Image Compression and Magnification Using Fractal Technique

Suman K. Mitra, C. A. Murthy and Malay K. Kundu
Machine Intelligence Unit,
Indian Statistical Institute,
203, B. T. Road,
Calcutta 700035.
INDIA.

ABSTRACT

New techniques for image compression and image magnification using the theory of fractals is proposed in this article. The fractal codes are designed assuming self similarity property of images. In particular, to generate the fractal codes, Genetic Algorithm is used which greatly decreases the search for finding self similarities in the given image. Obtained fractal codes are stored instead of the given image in the compression algorithm. Moreover magnification task is performed using the obtained fractal codes of the image instead of the original one resulting in a reduction in memory requirement. The article presents both theory and implementation of the proposed methods. A simple distortion measure scheme is also proposed to judge the image quality of the magnified image. Comparison with other compression and magnification techniques has not been reported here.

1. INTODUCTION

Fractal geometry has recently come into the limelight due to its use in various scientific and technological applications, specially in the field of computer based image processing. It is being successfully used for image data representation [1] [2] and as image processing tools [3]. In this connection, in image data compression, Iterative Function System (IFS) [1] has become very popular. IFS is a set of affine contractive maps which is computed using the self similarities present in the images. IFS can approximate a real image with a few number of maps. So, it is enough to store the relevant parameters of the maps instead of storing the whole image.

In the present work an attempt is made to show how IFS or fractal codes can simultaneously be used in image compression and image magnification. Image magnification is an important preprosessing task that has to be performed to bring data from different sources and scales to a common scale. Moreover the image regions which are not explicitly visible in the original may provide more information after magnification. Most commonly used magnification operator utilizes interpolation techniques. Here we have discussed a new magnification technique using IFS. There are several schemes to obtain IFS or fractal codes of an image. But most of them are very expensive in the sense of computational time. We have used here a computationally inexpensive method to obtain IFS using Genetic Algorithms (GAs) [4]. GAs [5][6] are optimization algorithms which reduce, significantly, the search space and time. To perform the compression task IFS is stored instead of storing the original image. The image can be reconstructed by using this IFS in an iterative sequence [1] [2]. Here we have proposed an algorithm for image magnification which utilizes only the stored IFS. So, IFS once computed for a given image can simultaneously be used for reconstructing the image as well as for magnifying the image.

The article reports the initial results of application of fractal codes in image compression and image magnification. Comparison with other methods has not been attempted here. A new distortion measure to judge the performance of image magnification has also been introduced. The overall performances of the algorithms are found to be satisfactory.

2. BASIC PRINCIPLES AND METHODOLOGY

2.1. Theoretical Foundation of IFS

Let I be a given image which belongs to X. Generally X is taken as the collection of compact sets.

Our intention is to find a set $\mathcal{F}$ of affine contractive maps for which the given image I is an approximate fixed point or attractor "A" of the set of maps $\mathcal{F}$ defined as follows :

$$\lim_{N\to\infty} \mathcal{F}^N(J) = A, \quad \forall J \in X,$$

and $\quad \mathcal{F}(A) = A, \quad$ where $\mathcal{F}^N(J)$ is defined as

$$\mathcal{F}^N(J) = \mathcal{F}(\ \mathcal{F}^{N-1}(J)\ ),$$

with $\quad \mathcal{F}^1(J) = \mathcal{F}(J), \quad \forall J \in X.$

Also the set of maps $\mathcal{F}$ is defined such that

$$d(\mathcal{F}(J_1), \mathcal{F}(J_2)) \leq s\ d(J_1, J_2);\ \forall J_1, J_2 \in X. \quad (1)$$

Here "d" is called the distance measure and "s" is called the contractivity factor of $\mathcal{F}$, with $0 < s < 1$.

$$\text{Let } d(I, \mathcal{F}(I)) \leq \epsilon \quad (2)$$

where ϵ is a small positive quantity. Now, by Collage theorem [1], it can be shown that

$$d(I, A) \leq \frac{\epsilon}{1-s} \quad (3)$$

Here, $(X, \mathcal{F})$ is called iterative function system and $\mathcal{F}$ is called the set of fractal codes for the given image I.

2.2. Image Coding by IFS

Let, I be a given image of size $w \times w$ and having the range of gray level values $[0, g]$. Thus the given image I is a subset of $\mathbb{R}^3$. The image is partitioned into n nonoverlapping squares of size, say $b \times b$, and let this partition be represented by $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_n\}$. Each $\mathcal{R}_i$ is named as range block where, $n = \frac{w}{b} \times \frac{w}{b}$. Let $\mathcal{D}$ be the collection of all possible blocks (within the image support) each of size $2b \times 2b$ and let $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_m\}$. Each $\mathcal{D}_j$ is named as domain block with $m = (w - 2b) \times (w - 2b)$. Let,

$$\mathcal{F}_j = \{f : \mathcal{D}_j \to \mathbb{R}^3; f \text{is affine contractive map}\}.$$

Now, for a given range block $\mathcal{R}_i$, let, $f_{i|j} \in \mathcal{F}_j$ be such that

$$d(\mathcal{R}_i,\ f_{i|j}(\mathcal{D}_j)) \leq d(\mathcal{R}_i,\ f(\mathcal{D}_j))\ \ \forall f \in \mathcal{F}_j,\ \forall j,$$

and let k be such that

$$d(\mathcal{R}_i,\ f_{i|k}(\mathcal{D}_k)) = \min_j\{\ d(\mathcal{R}_i,\ f_{i|j}(\mathcal{D}_j))\ \} \quad (4)$$

Also, let $\quad f_{i|k}(\mathcal{D}_k) = \widehat{\mathcal{R}}_{i|k}.$

Our aim is to find $\widehat{\mathcal{R}}_{i|k}$ for each $i \in \{1, 2, \cdots, n\}$ or in other words for each range block $(\mathcal{R}_i)$ we are to find appropriately matched domain block $(\mathcal{D}_k)$ and appropriately matched map $(f_{i|k})$. Thus $\{\mathcal{D}_k, f_{i|k}\}$ is called IFS or fractal code for $\mathcal{R}_i$.

2.3. Image Magnification using IFS

The transformation $f_{i|k}$ consists of two parts. The first part denotes the contraction to reduce the size of the domain block. The second part is a transformation which provides an approximation of the range block when applied to the contracted domain block. Thus $f_{i|k}$ can be looked as a mixture of two transformations, $f_{i|k} = t_{i|k}\ \mathcal{C}$, where, $\mathcal{C}$ is contraction operation and $t_{i|k}$ is the transformation for the contracted domain block.

We have, $I = \bigcup_{i=1}^{n} \mathcal{R}_i$. Thus by (2) we have,

$$d(\ \bigcup_{i=1}^{n} \mathcal{R}_i,\ \bigcup_{i=1}^{n} \widehat{\mathcal{R}}_{i|k}\ ) \leq \epsilon \quad (5)$$

Now, let $\mathcal{M}$ be a magnification operator such that

$$d(\ \bigcup_{i=1}^{n} \mathcal{R}_i,\ \bigcup_{i=1}^{n} \mathcal{M}(\mathcal{R}_i)\ ) \leq \epsilon_1 \quad (6)$$

By (5) and (6) we have,

$$d(\ \bigcup_{i=1}^{n} \mathcal{R}_i,\ \bigcup_{i=1}^{n} \mathcal{M}(\widehat{\mathcal{R}}_{i|k})\ ) \leq \epsilon_2 \quad (7)$$

Again, we have, $\widehat{\mathcal{R}}_{i|k} = f_{i|k}(\mathcal{D}_k) = t_{i|k}\ \mathcal{C}(\mathcal{D}_k)$.

Reconstruction of image using the operator $\mathcal{M}$ should be an inverse of the contraction operation using the operator $\mathcal{C}$. So,

$$d\ (\ \mathcal{M}\ (\widehat{\mathcal{R}}_{i|k})\ ,\ t_{i|k}(\mathcal{D}_k)\ ) \leq \epsilon_3 \quad (8)$$

Thus from (8), it is clear that there is no need of constructing the separate operator $\mathcal{M}$, for image magnification. Only the second part of the fractal codes need to be applied on the domain block to get an image which is very close to the given image I and having size twice that of the given image.

3. FIDELITY CRITERIA

To judge the performance of magnification algorithm we are proposing a new measure. The magnified images usually have some specific artifacts appearing on or near the edges. So, in our proposed performance measure, the errors are measured from edges. Both the images are first partitioned into blocks proportional to their respective sizes in such a way that both images contain equal number of blocks. The error is then measured block wise and finally the average error is computed. To detect the edges of each block we have used the scheme suggested by Ramamurthi et al.[7]. The edge blocks consist of value "0" and "1" where, "1" represents the presence of edge. Now it is expected that the original and the magnified blocks should have same type of edge distributions. In other words the expected run of "1" present in both the blocks should be same if normalized by their respective sizes. Thus the error measure is defined by the difference between the normalized expected "run" of "1" present in the given image and in the magnified reconstructed image.For simplicity the average of vertical and horizontal edge errors provides the final error measure.

4. IMPLEMENTATION

As a specific implementation of the proposed algorithm, 128×128, 8 bit/pixel image is considered. The GA based technique [4] is applied to get the IFS codes. Moreover a two level partition scheme [2] and a classification scheme [4] for range blocks are adopted to reduce the complexity of the IFS codes and to get quality magnified image. In two level partition scheme two types of range blocks are considere one is parent range block and other is child range block. In classification scheme, the image regions where gray level gradient is very low are kept fixed and the regions are called smooth. The image regions which are not smooth are called rough. The IFS codes for rough type range blocks are then computed. Range blocks of size 8×8 and 2×2 are considered to obtain GA based fractal codes [4] to perform image compression and image magnification tasks respectively. To get more compression we took larger block size in case of compression algorithm. On the other hand small range blocks are chosen so as to get the finer image detail which is very important for magnification. These fractal codes are then modified stepwise to get the images which are magnified by a factor which is multiple of two. In each step the error, in comparison to its previous step is measured successively. The obtained results for image compression and image magnification are given in Table 1 and Table 2 respectively. Figures 1 and 2 are respectively original and decoded images in image compression algorithm. Figures 3 and 4 are respectively two times and four times magnified images of the original image (Figure 1) obtained using the fractal codes.

TABLE I
TEST RESULTS OBTAINED IN IMAGE COMPRESSION ALGORITHM

Range block size	Domain block size	Compression Ratio	PSNR (in dB)
8×8 and 4×4	16×16 and 8×8	11.87	30.22

TABLE II
TEST RESULTS OBTAINED IN IMAGE MAGNIFICATION ALGORITHM

Image	Reconstructed Image Statistics					
	MF	Error	MF	Error	MF	Error
Lena	2	1.18	4	1.37	8	1.24

MF=Mgnification Factor

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Figure 1


Figure 2


Figure 3

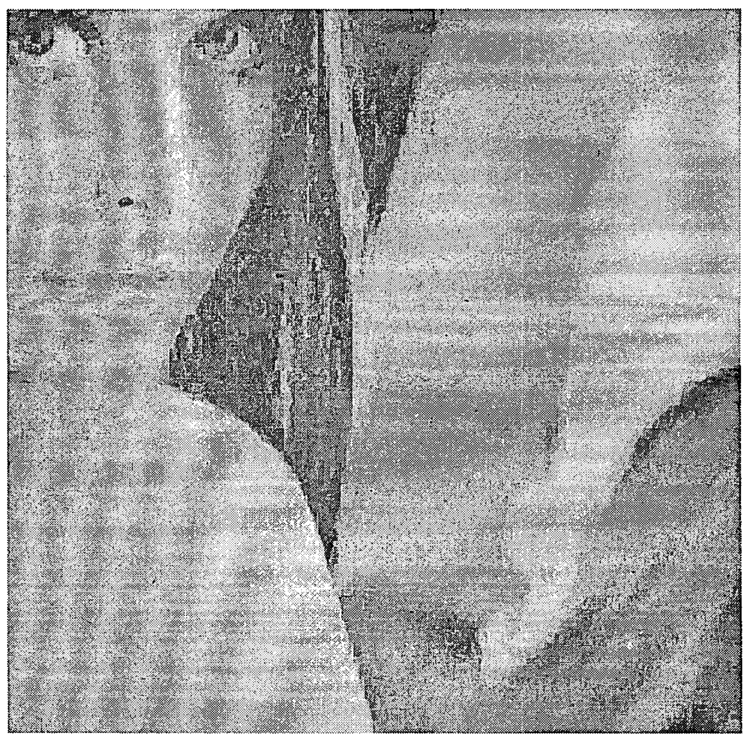
Figure 4