



## A pipeline architecture for computing the Euler number of a binary image <sup>☆</sup>

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### Abstract

Euler number of a binary image is a fundamental topological feature that remains invariant under translation, rotation, scaling, and rubber-sheet transformation of the image. In this work, a run-based method for computing Euler number is formulated and a new hardware implementation is described. Analysis of time complexity and performance measure is provided to demonstrate the efficiency of the method. The sequential version of the proposed algorithm requires significantly fewer number of pixel accesses compared to the existing methods and tools based on bit-quad counting or quad-tree, both for the worst case and the average case. A pipelined architecture is designed with a single adder tree to implement the algorithm on-chip by exploiting its inherent parallelism. The architecture uses  $O(N)$  2-input gates and requires  $O(N \log N)$  time to compute the Euler number of an  $N \times N$  image. The same hardware, with minor modification, can be used to handle arbitrarily large pixel matrices. A standard cell based VLSI implementation of the architecture is also reported. As Euler number is a widely used parameter, the proposed design can be readily used to save computation time in many image processing applications.

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## 1. Introduction

Topological properties that remain invariant under various transformations are useful in image characterization for matching shapes, recognizing objects, image retrieval from a database, and in many other image processing and computer vision applications. An important topological feature of a binary image is the *Euler number* (or genus), which is defined as the difference of the number of connected components (objects), and the number of holes [1,2]. Euler number remains invariant under translation, rotation, scaling, and rubber-sheet transformation of the image. Many critical image processing applications involve large amount of data, and at the same time demand quick real-time response. Euler number provides a simple and fast method of screening in such cases. Euler number of cell images is widely used in medical diagnosis [3]. It has recently been observed that Euler number is the most clinically useful feature that discriminates many cervical disorders [4]. Being a fundamental topological feature, it has numerous applications in image processing, e.g., optical character recognition, document image processing [5], reflectance-based object recognition [6], analysis of sandstone for geological applications [7], shadow detection [8]. Other topological properties using the convex deficiency and convex hull of the shape are used in conjunction with Euler number for classifying typewritten letters or binary silhouettes [9]. Recently, based on Euler number, the concept of Euler vector is introduced to characterize a gray-tone image [10].

The classical algorithm for computing the Euler number of a binary image is based on counting certain  $(2 \times 2)$  pixel patterns called bit-quads [2,11] over the entire image. Gray [11] used the fact that the Euler number of a region of space is locally countable [12]. The classical graph-theoretic definition of Euler number relating vertices, edges and faces is applied to an image followed by triangulation and then its Euler number is computed as the difference of the number of connected components and that of holes. Alternatively, Euler number is shown to be the difference of left-facing convexities and concavities in the image. Compu-

tation of these parameters needs counting of specific types of bit-quads in the image namely,

$$Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad Q_D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and their rotational equivalents. Let  $v$ ,  $t$ , and  $d$  be the patterns  $Q_1$ ,  $Q_2$ , and  $Q_D$  respectively in an image. Then the Euler number can be expressed as  $(\sum v - \sum t - 2 \times \sum d)/4$  under the 8-connectivity definition. The commercial image processing toolbox MATLAB uses the above quad counting algorithm for Euler number computation [13]. Dyer [14] proposed an algorithm to compute the Euler number of an image represented by a quad-tree. Samet and Tamminen [15] improved the algorithm further by using a new staircase type of data structure to represent the blocks that have already been processed. Juan et al. [16] considered a skeletonized version of the binary image and computed Euler number in terms of the number of terminal points  $T_p$ , and the number of three-edge points  $TE_p$ , as  $(T_p - TE_p)/2$ . The terminal and three-edge points are defined from pixel neighborhood relations. Chen and Yen [3] developed a parallel localized algorithm using square graphs to calculate the Euler number of a given binary image on a square grid. Chiavetta and Gesù [17] used connectivity graph representation of a binary image and developed an algorithm for computing the Euler number. The connectivity graph was derived from the discrete version of the cylindrical algebraic decomposition of the digital plane. The authors also described a parallel implementation of the same algorithm on a linear array network topology.

Rosenfeld and Kak [18] observed that Euler number can be computed from the run length representation of an image. If for each run  $r$ ,  $k(r)$  be the number of runs on the preceding row to which  $r$  is adjacent, then Euler number can be expressed as:  $E = \sum_r (1 - k(r))$ . Further results on 2D and 3D images are reported in [19]. Di Zenzo et al. [20] suggested a planar graph representation of an image from run descriptions. Next, the number of connected components  $C$  in the image is computed by applying a standard graph algorithm. The number of holes  $H$  is then computed from the Euler's

formula of a planar graph as  $H = 1 + m - v$ , where  $m$  = number of edges and  $v$  = number of nodes in the graph. Finally, Euler number is calculated as  $E = C - H$ . Dey et al. [21] have reported a divide-and-conquer algorithm that can be parallelized for computing the Euler number of a binary image.

In this work, we revisit the run-based expression of Rosenfeld and Kak [18], and present a novel hardware design for computing Euler number. We show that certain properties of runs and neighboring runs and their distributions in the pixel matrix can be exploited to compute the Euler number of a binary image very efficiently. Performance analysis of the algorithm indicates that the proposed technique outperforms significantly in speed, the existing bit-quad counting and quad-tree based methods [2,11,13–15]. Experimental results on a logo database show very favorable results. This run-based algorithm, with its inherent parallelism, provides the basis of building a pipeline architecture for on-chip computation of Euler number. Given a pixel matrix of size  $(N \times N)$  the upper and lower bounds on the value of the Euler number are also derived. This result allows us to design the architecture correctly. We further derive analytical expressions for performance measures of the proposed architecture to establish its efficiency. It is also shown that the circuit, with minor modifications, can handle a pixel matrix of arbitrary size. A standard cell based VLSI implementation of the architecture on a real technology is described and relevant data on circuit area and delay are reported. To the best of our knowledge, on-chip design of any run-based technique does not seem to be available to date.

The rest of the paper is organized as follows. Section 2 presents the formulation for computing Euler number based on runs, theoretical analysis of complexities, and experimental results on run distributions. Next in Section 3, a simple parallel version of the algorithm, the proposed pipeline architecture, and results on its performance analysis are reported. Section 4 describes VLSI implementation of the design for a  $256 \times 256$  image. Section 5 concludes the paper.

## 2. Proposed algorithm

### 2.1. Theme

Let the binary image be represented by a 0–1 pixel matrix of size  $(N \times M)$ , in which an object (background) pixel is denoted as 1 (0). In a binary image, a *connected component* is a set of object pixels such that any object pixel in the set is in the 8 (or 4) neighborhood of at least one object pixel of the same set. A *hole* is a set of background pixels such that any background pixel in the set is in the 4- (or 8-) neighborhood of at least one background pixel of the same set and this entire set of background pixels is enclosed by a connected component. The sets referred to in the definition are sets contained in the image. A *run* in any row (or column) of the pixel matrix is defined to be a maximal sequence of consecutive 1's in that row (or column). Let  $R(i)$  denote the number of such runs in the  $i$ th row (column).  $R(i)$  can be counted as the number of  $0 \rightarrow 1$  transitions in that row with a 0 padded at the start of the row (the 0 is to be padded to handle the case where there is a 1 at the start of the row). See Figs. 1 and 2 for illustration. There can be no holes in a single row, and the number of connected components in that row is same as the number of runs in that row.

**Fact 1.** *If the image  $I$  consists of a single row or a single column  $i$ , the Euler number  $E(I) = R(i)$ .*

**Fact 2.** *Euler number satisfies the local additive property. Given two images  $I_1$  and  $I_2$  with Euler numbers  $E(I_1)$  and  $E(I_2)$  respectively, the Euler number of the image  $I = I_1 \cup I_2$  is given by:  $E(I) = E(I_1 \cup I_2) = E(I_1) + E(I_2) - E(I_1 \cap I_2)$  (see [11,12]).*

The *union* ( $\cup$ ) of two images is defined as simple juxtaposition of  $I_1$  and  $I_2$  either vertically or horizontally, without any overlap. The *intersection* ( $\cap$ ) of  $I_1$  and  $I_2$  is the image formed by the last row (or column) of  $I_1$ , and the first row (or column) of  $I_2$ , if the images  $I_1$  and  $I_2$  are joined horizontally (or vertically). The intersection image is always two pixel row (or column) wide. Without any loss of gener-

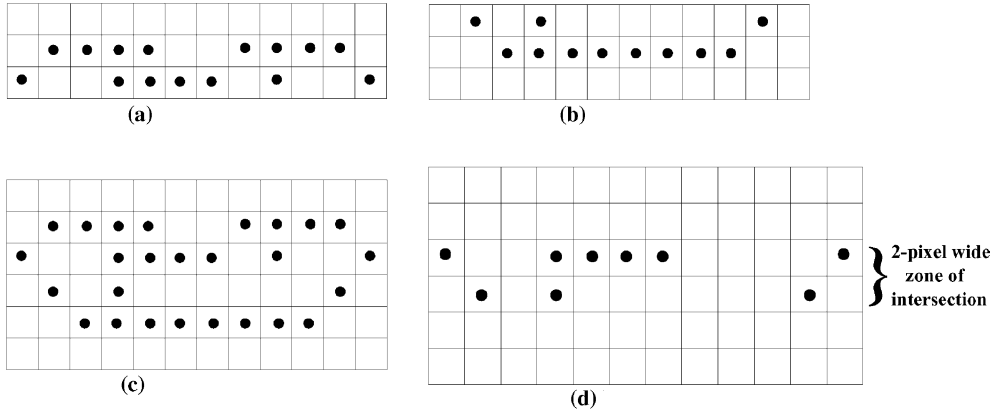


Fig. 1. Union and intersection of images. (a) Image  $I_1$ , (b) image  $I_2$ , (c) image  $I_1 \cup I_2$ , and (d) image  $I_1 \cap I_2$ .

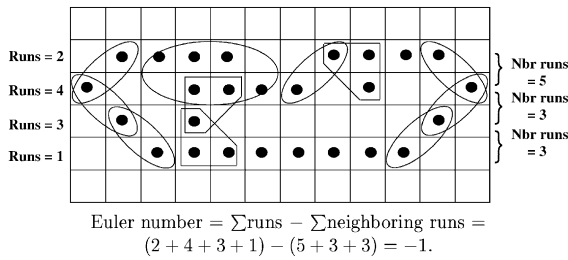


Fig. 2. Illustration of runs and neighboring runs.

ality, let  $I_1$  and  $I_2$  be joined horizontally. Thus, the last row of  $I_1$  will lie above the first row of  $I_2$ . See Fig. 1 for an example. Two runs appearing in two adjacent rows each, are said to be *neighboring* if at least one pixel of a run is in the 8- (or 4-) neighborhood of a pixel of the other run (we follow 8-neighborhood convention throughout). Clearly,  $(I_1 \cap I_2)$  will denote the image containing the last row of  $I_1$  and the first row of  $I_2$  and is a two-row wide image. See Fig. 1 for an example. No holes can be present in a two-row wide image, and also the number of connected components in a two-row wide image is the number of neighboring runs. So, we have the following observation:

**Fact 3.**  $E(I_1 \cap I_2) =$  the number of neighboring runs between  $I_1$  and  $I_2$ .

We now use Facts 1–3 iteratively to compute the Euler number of the entire image, as follows.

Let  $I_{i-1}$  be the partial image consisting of rows  $1, 2, \dots, (i - 1)$  of the pixel matrix. Let  $E(I_{i-1})$  be

the Euler number of  $I_{i-1}$ . The Euler number of the image consisting of only row  $i = R(i)$  (by Fact 1). The row  $i$  is now added to  $I_{i-1}$  to form the union image  $I_i$ . The intersection image is formed by the  $(i - 1)$ th and the  $i$ th row. Let the number of neighboring runs between them be  $O_i$ . Hence,

$$\begin{aligned}
 E(I_1) &= R(1), \\
 E(I_2) &= E(I_1) + E(2) - O_2 \\
 &= R(1) + R(2) - O_2, \\
 E(I_3) &= E(I_2) + E(3) - O_3 \\
 &= R(1) + R(2) + R(3) - (O_2 + O_3), \\
 &\dots \\
 E(I_N) &= E(I_{N-1}) + E(N) - O_i \\
 &= (R(1) + R(2) + R(3) + \dots + R(N)) \\
 &\quad - (O_2 + O_3 + \dots + O_N), \\
 &= \sum_{i=1}^N R(i) - \sum_{i=2}^N O_i,
 \end{aligned}$$

where,  $I_N$  denotes the entire image.

The above analysis proves the following known result [18], and provides the basis of our hardware implementation.

**Theorem 1.** The Euler number of a given binary image is the difference between the sum of the number of runs for all rows (or columns), and the sum of the neighboring runs between all consecutive pairs of rows (or columns).

## 2.2. Computation of run and neighboring run

A run of a row is a sequence of consecutive 1-pixels in that row. See Fig. 2 for an example.

**Lemma 1.** *The number of runs in any row is counted as the number of  $0 \rightarrow 1$  transitions in the row with an additional 0 padded at the start.*

**Proof.** The proof is trivial.  $\square$

Two runs in two adjacent rows are neighboring if at least one pixel of a run in a row is in 8-neighborhood of a pixel of a run in the other row. To count the number of neighboring runs, the occurrence of the start of a neighboring run is to be determined. A neighboring run between two rows can start only when there is an occurrence of a run ( $0 \rightarrow 1$  transition) in at least one of the two rows, and there is at least one 1-pixel in 8-neighborhood of the location where the run starts.

**Lemma 2.** *A neighboring run between two rows occurs if and only if there is a  $0 \rightarrow 1$  transition in at least one of the two rows and there is a 1 pixel in the 8-neighborhood of that location.*

**Proof.** A neighboring run cannot occur without a run in any of the two rows. Two such runs should have at least one 1 pixel in their neighborhood following 8-neighborhood relations. To count the number of neighboring runs between the  $(i - 1)$ th and  $i$ th rows, we have to do the following. A run starts in any  $j$ th column of the  $i$ th row, if there is a 0 in the  $(j - 1)$ th column and 1 in the  $j$ th column of the  $i$ th row. A run ends in any  $j$ th column of the  $i$ th row, if there is a 1 in the  $(j - 1)$ th column and 0 in the  $j$ th column of the  $i$ th row. The run start location  $s_j$  and run end location  $e_j$  are stored while counting the number of runs in the  $i$ th row. When a run occurs in any  $k$ th location in  $(i + 1)$ th row, if the pixel  $(i + 1, k)$  is in the 8 neighborhood of any of the pixels  $(i, s_j), \dots, (i, e_j)$ , then a neighboring run count is increased by one.  $\square$

See Fig. 2 for an illustration of the above lemma.

## 2.3. Algorithm

### Method

**Input:** An  $(N \times M)$  binary pixel matrix of an image  $I$ ;

**Output:** Euler number  $E(I)$ .

### Compute\_Euler\_Sequential

compute the number of runs  $R(1)$  present in the first row;

$E(I) = R(1)$ ;

**for** ( $i =$  row number 2 to  $N$ )

calculate the number of runs  $R(i)$  in the  $i$ th row;

**if** (there are runs between the  $(i - 1)$ th and  $i$ th rows)

(comments: see Lemma 2)

calculate the number of neighboring runs  $O_i$  between  $(i - 1)$ th and  $i$ th rows;

**endif**

(see Section 2.2)

$E(I) = E(I) + R(i) - O_i$ ;

**endfor**

**return**  $E(I)$  as the Euler number.

## 2.4. Space and time complexity

The lower bound of computation of Euler number is  $\Omega(N^2)$  by definition of the Euler number. In that sense, most of the algorithms are asymptotically optimal in that they compute Euler number in  $O(N^2)$ . But in image processing applications where huge data is involved and real time application is needed, the constant so often hidden in the  $O$ -notation becomes important. This has motivated several algorithms for computation of Euler number. So, we find out the exact number of image pixel matrix accesses for an  $N \times N$  matrix under the sequential *RAM* model for the previous algorithms and compare it to the run-based method.

The bit-quad counting technique [2,11] checks for bit-quads (i.e. 4 pixels) for each entry and also has to check for the convexity and concavity along the borders. This takes  $4N^2 + 4N$  accesses. Note that, irrespective of the matrix entries, these number of pixel accesses are required. So, the average case access would also be the same.

The Euler number computation based on the thinned version of the image [16] checks for two types of terminal points. For that, 8 neighbors of each pixel is to be accessed. That takes  $8N^2$ , add to that the time thinning takes and obviously this algorithm takes more pixel accesses than the bit-quad counting technique [2,11].

The Euler number computation based on the quad-tree [14,15] involves computing the quad-tree from the pixel entries followed by a traversal on the quad-tree. The time complexity  $T(n)$  for formation of the quad-tree involves the following recurrence:

$$T(n) = \begin{cases} 1 & \text{if } n = 1; \\ 4T(\frac{n}{4}) + n & \text{if } n \geq 2. \end{cases}$$

Solving this recurrence,  $T(n) = n \log_4 n + n$ . In our case  $n = N^2$ , so the number of accesses is  $N^2 \log_2 N + N^2$ . Now, traversal of a quad-tree for computing the Euler number takes linear time in the number of leaf nodes in the quad-tree [14,15]. The number of leaf nodes can be  $N^2$ . So, Euler number computation takes  $N^2 \log_2 N + N^2 + cN^2$ . This is obviously greater than the accesses required by the bit-quad counting technique.

The above analysis shows that the bit-quad counting algorithm is the best in terms of less number of pixel accesses. That might be the reason of the commercial image processing toolbox MATLAB using this algorithm [13].

Next, we compute the worst case and average case analysis of the proposed run-based technique to show its efficacy.

We require  $N \times N$  space to store all the pixels of the given image. To calculate the number of neighboring runs between two consecutive rows, we need to store for each run, the column numbers where a run starts or terminates. The worst case arises in a checkerboard situation when a row has the maximum number of runs, i.e., when every alternating pixel is an object pixel. For  $N$  columns, the maximum number of runs in a row is  $N/2$ . We calculate the number of neighborhood runs between two consecutive rows at a time. Since a run can be designated by its two ends, the total

space required =  $(N \times N) + 4 \times (N/2) \approx O(N^2)$ , i.e. linear in the number of pixels.

Computation of runs for all the rows needs  $2 \times (N \times N)$  pixel accesses. The number of runs  $R(i)$  in a row in the worst case is  $N/2$ . The number of pixel accesses required to check and store the two end points of all runs present in the matrix =  $P_R = 2 \times \sum_{i=1}^N R(i) = N^2$ . Checking whether a run in row  $(i-1)$  is in the neighborhood of a run of row  $i$ , requires 2 accesses. Also, a neighboring run is to be checked only after the occurrence of a run. So, determination of neighboring runs for all consecutive pairs of rows needs =  $2 \times \sum_{i=2}^N \sum_{j=1}^{R(i)} O_i = N \times (N-1) = (N^2)/2 - (N/2)$  accesses. Note that, the upper limit on the summation of  $O_i$  is  $R(i)$  (due to Lemma 2) and not  $N$ . Therefore, the total number of accesses is,  $2N^2 + N^2 + (N^2 - N) = 4N^2 - N$ .

For the average case analysis, we assume a random input of 0 and 1 with a probability of  $\frac{1}{2}$  each. As the number of accesses depend on the number of runs, we compute the expected number of runs in a row of width  $N$ . Let  $X_t$  be the number of 1 runs in the first  $t$  entries and  $Y_t$  be the number of 0 or 1 runs in the first  $t$  entries. Obviously,  $E(X_t) = E(Y_t)/2$ , where  $E(X)$  denotes the expectation of  $X$ . Now, the conditional expectation of  $Y_{t+1}$  when  $Y_t = c$  is  $E(Y_{t+1}|Y_t = c) = c + \frac{1}{2}$ , because a 0 or 1 can occur with probability  $\frac{1}{2}$  in the  $(t+1)$ th entry. Now,  $E(Y_{t+1}) = \sum_c (\frac{1}{2} c P(Y_t = c) + \frac{1}{2} (c+1) P(Y_t = c)) = \sum_c c P(Y_t = c) + \frac{1}{2} \sum_c P(Y_t = c) = E(Y_t) + \frac{1}{2}$ . Now, surely  $E(Y_1) = 1$  because either there is a 0 or 1. Therefore, according to the recurrence just deduced  $E(Y_N) = \frac{N+1}{2}$ . So,  $E(X_N) = \frac{E(Y_N)}{2} = \frac{N+1}{4}$ . As an example, for  $N=4$ , there are  $2^4 = 16$  possible entries of 0 and 1 combination. The frequency of a single run is 10 and the frequency of two runs is 5. So, the expected value of 1 runs is  $\frac{10 \times 1 + 2 \times 5}{16} = \frac{5}{4}$ . So, the number of accesses in the expected case is  $2N^2 + 2N(\frac{N+1}{4}) + 2(N-1)(\frac{N+1}{4}) = 3N^2 + \frac{N}{2} - \frac{1}{2}$ . Thus, it can be seen that the proposed method of Euler number computation requires fewer number of accesses than the bit-quad counting method.

In the proposed method, the number of pixel accesses depends on the distribution of the runs in the pixel matrix in contrast to the method in

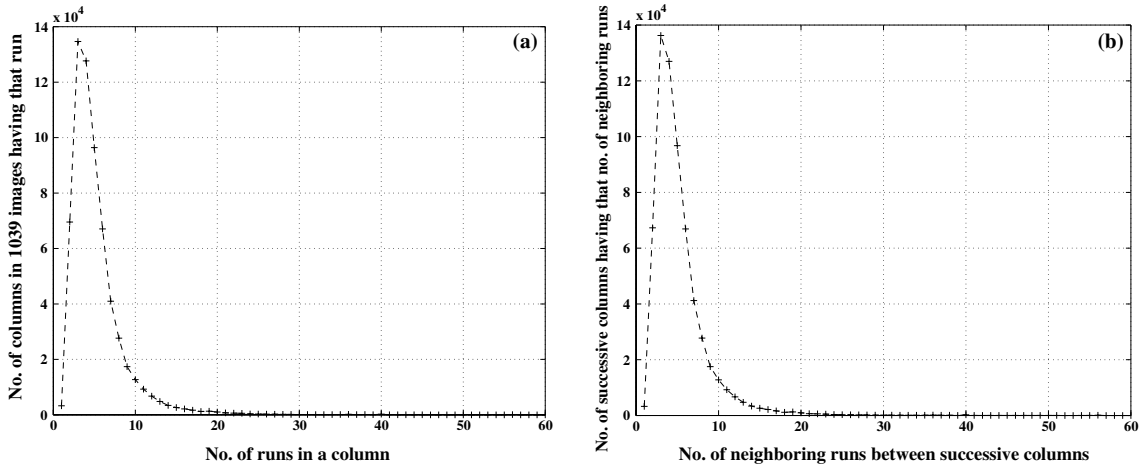


Fig. 3. Graphs showing (a) run and (b) neighboring run statistics.

[11]. It has been observed that for most of the images,  $R(i) \ll M$  and also  $O_i \ll M$ , and typically both  $R(i)$  and  $O_i$  have a value around 4. Thus, on the average, the number of pixel accesses will be much less compared to those of the other methods. Empirical evidence justifies the rationale. We have considered a database of 1039 logo images, and normalized each of them to the same size. From the experimental results, the expected value of the number of runs  $R(i)$  present in a column is observed to be 4.252741 and that of the neighboring runs between two consecutive columns,  $O_i$  is found to be 4.266170 (see Fig. 3(a) and (b)). Thus, the expected number of runs in actual images is nearly constant and much less than the expected case of  $\frac{N+1}{4}$  runs. Therefore, the number of accesses would be much less than that of the average case value. The above analytical expressions and experimental validation suggest that the proposed method of computing Euler number will outperform the existing techniques significantly.

We observed that the average CPU time required for computing the Euler number of a  $256 \times 256$  logo image by running the proposed algorithm is around  $11 \times 10^3 \mu\text{s}$  on a Sun Ultra-5\_10 Sparc workstation (233 MHz) with SunOS Release 5.7 Generic OS. On the same platform, the bit-quad based algorithm [2,11] takes around  $19.5 \times 10^3 \mu\text{s}$ .

### 3. Hardware implementation

Euler number satisfies the local additive property or additive set property [11]. This property allows us to split the image into smaller sub-images, and calculate the overall Euler number by combining the Euler numbers of the smaller subimages. An essential feature common to most of these methods is the addition and subtraction of the Euler numbers or some other parameters of the subimages [3,11,16–18,21]. Thus for fast computation of Euler number, the design of the adder is most vital. It may be noted that once the local property counting has been completed involving a set of pixels, the same pixels should not be required again to reduce CPU time. This indicates that in terms of precedence relations the addition process follows the local property counting and it can be overlapped with the next set of local property counting involving another set of pixels. This observation suggests that addition can be pipelined with the local property counting. In this section, the hardware design is reported. We also derive the upper and lower bounds of Euler number, which lead to estimation of the time complexity as well as the overall gate count. The design proposed here is then compared to other methods. The run-based sequential algorithm can be easily parallelized

soas to make it suitable for hardware implementation.

### 3.1. Parallel algorithm

#### Method

**Input:** An  $(N \times M)$  binary pixel matrix of an image  $I$ ;

**Output:** Euler number  $E(I)$ .

#### Compute\_Euler\_Parallel

```

for ( $i =$  row number 1 till end do in parallel)
    calculate the number of runs  $R(i)$  in the  $i$ th row;
    (by Fact 1, run of each row can be computed independently)
    calculate the number of neighboring runs  $O_i$ 
    between  $(i - 1)$ th and  $i$ th rows; (by Fact 3)

```

#### endfor

$E(I) = \sum_{i=1}^N R(i) - \sum_{i=2}^N O_i$ ; (by Theorem 1)

**return**  $E(I)$  as the Euler number.

### 3.2. Hardware architecture

To implement the proposed algorithm in hardware, two types of processing elements (PE) are required:  $P_1$  for identifying the start of a run in a row;  $P_2$  to signal the start of a neighboring run.

Using Lemma 1, the PE  $P_1$  (shown in Fig. 4(a)) is used to detect the transition from 0 to 1; the number of runs is equal to the number of such detections. A delay (D) flip-flop is initialized to 0 at the start of processing each row. It holds the value of the previous pixel for the purpose of checking a transition. The pixels in a row are pipelined into  $P_1$ . At any instant of time  $t_i$ , the  $i$ th pixel and  $(i - 1)$ th pixel of a row are checked for a  $0 \rightarrow 1$  transition. The maximum number of runs in a row having  $M$  columns is  $\lceil (M/2) \rceil$ .

In Fig. 4(b),  $P_1$  is shown within a box, and the remaining portion of the circuit constitutes the processing element  $P_2$ . It is used to detect the start of a neighboring run between two consecutive rows following Lemma 2. The PE  $P_2$  checks for the condition when a run in a row begins, and whether it is in the neighborhood of another run in its adjacent row. The pixels corresponding to the columns of two adjacent rows are fed to  $P_2$  in a pipeline. In addition, it receives data from the outputs of the D flip-flops of these two rows. At any instant of time  $t_i$ ,  $i$ th and  $(i - 1)$ th pixels of two consecutive rows are checked for a neighboring run. The maximum number of neighboring runs is  $2 \times \lceil (M/2) \rceil$ . To process an  $(N \times M)$  image in parallel, we require  $N$  pieces of  $P_1$ , and  $(N - 1)$  pieces of  $P_2$ .

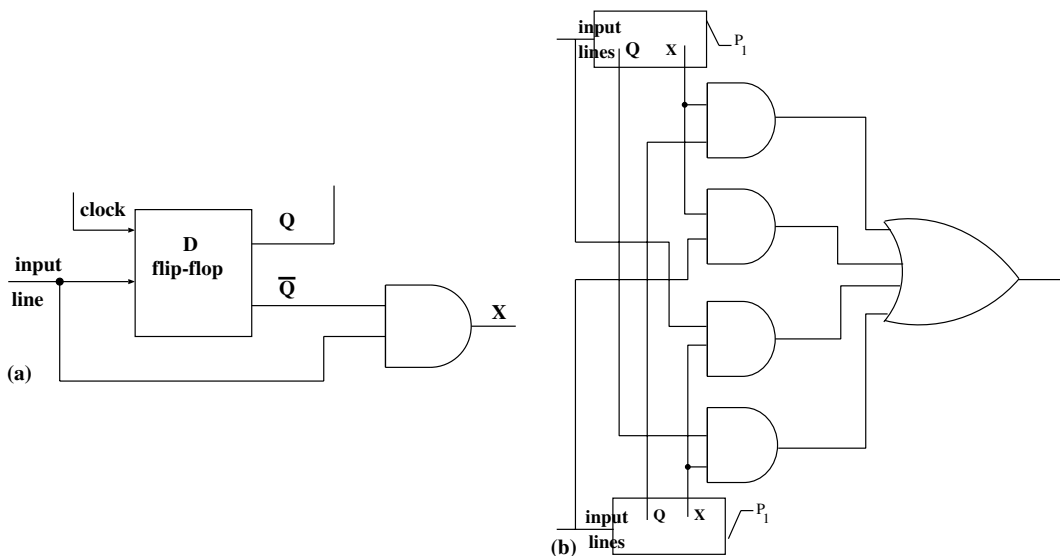


Fig. 4. Processing elements  $P_1$  (a) and  $P_2$  (b).

Table 1  
Truth table

	$P_1$	$P_2$	$p_1$	$p_2$	$(P_1 + p_1) - (P_2 + p_2)$	Sign-bit (s)	Data-bit (d)
1	0	0	0	0	0	0	0
2	0	0	0	1	-1	1	1
3	0	0	1	0	1	0	1
4	0	0	1	1	0	0	0
5	0	1	0	0	×	×	×
6	0	1	0	1	×	×	×
7	0	1	1	0	0	0	0
8	0	1	1	1	-1	1	1
9	1	0	0	0	1	0	1
10	1	0	0	1	×	×	×
11	1	0	1	0	×	×	×
12	1	0	1	1	×	×	×
13	1	1	0	0	0	0	0
14	1	1	0	1	-1	1	1
15	1	1	1	0	1	0	1
16	1	1	1	1	0	0	0

To compute Euler number of the whole image, we add the number of runs (calculated by summing up all  $0 \rightarrow 1$  transitions in all rows) and deduct from it the number of neighboring runs (calculated by adding up the number of all neighboring runs between consecutive rows). At any instant of time  $t_i$ , the output from  $P_1$  or  $P_2$  goes high or low indicating the start of a run or neighboring run respectively. A neighboring run implies the presence of a run in either or both of the corresponding row(s). Thus, a high (1) output from  $P_2$  is accompanied by a high output from either one

or both of the  $P_1$ 's connected to it. The interrelation of runs and neighboring runs is depicted as a truth table in Table 1, where  $\times$  represents a don't care entry.

### 3.3. Truth table and combinational circuit

In order to perform the addition process efficiently, we combine the local results in such a fashion so that the final subtraction can be avoided and a single adder tree can be designed to output the final result.

We take the outputs from the two modules generating  $P_1$  and two modules producing  $P_2$  for consecutive rows. To distinguish them, we use upper and lower case symbols ( $P_1, p_1, P_2, p_2$ ) in Table 1. The sum of the two  $P_1$  outputs is computed, and the sum of the two  $P_2$  outputs is then subtracted from it. It is easy to prove that the result is always either  $-1$ , or  $0$ , or  $1$ . This follows from the properties of runs and neighboring runs as mentioned in the previous subsection. The entries corresponding to rows 5 and 6 in the truth table are not feasible as  $P_2 = 1$  implies that either or both of  $P_1$  and  $p_1$  must be 1. The case as in row 10 does not appear as  $p_2 = 1$  and  $p_1 = 0$  implies the PE  $P'_1$  associated with  $p_2$  (and not included in this group of 4  $P_1$ 's and  $P_2$ 's, and as such not shown in the truth table) should be high. If  $P'_1$  and  $p_2 = 1$ , it implies the presence of a continuing run (not a run start) in the row associated with  $p_1$ ; otherwise  $p_2$  cannot be

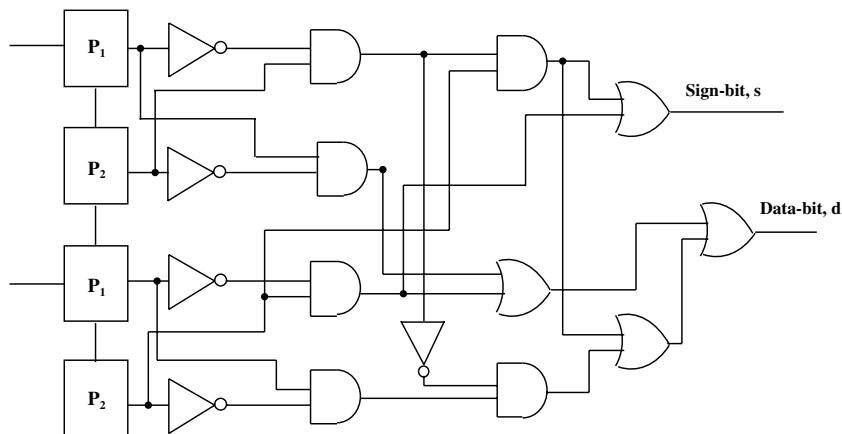


Fig. 5. Combinational circuit for finding the sign and data bit.

come high. If there exists a continuing run associated with  $p_1$  and also  $P_1 = 1$ , then  $P_2$  should be high, but that does not happen. Hence, this input combination never arises. Rows 11 and 12 are also inadmissible as in both the cases,  $P_1 = 1$  and  $p_1 = 1$  imply that  $P_2 = 1$ , which is not true. To represent  $-1, 0, 1$  in 2's complement, we require 2 bits (the sign bit and data bit). A combinational circuit  $C$  as shown in Fig. 5, is designed following the truth table (Table 1) to produce the sign and data bits from different input combinations of the two  $P'_1$ s and  $P'_2$ s.

### 3.4. Adder design and time complexity

The sum of the combination of the outputs from the two modules generating  $P_1$  and two modules generating  $P_2$  for consecutive rows lies in the range  $[-1, 1]$ . To design the adder circuit for Euler number computation, it is essential to determine the range of values Euler number can assume for an  $(N \times M)$  image, so that the width of the adders can be properly determined.

**Lemma 3.** *The Euler number of an  $(N \times M)$  binary image lies in the range  $[-M/2 \times \lceil(N/2)\rceil, M/2 \times \lceil(N/2)\rceil]$ .*

**Proof.** For an  $N \times M$  image, we require  $\lceil(N/2)\rceil$  pieces of combinational module  $C$ . Each  $C$ -module produces an output in the range  $[-1, 1]$ . Thus, each column can have a maximum sum of  $\lceil(N/2)\rceil$  and a minimum sum of  $-\lceil(N/2)\rceil$ . Since a run and a neighboring run for a row cannot begin in two consecutive columns, the Euler number should lie in the range  $[-M/2 \times \lceil(N/2)\rceil, M/2 \times \lceil(N/2)\rceil]$ . The adders should be so designed that they are able to handle this range of numbers. Alternatively, it can also be proved from the truth table entries as given in Table 1. The truth table entries that give rise to a sum of 1 are 3, 9 and 15 of Table 1; the entries that give rise to a sum of  $-1$  are 2, 8 and 14 of Table 1. To prove that the range of Euler number for an  $(N \times M)$  image lies in  $[-M/2 \times \lceil(N/2)\rceil, M/2 \times \lceil(N/2)\rceil]$ , it suffices to show that if any one of the entries 3, 9 and 15 of Table 1 occurs at any clock instant, the other two entries cannot occur at the next clock instant. Similar

cases arise for the entries 2, 8 and 14 of Table 1. If otherwise, then the sum would not lie in the range  $[-M/2 \times \lceil(N/2)\rceil, M/2 \times \lceil(N/2)\rceil]$ . If the entry 3 occurred at any clock instant  $t_i$ , then for  $p_1$  (see Table 1) to have a high (1) output, a 1 was fired at time instance  $t_i$  and a 0 at  $t_{i-1}$ , for a run ( $0 \rightarrow 1$  transition) to be detected. At time instance  $t_{i+1}$ , the 9th or 15th entry of the Table 1 cannot occur. If the 9th entry occurred, there should be an occurrence of a neighboring run, because with  $p_1$  going high at time instance  $t_i$  and  $P_1$  going high at time instance  $t_{i+1}$ ,  $P_2$  should also go high at time instance  $t_{i+1}$ . So, the 9th entry should have been as  $P_1 = 1, P_2 = 1, p_1 = 0$  and  $p_2 = 0$ . So, there is a contradiction, and as such the 9th entry cannot occur after the 3rd entry. Similarly, it can be shown that the pairs (3, 15), (9, 15), (2, 8), (2, 14) and (8, 14) cannot occur at consecutive time instances.  $\square$

The complete hardware design for a  $(16 \times 16)$  image is shown in Fig. 7. The output of  $C$  represents a 2-bit 2's complement number generated by two  $P'_1$ s and two  $P'_2$ s. An adder circuit is needed to sum up the outputs of all these  $C$ -modules to obtain the final result. We use a binary adder tree in pipeline [22] to accelerate the addition process. Addition of two 2's complement numbers may produce a 3-bit number. To implement such a scheme, we use at the leaf level of the adder tree, a set of 3-bit adders each with a sign bit extension. For each subsequent level in the tree, the adder size (width) is increased by one bit. The depth of the adder tree would be  $\lceil \log_2 N \rceil - 1$ . As the width of the adder in the tree increases by 1 at each level, the width of the adder  $A_r$  at the root, would be  $(3 - 1) + \lceil \log_2 N \rceil - 1 = \lceil \log_2 N \rceil + 1$ . Finally, a sequential full adder  $FA$  is used to accumulate the sum. The range of numbers  $FA$  should be able to handle is  $[-M/2 \times \lceil(N/2)\rceil, M/2 \times \lceil(N/2)\rceil]$  (see Lemma 3) Thus, the number of bits  $T$  required for the adder  $FA$  would be  $= \lceil \log_2 \{M \times \lceil(N/2)\rceil + 1\} \rceil$ . With the assumption that  $M \approx N$ ,  $T$  is  $O(\log_2 N)$ . The number of the stages of the pipeline is two more (one for the  $PE$ s  $P_1, P_2$  and  $C$  and the other for the adder  $FA$ ) than the depth of the adder tree. Therefore, the stages of the pipeline equals  $(\lceil \log_2 N \rceil - 1) + 2 =$

$\lceil \log_2 N \rceil + 1$ . The clock period of the linear pipeline is determined by the sum of the longest pipeline stage and the delay of the latches, and hence is equal to  $T + \delta$ , where  $T$  is the time taken by the adder  $FA$  and  $\delta$  is the time delay of the latches. The number of clock cycles required by the linear pipeline to perform the entire addition is  $(\lceil \log_2 N \rceil + 1) + (M - 1) = \lceil \log_2 N \rceil + M$ . Therefore, total time needed to produce the final output in 2's complement form is  $(T + \delta) \times \{M + \lceil \log_2 N \rceil\}$  and  $M$  being approximately equal to  $N$ , the time taken is  $O(N \log N + 2 \times \log_2^2 N) \approx O(N \log N)$ . The performance measures e.g., speed up, efficiency and throughput [22] for the linear pipeline designed here are given below.

*Speed-up,  $S_k$* : The speed-up is  $nk/k + (n - 1)$ , where  $n$  is the number of tasks and  $k$  is the number of stages of the pipeline. So, in our case for an  $(N \times N)$  image,

$$S_k = \frac{N(\lceil \log_2 N \rceil + 1)}{(\lceil \log_2 N \rceil + 1) + (N - 1)} = \frac{1 + \lceil \log_2 N \rceil}{1 + \frac{\lceil \log_2 N \rceil}{N}}$$

*Efficiency,  $\eta$* : The efficiency is the ratio of speed-up and the number of stages. So, here the efficiency is  $\frac{1}{1 + \frac{\lceil \log_2 N \rceil}{N}}$ .

*Throughput,  $w$* : The throughput is defined as the number of tasks that the pipeline can complete per unit time and is the ratio of the efficiency and the clock period of the pipeline. Thus,

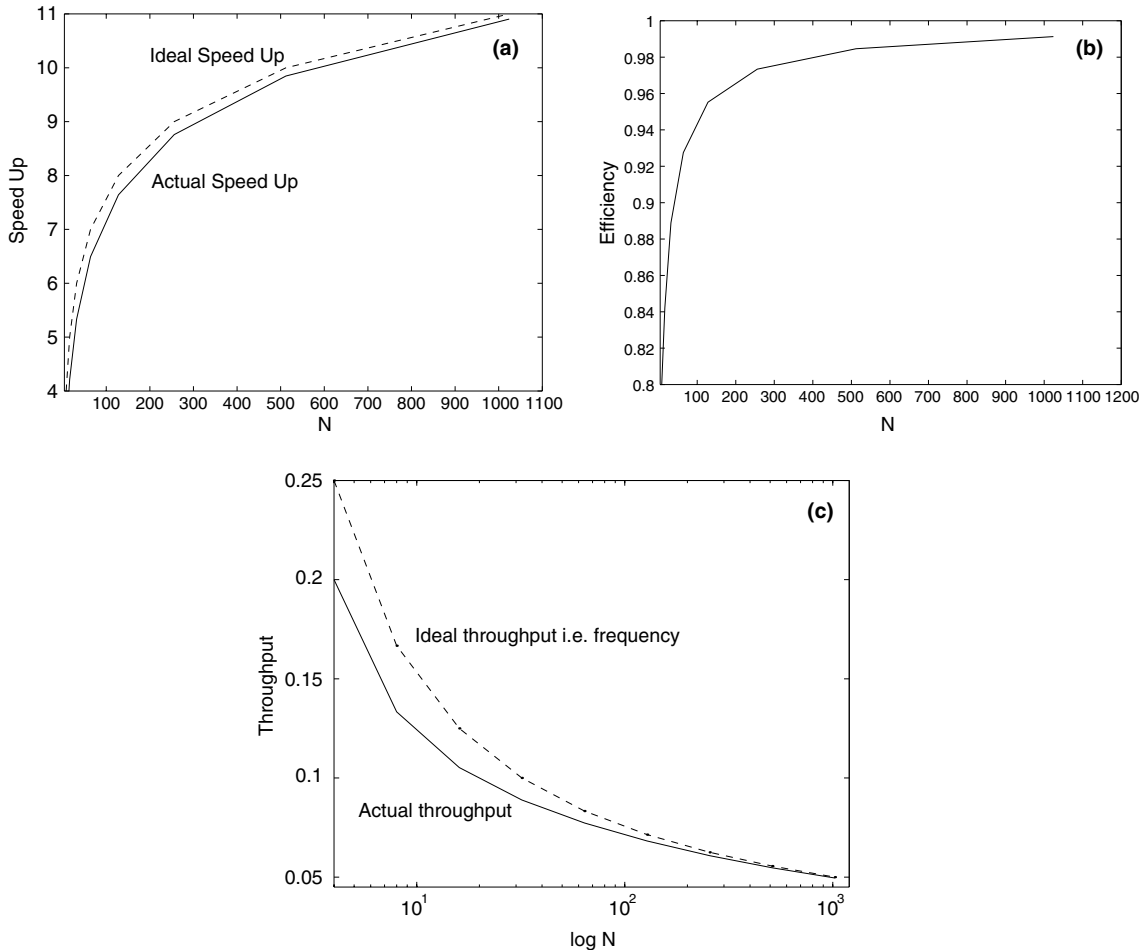


Fig. 6. Performance measures of the pipeline. (a) Speed-up of the pipeline, (b) efficiency of the pipeline, and (c) throughput of the pipeline.

$$w = \frac{\eta}{T + \delta}$$

$$= \frac{1}{(\lceil \log_2 \{N \times \lceil (N/2) \rceil + 1 \} \rceil) \left(1 + \frac{\lceil \log_2 N \rceil}{N}\right)},$$

neglecting  $\delta$ .

The graphs in Fig. 6(a)–(c) show speed-up, efficiency, and throughput, with the variations in the image size  $N$  respectively.

If the final adder  $FA$  performs a carry look-ahead addition [23] taking  $O(\log T)$  time, then the clock period of the pipeline can further be reduced and would be determined by the delay of  $A_r$ .

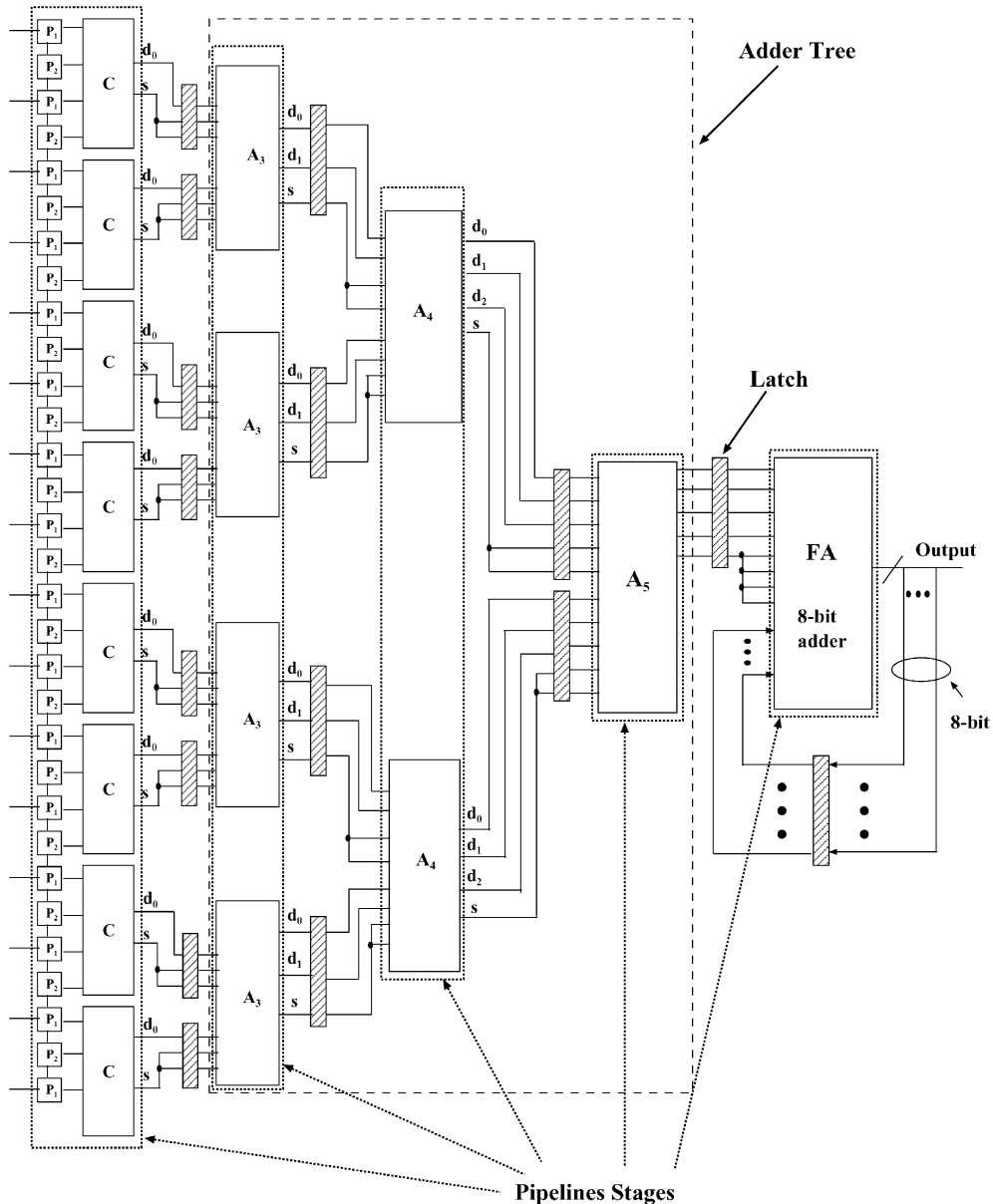


Fig. 7. A sample circuit for calculating the Euler number of a  $(16 \times 16)$  image.

### 3.5. Circuit cost and gate count

We require the following components to implement the hardware architecture for processing an  $(N \times M)$  binary image:

- (1)  $N$  pieces of processing element  $P_1$ , each having one edge triggered D-flip-flop and one 2-input AND gate. The gate count is  $O(N)$ , with constant number of gates required for each  $P_1$  and D-flip-flop.
- (2)  $N - 1$  pieces of processing element  $P_2$ , each having four 2-input AND gates and one 4-input OR gate. Here, also the gate count is  $O(N)$ .
- (3)  $\lceil N/2 \rceil$  pieces of the combinational circuit  $C$ , each having four inverters, six 2-input AND gates and five 2-input OR gates. As each piece of combinational circuit  $C$  requires constant number of gates, the gate requirement is  $O(N)$ .
- (4)  $\lceil N/4 \rceil$  pieces of 3-bit full adder,  $\lceil N/8 \rceil$  pieces of 4-bit full adder, ..., one piece of  $\lceil \log_2 N \rceil$ -bit full adder. The full adders require constant number of gates for each full bit adder. So, the gate count is of the order of the following:

$$\begin{aligned} & 3\lceil N/4 \rceil + 4\lceil N/8 \rceil + \dots + \log_2 N \\ & \approx \frac{3N}{2^2} + \frac{4N}{2^3} + \dots + \log_2 N \frac{N}{2^{\log_2 N}} \\ & \approx \frac{13N - 4}{16} + \frac{\log_2 N}{2} = O(N). \end{aligned}$$

- (5) One piece of  $\lceil \log_2 \{M \times \lceil N/2 \rceil + 1\} \rceil$ -bit full adder. The gate count is  $O(N)$ .
- (6) Latches between the two adjacent stages of the pipeline. The total gate count =  $\frac{6N}{4} + \frac{8N}{8} + \dots + \lceil \log_2 N \rceil + 1 + T \approx O(N)$ .

Hence, the total gate count is  $O(N)$ .

The complete circuit for a  $(16 \times 16)$  image with appropriate adder blocks is shown in Fig. 7.

### 3.6. Comparisons with other methods

The various formulations of computing Euler number require additions of two or more proper-

ties and finally their subtraction. The formulation in [11] given as  $(\sum v - \sum t - 2 \times \sum d)/4$  requires addition and subtraction of three properties viz.  $v$ ,  $t$  and  $d$ , pertaining to bit-quads  $Q_1$ ,  $Q_2$  and  $Q_D$ . Three types of processing elements would be required to detect bit-quads  $Q_1$ ,  $Q_2$  and  $Q_D$ . Also, adder complexity would be higher as more variables are to be dealt with in comparison to the run based method. The formulation has a divide-by-4 operation. So, it is obvious that the width of the adder would be more and shift registers will be needed for the division operator. This bit-quad formulation for computing the Euler number has also another drawback regarding border pixels. Any implementation of this bit-quad algorithm has to deal with the convexity on the border by specially employing separate processing elements for the border pixels other than the ones required for bit-quad checking. The parallel implementation based on Euler number computation of the thinned version [16] does not hold promise as parallel thinning in itself is a non-trivial task [24]. The hardware implementation of quad-tree based algorithms [14,15] are complicated as the sizes of the blocks represented by the leaf nodes may be unequal. Further, the number of leaf nodes may vary widely for different image samples. The best known parallel algorithm has been proposed in [17] that uses the connectivity graph (CG) derived from the cylindrical algebraic decomposition of the Euclidean plane. The authors describe a parallel implementation of their algorithm on a linear array network topology that uses the CG as the image data structure and performs a parallel searching of the sub graphs in CG. The estimated time complexity is  $O(\lceil M/5 \rceil / 16 (1 + \log_2 M))$ , where  $M$  is the number of arcs in the CG. A careful analysis reveals that the number of arcs  $M$  in CG can be linear in the number of pixel entries (which is  $N^2$ ) i.e.,  $M$  can be  $O(N^2)$  and so does the time complexity. Our proposed algorithm with  $O(N \log N)$  time complexity and  $O(N)$  gates thus compares favorably against existing algorithms.

### 3.7. Handling large images

Given an architecture for computing the Euler number of an image matrix of size  $N \times N$ , the

Euler number of a larger image of size  $K \times K$ , where  $K = x \times N$  can be easily determined. The matrix is partitioned into several  $N \times K$  sub-blocks as  $B_1, B_2, \dots, B_x$ , where each  $B_i$  is an  $N \times K$  matrix.

The Euler number of each such block of size  $N \times K$  is computed using our proposed architecture for an  $N \times N$  block by changing the size of the adder FA. Let  $E_i$  be the Euler number of a block  $B_i$ .

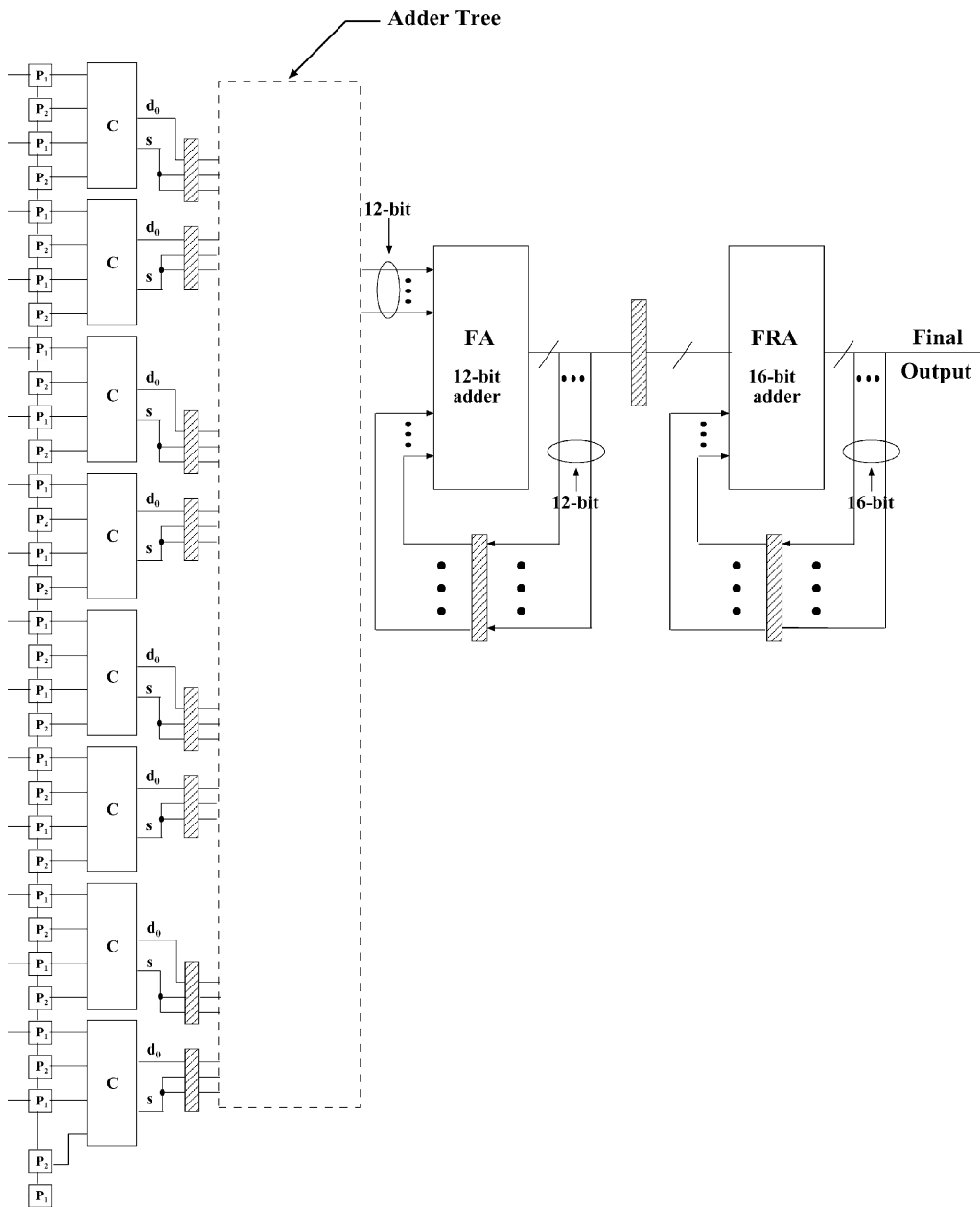


Fig. 8. A scalable circuit for calculating the Euler number of a  $(256 \times 256)$  image using the circuit for a  $(16 \times 16)$  image.

The Euler number of the overall image can now be easily determined by using the following algorithm based on Facts 2 and 3 as mentioned in Section 2.1.

#### Method

**Input:** A  $(K \times K)$  binary pixel matrix of an image  $I$ , where  $K = x \times N$ ;

**Output:** Euler number  $E(I)$ .

#### Compute\_Euler\_General

$E(I) = 0$ ;

$O_i = 0$ ;

**for** ( $i =$  block number 1 to  $x - 1$ )

calculate the Euler number,  $E(B_i)$  of the block  $B_i$  using *Compute\_Euler\_Parallel*;

calculate the number of neighboring runs  $O_i$  between last row of block  $B_i$  and first row of block  $B_{i+1}$ ; (by Fact 3)

$E(I) = E(I) + E(B_i) - O_i$ ; (by Fact 2)

**endfor**

compute the Euler number,  $E(B_x)$  of the last block  $B_x$  using *Compute\_Euler\_Parallel*;

$E(I) = E(I) + E(B_x)$ ; (by Fact 2)

**return**  $E(I)$  as the Euler number.

The adder tree can handle an image matrix with  $N$  rows, as it deals with only one column at a time. The adder  $FA$  adds up values from all columns. When the number of columns changes from  $N$  to  $K$ , the width  $W$  of the adder  $FA$  should be made equal to  $\lceil \log_2 \{K \times \lceil (N/2) \rceil + 1\} \rceil$ . To calculate the number of neighboring runs ( $O_i$ ) between the last row of block  $B_i$  and the first row of block  $B_{i+1}$ , we require processing elements  $P_1$  and  $P_2$ . The output of  $P_2$  is fed as an input of the last  $C$ -module in the column (Fig. 8). The outputs of  $FA$  corresponding to each block of size  $N \times K$  is pipelined to the final sequential adder  $FRA$  (Final Root Adder) (see Fig. 8). Using Lemma 3, it can be deduced that the number of bits  $T$  required for the adder  $FRA$  would be  $\lceil \log_2 \{K \times \lceil (K/2) \rceil + 1\} \rceil$ . The clock period of the linear pipeline is obviously  $T + \delta$ , where  $\delta$  is the delay of the latches. The number of stages  $S_N$  of the linear pipeline in the scalable circuit is one more than the earlier case and is  $(\lceil \log_2 N + 2 \rceil)$ . Therefore, the total time needed to compute the Euler number scalably is  $(T + \delta) \times (S_N) \approx O(Nx \log_2(Nx))$ . The speed-up, efficiency, throughput are as follows:

**Speed-up  $S_k$ :** The number of tasks is obviously now  $Kx = \frac{K^2}{N}$  and the number of stages is  $S_N$ . So, the speed-up is

$$\frac{\frac{K^2}{N} \times (\log_2 N + 2)}{\lceil \log_2 N \rceil + \frac{K^2}{N} + 1} = \frac{Nx^2(\lceil \log_2 N \rceil + 2)}{\lceil \log_2 N \rceil + Nx^2 + 1}.$$

The ideal speed-up is obviously  $\lceil \log_2 N + 2 \rceil$ .

**Efficiency  $\eta$ :** The efficiency is the ratio of speed-up and the number of stages and is  $\frac{Nx^2}{\lceil \log_2 N \rceil + Nx^2 + 1}$ .

**Throughput  $w$ :** The throughput is

$$\frac{Nx^2}{(\lceil \log_2 N \rceil + Nx^2 + 1)(\lceil \log_2 (Nx \times \lceil \frac{Nx}{2} \rceil + 1) \rceil)}.$$

It can be seen that for a fixed  $N$ , speed-up and efficiency of the pipeline increases with  $x$  and throughput decreases with  $x$ . This is a desirable feature of the pipeline.

The circuit for computing the Euler number of an image of size  $(256 \times 256)$  using the circuit module for a  $(16 \times 16)$  image, is shown in Fig. 8. We have assumed sequential full adders for our complexity analysis for both the normal and scalable cases. It can be observed that the time complexity can further be improved by using carry-save addition [23].

#### 4. VLSI implementation of the architecture

The proposed architecture has been designed on-chip using Mentor Graphics Leonardo Spectrum, Modelsim, and IC Station run on a SUN Blade 2000 Workstation. The design is coded with VHDL, and after simulating and verifying it using Modelsim, all the VHDL modules are synthesized to generate the Verilog netlist using 0.18 micron technology. The Verilog netlist file is then fed to the tool IC Station, which produces the final chip layout using standard cell design style. All the geometric information of the layout is produced in Graphic Design System-II (GDS-II) code that is needed by foundry for fabrication of the chip. The synthesized netlist report is presented in Table 2 for a  $256 \times 256$  image. The internal zone area is  $400,980.9 \mu\text{m}^2$ . The overall chip  $X$ -dimension is  $607.9 \mu\text{m}$  and the  $Y$ -dimension is  $671.2 \mu\text{m}$ . The critical delay in the circuit is 2.29 ns. The on-chip

Table 2

Synthesis report of the circuit for processing a  $(256 \times 256)$  image

Number of ports	274
Number of nets	5362
Total number of gates	138,594

time to compute Euler number of a  $256 \times 256$  binary logo image turns out to be  $0.6 \mu\text{s}$ .

## 5. Conclusions and discussions

A run-based algorithm for computing the Euler number of a binary image is formulated and its performance is analyzed. The algorithm is based on certain combinatorial and statistical properties of runs present in the pixel matrix of the image. Analytical and experimental studies on a logo database show that the proposed algorithm outperforms existing methods based on bit-quad or quad-tree significantly. A new hardware implementation using pipeline architecture for fast on-chip computation of Euler number is also reported. The hardware design uses  $O(N)$  gates to compute the Euler number of an  $N \times N$  image in  $O(N \log N)$  time. This improves on the best known parallel implementation of  $O(N^2)$  on a linear array network topology. The basic module can be used to handle arbitrarily large-sized pixel matrices. The architecture has been implemented in VLSI and relevant chip data on area and speed are reported. It has been observed that the on-chip computation is extremely fast and hence, the design will be useful to many real-time applications. Design of algorithms for applicability to higher dimensions needs further investigation.

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