SELECTION OF MAPPING FUNCTION FOR OPTIMAL ENHANCEMENT USING FUZY SET THEORETIC MEASURE

Malay K. Kundu, Member IEEE and Sankar K. Pal, Senior Member IEEE

Electronical and Communication Sciences Unit
Indian Statistical Institute
203 Barrackpore Trunk Road
Calcutta 700 035
India.

Abstract

Algorithm for selection of mapping function appropriate for object enhancement of a given image is described. The algorithm does not need iterative visual interaction and prior knowledge of image statistics in order to select the transformation function for its optimal enhancement. Quantitative measure for evaluating enhancement quality has been provided based on fuzzy Geometry. The concept of minimising fuzziness (ambiguity) in both grayness and in spatial domain, as used by Pal and Rosenfeld [4], has been adopted.

I. Introduction

The process 'Image enhancement' is used to improve the image quality of an image, so that its usefulness is increased for subsequent processing and analysis operations. The methods and objectives of this process may vary with applications. When images are enhanced for human viewer, objective may be to improve perceptual aspect: image quality, intelligiblity or visual appearance. On the other hand in an application such as object identification by machine, the image enhancement process such removal of noise, deblurring of object edges and highlighting specified object features greatly improves the perforamnce as well as success of the process. Because the objective of image enhancement is dependent on the application context, and the criteria for enhancement are often subjective or too complex to be easily converted to useful objective measures, image enhancement algorithm tends to be simple qualitative and adhoc [1-3]. In addition in any given application, an algorithm that perform well for one class of problems may not perform well for other classes. Based on the goals, the enhancement techniques can broadly be grouped in to the following categories,

- noise cleaning.
- feature enhancement (contrast, edge etc.).
- noise cleaning plus feature enhancement.

In the present paper our discussion will be restricted to 'Contrast enhancement'.

When an image is processed for visual interpretation it is ultimately upto the viewers to judge its quality for a specific application and how well a particular method works. The process of evaluation of image quality therefore becomes subjective which makes the definition of a good processed image an elusive standard for comparison of algorithm performance. Again, it is customary to have an iterative process with human interaction in order to select an appropriate operator for obtaining such a desired processed output.

For example, let us consider the case of contrast enhancement using nonlinear functional mapping. Any kind of nonlinear function will not produce a desired (meaningful) enhanced version [2, p.10-13]. The questions that automatically arise are "given an aribitrary image which type of nonlinear functional form will be best suited without having the prior knowledge on image statistics (e.g., in robot vision and remote applications where frequent human interaction is not possible) for highlighting its object ?" and "having known the enhancement function how one can quantify the enhancemnt quality for obtaining the optimal one ?" Regarding the first question; even if we are given the image statistics, it is possible only to estimate approximately the function required enhancement and the selection of exact functional form still needs human interaction in iterative process. The second question, needs on the other hand, individual judgement which makes the optimal decision subjective.

The present work is an attempt to demonstrate an application of the theory of fuzzy sets to avoid such human iterative interaction and to make the task of subjective evaluation objective. An algorithm is formulated here which minimises (optimises) two types of ambiguities (fuzziness) namely, ambiguity in grayness and ambiguity in geometry of image containing an object.

It is to be mentioned here that the said ambiguity measures have been found, recently, by Pal and Rosenfeld [4] to obtain both fuzzy and nonfuzzy optimum segmentation for object background classification of an image. Their algorithm used Zadeh's Standard S-function [5] to compute "bright image" membership plane and to compute its ambiguities. By changing cross-over point of the S function for a fixed bandwidth, an optimum membership plane for which ambiguity is minimum was obtained.

The proposed algorithm has three parts. Given an input image X and a set of nonlinear transformation functions, it first of all enhances the iamge with a particular enhancement function with its varying parameters. The second phase consists of measuring both spatial ambiguity and grayness ambiguity of the various enhanced various X' using the algorithm in [4] and to check if these measures posses any valley (minimum) with change in parameters. The same procedure is repeated in the stage for other functions. Among all the valleys, the global one is selected. The corresponding function with the prescribed parameter values can be regarded as optimal one, and the value of ambiguity corresponding to the global minimum can be viewed as a quantitative measure of enhancement quality.

The nature of nonlinearity of the optimal enhancement function was further justified from the point of bounds [7] of S-type membership functions. Effectiveness of the algorithm is demonstrated on various unimodal, bimodal, skewed and multimodal images when different nonlinear functions are considered as enhancement operators.

It is further to be noted that although the geometrical ambiguity measure [4] was formulated only for single compact object, the present experiment has been extended to other types of objects also to investigate the effect of the said ambiguity measure.

II. Grayness ambiguity and spatial ambituity [4]

A gray tone image X of L levels and dimension MxN can be considered as an array of fuzzy singletons, each having a value of membership denoting its degree of brightness relative to some brightness level l; l = 0,1,2... L-1. In the notation of fuzzy sets, it can be

0,1,2... L-1. In the notation of fuzzy sets, it can be represented as

$$X = \left\{ \left(\mu(x_{mn}), \; x_{mn} \right) = \mu_{mn}/x_{mn} \; : m = 1,2,\ldots M.; \; n = 1,2,\ldots N \right\} \tag{1}$$

where x_{mn} is the (m,n)th pixel intensity. $0 \leq x_{mn} \leq L-1$. $\mu_{mn}(=\mu(x_{mn}))$ denotes the grade of posssessing some property (e.g., brightness, darkness, edginess, smoothness) by the (m,n)th pixel intensity x_{mn}.

Entropy of image X may be computed as

$$H(X) = \frac{1}{MN \ln 2} \sum_{l} T_e(l)\, h(l) \tag{2}$$

with $T_e(l) = -\mu(l) \ln 2\, \mu(l) - (1-\mu(l)) \ln(1-\mu(l))$ and h(l) denoting the frequency of level l.

Similarly, the area, perimeter and compactness of X [6] may be computed as

$$a(X) = \sum_m \sum_n \mu_{mn} = \sum_l u(l)\, h(l) \tag{3}$$

$$p(X) = \sum_{m=1}^{M} \sum_{n=1}^{N-1} \left| \mu_{mn} - \mu_{m,n+1} \right| + \sum_{n=1}^{N} \sum_{m=1}^{M-1} \left| \mu_{mn} - \mu_{m+1,n} \right| \tag{4}$$

$$\text{and} \quad comp(X) = a(X) / p^2(X) \tag{5}$$

It is thus seen from the above measures that H(X) considers global information and provides an average amount of fuzziness of X i.e. the degree of difficulty (ambiguity) in deciding whether a pixel would be treated as black (dark) or white (bright). The difficulty is minimum when μ_{mn} = 0 or 1 (i.e. the image its crisp with either fully black or white pixels) and maximum when μ_{mn} = 0.5 (semi bright pixels). Other grayness ambiguity measures e.g., index of fuzziness and index of crispness are available in [4].

The measure comp(X), on the other hand, takes into account the local information and reflects and amount of fuzziness in geometry (spatial domain) of X. Among all possible fuzzy disks, the compactness is the smallest for its crisp version.

Combining the two types of ambiguities described above a composite measure was defined as

$$\Theta(X) = H(X).Comp(X) \tag{6}$$

which involves fuzziness both in gray level and in spatial domain of image. Thus, a μ_{mn} plane having minimum Θ value implies that the image X has minimum ambiguity (fuziness) as far as its grayness and geometry of its object together are concerned.

For computing the ambiguity measures (equation 2-5) of an image X, μ_{mn} is considered here to be "bright image" subset and is obtained from x_{mn} using Zadeh's standard S-function [5] as follows :

$$\mu_{mn} = \begin{cases} 0 & , \; x_{mn} \leq a \\ 2\left[(x_{mn}-a)/(c-a)\right]^2, & a \leq x_{mn} \leq b \\ 1-2\left[(x_{mn}-c)/(c-a)\right]^2, & b \leq x_{mn} \leq c \\ 1 & x_{mn} \leq c \end{cases} \tag{7}$$

with b = (a + c)/2.

b is the crossover point i.e., at x_{mn} = b, $\mu_{mn}(x_{mn})$=0.5 and $\Delta b = b - a = c - b$ is the bandwidth. μ_{mn} represents the degree of brightness of (m,n)th pixel intensity x_{mn}.

III. Gray level rescaling

The gray level rescaling is one of the most widely used techniques for contrast enhancement which decreases the blurring and at the sme time reveals the features of interest. Each pixel is directly requantized or mapped here to a new gray level in order to improve the contrast of an image. In many pictures, the gray level difference between object and background is so small that it becomes difficult visually to discriminate them; enhancement is then required to increase such difference.

The simplest form of the functional mapping may be epressed as

$$x'_{mn} = x_{max} \;\; f(x_{mn})$$

where x_{mn} = gray value of the (m,n)th pixel in the input image (original)

x'_{mn} = transformed gray value of the (m,n)th pixel (enhanced)

$f(x_{mn})$ = prespecified mapping (transformation) function.

$$0 \leq f \leq 1$$

x_{max} = maximum gray level of input dynamic range.

Some of the most popular [3] mapping functions are shown in Fig.1.

The transformation function like Fig.1(a) when applied on an image, makes the dark area (lower range of gray level) stretched and the bright area compressed, resulting in an increase in the contrast within the darker area of the image. Similarly, the application of a mapping function similar to Fig.1(c) will produce the effects exactly opposite to that of Fig.1(a).

The mapping function as in Fig.1(b) will result in stretching of the middle range gray levels. The curve in Fig.1(d) (which is also known as gamma correcting curve for display nonlinearity), when used as mapping function, will compress drastically the midrange values, and at the same time it will stretch the gray levels of the upper and lower ends.

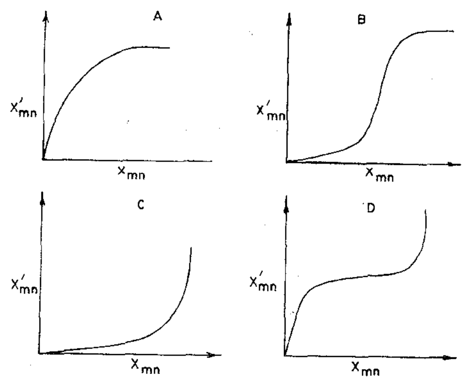


Figure 1. Commonly used mapping functions for image contrast enhancement.

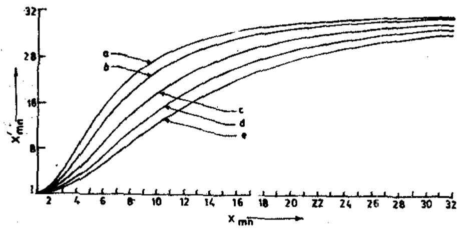


Figure 2(a). The mapping function corresponding to equation (8). a:k=100, b:k=66.6, c:k=50, d:k=40, e:k=33.3.

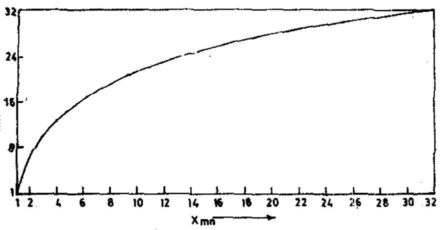


Figure 2(b). The mapping function corresponding to equation (9).

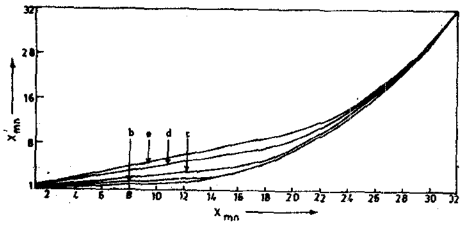


Figure 2(c). The mapping function corresponding to equation (10). a:A=0.064, B=0.1, D=11. b:A=0.074, B=0.15, D=13. c:A=0.080, B=0.25, D=15. d:A=0.084, B=0.4, D=17. e:A=0.093, B=0.5, D=19.

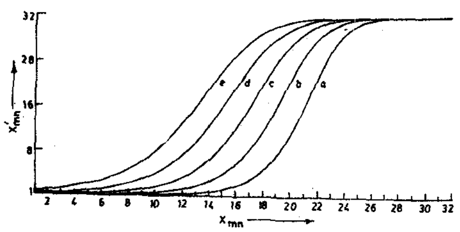


Figure 2(d). The mapping function corresponding to equation (11). a:F_d=12, b:F_d=14, c:F_d=16, d:F_d=18, e:F_d=20.

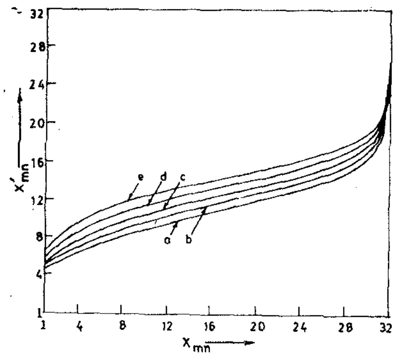


Figure 2(e). The mapping function corresponding to equation (12). a:F_d=22, b:F_d=21, c:F_d=20, d:F_d=19, e:F_d=18.

In our experiment we have simulated these mapping operators with the help of different nonlinear functions in order to investigate their relative enhancement performance on different images. The following forms of nonlinear function (monotonically nondecreasing) are used to represent (approximately) the curves in Fig.1. The mapping function in Fig.1(a) is represented by

$$\text{either } f(x_{mn}) = \frac{a x_{mn}^2}{1+a x_{mn}^2} = \frac{x_{mn}^2}{\frac{1}{a}+x_{mn}^2} = \frac{x_{mn}^2}{k+x_{mn}^2} \tag{8}$$

$$\text{or, } \quad f(x_{mn}) = b \log(x_{mn}) \tag{9}$$

where the parameters a & b are positive constants. The mapping function in Fig.1(c) is represented as

$$f(x_{mn}) = A\left[F(x_{mn})\right]^2 + Bx_{mn} + C \tag{10}$$

$$0 < A, B, C < 1$$

$$\text{where } F(x_{mn}) = x_{mn} - D \quad \text{for} \quad x_{mn} > D$$

$$= 0 \quad \text{for} \quad x_{mn} \le D.$$

$$x_{min} < D < x_{max}$$

where x_{min} and x_{max} are the minimum and maximum gray levels in the image.

The function in Fig.1(b) can be expressed as

$$f(x_{mn}) = \left[1 + \left(\frac{x_{max} - x_{mn}}{F_d}\right)^{F_e} \right]^{-1} \tag{11}$$

and that of Fig.1(d) as

$$f(x_{mn}) = \frac{1}{x_{max}} \left[x_{max} - F_d\left\{\left(\frac{x_{max}}{x_{mn}} + \beta\right) - 1\right\}^{-F_e} \right] \tag{12}$$

where F_e & F_d are the positive constants and β is the value of $f(x_{mn})$ for $x_{mn} = 0$. The functional forms for equations (8)-(12) are shown through Figs.2(a-e) respectively.

IV. Algorithm

The block diagram of the proposed algorithm is shown in Fig.11. Given an input image X, it is first of all transformed (enhanced) by one of the nonlinear

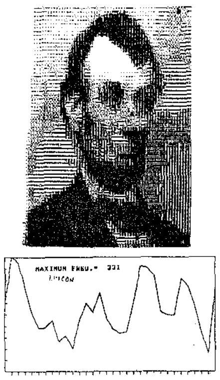


Fig. 3.

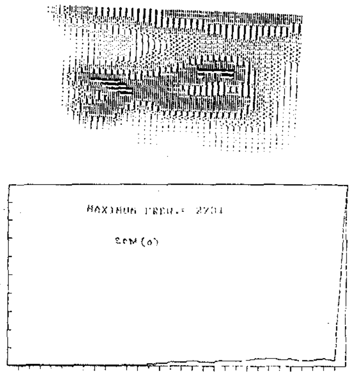


Fig. 4.

Figure 3. Lincon image and its gray level histogram.

Figure 4. Chromosome image and its gray level histogram.

Table 1 : Ambiguity measures for different input images

Image	Entropy (H)	Compactness (comp)	Product of H and comp (Θ)
Lincon	0.481	0.00428	0.00128
Chromosome	0.275	0.0892	0.00988
Cell	0.541	0.014	0.00484
Biplane	0.304	0.0150	0.00219

mapping functions f^q_i (Figs.2(a)-2(e)). The μ values of the transformed image X' are computed with Zadeh's standard S-function (equation 7) in order to compute its I(X') value. Here I stands for either entropy (equation2) or compactness (equation 5) or Θ (equation 6). The same procedure is repeated for different parameters q of the function f^q_i. It is then checked whether these ambiguity measures possess any valley or not.

Similar checking is adopted for other mapping functions under consideration. In the final stage, a global minimum (valley) is determined. The corresponding mapping function with specific q value can be regrded as optimum enhancement function of the image X. The image $X' = f^q_i(x_{mn})$ thus provides the optimum enhanced output, given a set of functions. The concept of optimum enhancement is explained below.

Optimum Fuzziness and Object Enhancement

Let us consider, for example, an image X consisting of an object in a background. An enhancement transformation (viz..equations 8-11) when applied on X, attempts to enlarge the gaps in levels between two regions and at the same time to reduce the difference in levels within a region. As a result, the μ_{mn} values of the enhanced image would tend to either 0 or 1 ; thus making a decrease in value of H(X). It is to be noted that the above decrease in H(X) (fuziness in grayness) does not ensure proper enhancement of the object. In other words, unless the transformation function is able to discriminate the object geometry (or boundary) from background, the H(X) value even if they decrease will not reflect its meaningful enhancemnt. This leads one to determine the appropriate functional form such that the corresponding enhanced image would result in minimum number of pixels with $\mu_{mn} \simeq 0$ or 1 ; thus contributing least

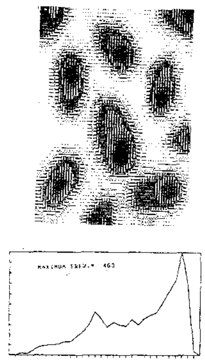


Fig. 5.

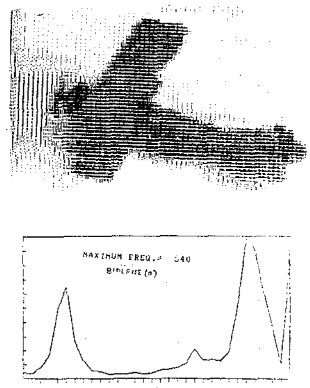


Fig.6.

Figure 5. Cell image and its gray level histogram.

Figure 6. Biplane image and its gray level histogram.

Table 2 : Value of fuzzy measures at valley points, using different mapping functions.

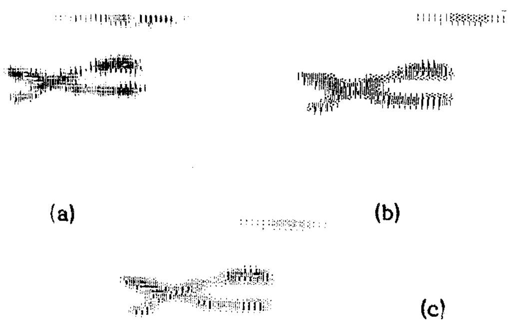
Mapping function	Entropy(H)	Compactness	Product(Θ)
Input image : LINCON			
eq. (8)	No valley	No valley	No valley
eq.(10)	.32	No valley	No valley
eq.(11)	.12	.00355	.00108
eq.(12)	No valley	No valley	No valley
Input image : CHROMOSOME			
eq.(8)	No valley	.022	No valley
eq.(10)	.205	No valley	.0058
eq.(11)	No valley	.081	No valley
eq.(12)	No valley	No valley	No valley
Input image : CELL			
eq.(8)	No valley	No valley	No valley
eq.(10)	.51	.00165	.0017
eq.(11)	.175	No valley	No valley
eq.(12)	No valley	No valley	No valley
Input image : Biplane			
eq.(8)	No valley	.00155	No valley
eq.(10)	No valley	.008	No valley
eq.(11)	.302	.00031	No valley
eq.(12)	No valley	No valley	No valley

towards H(X). This may be treated as optimum value of gray level fuzziness in the sense that the value obtained from any other transformation function will be greater than this.

Similar is the case with the spatial ambiguity mesure comp(X) where modification of enhancement function will result in different μ_{mn} planes with varying 'compactness'. comp(X) will reach a valley (optimum) only when there is an appropriate enhancement of the

(a) Figure 7 (b)

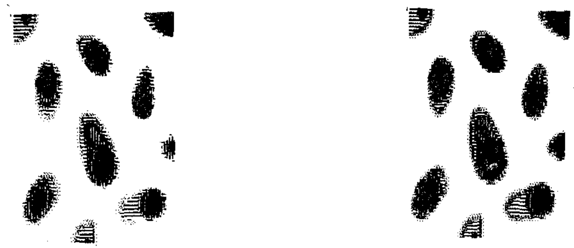
Fig.7. Enhanced (optimal) image output
(a) when equation (10) is used on Fig.3
(b) when equation (11) is used on Fig.3



(a) (b) (c)

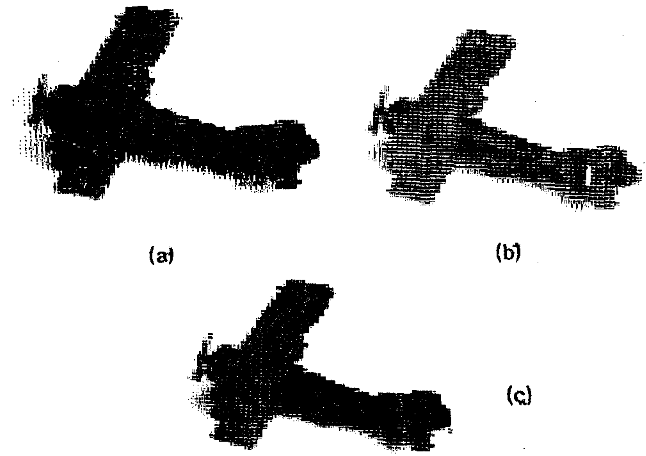
Figure 8

Fig.8. Enhanced (optimal) image output
(a) when equation (8) is used on Fig.4
(b) when equation (10) is used on Fig.4
(c) when equation (11) is used on Fig.4



(a) Figure 9 (b)

Fig.9. Enhanced (optimal) image output
(a) when equation (10) is used on Fig.5
(b) when equation (11) is used on Fig.5



(a) (b) (c)

Figure 10

Fig.10. Enhanced (optimal) image output
(a) when equation (8) is used on Fig.6
(b) when equation (10) is used on Fig.6
(c) when equation (11) is used on Fig.6

object geometry of X. For any other choices of enhancement function, either a part of the object will be treated as background or a part of background will merge with object. In the earlier one, both area and perimeter will decrease but the decrease is more for the denominator ; thus resulting in a value greater than the optimum one. The later case, on the other hand, involves faster increase of the numerator than that of the denominator and gives rise to the same result as the earlier one.

V. Results and discussion

To study effects of various types of enhancement (mapping) function on different types of images, we have considered here 4 different types of images namely, Lincon, Chromosome, Cell and Biplane which have multimodal, highly right skewed, partially skewed and bimodal histograms as shown in Figs.3-6. The different mapping (enhancement) functions indicated by Eqs.(8),(10),(11) and (12) and their varying forms (for different values of parameters) were applied on each image. For every form of the mapping functions the three different fuzzy set theoretic measures such as entropy (H(X)), entropy-compactness product (Θ(X)) and compactness (comp(X)) were calculated using the algorithms described in the Section II. Table 1 shows the values of these measures for input images. Table 2 shows the values of those measures at valley points when different mapping function (Eqs.(8),(10),(11),(12)) are applied on four different input images.

Quantitative selection of a nonlinear function for optimal enhancement of image is explained through fuzzy set theoretic measures. The algorithm does not need iterative visual interation and prior knowledge of image statistics in order to select thefunction. Measures like entropy, compactness and their product have been used so that ambiguity (fuzziness) in both grayness and in geometry of regions in an image can play the role in taking decision. Again, the concept of fuzzy compactness as originated by Rosenfeld [6] was developed considering a single object in an image. Here we have considered single object viz. Biplane and Chromosome, multiple object like Cell and single object with multiple disjoint region such an Lincon to demonstrate its effectiveness.

Functions like Eqs.(10) and (11) are found to provide optimal enhancement for all types of images considered here as far as minimisation of at least one of the fuzzy measures is concerned. For Cell image its optimal enhancement produced by Eq.(10) is reflected by all fuzzy measures (Fig.9a). Similar is the case for Lincoln image when Eq.(11) Fig.(7b) is considered. Visual discrimination of enhancement quality produced by grayness ambiguity minimisation spatial ambiguity minimisation is found to be well characterised by the respective measures.

Since the entropy (Eq.2) depends on the nature of histogram of an image, function like Eq.10 (Fig.2c) was able to produce valley in the behaviour of H measure only for the images of Lincoln. Chromosome and Cell; because the appropriate selection of parameter D made its cross-over point lie around thresholds between object and background. For the same reason, Eq.11 (Fig.2d) has been able to produce valley in H measure for Lincoln, Cell and Biplane images; the Chromosome image having right skewed histogram got discriminated. However, minimising both spatial and grayness ambiguity is a good indication for acceptance of an enhanced image.

For Chromosome image global mimimum among the valleys is obtained using Eq.8 and for Biplane image same is obtained using Eq.11. The camparitive results are shown in Fig.8 and Fig.10 respectively.

Equation 8 showed comparitively inferior performance, because the test images do not have left skewed histogram. Equation 12 increases the ambiguity in grayness of pixels and thus could not produce enhancement of any image. Bound functions [7] are also found to be an indication for discarding enhancement functions.

It is to be mentioned here that four distinct enhancement functions have been considered here in order to cover of the enhancement functions normally used [3].

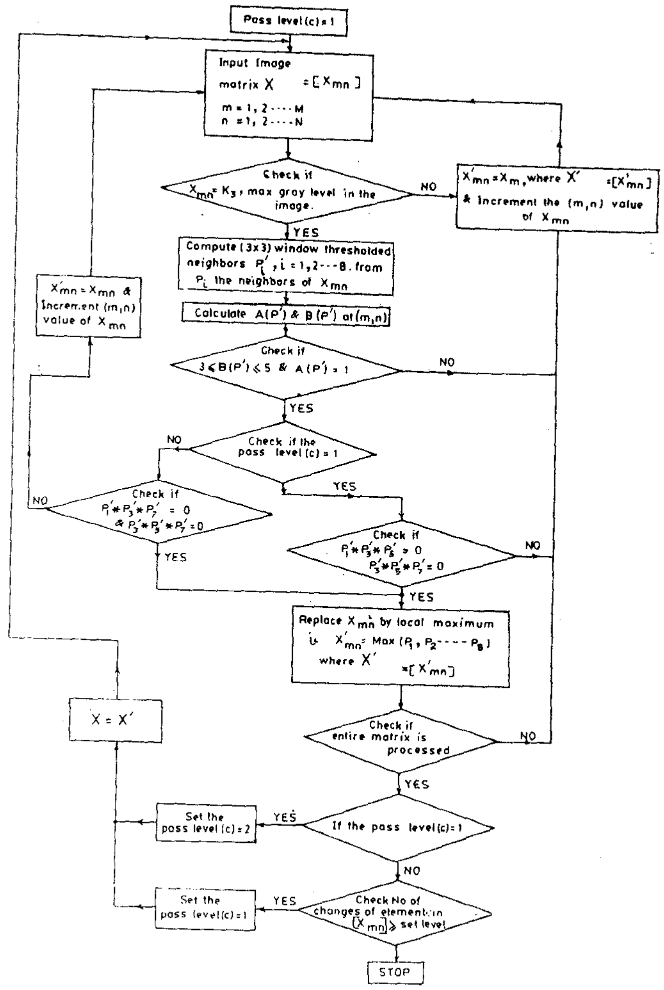


Figure 11. Block diagram of proposed algorithm.

References

1. R.C. Ganzalez and P. Wintz. Digital Image Processing. Addison Wesley. Reading. Mass. 1977.
2. M.P. Ekstrom. Digital Image Processing Techniques, Academic Press. N.Y., 1984.
3. D.C.C. Wang, A.H. Vagnucci and C.C. Li. 'Digital image enhancement : a survey', Computer Vision, Graphics and Image Processing, 24, p.368-381, 1983
4. S.K. Pal and A. Rosenfeld. 'Image enhancement and thresholding by optimization of fuzzy compactness', Pattern Recognition Letters, 7, p.77-86, 1988.
5. L.A. Zadeh. 'Calculas of fuzzy restrictions', in L.A. Zadeh et.al. Eds. Fuzzy sets and their application to cognitive and decision processes. Academic Press. London, p.1-39, 1975.
7. M.K. Kundu and S.K. Pal. 'Automatic selection of object enhancement operator with quantitative justification based on fuzzy set theoretic measures', Pattern Recognition Letters. 11, p.811-829, 1990.