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# A Technique For Image Magnification Using Partitioned Iterative Function System

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**Abstract**

A new technique for image magnification using the theory of fractals is proposed. The technique is designed assuming self transformability property of images. In particular, the magnification task is performed using the fractal code of the image instead of the original one resulting in a reduction in memory requirement. To generate the fractal codes, Genetic Algorithm with elitist model is used which greatly decreases the search for finding self similarities in the given image. The article presents both theory and implementation of the proposed method. A simple distortion measure scheme and a similarity criterion based on just noticeable difference have also been proposed to judge the image quality of the magnified image. Comparison with one of the most popular magnification techniques, the nearest neighbor technique, is made.

*Keywords:* Image Magnification, Iterated Function System (IFS), Partitioned Iterative Function System (PIFS), Genetic Algorithms (GAs).

## 1 Introduction

Image magnification ideally is a process which virtually increases image resolution in order to highlight implicit information present in the original image, not evident as such. It can be looked upon as a scale transformation. Image magnification is used for various application like matching of images captured using different sensors (having different capturing resolutions), satellite image analysis [1, 2], medical image display etc. Normally the image is represented in the form of a two dimensional array of pixel values (matrix form), and it requires large memory space. The memory requirement for storage or bandwidth requirement for transmission is greatly reduced when different coding schemes are used. The actual requirement (memory/bandwidth) is dependent on the size of the image and the method used for coding. Conventional magnification operation is generally performed on an image represented in the form of a matrix (normal form). Before applying magnification technique, any coded image has to be converted into normal form through decoding process which requires some computational cost. So it should beneficial if the magnification could be done during decoding itself. Moreover bandwidth requirement of an image transmission system would be reduced further if image of smaller size is transmitted but at the receiving end a magnified version is generated. With this problem in mind an attempt is made to propose a new magnification technique which can be applied directly on the coded version of the image.

Fractal image coding technique is one of the efficient approximate image coding techniques currently available. In image coding, the reconstructed image produced is usually subjectively very close to the original image. Actually the codes of an image must implicitly carry all the spatial information associated with the image. Besides the spatial information, the fractal codes carry the information of the self similarities present in the image. These self similarity property is also exploited in the proposed image magnification technique. We call it as fractal image magnification technique.

Fractal geometry has recently come into the limelight due to its uses in various scientific and technological applications, specially in the field of computer based image processing. It is being successfully used for image data representation [3, 4] and as image processing tool [5, 6]. In this connection, the use of Iterative Function System (IFS) and Collage theorem [3] have shown a remarkable improvement in the quality of processing compared to that obtained using existing image processing techniques.

A fully automated fractal image compression scheme, known as Partitioned Iterative Function System (PIFS) of digital images was first proposed by Jacquin [7]. The basic idea of fractal image compression or to find the fractal codes of an image is to approximate small blocks, called range blocks, of the given image from large blocks, called domain blocks, of the same image. Thus, to find the fractal codes for a given image, a mathematical transformation for each range block is to found, which, when appropriate domain block gives rise to an approximation of the range block. This set of transformations, obtained by partioning the whole image is called Partitioned Iterative Function System (PIFS). In this scheme the self similarity of the image blocks are obtained locally so the scheme is also known as local iterative function system [8].

Several researchers have suggested different algorithms with different motivations to obtain PIFS for a given image. We have already suggested a faster algorithm, to obtain PIFS, using Genetic Algorithms (GAs) [9, 10]. GAs [11, 12, 13] are optimization algorithms which are modeled according to the biological evolutionary processes. These optimization techniques reduce the search space and time significantly.

In the present work an attempt is made to use fractal codes as an input to an image magnification system. Some of the popular techniques of digital magnification of images are nearest neighbor, bilinear and bicubic interpolations. All these techniques are based on surface interpolation. usually, in the interpolation techniques, the global information is often ignored. The local or semi global information is generally exploited. In the proposed scheme, the magnification task has been performed by using fractal codes where both the local and semi global information are used. The scheme is nothing but a decoding scheme of fractal codes which gives rise to a magnified version of the original image. The article reports the initial results of magnification by a factor which is multiple of two. The technique uses fractal codes which are obtained by a GA based technique [9, 10]. Comparison with the nearest neighbor image magnification method has also been reported here.

In the magnification techniques the distortion due to blocking which is a local phenomenon is very usual. To quantify the amount of distortion, the widely used distortion measure is the mean squared error (MSE) or some other form of it. MSE is a global measure which fails to account properly the local distortion due to blocking. But the blocking effects are very much sensitive to the human visual system. So, to quantify the global and the local distortions simultaneously, a new distortion measure (fidelity criterion) is introduced.

In the process of magnification, the magnified image should be visually similar to the original one. Beside the visual judgment, we have proposed here a similarity criterion based on just noticeable difference (JND). As the sizes of the magnified image and the original image are different, the similarity between them can't be judged by inspecting the pixel values alone. Hence the JND based scheme is proposed in this regard.

Theory and key features of IFS, magnification using PIFS and GA are outlined in Section 2. The methodology of using fractal codes for magnification of a given image is described in Section 3. A new fidelity criterion to judge the performance of the proposed algorithm is discussed in Section 4. JND based similarity criterion is discussed in Section 5. Section 6 presents implementation and the results. Discussion and conclusions are provided in Section 7.

## 2 Theory and Basic Principles

The detailed mathematical description of the IFS theory, Collage theorem and other relevant results are available in [3, 14, 15]. Only the salient features are discussed here. The theory of IFS in image coding and PIFS in image magnification are described in the following subsections. The basic principle of Genetic Algorithms is also described.

### 2.1 Theoretical Foundation of IFS

Let $I$ be a given image which belongs to the set $X$. Generally $X$ is taken as the collection of compact sets. Our intention is to find a set $\mathcal{F}$ of affine contractive maps for which the given image $I$ is an approximate fixed point. The fixed point or attractor "$A$" of the set of maps $\mathcal{F}$ is defined as follows :

$$\lim_{N\to\infty} \mathcal{F}^N(J) \;=\; A, \quad \forall J \in X,$$

and $\qquad \mathcal{F}(A) \;=\; A, \quad$ where $\mathcal{F}^N(J)$ is defined as

$$\begin{aligned} \mathcal{F}^N(J) \;&=\; \mathcal{F}(\;\mathcal{F}^{N-1}(J)\;), \text{with} \\ \mathcal{F}^1(J) \;&=\; \mathcal{F}(J), \quad \forall J \in X. \end{aligned}$$

Also the set of maps $\mathcal{F}$ is defined as follows:

$$d(\mathcal{F}(J_1), \mathcal{F}(J_2)) \leq s\; d(J_1, J_2);\; \forall J_1, J_2 \in X \quad \text{and} \quad 0 \leq s < 1. \tag{1}$$

Here "$d$" is called the distance measure and "$s$" is called the contractivity factor of $\mathcal{F}$.

$$\text{Let} \;\; d(I, \mathcal{F}(I)) \leq \epsilon \tag{2}$$

where $\epsilon$ is a small positive quantity. Now, by Collage theorem [1], it can be shown that

$$d(I, A) \leq \frac{\epsilon}{1-s} \tag{3}$$

Here, $(X, \mathcal{F})$ is called iterative function system and $\mathcal{F}$ is called the set of fractal codes for the given image $I$.

#### 2.1.1 Image Coding Using PIFS

Let, $I$ be a given image having size $w \times w$ and the range of gray level values be $[0, g]$. Thus the given image $I$ is a subset of $\mathbb{R}^3$. The image is partitioned into $n$ non overlapping squares of size, say $b \times b$, and let this partition be represented by $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_n\}$. Each $\mathcal{R}_i$ is named as range block where, $n = \frac{w}{b} \times \frac{w}{b}$. Let $\mathcal{D}$ be the collection of all possible blocks (within the image support) which is of size $2b \times 2b$ and let $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_m\}$. Each $\mathcal{D}_j$ is named as domain block with $m = (w - 2b) \times (w - 2b)$. Let,

$$\mathcal{F}_j = \{f : \mathcal{D}_j \to \mathbb{R}^3 \ ; \quad f \text{ is an affine contractive map}\}.$$

Now, for a given range block $\mathcal{R}_i$, let, $f_{i|j} \in \mathcal{F}_j$ be such that

$$d(\mathcal{R}_i,\ f_{i|j}(\mathcal{D}_j)) \ \leq\ d(\mathcal{R}_i,\ f(\mathcal{D}_j)) \quad \forall f \in \mathcal{F}_j,\ \forall j.$$

Now let $k$ be such that

$$d(\mathcal{R}_i,\ f_{i|k}(\mathcal{D}_k)) \ = \ \min_j\{\ d(\mathcal{R}_i,\ f_{i|j}(\mathcal{D}_j))\ \} \tag{4}$$

Also, let $\qquad f_{i|k}(\mathcal{D}_k) = \widehat{\mathcal{R}}_{i|k}.$

Our aim is to find $\widehat{\mathcal{R}}_{i|k}$ for each $i \in \{1, 2, \cdots, n\}$ or in other words for each range block $(\mathcal{R}_i)$ we are to find appropriately matched domain block $(\mathcal{D}_k)$ and appropriately matched map $(f_{i|k})$. Thus $\mathcal{W}_i \ = \ \{\mathcal{D}_k, f_{i|k}\}$ is called fractal code for $\mathcal{R}_i$ and the set $\mathcal{W} \ = \ \{\mathcal{W}_i\ ,\ i = 1(1)n\}$ is called the PIFS of the given image $I$. Figure 1 illustrates the mapping of domain blocks to the range blocks.

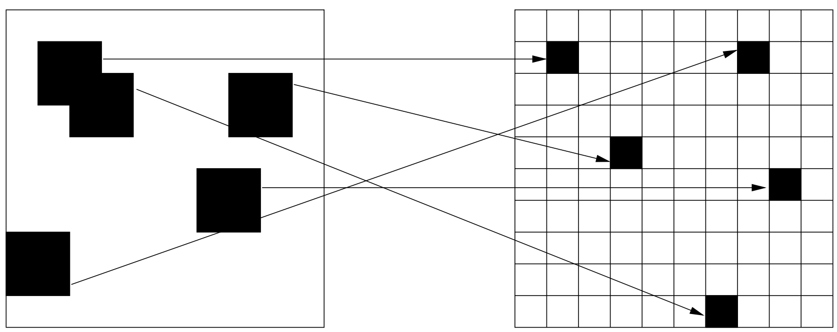

Figure 1: Mapping for an IFS scheme

## 2.2 Image Magnification Using PIFS

The affine contractive transformation $f_{i|k}$ is constructed using the fact that the gray values of the range block are scaled and shifted version of the gray values of domain block. The transformation $f_{i|j}$, defined on $\mathbb{R}^3$, is such that $f_{i|j}(\mathcal{D}_j)$ approximate $\mathcal{R}_i$. $f_{i|j}$ consists of two parts, one for spatial information and the other for information of gray values. The first part indicates which pixel of the range block corresponds to which pixel of domain block. The second part is to find the scaling and shift parameters for the set of pixel values of the domain blocks to the range blocks.

The first part denotes the shuffling the pixel positions of the domain block and can be achieved by using any one of the eight transformations (isometries) on the domain blocks[7, 9]. Once the first part is fixed, second part is estimation of a set of values (gray values) of range blocks from the set of values of the transformed domain blocks. These estimates can be obtained by using the least square analysis of two sets of values [9, 10].

The second part is obtained using least square analysis of two sets of gray values once the first part is fixed. Moreover the size of the domain blocks is double that of the range blocks.

But, the least square (straight line fitting) needs point to point correspondence. To overcome this, one has to construct contracted domain blocks such that the number of pixels in the contracted domain blocks become equal to that of range blocks. The contracted domain blocks are obtained by adopting any one of the two techniques. In the first technique, the average values of four neighboring pixel values in a domain blocks are considered as the pixel values of the contracted domain blocks [7]. In the other scheme, contracted domain blocks are constructed by taking pixel values from every alternative rows and columns of the domain blocks [9, 10]. In the present article we have adopted the first one.

Thus $f_{i|k}$ can be looked upon as mixture of two transformations, $f_{i|k} = t_{i|k}\mathcal{C}$ , where, $\mathcal{C}$ is contraction operation and $t_{i|k}$ is transformation for rows, columns and gray values.

Here we have, $I = \bigcup_{i=1}^{n} \mathcal{R}_i$ and using (2) we have,

$$d( \bigcup_{i=1}^{n} \mathcal{R}_i, \bigcup_{i=1}^{n} \widehat{\mathcal{R}}_{i|k} ) \leq \epsilon \tag{5}$$

Now, let $\mathcal{M}$ be a magnification operator such that

$$d( \bigcup_{i=1}^{n} \mathcal{R}_i, \bigcup_{i=1}^{n} \mathcal{M}(\mathcal{R}_i) ) \leq \epsilon_1 \tag{6}$$

where $\epsilon_1$ is a small positive quantity. Now by (5) and (6) we have,

$$d( \bigcup_{i=1}^{n} \mathcal{R}_i, \bigcup_{i=1}^{n} \mathcal{M}(\widehat{\mathcal{R}}_{i|k}) ) \leq \epsilon_2 \tag{7}$$

where $\epsilon_2$ is a small positive quantity. Again, we have,

$$\widehat{\mathcal{R}}_{i|k} = f_{i|k}(\mathcal{D}_k) = t_{i|k}\mathcal{C} (\mathcal{D}_k).$$

So,

$$d( \bigcup_{i=1}^{n} \mathcal{R}_i, \bigcup_{i=1}^{n} \mathcal{M}\ t_{i|k}\ \mathcal{C}(\mathcal{D}_k) ) \leq \epsilon_2. \tag{8}$$

Now, reconstruction of images using the operator $\mathcal{M}$ should be an inverse of contraction operation using the operator $\mathcal{C}$. So, by (7) and (8)

$$d\ (\ \mathcal{M}\ (\widehat{\mathcal{R}}_{i|k})\ ,\ t_{i|k}(\mathcal{D}_k)\ ) \leq \epsilon_3. \tag{9}$$

Hence, by (7),

$$d\ (\ \bigcup_{i=1}^{n} \mathcal{R}_i\ ,\ \bigcup_{i=1}^{n} t_{i|k}(\mathcal{D}_k)\ ) \leq \epsilon_4. \tag{10}$$

Both $\epsilon_3$ and $\epsilon_4$ are small positive quantities.

Thus from (10), it is clear that there is no need of constructing the magnification operator $\mathcal{M}$, only the second part of the fractal codes has to be applied on the domain block to get an image which is very close to the given image $I$ and this image has size double that of the given image.

### 2.3 Genetic Algorithms

Genetic Algorithms (GAs) [11, 12, 13] are highly parallel and adaptive search and machine learning processes based on a natural selection mechanism of biological genetic system. Parallelism of GAs depend upon the machine used for computations. GAs help to find the global optimal solution without getting stuck at local optima as they deal with multiple points (called, chromosomes) simultaneously. To solve the optimization problem, GAs start with the chromosomal (structural) representation of a parameter set. The parameter set is coded as a string of finite length called chromosome or simply string. Usually, the chromosomes are strings of 0's and 1's. If the length of chromosome (string) is $l$ then total number of chromosomes is $2^l$. To find a near optimal solution, three basic genetic operators, i)Selection, ii)Crossover and iii)Mutation are exploited in GAs.

In selection procedure the objective function values or the fitness function values of each individual string is responsible for its selection as a new string in the next mating pool. We have used the elitist model of GAs where the worst string in the present generation is replaced by the best string of the previous generation. The crossover operation on a pair of strings emulates the mating process of natural genetic system. This process is very often in natural genetic system and thus, a high probability is assigned to indicate the occurrence of this operation. In mutation operation every bit of every string is replaced by the reverse character (i.e. 0 by 1 and 1 by 0) with some probability. Usually a low probability is assigned for mutation operation and the occurrence of this operation is guided by this probability. We have used the varying mutation probability scheme [16] to guide the mutation operation in the present work. Starting from the initial population (of strings) a new population is created using three genetic operators as described above. This entire process is called an iteration. In GAs a considerable number of iterations are performed to find the optimal solution. The string which possesses optimal fitness value among all the strings is called the optimal string. The optimality of the fitness value of strings is problem dependent. If the problem is a minimizing problem, the lowest fitness value is taken as the optimal one and the maximum fitness value is selected as the optimal one if the problem is a maximization problem. The convergence of GAs to an optimal solution is assured as the number of iterations increases [17].

The methodologies to obtain magnified images from fractal codes are now discussed below.

## 3 Methodology

So far we have discussed how to apply the fractal codes or IFS to get a magnified image which is double in size that of the given image. On successive applications of this proposed algorithm, magnification by factor 4, 8, 16 etc. can also be achieved. But the first task is to obtain the fractal codes or IFS for a given image.

### 3.1 Construction of PIFS for magnification

The size of the range blocks plays an important role in image compression as well as magnification. If small blocks are taken, the finer details of the image are preserved and restored in the decompressed image but the compression ratio will be less. On the other hand more compression will be achieved, at the cost of finer image details, if larger range blocks are considered. Thus a trade off has to be made to get good quality decompressed image as well as considerable amount of compression. But the main task in magnification is only to restore all the image details and almost no emphasis is given on the amount of compression achieved. So, in this case, small range blocks are considered to keep track with every minute details of the original image.

In the proposed algorithm, to obtain the fractal codes of small range blocks of a given image, the blocks are first classified into two groups using a simple classification scheme [10]. The groups are formed according to the variability of the pixel values in the blocks. If the variability of a block is low *i.e.*, if the variance of the pixel values in the block is below a fixed value, called threshold, we call the block as smooth type range block. Otherwise we call it a rough type range block. The threshold value to separate the range blocks into two types is obtained from the valley in the histogram of the variances of pixel values of the blocks. All the pixel values in a smooth type range block are replaced by the mean of its pixel values. So, it is enough to store only the discretized mean value. On the other hand for each rough type range block, the appropriately matched domain block as well as appropriately matched transformation from eight possible isometric transformations [7] have to be searched out. To solve this search problem a GA based technique [9, 10] is adopted. GA is a search technique which finds out the optimal solution faster than the exhaustive search technique [10].

### 3.2 GA to Find PIFS

The parameters which are to be searched using GA are location (starting row and starting column) of domain block and its eight possible isometric transformations [7]. The realization of the first is two integer values between 1 and $w - 2b$ and the second can take any value between 1 and 8. Binary strings of length $l$ are introduced to represent the parameter set. Here $l$ is chosen in such a way that the set of $2^l$ binary strings exhausts the whole parametric space.

A string indicates the location and the isometric transformation of a domain block. In fractal codes we are to find an appropriate domain block and an appropriate transformation for each range block. In other words we are to find the appropriate string for each range block. Out of $2^l$ strings a few strings are selected randomly to start the GA. Starting with the initial mating pool and using the three basic operations, new populations are generated in each iteration of the GA. After a large number of iterations, the GA will produce a near global optimal solution. To obtain the appropriate string in each step we are to calculate the fitness function of each string in each iteration. Mean square error (MSE) is used as fitness function of a string. In each mating pool, the strings first under go crossing over operation pairwise and the mutation operation is applied in each bit of each string. Using the fractal codes or PIFS by GA, a magnification of order two is achieved.

In the next subsection the technique for successive magnification has been described.

### 3.3 Successive Magnification

In the case of successive applications of the algorithm, or to obtain magnification of order more than two, the fractal codes need not be computed afresh. The fractal codes that are used in a step (of order which is multiple of two) are obtained by modifying the fractal codes obtained in the previous step. In particular, the transformations $t_{i|k}$ are modified by using the image that is already obtained in the previous step. The locations of the appropriately matched domain blocks are kept fixed in all the steps. Only the size of the domain blocks is increased in the modification process. Thus, in particular, only the gray level transformation in $t_{i|k}$ is to be modified. The gray level transformations are obtained using least square technique.

In this technique a straight line is fitted with two sets of gray level values of which one is from range block and other is from contracted domain block. In the successive magnification scheme these two sets are enlarged. These enlarged sets are divided into several parts and separate straight lines are fitted for each parts using the same least square technique. The sets are divided into 4 parts for achieving magnification by a factor 4 and divided in to 16 parts for the magnification by a factor 8, and so on. So, the number of fractal codes, in a step, becomes 4 times larger than its counter part in the previous step. So, it is enough to find the fractal codes in the case of magnification by a factor 2 and in other cases these codes are modified accordingly. The modified codes are then used for achieving magnification greater than two.

The next subsection provides a discussion on the magnification by any order.

### 3.4 Magnification By Any Order

In this article fractal image magnification algorithm is implemented with magnification factors which are multiples of 2. But in practice one may need to magnify the given image by other factors too. One way of performing the magnification of order $k$ is to select the domain blocks which are $k$ times larger than that of the range block. But if $k$ is large *i.e.* if the size of the domain blocks are very large compared to that of range blocks, the similarity patterns between range blocks and domain blocks will not be observed. In that case true magnification will not be possible. To avoid this, magnifications of order 4, 8 and 16 are obtained by modifying the fractal codes of range block size $2 \times 2$ and using magnified image obtained in the previous step (*e.g.* magnification of order 4 is obtained using the twice magnified image and the fractal codes.) Similarly to obtain magnification of order 3 one has to consider the domain blocks which are three times larger than that of range blocks. Hence, by modifying these codes, magnification of order which are multiple of three can be obtained.

On the other hand, magnification by factors which are not multiples of two or three can also be achieved by considering the normalized distance between the range blocks and their matched domain blocks in the original image. The normalized distances are stored in the PIFS codes for each range block. Now, to achieve magnification of order $k$, from PIFS codes, the location of the matched domain blocks for range blocks may be obtained by multiplying the normalized distances by $k$. Once the matched domain block is fixed the rest is same as magnification by factor two. While performing the magnification task, if the distance between the range block and the domain block appears to be fractional, one has to discretize it.

The PIFS codes provide some loss of information in the reconstructed image. So, the image quality, in magnification, will be decreased with the increasing value of $k$. This problem can be handled by splitting the magnification of $k$ factor task into several steps and by modifying the PIFS codes in each step. Though some loss would be incurreded, the overall gain will be in terms of storage requirements as instead of the original image the codes are used, for the task to perform.

In the next section we shall discuss the evaluation criteria to judge the performance of the proposed algorithm.

## 4 Fidelity Criterion

It is necessary to judge the performance of the proposed fractal based image magnification algorithm for understanding its properties and fruitfulness. For this purpose one has to measure the distortion between the given image and the reconstructed image. To quantify the amount of distortion, the widely used distortion measure is peak signal to noise ratio (PSNR) which is a function of Mean Squared Error (MSE). MSE or PSNR examines the similarities between two images. But MSE is a size dependent measure *i.e.*, the two images, under consideration, should have the same size. Moreover it is a global measure which is the average of pixel to pixel difference. It does not accurately indicate the large and significant local distortions due to blocking or blurring [20] as it deals with the average distortions. But the blocking effects are very much sensitive to the human visual system. So, one has to think of a size independent measure which reflects local as well as global distortions and judges the performance of magnification methodology. A new fidelity criterion whose performance is also similar to that of visual judgment is introduced to find out the distortion between the given image and the magnified reconstructed image. The overall performances of the proposed algorithm is found using this new distortion measure.

### 4.1 Edge Based Distortion Measure

The images that are obtained from the codes usually have specific artifacts such as blocking, ringing and blurring. Actually these artifacts are reflected more prominently in the high frequency component of the image and are very sensible to the human visual system [18]. So, in our proposed distortion measure, we have tried to measure the dissimilarities in edge pattern. For simplicity only the vertical and horizontal edges have been considered.

Both the images are first partitioned into blocks proportional to their respective sizes in such a way that both images contain equal number of blocks. The error is then measured block wise and finally the average error is computed. To detect the edges of each block we have used the scheme suggested by Ramamurthi et al. [19]. The edge blocks consist of value "0" and "1" where, "1" represents the presence of edge. Now it is expected that the original and the magnified blocks should have same type of edge distributions. In other words the expected run of 1's present in both the blocks should be same if normalized by their respective sizes. Thus the distortion measure is defined by the difference between the normalized "expected run" of 1's present in the given image and in the magnified reconstructed image. The vertical and horizontal edges are considered separately and then averaged to give rise to the final distortion measure of a block. The algorithm of the proposed distortion criterion is described below.

#### 4.1.1 Description of the algorithm

**Step 1 :** Partition the images, $I_1$ and $I_2$ (with size of $I_1$ less than size of $I_2$) into square blocks such that the number of partitions is same in both the images. Let $p_1$ and $p_2$ (with $p_1 < p_2$) be the sizes of the square blocks for the images $I_1$ and $I_2$ respectively. Let these blocks be $B_{11}, B_{12}, \cdots, B_{1n}$ and $B_{21}, B_{22}, \cdots, B_{2n}$.

**Step 2 :** From $B_{ij}, i = 1, 2$ and $j = 1, 2, \cdots, n$ compute gradient matrices or the edge images. Let $G^h_{ij}$ and $G^v_{ij}$ be respectively the horizontal and the vertical gradient matrix. The elements of the gradient matrices are all either 0 or 1. The gradient matrices are defined as follows

$$
\begin{aligned}
G_{ij}^{h}(m,n) &= 0 \quad if \quad \frac{|g_{m,n}-g_{m,n+1}|}{\frac{g_{m,n}+g_{m,n+1}}{2}} < T \\
&= 1 \quad if \quad Otherwise.
\end{aligned}
$$

and

$$
\begin{aligned}
G_{ij}^{v}(m,n) &= 0 \quad if \quad \frac{|g_{m,n}-g_{m+1,n}|}{\frac{g_{m,n}+g_{m+1,n}}{2}} < T \\
&= 1 \quad if \quad Otherwise.
\end{aligned}
$$

Here $g_{m,n}$ = Gray level value of $(m,n)th$ pixel in a block and $T$ = A prefixed threshold value.

**Step 3 :** Find the expected run of 1's present in both horizontal and vertical directions in both $G_{ij}^{h}$ and $G_{ij}^{v}$. Let $L$ be the random variable denoting the number of run of 1's in a particular gradient matrix in a particular direction. Compute $E_{ij}^{hh}(L)$ , $E_{ij}^{hv}(L)$ , $E_{ij}^{vh}(L)$ , $E_{ij}^{vv}(L)$. Here expected run of 1's is defined as

$$E(L) = \sum_{k} k \frac{Number\ of\ times\ the\ run\ of\ length\ k\ appears}{Total\ number\ of\ runs\ (of\ all\ possible\ lengths)\ present}.$$

Now compute

$$E_{ij}(L) = \frac{E_{ij}^{hh}(L) + E_{ij}^{hv}(L) + E_{ij}^{vh}(L) + E_{ij}^{vv}(L)}{4}.$$

**Step 4 :** Normalize $E_{ij}(L)$ by respective block size.

$$
\begin{aligned}
E_{ij}(L) &= \frac{E_{ij}(L)}{p_1 \times p_1} \quad if \quad i = 1 \\
&= \frac{E_{ij}(L)}{p_2 \times p_2} \quad if \quad i = 2
\end{aligned}
$$

**Step 5 :** Compute the final error measure $E$ between the two given images $I_1$ and $I_2$. $E$ is defined as

$$E = \frac{1}{n} \sum_{j=1}^{n} \{E_{1j}(L) - E_{2j}(L)\}^2.$$

The next section discuses the JND based similarity criterion.

## 5 JND Based Similarity Criterion

Just noticeable difference (JND) measures the amount of change in gray value of a pixel in comparison of its surrounding pixels. Usually, JND is used to evaluate the edges present in an image [22]. Here we have proposed a similarity measure based on JND to judge the similarity between two images having unequal size. In particular, the proposed similarity measure judges the performance of the proposed PIFS based image magnification technique. JND is basically the difference in contrast of an object from its background and it plays an important role in human visual system. The human visual system (HVS) model [20] deals mainly with three factors, the luminance level, spatial frequency and signal content. Out of these, the perceived luminance is a nonlinear function of incident luminance. According to Weber's Law [21], if the luminance $(L_B + \triangle L)$ of an object is just noticeably different from its background luminance $L_B$, then $\triangle L / L_B$ = constant. Therefore, the just noticeable difference (JND) $\triangle L$ increases with the increase in $L_B$.

In the present case, we have developed a criterion based on JND, to judge the similarity between two images, which are unequal in size. In particular, first the average values of $\triangle L$ from both the images are computed. Next percentage of similarity present is computed using average values of $\triangle L$. We call this percentage of similarity based on JND as JND similarity. Note that, JND similarity $\rightarrow 100$, implies complete similarity between two images. The computation of $L_B$ and $\triangle L$ are carried out as described in [22].

# 6 Implementation and Results

To find the fractal codes for a given image the search is to be made for all possible domain blocks as well as eight possible isometric transformations [7]. To reduce the search space and time Genetic Algorithm is used instead of exhaustive search. The search space reduction is achieved since near optimal solutions are usually satisfactory and, intuitively, solutions which are far away from the expected are rejected in a probabilistic manner. This is the reason for GA to perform well for optimization problems. The good performance of GAs to find fractal codes of a given image has already been shown by Mitra et. al.[10]. The results are quite satisfactory and at least 20 times reduction in the search space is achieved.

For the specific implementation of the proposed algorithm, a part of the original "Lena" image (Figure 2(a)) is treated as the original input image. The input image is a $128 \times 128$, 8 bit/pixel image. The GA based technique [10] is applied to generate the fractal codes. Moreover, a simple classification scheme [10] for range blocks have been adopted to retain the regions where the gray level variation is minimum. In the classification scheme, the range blocks are grouped into two classes *viz.*, "smooth" and "rough". Every pixel value of a smooth range block is replaced by the average of all the pixel values. For each rough type range block the GA based technique [10] is used to find fractal codes. In the case of magnification algorithm, small range blocks of size $2 \times 2$ are considered for the computation of fractal codes. It is true that compression ratio will be reduced by considering small range blocks but, the finer details of rough type range blocks will be retained. The main aim of a magnification task is to magnify the image keeping all the image details. So, we have considered small range blocks. Now, Using the obtained fractal codes, in the way described in Section 3, an image of size $256 \times 256$ is reconstructed. The reconstructed image is two times magnified than the original image. This image is found to be very close to the original image which is judged by the error measure and the similarity measure as described in Section 4 and 5 respectively. The fractal codes are then modified stepwise, as described in Section 3.3, to get the images which are 4 times and 8 times magnified than the original one. In each step, the error, in comparison to its previous step, is measured successively. Also the similarities of magnified images are judged, successively, by the JND based similarity criterion.

The proposed algorithm is also compared with the nearest neighbour technique for image magnification in terms of proposed distortion measure and similarity measure. Nearest neighbour is the simplest method of digital magnification. Given an image of size $w \times w$, to magnify it by a factor $k$, every pixel in the new image is assigned the gray value of the pixel in the original image which is nearest to it. This is equivalent to repeating the gray values $k \times k$ times to obtain the magnified image. The resultant image for large magnification factors will have prominent block like structures due to lack of smoothness. The other techniques of digital image magnification are basically interpolation methodologies which are based on linear, bilinear, cubic or bicubic interpolation [23, 24, 25, 26].

The proposed algorithm has also been implemented on a "Low Flying Aircraft" (LFA) image having size $128 \times 128$ and range of gray level values 0 to 255. Other parameters of the algorithm are kept fixed as in the case of “Lena” image. All the results obtained are presented in Table 1 and Table 2.

Table 1: The results obtained in terms of **Distortion** of the Image magnification Algorithms

| Image | Reconstructed Image Statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | MF | Distortion | | MF | Distortion | | MF | Distortion | |
| | | Fractal | NN | | Fractal | NN | | Fractal | NN |
| Lena | 2 | 1.18 | 1.62 | 4 | 1.37 | 3.11 | 8 | 1.24 | 6.15 |
| LFA | 2 | 2.32 | 2.37 | 4 | 2.85 | 4.59 | 8 | 2.43 | 9.11 |

MF=Magnification Factor and NN=Nearest Neighbour

Table 2: The results obtained in terms of **similarity** of the Image magnification Algorithms

| Image | Reconstructed Image Statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | MF | Similarity (%) | | MF | Similarity (%) | | MF | Similarity (%) | |
| | | Fractal | NN | | Fractal | NN | | Fractal | NN |
| Lena | 2 | 63.47 | 64.39 | 4 | 57.75 | 49.06 | 8 | 54.50 | 44.38 |
| LFA | 2 | 82.05 | 68.22 | 4 | 72.46 | 51.83 | 8 | 64.40 | 47.03 |

MF=Magnification Factor and NN=Nearest Neighbour

The original and decoded images of “Lena” are shown in figures 2 and 3 respectively. Figure 4, 6 and 8 are respectively the two times, four times and eight times magnified images of “Lena” using the proposed fractal based technique while figures 5, 7 and 9 are respectively two times, four times and eight times magnified images of “Lena” using nearest neighbour technique. Figures 10 and 11 are respectively the original and decoded image of “LFA”. The results of fractal based magnification of “LFA” image are shown in figures 12, 13 and 14 while figure 15 is the eight times magnified image of “LFA” using nearest niegbour technique.

From Table 1, it is evident that in terms of the proposed error criterion the performance of the proposed fractal based image magnification scheme is better than that of the nearest nighbour technique. The results presented in Table 2 are showing that in terms of the similarity criterion, the nearest neighbour technique is better than that of the fractal based technique but the later appears to be better for magnification factor more than two. Comparing figures 4 with 5, 6 with 7 and 8 with 9, visually, one can find some ringing and blurring are present in the case of nearest nighbour technique for magnification of order more than two. On the other hand, in the case of proposed fractal based magnification a few block effects have been observed.

# 7 Discussion and Conclusions

The most important advantage of the proposed technique of fractal image magnification is that it utilizes the coded (fractal) version of the input image instead of the original image. Therefore it is cost effective in the sense of storage space and time as no decoding is performed at the receiving end in case of transmission of the codes.

Another advantage of fractal image magnification is that it magnifies the image by expanding the fractal codes or the transformations which may be looked upon as independent of image resolution. The only error involved with it is the problem of discretization. Thus the structure

Figure 2: Original "Lena" image

Figure 3: Decoded "Lena" image

and the shape of the image remains almost same. In a sense, it is like interpolation resulting in a sharper expanded image. Other image magnification schemes use pixel replication to expand image. Pixel replication makes an image blocky, blurry and patchy after a certain extent of expansion.

The size of the range blocks considered plays a vital role in fractal image compression and fractal image magnification (ref. section 3.1). In particular these two algorithms are in opposite direction from the point of view of range block size. So, one can think of an optimal range block size for which good quality magnified images can be reconstructed from the fractal codes and at the same time considerable amount of compression (in terms of compression ratio) can be achieved. To solve this problem one can think of quadtree partitioning of the images instead of square partitioning while generating the fractal codes [27]. Another scheme, to obtain the PIFS codes of an image, has been suggested by Thomas et al. [28] can also be adopted in this connection. In this scheme they have considered irregular shaped range blocks. Automatically the matched domain blocks are just the scaled and magnified versions of these irregular shaped range blocks.

In the present article we have introduced a new distortion measure or fidelity criterion to judge the performance of the proposed algorithm. There are other methods which are non-parametric statistical tests [29] for the same purpose. The common tests for examining the degree of association between two distributions whose distribution functions are unknown are Sign test, Wald Wolfowitz Run test, Wilcoxson test and Kolmogorov Smirnov test. Another evaluating criteria based on fractal dimension has already been suggested by Lalitha et al. [30]. But the most important feature which should be considered while examining the distortion between two images is the edge distribution of the images as the edges are very sensitive to human eyes. But neither the fractal based evaluating criteria nor the statistical tests take care of distortions present in the edges. The main advantage of the proposed error criterion is that it takes care of distortions in the edges. Thus, one of the important tasks is to find the proper edges in the images for the implementation of the proposed distortion

Figure 4: Two times magnified “Lena” using Fractal technique

Figure 5: Two times magnified “Lena” using Nearest Neighbour

Figure 6: Four times magnified “Lena” Fractal technique

Figure 7: Four times magnified “Lena” using Nearest Neighbour

Figure 8: Eight times magnified "Lena" Fractal technique

Figure 9: Eight times magnified "Lena" using Nearest Neighbour

Figure 10: Original “LFA” image

Figure 11: Decoded “LFA” image

Figure 12: Two times magnified “LFA” using Fractal technique

Figure 13: Four times magnified "LFA" Fractal technique

measure. We have used a very simple technique for the detection of edges though one may suggest more complex techniques for it.

The other measure proposed for judging the performance of the proposed fractal based image magnification technique is JND based similarity criterion. This measure also takes care of the distribution of edges as JND is basically the change in contrast of an object with respect to its background. But one disadvantage of this similarity measure is that it deals with the change in pixel values ignoring the edge pattern.

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Figure 14: Eight times magnified “LFA” Fractal technique

Figure 15: Eight times magnified “LFA” using Nearest Neighbour

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## Summary

Image magnification is a process which increases image size, keeping all the image details unaffected, in order to highlight implicit information present in the image, not evident as such. Image magnification is used for various applications like, satellite image analysis, medical image display. Generally, the imge is represented in the form of a two dimensional array of pixels values (matrix form) and it requires huge memory space. The memory requirement of storage is greatly reduced by using some form of image compression. But for performing image processing tasks, the coded image is usually brought back to its normal form. It is increasingly the case that the coded form of the image is used, instead of the normal form, as the input to perform different image processing tasks. With this aim in mind an attempt is made, in the present article, to propose a new magnification technique which can be applied directly on the coded form of the image. In particular the proposed algorithm is using that coded form of the image which is obtained by fractal image compression technique.

Recently, fractal based image compression is very popular and there are many techniques available in the literature for finding fractal code of an image. The encoding process involves in computing a set of linear contractive maps from the target image. In the decoding process, the obtained set of maps is applied on an arbitrary image in an iterative way to result in an image which is very close to the target image. The set of maps is called fractal code or Partitioned Iterative Function System (PIFS) code of the image. In the process of iterative sequence, PIFS code converges to a fixed image which is very close to the target image. Computational task for finding PIFS code of an image is usually time consuming. But we have used here a cost effective Genetic Algorithm (GA) based technique to find PIFS code to propose a new magnification technique.

Conventional magnification techniques are basically interpolation methodologies which are based on linear, bilinear, cubic or bicubic interpolations. In the proposed algorithm magnified version of an image is obtained using the reconstruction of the fractal code of that image. No magnification operator like interpolation is needed. Only the fractal code or in otherwords the set of affine contractive maps is needed to magnify the image. We have proposed here the technique of magnification of orders which are multiple of two. The technique can be extended to the case of magnification by any order.

We have also proposed two techniques to judge the performance of the proposed magnification algorithm. The main task involves here to measure the distortion or similarity between the given image and the magnified image. The commonly used distortion measure is peak signal to noise ratio (PSNR) which is a function of mean squared error (MSE). The MSE is a global measure and also it is image size dependent. The proposed techniques are not only image size independent but also utilizes both global and local information. The first technique is a distortion measure based on the edge distribution of the images and indicates the influence of artifacts like blocking, blurring and ringing which may appear due to magnification. The other one is a similarity measure based on just noticeable difference (JND) which is nothing but change in luminance of an object pixel with respect to its background pixels. The overall performance of the proposed magnification technique is found to be satisfactory both qualitatively and quantitatively. Comparison with one of the most popular magnification techniques, the nearest neighbor technique, is made.

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