

Cryptography



with the

space-time

constraints

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PRINCIPLES OF CRYPTOGRAPHY

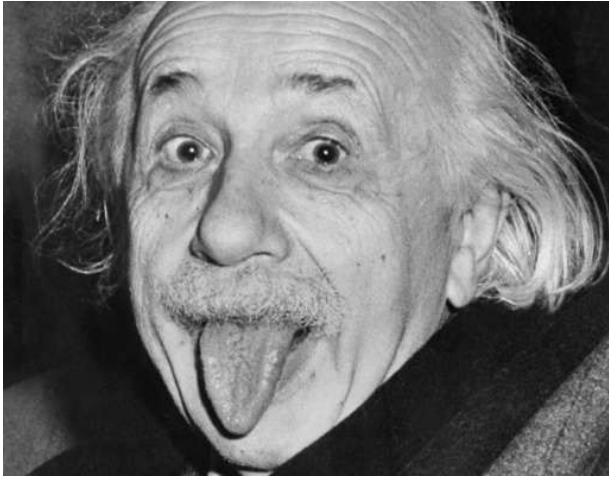
- 1. Definitions.**
 - 2. Assumptions.**
 - 3. Security Proofs.**
-

Assumptions

Example:

1. Computational Assumptions.
 - a. Probabilistic polynomial time adversary.
 2. Assumptions, inspired from the physical laws.
 - a. **No cloning principle** in quantum information theory.
 - b. **No superluminal signalling(NSS)** principle from the special theory of relativity.
-

NSS PRINCIPLE

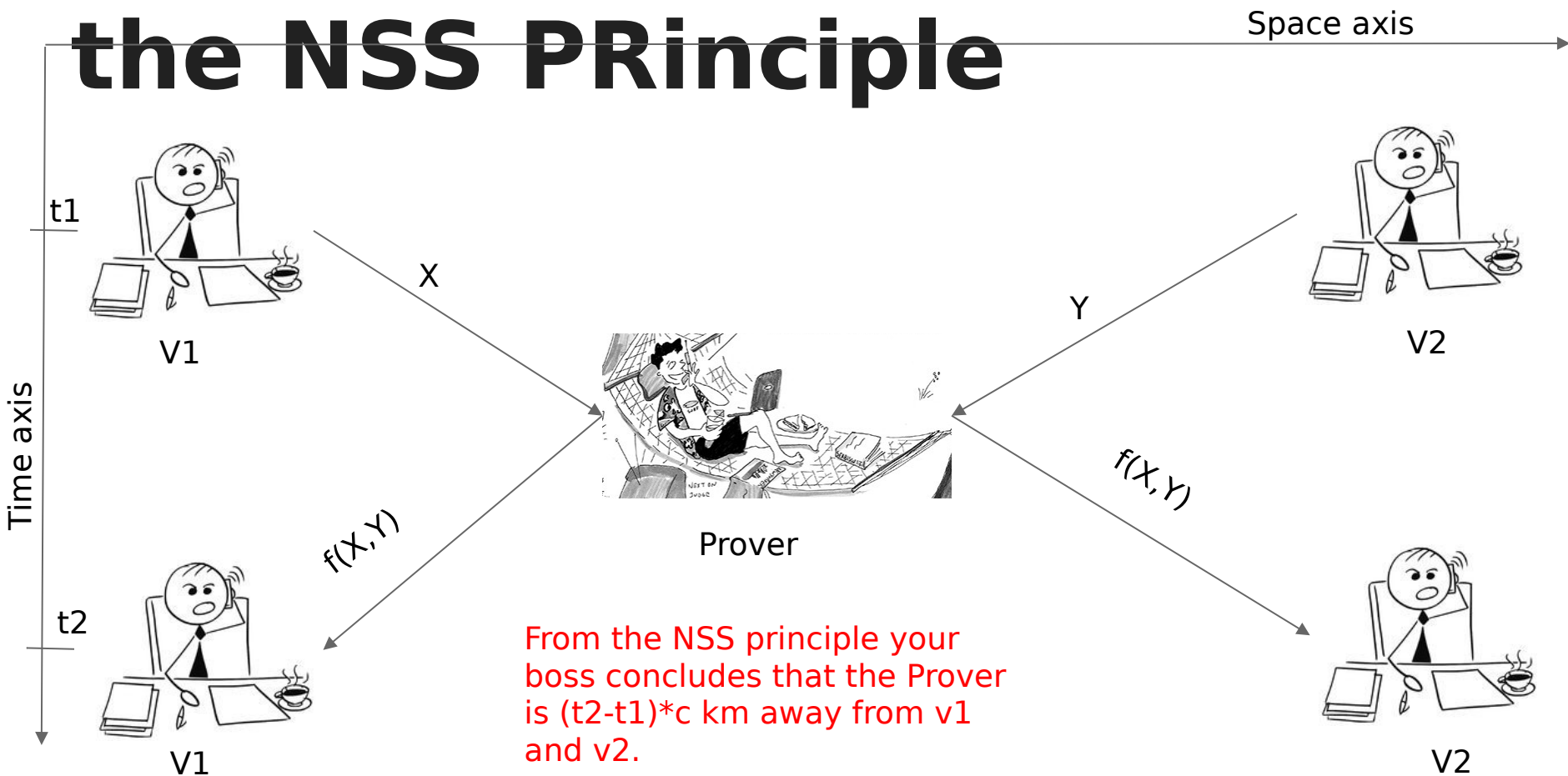


No physical carrier of information can travel faster than the speed of light.

Location verification using the NSS PRinciple



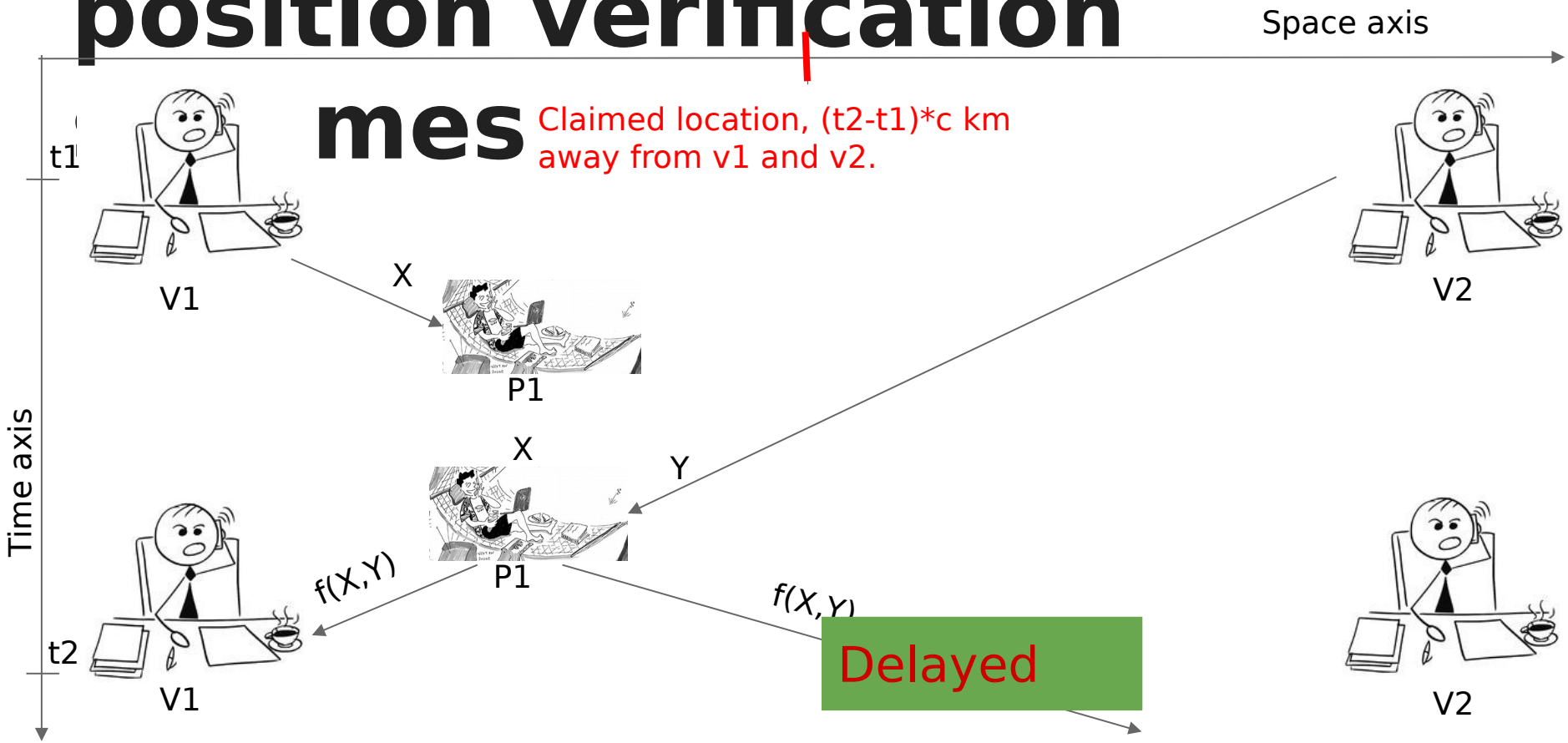
Location verification using the NSS PRinciple



APPLICATIONS

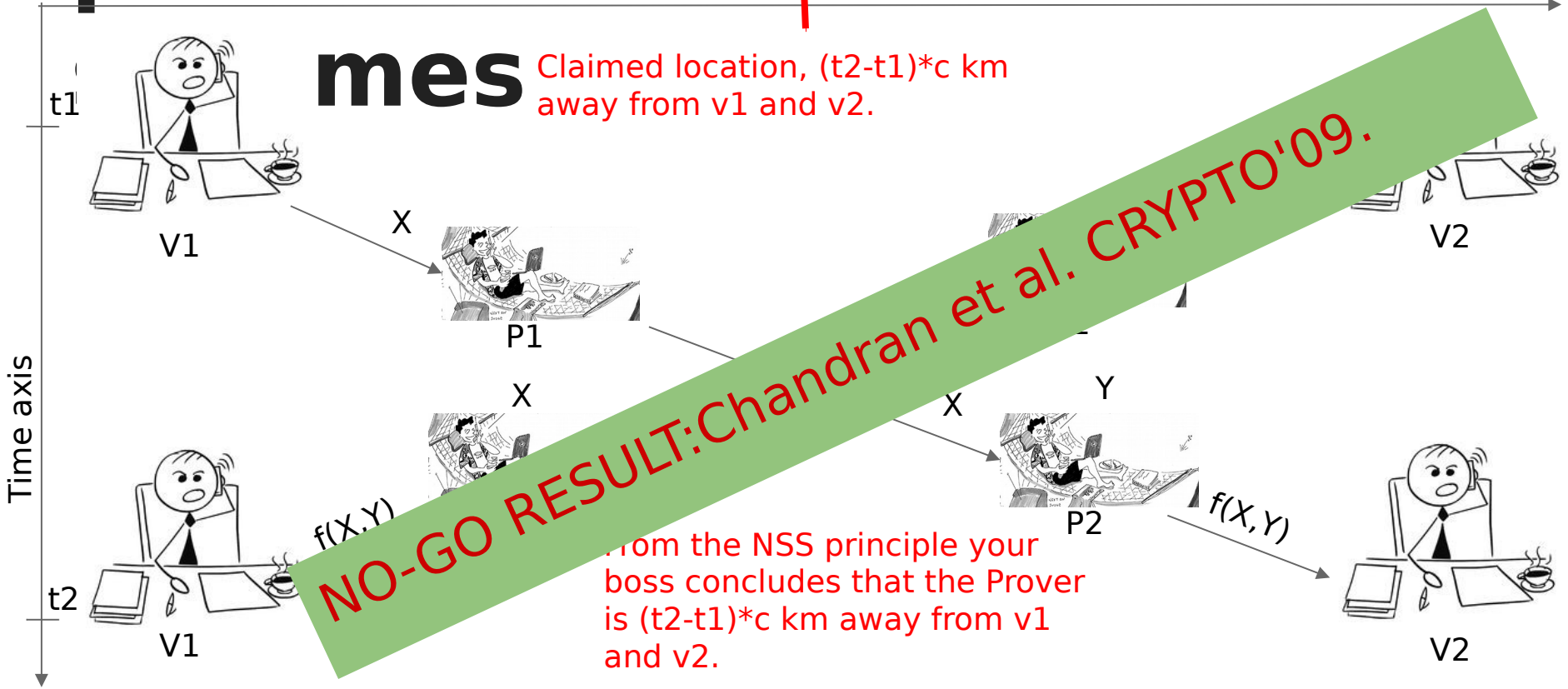
1. **Position verification** : a player Alice wants to convince the (honest) verifiers that she is located at a particular point.
2. **Position based encryption** : Alice wants to send a message to Bob at a specified position with the guarantee that it can only be read if BOB is located at that particular position.
3. **Position based authentication** : guarantee that a received message originated from a

Attack on the classical position verification



Attack on the classical position verification

Space axis



Location verification in the quantum world

Space axis

Time axis

t1



V1

$X, |\psi\rangle$

$f: \{0,1\}^n \rightarrow \{0,1\}$

$|\psi\rangle \equiv$ qubit and it cannot be copied.



Prover

Y



V2

If $f(X, Y) = 1$
 $|\psi\rangle$

t2



V1

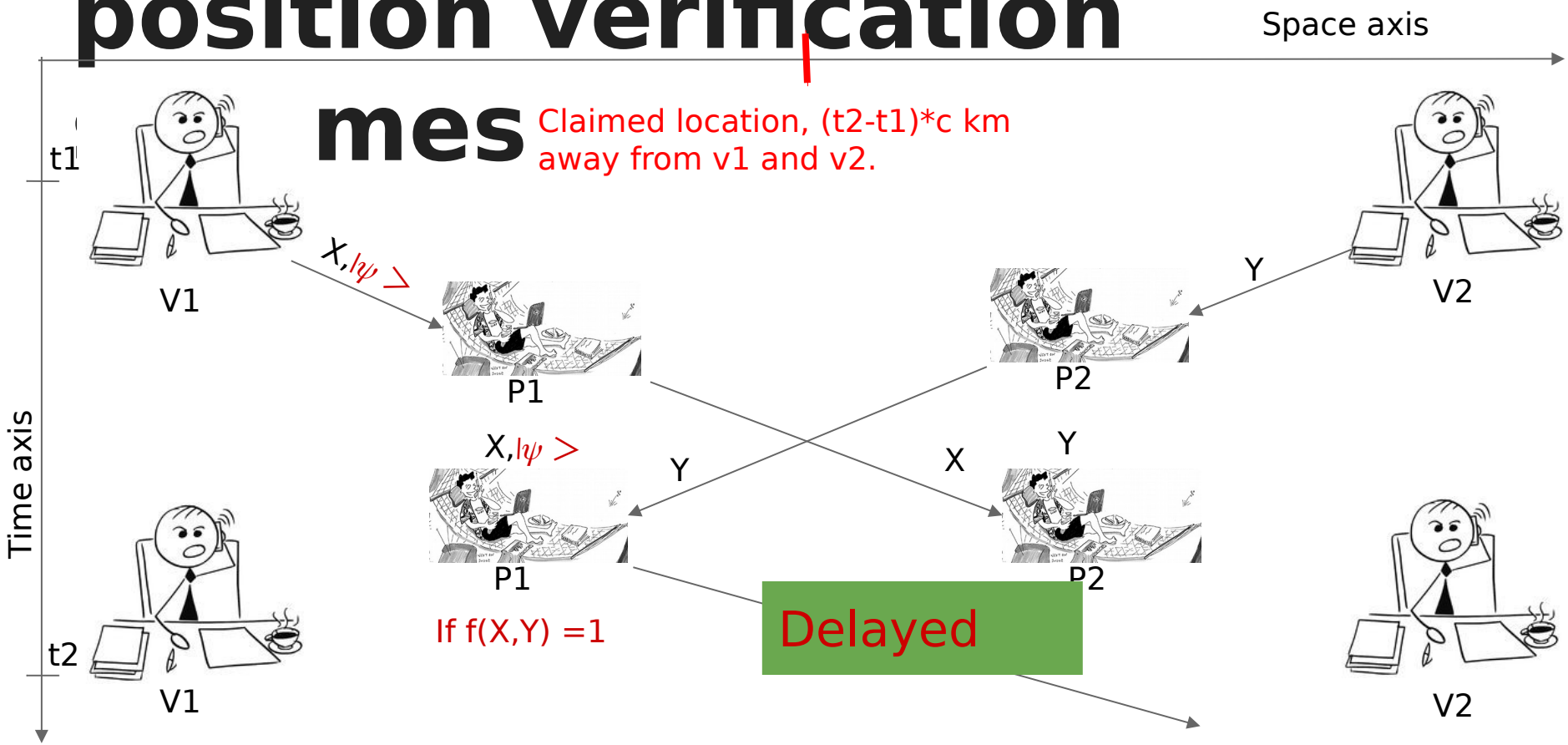
If $f(X, Y) = 0$
 $|\psi\rangle$

From the NSS principle your boss concludes that the Prover is $(t2-t1)*c$ km away from v1 and v2.



V2

Attack on the classical position verification



Existing results on the location verification in the quantum world

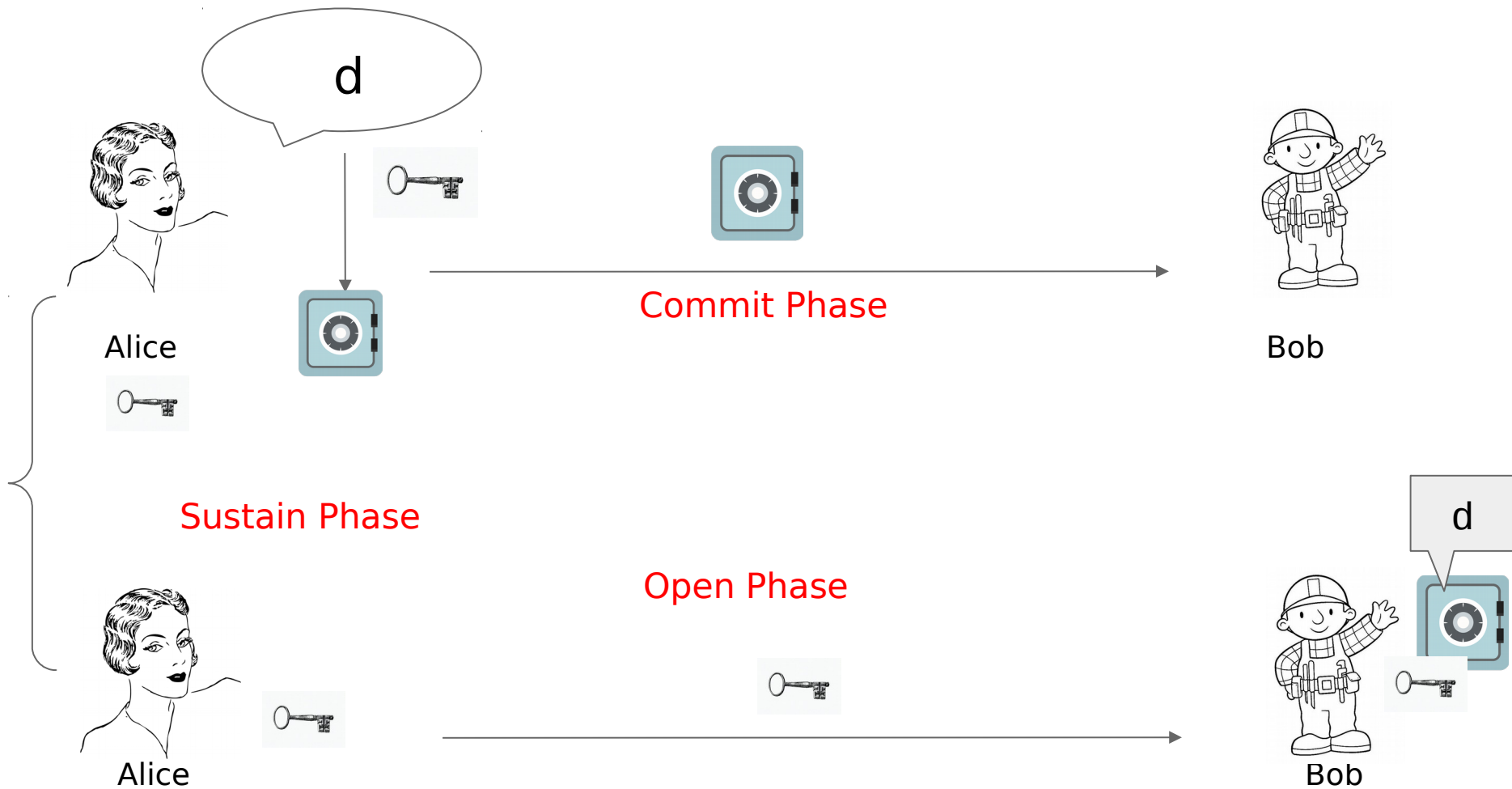
1. If the adversaries **do not have** access to **entanglement** then it is possible to have a **secure location verification scheme**. (Buhrman et al. Crypto 2011.)
2. If the adversaries can **pre-share exponential** (in n) number of **EPR-pairs** then **no secure location verification** scheme is possible. (Buhrman et al. Crypto 2011.)
3. In the **quantum random oracle model** it is **possible** to design a **secure location verification** scheme. (Unruh, Crypto 2014.)

**What
about the
other
primitive
s?**

Bit Commitment



Bit Commitment



Security Requirement for the Bit Commitment

- Security for Alice (**Hiding Property**): Bob shouldn't have any information about the commitment before reveal phase.

$$\Pr[\text{Bob guesses } d] \leq \frac{1}{2} + \epsilon$$

- Security for Bob (**Binding Property**): Alice can't change the commitment after the commit phase.

$$\Pr[\text{Alice successfully reveals } 0|Y] + \Pr[\text{Alice successfully reveals } 1|Y] \leq 1 + \epsilon. \text{ (Sum Binding)}$$

EXISTING RESULTS ON THE BIT COMMITMENT

- In the classical setting **information theoretically** secure bit commitment protocol is **impossible**.
 - Ben-Or et al., STOC 1988.

- In the quantum setting **information theoretically** secure bit commitment protocol is also **impossible**.
 - Mayers, D., Physical Review Letters, 1997.
 - Lo, H.,K., Chau, H.,F., Physical Review Letters, 1997.

A brief overview of the classical no-go



Alice

Perfect hiding

Perfect



Alice

,TA

Reveal Phase



Bob

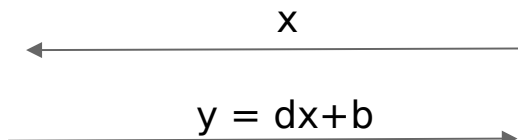
Binding perfect hiding implies, Alice can reveal $d \neq d'$ and Bob can not reveal them.

Can relativistic assumptions help here?

BIT COMMITMENT WITH MULTIPLE ALICE (Ben-OR et al., Stoc 1988)



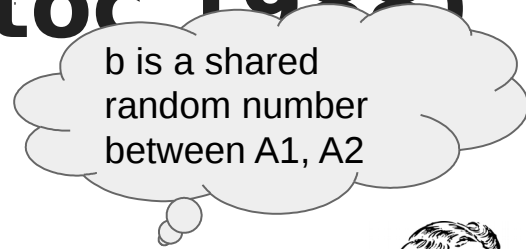
A1



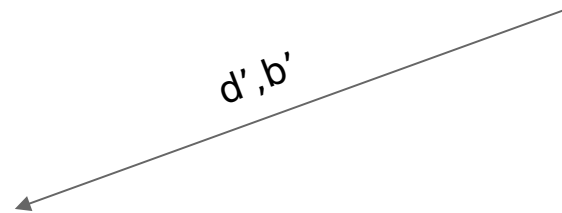
Commit Phase



Bob



A2



Open Phase

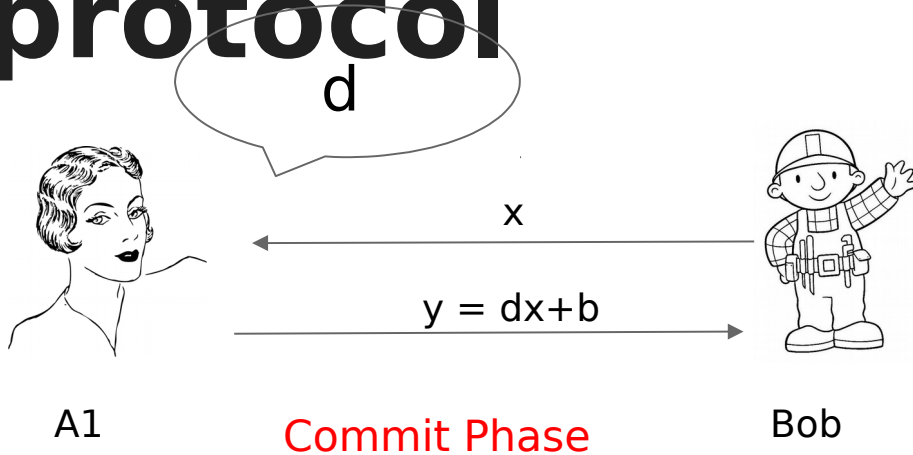


Bob

Bob computes,
 $y' = d'x + b'$

Bob accepts if $y = y'$

Hiding property of the protocol



b is a shared random number between A1, A2

Bob has no information about b.

Hence, Bob will not have any information about d from y.

The protocol is hiding.

Binding property of a protocol

$\Pr[A2 \text{ successfully reveals } 0|Y] + \Pr[A2 \text{ successfully reveals } 1|Y]$



A1



Commitment

This protocol is a secure bit-commitment protocol. Alice (A1) and Bob (A2) are **not** allowed to communicate.



A2

For revelation

Commitment

For revelation

A2 can only

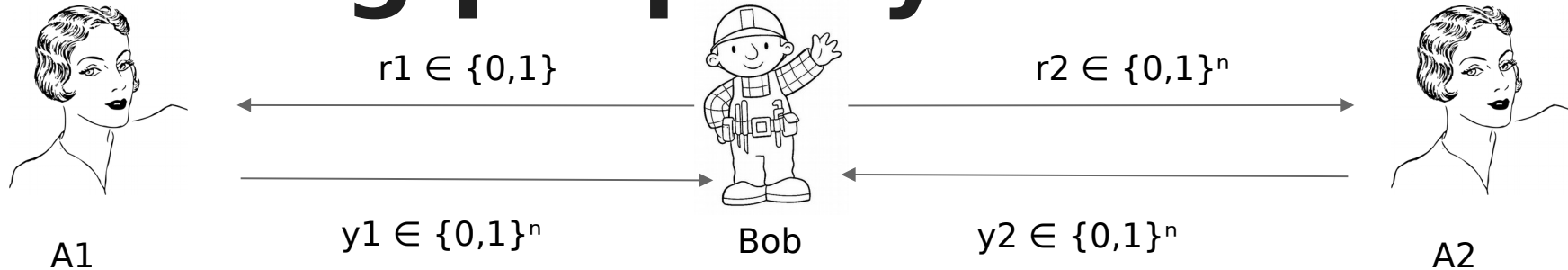
Is the non-communication assumption enough for a secure bit-commitment protocol?

needs to send $y = x + b'$.

he **knows** x .

Otherwise, A2 succeeds with probability $2/q$.

Another look at the sum binding property



A1, A2 win iff $y1 - y2 = r1.r2$

CHSH Game

Lemma: If Win. Prob. of CHSH = $\frac{1}{2} + \delta$ then the bit commitment protocol is 2δ -binding.

Upper bound on The value of δ

- In the classical setting $\delta \leq 1/2^n$. It is a tight bound.
- In the quantum setting $\delta \leq 1/\sqrt{2^n}$. It is a tight bound.

Can we do better?

There are other physical theories where one can get $\delta = 1$. But we do not have any experimental evidence.

How to implement non-communication assumption



A1

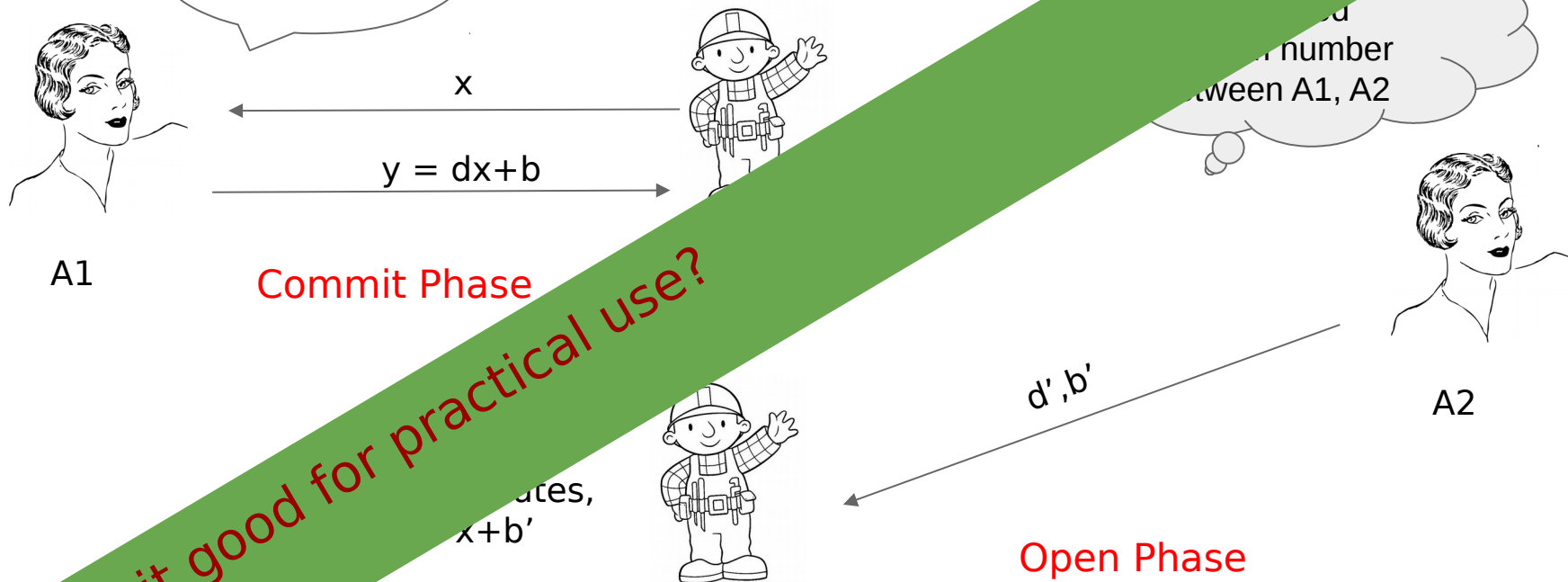
Put the **D** distance apart.



A2

NSS Principle: It will take at least **D/c seconds** to send any information from **A1 to A2**.

BIT COMMITMENT WITH MULTIPLE ALICE (Krent, a., prl 1999)



Is it good for practical use?

- Bob receives d', b' before D/c seconds.
- $y = y'$.

Open Phase

Issues with the Kent, A., PRL 1999 protocol

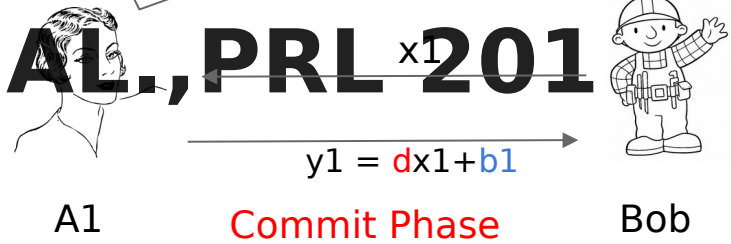
1. Needs multiple agents of Alice.

2. For practical purpose, needs two agents of Bob.

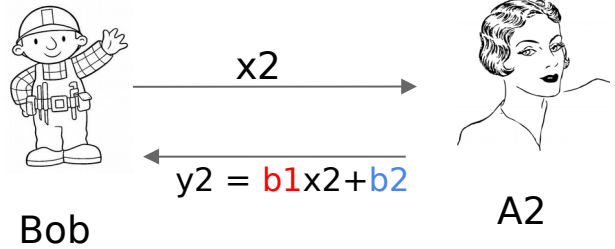
3. If the sustain phase is **one year long**, then **A1, A2 should be separated by one light year.**

INCREASING THE SUSTAIN TIME KEEPING D FIXED (LUNGH ET

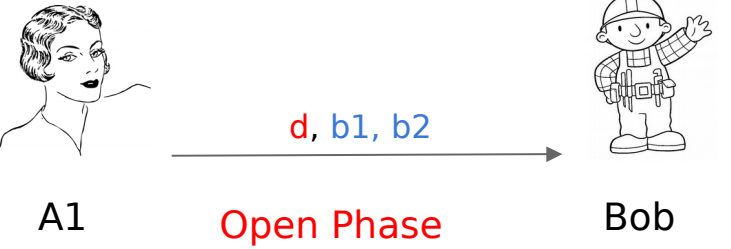
AL., PRL 201



Time gap $t < D/c$.



Time gap $t < D/c$.



Total sustain time: $2t$.

Security of k-round Ichi

et al. protocol

Hiding Property: **Perfectly hiding** because Bob doesn't
look before the opening phase

Binding Property: **Nontrivial**.

- $\epsilon \leq \exp(-n/2^k)$ [Lund et al., 2015].
 - Still impractical
- $\epsilon \leq 2k\sqrt{2^{-n}}$ [Makraborty et al. PRL, 2015, Fehr et al. Crypt 2015.]

These are true against the **classical adversaries**.

The problem is open against the quantum adversaries.

**What
about the
other
primitive
s?**

EXISTING RESULTS

1. Oblivious transfer (OT) is not possible in the relativistic setting.
2. OT is possible under non-communication assumptions.
3. Single-round Zero-Knowledge proofs are possible in the relativistic setting.

Open problems

1. **Security** against **quantum adversaries** for the multi-round bit commitment protocols.
2. Increase the efficiency of the multi-round relativistic bit commitment protocols.
3. Design secure multi-round **zero-knowledge proofs**.

**THANK
YOU FOR
YOUR
ATTENTI
ON**