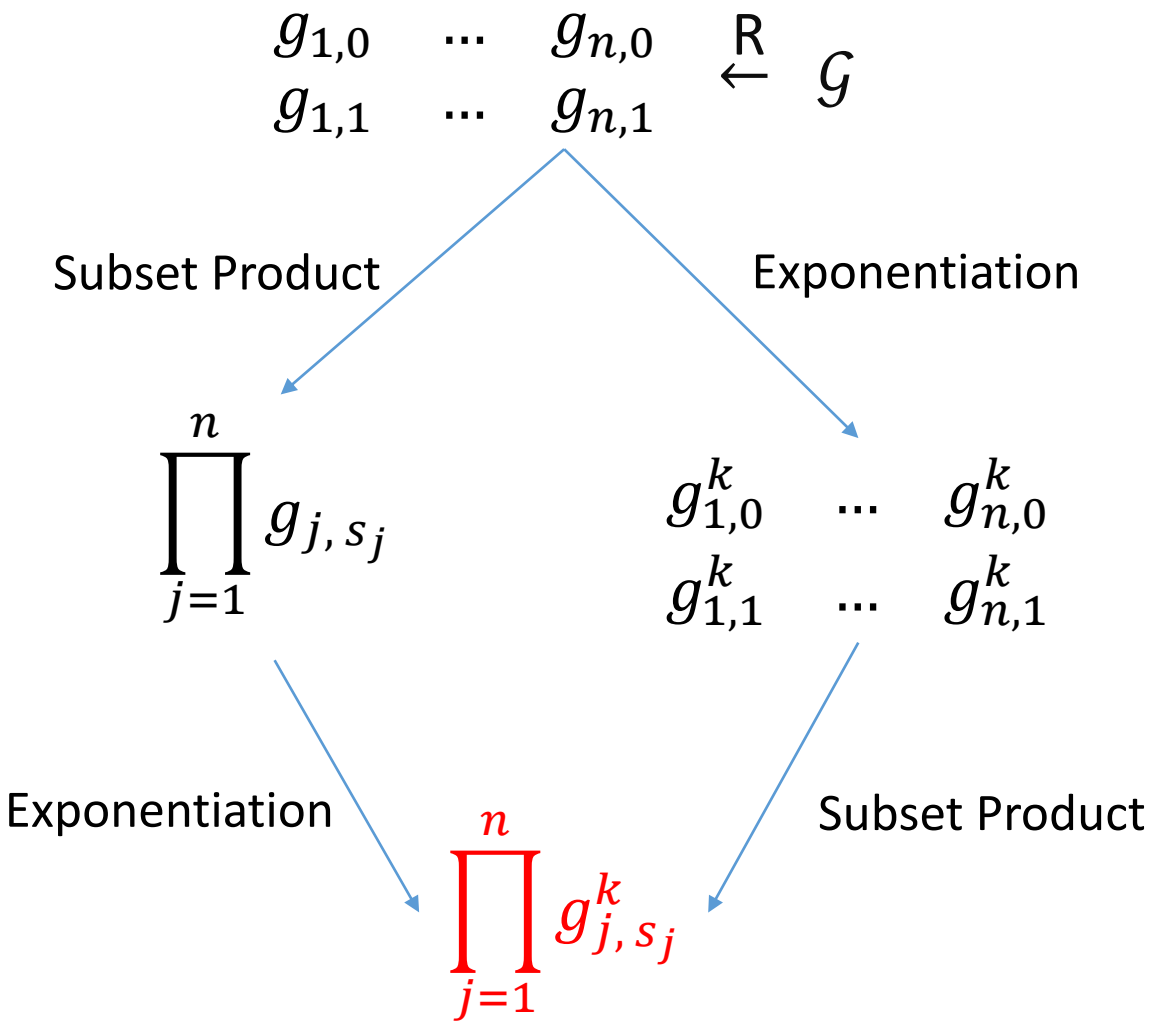
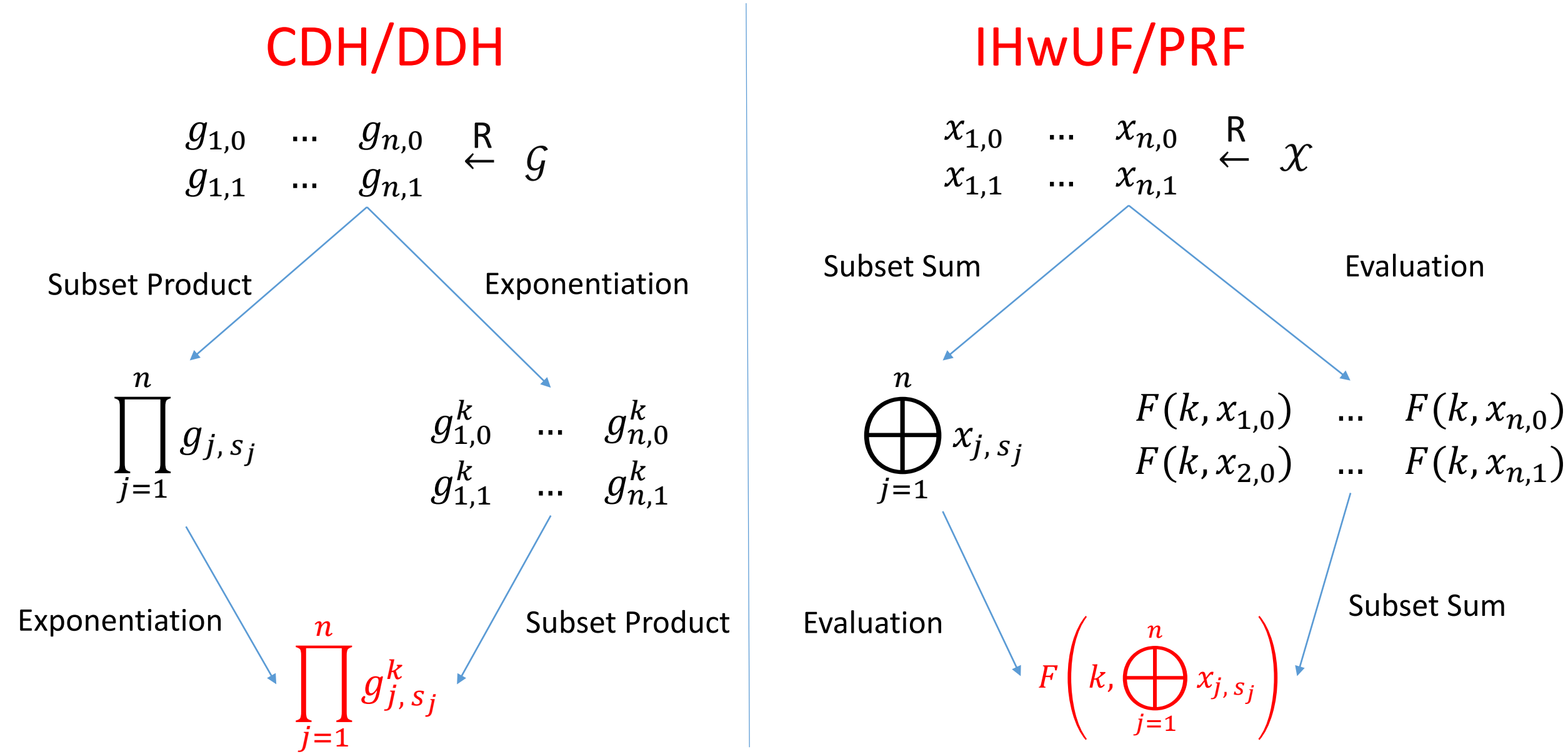
Abstracting a CDH/DDH-based protocol

CDH/DDH



IHwUF/PRF

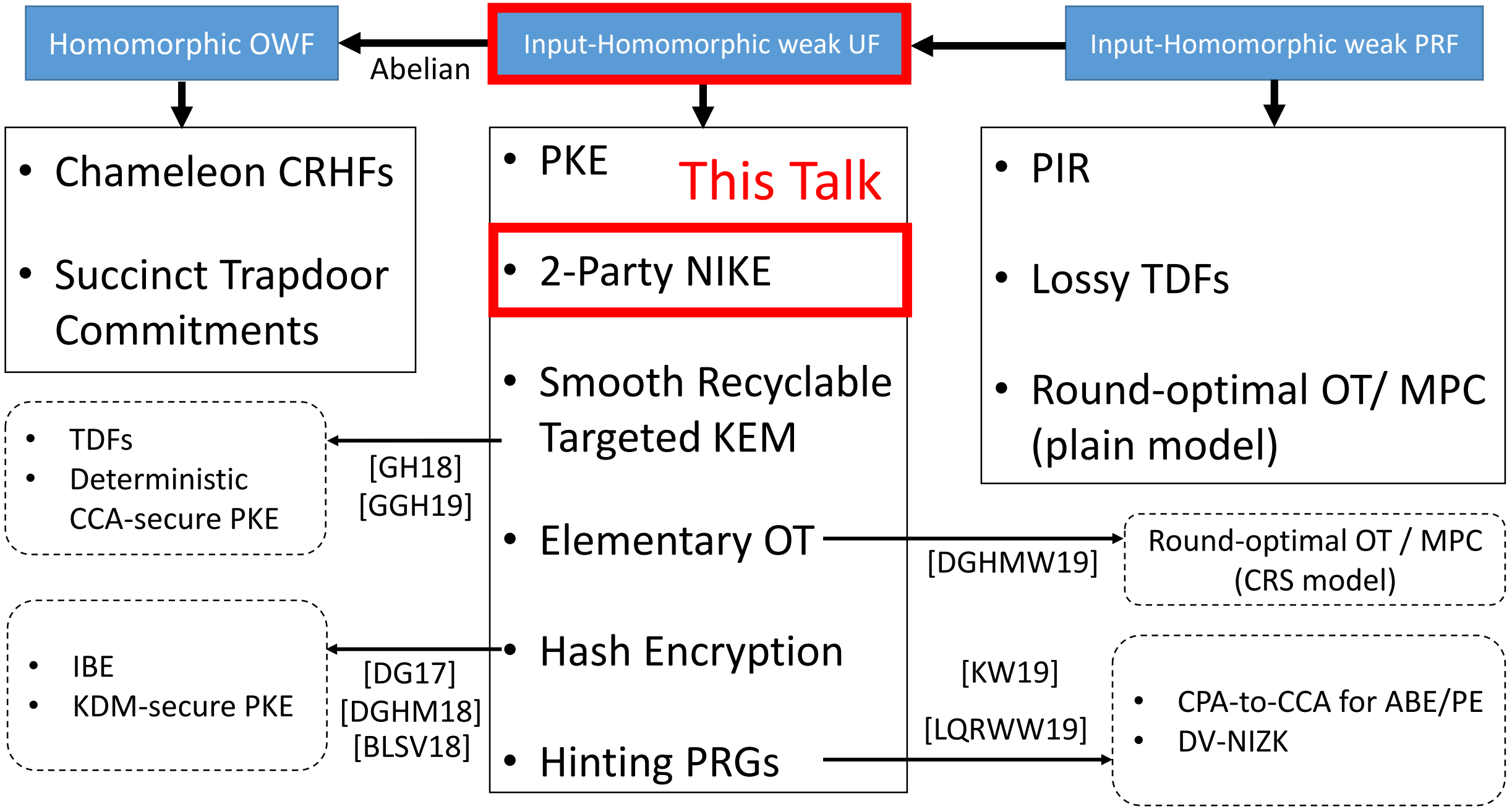
Abstracting a CDH/DDH-based protocol



Outline

- Our results
- An alternative approach to designing primitives
- A cryptoplexity hierarchy in Cryptomania
- Techniques
- **Summary and open questions**

All implications hold for bounded homomorphism



Open Questions

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  - HOWFs and IHwUFs?

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N. Alamati, L. De Feo, H. Montgomery, **S. Patranabis**.
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In ASIACRYPT 2020 (Conditionally accepted).

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Thank you!

eprint.iacr.org/2019/108
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eprint.iacr.org/2020/606