A novel method for image thresholding using interval type-2 fuzzy set and Bat algorithm

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Abstract

In this paper, we propose a novel image thresholding method based on the interval type-2 fuzzy set (IT2FS). The interval type-2 fuzzy membership function (IT2FMF) is generated from a bag of type-1 fuzzy membership functions (T1FMFs) chosen adaptively based on the image characteristics for a given problem. An evolutionary algorithm called Bat algorithm is used to enhance the computational efficiency of the proposed thresholding technique. It is expected that the IT2FS based threshold technique will be better than that of the methods based on type-1 fuzzy set due to superior uncertainty handling capacity of the former technique. This fact is experimentally verified using benchmark dataset. The performance and robustness of the proposed method under different noise corruptions are measured statistically on the dataset by Modified Cramer-Rao Bound. The segmentation performance of the proposed method is compared experimentally with that of the state-of-the-art methods based on fuzzy and non-fuzzy frameworks. It is observed that the proposed method can achieve a higher segmentation accuracy in comparison to other state-of-the-art methods when they are benchmarked against the Modified Cramer-Rao Bound. Also, on average the misclassification error(ME) of the proposed method in segmentation is found to be minimum in comparison to the state-of-the-art methods.

Keywords: Interval type-2 membership function, uncertainty handling, image thresholding, Bat algorithm, Modified Cramer-Rao Bound.

1. Introduction

Image thresholding is one of the most commonly used methods of two class object background segmentations. In a two class image segmentation process based on thresholding, it is expected that the process should be capable of making two distinct regions. The regions should be made in such a way that their within class variation is negligible in comparison to across the region variations. Also, the location of the boundary between the two regions should be as accurate as possible. In order to locate the segmentation boundary accurately, it is necessary that the process should be capable of reducing uncertainty arising due to gray level and spatial ambiguity in an image. The ambiguities occur as a result of digitization with finite steps. The segmentation using thresholding is one of the basic step of image analysis having a wide variety of applications for non-destructive testing, text binarization, medical image processing etc. Various applications of thresholding can be found in [1].

The literature on the approaches used for finding accurate threshold value in a segmentation process is quite rich. Conventional thresholding methods without the uncertainty handling capacity can be classified mainly into two categories, parametric and nonparametric methods[2]. Parametric methods [3, 2] assume a statistical distribution associated with the gray level of each class. On the contrary, nonparametric methods [4, 5, 6, 7, 8, 9, 10, 11] are distribution-free approaches and they determine the optimal threshold by optimizing certain objective function. The non-parametric method by Kundu et al. [12] used human psychovisual phenomena (HVS) for threshold selection.

Both the conventional parametric and non-parametric methods involve crisp decision (i.e yes or no, black and white, 0 and 1 etc) about the object and background regions. They ignore the uncertainties present in a digital image due to gray level and spatial ambiguity in the image. So their performance becomes poor when the boundaries between the regions are not distinct (abrupt change), rather it changes gradually from one region to another (slant change). In view of that, uncertainty handling model based on fuzzy set as introduced by Zadeh [13] becomes a popular tool for segmentation applications. Pal et al. [14, 15] used fuzzy entropy method for thresholding and

image enhancement. Also Ye et al. [16]and Huang et al. [17] proposed the fuzzy entropy based thresholding algorithm where they minimized the entropy to obtain the threshold value. Different methods using the fuzzy set for image thresholding were also proposed in [18]and [19].

In a conventional fuzzy set, which is also called a type-1 fuzzy set, each element can have any membership value between 0 and 1. In the true sense of fuzzy set as a tool for uncertainty handling, the membership value should also be a fuzzy number. Since the type-1 membership values are totally crisp, the type-1 set does not follow the ideal property of a true uncertainty handling mechanism. This problem is the motivation behind the concept of type-2 set as an extension of the type-1 fuzzy set(T2FS) [20]. In a type-2 fuzzy set, the uncertainty at a point is represented by a range of values rather than a single value. The range of values is called the primary memberships of the point. Again, each point in the range of values has its own membership values called the secondary membership. Although type-2 fuzzy sets can give more freedom to the decision-making process, but three-dimensional natures of the type-2 membership functions make them very difficult to compute practically [21, 22]. Due to the complex nature of type-2 fuzzy sets, interval type-2 fuzzy sets(IT2FS) are introduced as a practical extension of general type-2 fuzzy sets, with a uniform secondary membership function [23]. Tizhoosh [24] proposed an image thresholding method using interval type -2 set and a measure of ultrafuzziness. Pagola et al.[25] proposed an approach for estimating the IT2FMF from a group of conventional type-1 membership functions and used it for the image thresholding applications.

Though many researchers used the IT2FMF for image segmentation, in any IT2FMF the main concern is to find the proper intervals associated with it. The researchers estimated the intervals associated with the IT2FMF by various methods. Tizhoosh [24] generated the intervals by blurring a type-1 membership function. Obviously, the accuracy of the IT2FMF depended on the particular membership function i.e on the opinion of the expert. That means, the choice of T1FMF for generating IT2FMF is individualistic or ad-hoc. Again, the blurring operation may lead to disappointing results [26]. On the other hand, Pagola et al. [25] combined the opinion of different experts (i.e different type-1 membership functions) to develop the IT2FMF. Here, the

intervals were generated due to the difference of opinions between the experts.Tahayori et al. [27] used fuzzy information-theoretic kernels for generating the IT2FM from a set of T1MFs. A procedure for generating a set of extended IT2FMFs from data for an interval type-2 linguistic variable was proposed by Liao [28]. Here, the parameters of the IT2FMF were obtained by minimizing the mean squared error (MSE) between the membership values obtained by fuzzy c-means variant(FCMV) [29] and those predicted by the extended IT2MFs. Ideally, for reducing uncertainty in locating the segmentation boundary, one should also consider the statistics of the image at hand for computing the IT2FMF. That means the intervals associated with it should be estimated adaptively. This shot of adaptation for computing the IT2FMF have not been followed by any other researchers including Tizhoosh and Pagola. Rather, they preassumed the intervals independent of the image at hand. Hence, they required opinions of the external experts. This is the motivation behind our current investigation to generate IT2FMF adaptively to find a proper threshold value. This would be based on an accurate and exact estimation of an IT2FMF adaptively from a bag which contains different T1FMFs. The bag of type-1 membership functions is used, as we do not know the suitable function for the particular image at hand. To select the proper set of membership functions efficiently from the bag, we have to used an evolutionary algorithm in the proposed method. Similar methodology can be used for generating the extended IT2MFs proposed by Liao [28] depending on the image at hand for image thresholding application.

Nowadays, apart from genetic algorithm [30], several evolutionary algorithms have been used by researchers for optimization[31, 32, 33, 34]. In the proposed method, we use nature inspired Bat Algorithm (BA), a relatively new stochastic evolutionary algorithm [35] for this search operation. The BA has gained popularity due to its capability of handling the optimization problem more efficiently than other evolutionary algorithms [36, 16]. The BA algorithm has been used in many optimization problem applications and can be found in [37].

The paper is organized as follows. Section 2 describes the proposed method and novelty of the proposed method. Section 3 describes the different type-1 membership functional forms used for generation of IT2FMF and image thresholding. The theory

of interval type-2 fuzzy set and the measure of uncertainty are described in Section 4. The theory of quantitative performance measure and modified Cramer-Rao bound is discussed in Section 5. The procedure for generating interval type-2 fuzzy membership function from a bag of type-1 membership functions are described in Section 6 . The optimization problem and the theory of Bat algorithm are discussed in Section 7. The implementation of an efficient approach to generate the IT2FMF for thresholding are discussed in Section 8. Section 9 describes the results, comparison with state-of-the-art methods and the statistical measure of the performance.

2. Proposed method

Here we propose an image thresholding method based on IT2FS with adaptive estimation of the IT2FMF. The uncertainty in the image is reduced to get the optimal threshold value by minimizing the IT2FS entropy. The exact IT2FMF is obtained when interval type-2 fuzzy entropy produces the minimum value i.e the IT2FS with minimum uncertainty. In our method, for building the IT2FS, the intervals of the IT2FMF are estimated from a combination of a bag of type-1 membership functions depending on the image statistics. These membership functions are generally used for image thresholding, but all the functions are not suitable for a particular image at hand. Naturally, the exact IT2FMF depends on the suitability of the type-1 membership functions and the corresponding parameters associated with them for the image at hand.

The whole operation of generating the IT2FMF for thresholding involves the search of a proper combination of T1FMFs with the corresponding optimal set of parameters. The parameters should be optimal in the sense that they should generate the IT2FMF with minimum uncertainty. The parameters are discrete real variables or integers which are bounded within a range. So, to get an optimal parameter set, we have to search from a large combinations of real or integer values within the range. Thus, the search operation is computationally expensive. To reduce the computational complexity and increase the efficiency of the search operation, we have to use an evolutionary algorithm to find the optimal set of parameters with real or integer values [38]. As the search involves the minimization of uncertainty, the interval type-2 function entropy function

would be the objective function. The schematic diagram of the proposed method is shown in Figure 1.

2.1. Novelty of the proposed method

The novelties of the proposed method are described below

(1) : In a conventional fuzzy set, choice of the membership function is individualistic or ad-hoc. So there may be a quite substantial error in the measure of uncertainties. To reduce the error it is desirable to adopt a mechanism so that the most suitable membership function can automatically be selected. That mechanism is proposed while computing appropriate IT2FMF by automatic means from a bag of type-1 membership functions, depending on the image statistics. This is implemented by proposing a suitable algorithm based on evolutionary Bat algorithm for choosing appropriate T1FMFs with suitable parameter values. The technique greatly reduces the error in uncertainty representation and increases the accuracy of thresholding as evident from the results.

(2) : In the proposed method the IT2FMF is a combination of a bag of T1FMFs. Hence, it also reduces the the uncertainties due to gray level and spatial ambiguities in an image like conventional type-1 fuzzy set.

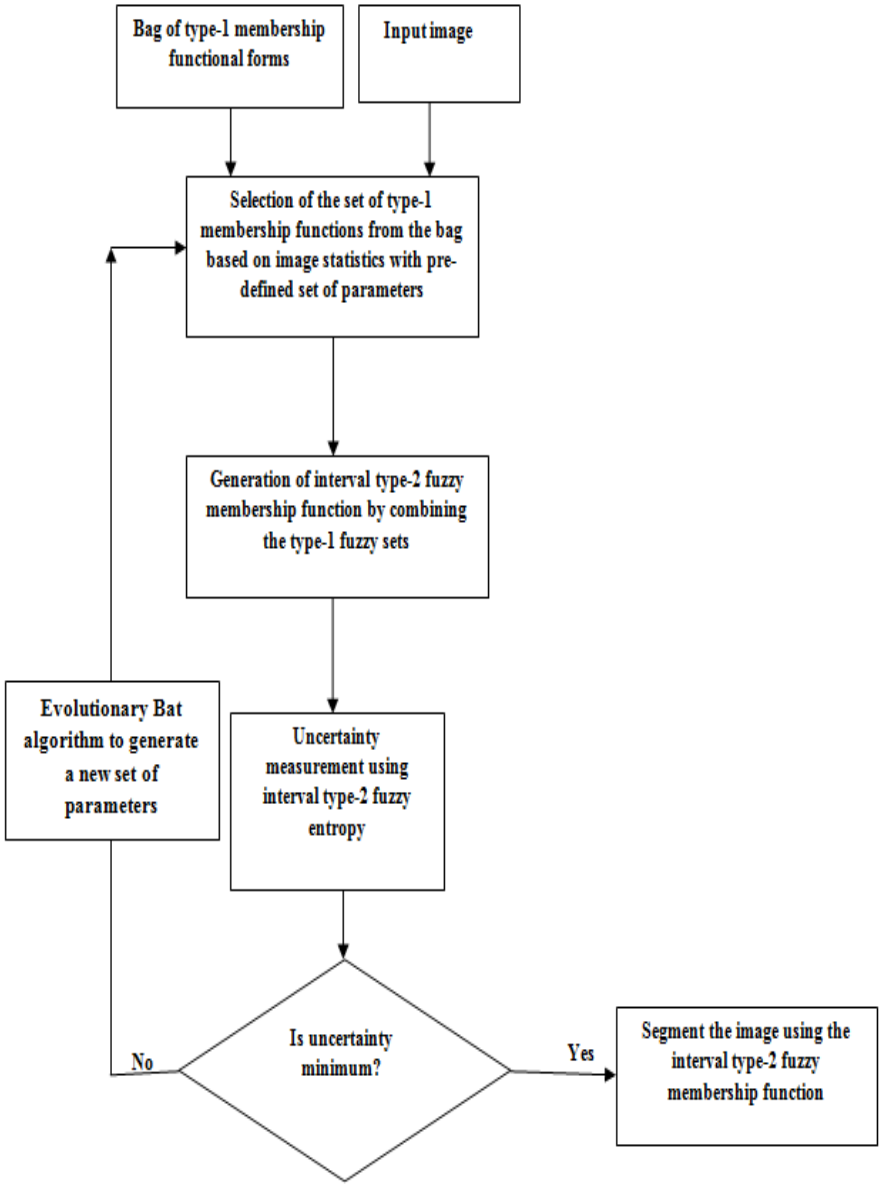

Figure 1: Schematic diagram of the proposed method.

3. Various models of type-1 membership functional forms used for generation of interval type-2 membership functions:

There are several type-1 membership functional forms which are commonly used for generating type-2 membership functions. In the proposed method the bag contains the four functions. These four types of membership functions also segment the object and background of an image. The four functions are described below.

1) Restricted Equivalence function (REF): REF function was proposed by Bustince et al. [39]. A function REF $:[0,1]^2 \rightarrow [0,1]$ is defined as

(1) $REF(x,y) = REF(y,x)$ for all $x,y \in [0,1]$

(2) $REF(x,y) = 1$ if and only if $x = y$

(3) $REF(x,y) = 0$ if and only if $x = 1$ and $y = 0$ or $x = 0$ and $y = 1$

(4) $REF(x,y) = REF(c(x),c(y))$ for all $x,y \in [0,1]$, c being a strong negation

(5) For all $x,y,z \in [0,1]$ if $x \leq y \leq z$,then $REF(x,y) \geq REF(x,z)$ and $REF(y,z) \geq REF(x,z)$

The membership function by REF is

$$\mu_1(q) = \begin{cases} (REF\left(\frac{q}{L-1}, \frac{m_b(t)}{L-1}\right) & \text{if } q \leq t \\ (REF\left(\frac{q}{L-1}, \frac{m_o(t)}{L-1}\right) & \text{if } q > t \end{cases} \tag{1}$$

$REF(x,y) = 1 - |x-y|^{k1}$ Here $m_b(t) = \frac{\sum_{q=0}^{t} qh(q)}{\sum_{q=0}^{t} h(q)}, m_o(t) = \frac{\sum_{q=t+1}^{L} qh(q)}{\sum_{q=t+1}^{L} h(q)}$

The generation of optimal threshold point from the type-1 fuzzy sets of REF is very simple and time efficient [40]. L denotes the level of pixels and h(q) denotes the frequency of the pixel with intensity q and $k1 > 0$. The threshold value is denoted by t.

2) Membership functions proposed by Huang [17]:

$$\mu_2(q) = \begin{cases} \frac{1}{1+(|q-m_b(t)|/C)^{k2}} & \text{if } q \leq t \\ \frac{1}{1+(|q-m_o(t)|/C)^{k2}} & \text{if } q \geq t \end{cases} \tag{2}$$

where $k2 > 0$ and C is the constant and its value is taken as 20 in the proposed method.

3) Membership function used by Tizhoosh : This membership function defined in [24] as

$$\mu_3(q) = \begin{cases} 1 - \left(\frac{q}{t}\right)^{k3} & \text{if } q \leq t \\ 1 - \left(\frac{L-q}{L-t}\right)^{k4} & \text{if } q \geq t \end{cases} \tag{3}$$

where $k3, k4 > 0$.

4) Exponential membership function: The exponential membership function for thresholding is given by

$$\mu_4(q) = \begin{cases} e^{-\left(\frac{|q-m_b(t)|}{L}\right)^{k5}} & \text{if } q \leq t \\ e^{-\left(\frac{|q-m_o(t)|}{L}\right)^{k5}} & \text{if } q \geq t \end{cases} \tag{4}$$

where $k5 > 0$.

The range of the $k1, k2, k3, k4, k5$ are shown in Section 7.1.

All the membership functions above perform the segmentation operation. But, different membership functions produce different membership values for the same intensity value. Hence, different membership values give the different type-1 fuzzy set for the same image. Obviously, each of the type-1 fuzzy set does not produce the desired segmentation result where uncertainty is represented by a single point which depends on a single membership function. Since it is very difficult to decide the exact T1FMF, it is better to define a fuzzy set which is made up from more than one T1FMFs. Hence, we use a bag of functions rather than a single function. Here, the intervals of the IT2FMF are originated due to the difference of the membership values of a pixel by the type-1 membership functions. More the interval, the higher will be the uncertainty. Hence, it represents the uncertainty due to different membership values. Also, each T1FMF value represents the uncertainty present in the image.

In the next two sections, we will discuss the theory of IT2FS and the theory of performance measure of the proposed method.

4. Interval type2 fuzzy sets :

4.1. Interval type-2 fuzzy membership functions

In a type-2 fuzzy set(T2FS), the MF(membership function) is itself fuzzy[20]. A type-2 set denoted by $\widetilde{A}$ is characterized by type-2 membership function $\mu_{\widetilde{A}}(x,u)$, where $x \in X$,i.e

$$\widetilde{A} = \{((x,u), \mu_{\widetilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{5}$$

in which $0 \leq \mu_{\widetilde{A}}(x,u) \leq 1$. $\widetilde{A}$ can also be expressed as

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\widetilde{A}}(x,u)/(x,u), J_x \subseteq [0,1] \tag{6}$$

where $\mu_{\widetilde{A}}(x,u)$ is the secondary membership function and J_x is the primary membership of x which is the domain of secondary membership function. Here $\int\int$ denotes union over all admissible x and u. For discrete universes of discourse $\int$ is denoted by $\sum$. When all $\mu_{\widetilde{A}}(x,u) = 1$ in the Eq.6 ,then $\widetilde{A}$ becomes interval type-2 fuzzy set(IT2FS) [26]. That means in IT2FMF, the secondary grade which is the amplitude of secondary MF becomes one. Uncertainty in the primary membership of an interval type-2 fuzzy set $\widetilde{A}$, consists of a bounded region called the footprint of uncertainty(FOU). It is the union of all primary membership, i.e.

$$FOU(\widetilde{A}) = \bigcup_{x \in X} J_x \tag{7}$$

As secondary grades of IT2FS carry no new information, the FOU is a complete description of an IT2FS. The upper membership function (UMF) and lower membership function(LMF) of $\widetilde{A}$ are two T1FMFs that bound the $FOU(\widetilde{A})$. The UMF denoted by $\overline{\mu}_{\widetilde{A}}(x), \forall x \in X$ is associated with the upper bound of $FOU(\widetilde{A})$ and LMF denoted by $\underline{\mu}_{\widetilde{A}}(x), \forall x \in X$ is associated with the lower bound of $FOU(\widetilde{A})$. Mathematically they can be expressed as

$$\overline{\mu}_{\widetilde{A}}(x) \equiv \overline{FOU(\widetilde{A})} \forall x \in X \tag{8}$$

$$\underline{\mu}_{\widetilde{A}}(x) \equiv \underline{FOU(\widetilde{A})} \forall x \in X \tag{9}$$

In IT2FS J_x can be expressed as

$$J_x = \{(x,u) : u \in [\underline{\mu}_{\widetilde{A}}(x), \overline{\mu}_{\widetilde{A}}(x)]\} \tag{10}$$

and $FOU(\widetilde{A})$ can be expressed as

$$FOU(\widetilde{A}) = \bigcup_{x \in X} [\underline{\mu}_{\widetilde{A}}(x), \overline{\mu}_{\widetilde{A}}(x)] \tag{11}$$

The footprint of uncertainty is very important for an interval type-2 fuzzy membership representation. Shape of FOU not only represents the nature of the uncertainties but

also provides a very verbal description of the entire domain of support for all secondary grades of an interval type-2 membership function.

4.1.1. Uncertainty measure in IT2FS

In IT2FS, the FOU represents the uncertainty present in the set. In the FOU, the lower membership function $\underline{\mu}_{\widetilde{A}}$ and upper membership function $\overline{\mu}_{\widetilde{A}}(x)$ play an important role for uncertainty measure in IT2FS. In this paper, we use the uncertainty measure proposed by Kacpryzk and Smidtz [41]. It is defined as

$$\xi_k(\widetilde{A}) = \frac{1}{N}\sum_{i=1}^{N}\frac{1-max(1-\overline{\mu}_{\widetilde{A}}(x),\underline{\mu}_{\widetilde{A}}(x)))}{1-min(1-\overline{\mu}_{\widetilde{A}}(x),\underline{\mu}_{\widetilde{A}}(x)))} \tag{12}$$

Where N is the cardinality of the fuzzy set.

5. Performance measure of the proposed method

5.1. Misclassification Error(M.E)

The quantitative performance of the proposed thresholding method and all the methods compared here are measured using widely used segmentation evaluation metric called Misclassification error [1] which is given by

$$M.E = 1 - \frac{|T_o \cap T_t| + |G_o \cap G_t|}{|T_o| + |G_o|} \tag{13}$$

where B_o and F_o denotes the background and foreground of the original image(ground truth).B_T and F_T denotes the background and foreground of the tested image. The $|.|$ is the cardinality of the set. M.E nearer to 1 means wrongly classified image and nearer to 0 means correct classification.

Though we measure the performance of all the methods considered here using M.E, one can find in [42, 43] that performance of segmentation evaluation methods varies widely depending on image content. Therefore it is better to find out the statistical performance bound of segmentation methods associated with a particular image content. The bound should indicate the maximum achievable performance of a segmentation algorithm on the particular image. Hence, to statistically bound the performance of

our image thresholding method a modified Cramer-Rao bound(CRB) on Mean Square Error(MSE) for segmentation proposed by Peng et al. [44] was used. In the next subsection, we discuss it.

5.2. The modified Cramer-Rao bound for MSE:

For finding the modified Cramer-Rao bound of the MSE we adopt the procedure in [44] where it is assumed that the output pixel level of a segmentation algorithm is a biased estimate of true pixel level. To determine the MSE bound for the test image, the segmentation problem is first modeled using varying-coefficient model(VCM) [44]. In this model the pixel intensity value y(x), where x are the pixel indices and ordered through zig-scanning, starting from top left to bottom right of an image with N pixels can be represented as

$$y(x) = \mathbf{h}^T.\varphi(x;\mathbf{B}) + w(x) \tag{14}$$

Where T denotes the transpose of matrix. Here pixel intensity y(x) includes the contribution from all the basis regions in an image i.e the regions that cannot be further segmented. The $w(x)$ represents the noise in the image. The $\mathbf{h}(x) = [h_1(x), h_2(x)...h_M(x)]^T$,where an image is divided into M basic regions and $h_i(x)$ is the membership value of x in ith basic region. This membership value can be interpreted as the degree to which the pixel x belongs to the ith basic region $\varphi(x;\mathbf{B}) = [\varphi(x;\beta_1), \varphi(x;\beta_2), .., \varphi(x;\beta_M)]$, $\varphi(x;\beta_1)$ represents the intensity of pixel at ith region and $\mathbf{B} = [\beta_1^T, \beta_2^T, ..\beta_M^T]$.Here $\varphi(x;\beta_i) = \sum_{i=1}^{m} \beta_{il}\zeta_l(x)$ where $\zeta_l(x)$ are B-spline basis functions, β_i is the coefficient vector, m is the number of knots in the image and l is the index of the knots which are ordered through zig-scanning starting from top left to bottom right of an image. The MSE of an estimator $\hat{h}(x)$,which is the output membership value of a segmentation algorithm and biased estimator of the true pixel label $h(x)$ is given by

$$MSE = E(\| \hat{\mathbf{h}} - \mathbf{h} \|^2) \tag{15}$$

The Cramer-Rao bound for biased estimation of MSE [44]for an image is given by

$$CRB_{MSE} = \frac{1}{MN}\sum_{1}^{M} Tr\{(E_{\mathbf{B}}[\mathbf{J}_F(\mathbf{h}_i)])^{-1} - (E_{\mathbf{B}}[\mathbf{J}_F(\mathbf{h}_i)])^{-1}((E_{\mathbf{B}}[J_{\mathbf{F}}(\mathbf{h}_i)])^{-1} + Cov(\mathbf{h}_i))^{-1} ((E_{\mathbf{B}}[\mathbf{J}_F(\mathbf{h}_i)])^{-1}\} \quad (16)$$

Where $Tr(\Gamma)$ represents the trace of the matrix Γ. Here $(E_{\mathbf{B}}[\mathbf{J}_F(\mathbf{h}_i)]$ denotes the expectation of $\mathbf{J}_F(\mathbf{h}_i)$, where $\mathbf{J}_F(\mathbf{h}_i)$ is the Fisher information matrix of $\mathbf{h}_i$ and $Cov(\mathbf{h}_i)$ is the covariance of $\mathbf{h}_i$. Here we assume the estimator of h to be biased and derive the bound on MSE of image thresholding results. The bound provides a valid performance prediction of the image thresholding algorithms and a benchmark of the thresholding results. The N is the number of the pixels in the image.

In the next section we will show the generation of IT2FMF from a bag of type-1 membership functions.

6. Generation of interval type-2 membership function from a bag of type-1 membership functions:

Here we propose a method to generate an IT2FMF from a bag which has $n(n > 1)$ number of type-1 fuzzy membership functions. That means, all the type-1 membership functions in the bag have contributions in making the IT2FS. For this, we follow the same strategy as used by Pagola et al. [25], but with the modification. Mathematically, here the IT2FS is expressed as

$$\widetilde{A} : FS(X) \times FS(X)...\text{n times} :\rightarrow IT2FS(X) \quad (17)$$

Where $FS(X)$ is the type-1 fuzzy set and

$$\widetilde{A}(\mu_1, \mu_2, ..., \mu_n)(x) = \{x, [\underline{\mu}(x), \overline{\mu}(x)] | x \in X\} \quad (18)$$

$$\underline{\mu}(x) = t - norm(w_1\mu_1(x), w_2\mu_2(x), ..., w_n\mu_n(x)) \quad (19)$$

$$\overline{\mu}(x) = t - conorm(w_1\mu_1(x), w_2\mu_1(x), ..., w_n\mu_n(x)) \quad (20)$$

Here we use minimum t-norm and maximum t-conorm. The parameters $w_1, w_2, ..., w_n$ represent the weights of the membership functions under the constraint $\sum_{i=1}^{n} w_i = 1$. The interval $(\overline{\mu}(x) - \underline{\mu}(x))$ at x produces due to the difference between $min(\mu_1(x), \mu_2(x), ..., \mu_n(x))$ and $max(\mu_1(x), \mu_2(x), ..., \mu_n(x))$. This interval represents the uncertainties at the point x. If we consider x to be a pixel intensity of an image then each of the $\mu_i(x)$ for $i = 1, .., n$ represents the uncertainty at x due to the gray level and spatial ambiguity of the image. Again the uncertainty at x due to different $\mu_i(x)$'s produces the interval $[\overline{\mu}(x) \quad \underline{\mu}(x)]$. When the length of the interval is high, uncertainty at the point x is high and vice versa.

In the proposed method we take $n = 4$ i.e the bag contains four type-1 membership functions. Though we use four different types of type-1 membership functions to generate the IT2FMF, all of them are not equally important for a particular image. Since we do not know the exact function which will be suited for a given image, it seems appealing and convenient to provide weights to each membership functions. Thus, four weight parameters w_1, w_2, w_3, w_4 are multiplied with μ_1, μ_2, μ_3 and μ_4 respectively . The weights are to be chosen according to the importance (suitability) of a function for a particular image and the weights depend on the particular image statistics. The combination should enable to generate the exact IT2FMF with minimum uncertainties. That means, the uncertainties will get minimized if the optimal set of parameters related with $\mu_1, \mu_2, \mu_3, \mu_4$, and w_1, w_2, w_3, w_4 are found. When the uncertainties are minimum, the image is segmented into object and background properly. The parameters have discrete integer or real values bounded within a range. So, finding the optimal set of parameters to generate the IT2FMF with minimum uncertainties is time-consuming. Thus, to speed up the process of generating the IT2FMF from the bag of type-1 membership functions, we define an optimization problem. The optimization problem is solved by evolutionary Bat algorithm. In the next section, we will discuss the optimization problem to speed up the generation of the IT2FMF and the Bat algorithm.

7. Optimization problem and evolutionary Bat algorithm(BA)

7.1. Optimization problem

The optimization problem for generating the exact IT2FMF by the Eq(19) and Eq(20) with parameter set $p = (k_1, k_2, k_3, k_4, k_5, t, w_1, w_2, w_3, w_4)$ can be written as

$$\begin{cases} Minimize : \xi_k(\widetilde{A}(x)) \text{ where } \widetilde{A}(x) \text{ is an IT2FMF and it depends on parameter set p,} \\ Subject\,to : \ 0.40 \le k_1 \le 0.50,\ 2 \le k_2, k_3, k_4 \le 3, 1 \le k_5 \le 2,\ q_{min} \le t \le q_{max} \\ \qquad and \sum_{i=1}^{4} w_i = 1 \quad \text{where } 0 \le w_1, w_2, w_3, w_4 \le 1 \end{cases}$$

where the q_{min} and q_{max} are minimum and maximum gray level intensity of the image.

In our proposed method we use the BA to solve the optimization problem. It is to be noted that though we choose the four functions mentioned in Section3, the proposed method can be applied to any other bag which contains any number of type-1 membership functions ($n > 1$) and any set of type-1 membership functions. But, as more number of functions makes the procedure computationally more expensive and less number of functions may not represent the uncertainties properly, we are restricted to $n = 4$. In the following section, we will discuss the theory of BA in determining an optimal set of parameters.

7.2. Bat algorithm

7.2.1. Basic steps of Bat algorithm

Yang [35] invented a swarm intelligence optimization algorithm through simulating the interesting behavior of a bat, which combines major advantages of simulated annealing and particle swarm optimization. Three generalized rules are used for implementing the bat algorithm:

1) All bats use echolocation to sense distance, and they also know the difference between food/prey and background barriers. In echolocation system, bats emit high frequency and listen to returning echoes. The differences between the emitted frequencies and the echoed frequencies are used to determine the distance and location of the prey.

2) Bats fly randomly with velocity at a position with a fixed frequency, varying wavelength and loudness to search for prey. They can automatically adjust the wavelength

of their emitted pulses and adjust the rate of pulse emission depending on the proximity of the target.

3) Although the loudness can vary in many ways, here it is assumed that the loudness varies from a large (positive) to a minimum constant value.

Main steps of the algorithm [45] are given below

1) Initialization;

Repeat

2) Generation of new solutions;

3) Local searching;

4) Generation of a new solution by flying randomly;

5) Finding the current best solution; until requirements are met.

7.2.2. Movements of virtual Bats

In BA algorithm, initialization of the bat population is performed randomly. Generating new solutions are performed by moving virtual bats according to the following equations:

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{21}$$

$$v_i^t = v_i^{t-1} + (p_i^t - p^*) f_i \tag{22}$$

$$p_i^t = p_i^{t-1} + v_i^t \tag{23}$$

Where v_i^t and p_i^t represents the velocity and position of the ith bat at an iteration t. The frequency of the bat is given by f_i. Initially, each bat is randomly assigned a frequency f which is drawn uniformly from $[f_{min} \quad f_{max}]$. The $\beta \in [0 \quad 1]$ is a random value drawn from a uniform distribution. Here p^* is the current global best location (solution) which is located after comparing all the solutions among all the bats. After the selection of a solution from the current best solutions, a new solution for each bat is generated locally using a random walk according to the equation:

$$p_{new} = p_{old} + \eta A^t \tag{24}$$

Where $\eta \in [-1 \quad 1]$ is a random number subject to Gaussian distribution. The η acts as a step size to generate new location (i.e new solution). The A^t is the average loudness of all the best solutions at this time step. The p_{old} is obtained from Eq23.

In the bat algorithm the loudness A_i and pulse rate emission r_i are updated at each iteration t. The loudness decreases when the bat founds the prey and the corresponding pulse emission increases. The loudness A_i and the rate of pulse emission r_i change according to the following equations.

$$A_i^{t+1} = \alpha A_i^t \qquad r_i^{t+1} = r_i^0[1 - e^{-\gamma t}] \tag{25}$$

Where α and γ are constants and $0 < \alpha < 1$, $\gamma > 0$. When $t \to \infty$, $A_i^t \to 0$ and $r_i^t \to 0$. The α is similar to the cooling factor of a cooling schedule of a simulated annealing. The loudness and emission rate are updated only when the new solution is obtained, that means the bats move towards the optimal solution.

The update of the velocities and positions of bats have some similarity to the procedure in the standard particle swarm optimization as f_i controls the pace and range of the movement of the swarming particles [35]. If we replace the variations of the frequency f_i by a random parameter and set $A_i = 0$ and $r_i = 1$, the bat algorithm essentially becomes the standard Particle Swarm Optimization (PSO) [35]. So, it can be said that BA is a generalized version of PSO and a balanced combination of the standard particle swarm optimization and simulated annealing.

8. Implementation

We have already stated that we used the BA to implement the generation of the IT2FMF from a bag of T1FMF depending on the image characteristics and define the optimization problem in Section 7.1. In the following subsections, we will discuss the parameters of the BA and the implementation of BA in determining the optimal set of parameters to generate the exact IT2FMF.

8.1. Representation of the parameters and domain of the parameters:

In BA each bat p_i, $i = 1, 2, ..., n$ in the bat population is represented by a string of parameters. Here in our method we have 10 parameters $(k_1, k_2, k_3, k_4, k_5, t, w_1, w_2, w_3, w_4)$

for the generating the IT2FMF. The domains of the ten parameters are shown in Section 7.1. Though an optimal solution in any evolutionary algorithm is obtained by an infinite number of population and an infinite number of generations, here for the practical purpose we restricted to the initial bat population as 300 and the total number of iterations as 30.

8.2. Reduction of the randomness of the parameters

The uniform distribution is assumed when generating the initial population of the bats randomly. The parameters are bounded by some values (shown in Section 7.1). The values of the parameters (i.e the bat) outside the bound are the infeasible solution. But, the bound violation may occur due to the randomness of the parameters related to new solution generation, and random walk operation of the bats. To handle the randomness we use the strategy of repairing the bound violated parameters by Liao [38]. In this strategy, the parameter is repaired either by taking the bound value that is being violated or replacing it with a newly generated value within the bound.

8.3. Evaluation function:

We need an evaluation function to quantify the desired image thresholding result. The present algorithm considers the evaluation function as the fitness function of BA. Here use the interval type-2 fuzzy entropy given in the Eq.12 as an evaluation function. The entropy of an image X considers the global information and provides an average amount of fuzziness in the grayness of X, i.e., the degree of difficulty (ambiguity) in deciding whether a pixel would be treated as an object (dark) or a background (white). Therefore the concept of minimization of these ambiguity measures may be considered as the basis of a fitness (evaluation) function. The objective of the BA is to construct the exact IT2FMF with least entropy. So the entropy is used as the fitness function, where BA will try to find out the minimum value of it by tuning the parameters.

8.4. An efficient approach for estimating the IT2FMF using Bat Algorithm :

In our proposed method, the input image is first mapped into the type-1 fuzzy sets by a bag of T1FMFs with a set of parameters. The weights are assigned to the T1FMFs

Algorithm 1: Algorithm for efficient approach for estimating IT2FMF using Bat algorithm

Input: Image of dimensions $R \times S$, Four fuzzy type-1 membership functions $\mu_j, j = 1,2,3,4$

Output: The IT2FMF with footprint of uncertainty and the segmentation result

Initialisation:Initialize the population of P bats i.e the set of position vectors $p_i = (k_1, k_2, k_3, k_4, k_5, t, w_1, w_2, w_3, w_4)_i$. Also initialize the frequency f_i, the velocity v_i, the loudness A_i and pulse emission rate r_i where $i = 1,2,..,P$. The maximum number of iterations is M1 and iteration number $iter = 0$.

repeat
  for each bat p_i in the current population **do**
    generate new solution i.e the new parameter vector by updating f_i, v_i and p_i with the help of p^* (Eq21, Eq22 and Eq23);
    if ($rand > r_i$) **then**
      select a local solution around the best solution. Here *rand* is a random variable;
    end if
    Generate a new solution p_{inew} by flying randomly (Eq24);
    Handle the bound violation by the parameters and map the image into T1FSs using p_{inew} and generate the IT2FMF from the T1FMFs $\mu_j, j = 1,2,3,4$ by t-norm and t-conorm.(Eq19 and Eq20);
    Calculate the evaluation function interval type-2 fuzzy entropy ξ_k.(Eq 12);
    if ($rand < A_i$ and $\xi_k(p_{inew}) < \xi_k(p^*)$) **then**
      Update the solution as $p_i = p_{inew}$ with increase of r_i and the decrease of A_i;
    end if
  end for
  $iter = iter + 1$;
  Find the current best solution p^* with minimum entropy among all the bats in the current population;
until $iter = M1$ or no change in evaluation function
Return the IT2FMF, the FOU and the segmentation results by the parameter set of the best solution;

on the basis of the image statistics. The type-1 fuzzy sets are combined using t-norm and t-conorm operations. The BA helps to speed the total process to find out the optimal combination which produces minimum interval type-2 fuzzy entropy. Thus the IT2FMF maps the image pixels to IT2FS with least entropy and segments the image into object and background.

In the proposed method, the BA tries to optimize the position of the virtual bat such that it is nearer to the optimal solution (position). For this, the position, the velocity, the frequency, the loudness and the pulse emission rate of bats in the population are initialized first. Each bat position p_i is a vector of the 10 parameters described in Section 7.1. For each position, the IT2FMF is produced. Then, the best position is found from the bat population. Here, the best position is the position with minimum interval type-2 fuzzy entropy. It is followed by the process of making of the new solutions (positions) for each bat, which are generated by adjusting the frequency, the velocity and using the current best solution. The loudness and the pulse emission rate of each bat are also updated. The process is repeated until it reaches the termination criteria. The current best solution represents the optimal set of parameters of type-1 membership functions to generate the IT2FMF. The step by step procedure is described in Algorithm 1.

9. Experimental results & discussion:

To illustrate the performance of the proposed method we tested our method on different types of images. We applied our method on the benchmark dataset of the camera captured natural images from Weizmann [46],and Berkley [47]. Several nondestructive tests(NDT) images from Mehmetsezgin [48] which were described in [1] were also used for the experiment. The size of the images varies from 200×200 to 500×500. We compared the performance of proposed method both qualitatively and quantitatively with non-fuzzy statistical methods like Kapur [7], Otsu [4], Cai [5]; fuzzy type-1 entropy based method like Ye [16] and Hung [17] and interval type2 fuzzy methods of Pagola [25] and Tizhoosh [24]. All the methods were performed using MATLAB2013 with 4GB RAM.

9.1. Quantitative estimation and accuracy of the proposed thresholding method

Table 1: Performance of different methods on 25 NDT images from [48]

Methods	*Cai* [5]	*Kapur* [7]	*Otsu* [4]	*Huang* [17]	*Ye* [16]	*Tizhoosh* [24]	*Pagola* [25]	*Proposed*
Average M.E	0.1132	.2042	0.2354	0.1969	0.0514	0.1418	0.1273	0.0480
Average Rank	3.73	5.82	6.37	5.402	2.77	4.95	4.73	2.23

For quantitative performance measure, we used NDT data set of 25 images from [48]. NDT means to detect an object and quantify its possible defects without harmful effects on it by special equipment and methods. An effective way to extract the regions is to find out proper threshold value of the image. For quantitative performance measure, we took the average performance of 30 runs of the proposed Algorithm1. The average M.E(Eq13) and the average rank of the methods are shown in Table 1. For an image, the method with the lowest M.E was given rank 1 and all other ranks were calculated accordingly.

We also applied the eight methods on all the images of the data set [46]. The dataset contains 100 dissimilar natural scene images for two class segmentation. In Figure 2, the box plot diagram summarizes the Misclassification Error statistics of individual thresholding methods applied on the 100 images from[46]. It can be seen that the lowest median was achieved by the proposed method followed by Pagola and Cai. Also, the number of outlines of the proposed method was less compared to the other methods. The interquartile range was also quite low for the proposed method.

It is observed from the table 1 and box plot in Figure2 that the quantitative performance of the proposed method, on average, was much better both on NDT and camera captured natural images, than that of the other seven methods compared here. The observation can be explained in the following way. The high uncertainties exist in the NDT images as their boundaries are not well defined. Similarly uncertainties present in camera captured natural scene due to digitization from analog domain. But the conventional nonparametric methods (Kapur, Otsu, and Cai) have no uncertainty handling capacities. So the hard decision about a pixel by the conventional methods

failed to produce the desired performance. Again, the fuzzy type-1 membership functions of Ye and Huang could not represent the uncertainty properly with their crisp membership values. Also, the choice of the membership functions of Huang was individualistic, and it did not take into account the characteristics of the image at hand. So, there were high chances of error in the representation of uncertainties. Though Ye used Bat algorithm to find the proper type-1 membership function, the method lacked the proper representation of uncertainty. Therefore, the method performed poorly on the NDT images and the wide variety of images from [46]. On the contrary, the proposed method has the uncertainty handling capacity of IT2FS based on the image statistics. The importance of a T1FMF to estimate the IT2FMF for the image was determined automatically by tuning the weight parameters. It is to be noted no prior knowledge was provided about the image to estimate the IT2FMF. Tizhoosh used IT2FS for uncertainty handling, but the IT2FMF was made by blurring a single type-1 membership function and the intervals were pre-assumed. So the interval type-2 membership value of the Tizhoosh mainly depended upon the corresponding type-1 membership function. So, his type-2 membership function might not represent the uncertainties properly for all types of images considered here. The IT2FMF used by Pagola was made from a set of type-1 membership functions, but no criteria were given to select them for an image at hand. So he ignored the image characteristics to generate the IT2FMF. Since in our method the IT2FMF were computed adaptively based on the image statistics, the proposed method outperformed both the Tizoosh and Pagola's method.

9.2. Statistical validation by the modified Cramer-Rao bound for MSE

To show the robustness of the proposed method under different noise corruptions, we measured the lower bound for MSE on the NDT image dataset. Here we assumed that each image included two non overlapping basic regions ($M = 2$), one was object region and other was background region. The two regions were obtained from the ground truth for the image provided in the dataset. The B-spline function used here was the cubic B-spline. The knots were deployed at every 8 pixels in both horizontal and vertical directions. White Gaussian noise was added to an image with zero mean

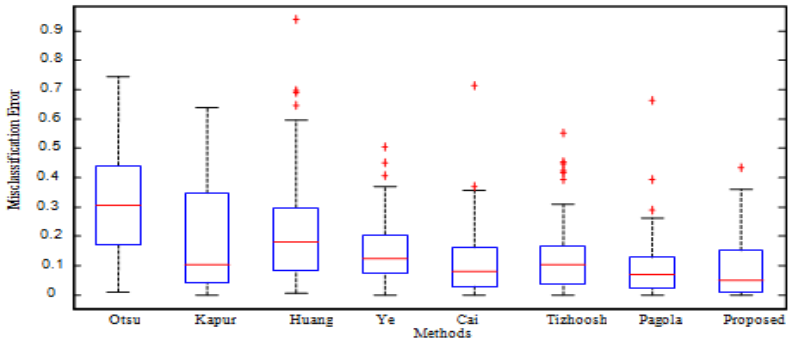

Figure 2: Box plots of Misclassification Errors of different methods on the data set [46].

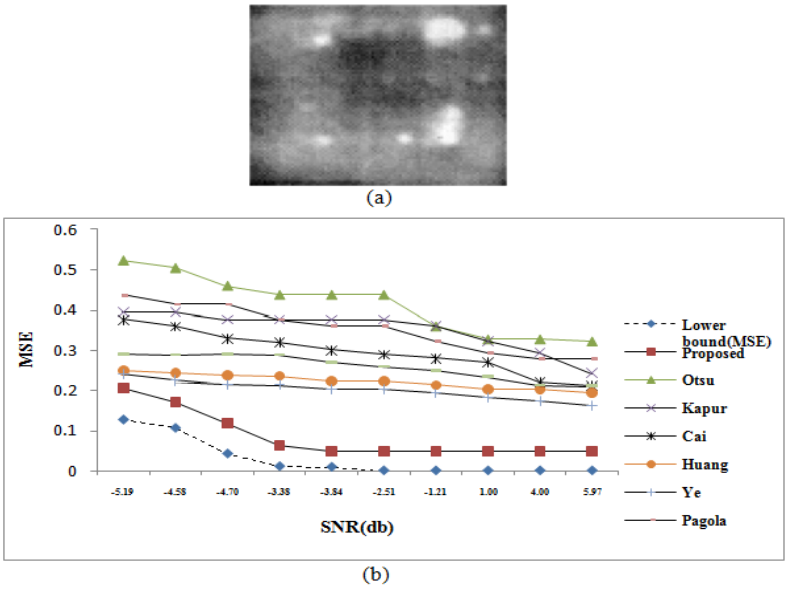

Figure 3: (a) Original NDT test image from [48].(b) MSEs of various segmentation methods on the image at different SNRs and modified Cramer-Rao lower bound of MSE for the image.

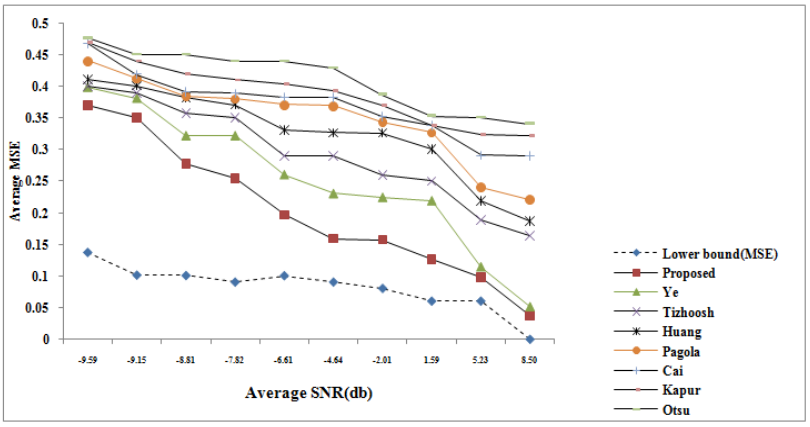

Figure 4: Average lower MSE bound and average MSEs of various methods at different average SNRs on the NDT images from [48] .

and variance σ_2. This procedure was used for all the 25 NDT data in the dataset. The MSE bound of a sample test image for the proposed method and all the methods compared here is shown in Figure 3. The lower bound indicates the maximum achievable segmentation performance for the particular image in terms of MSE. The average lower bound of MSE and average individual MSE at different SNRs are shown in Figure 4. From the result, it can be seen that the proposed method yielded better results in terms of smaller MSE, especially in the relatively higher SNRs followed by Ye. This measure also indicates that the proposed method was more robust against the addition of noise compared to the other methods used here. The uncertainty increases in an image due to the addition of noise and the noise also changes the statistics of the image. As, in the proposed method the uncertainty was handled by adaptive IT2FS, the IT2FMF changed accordingly depending on the image statistics. So the method was more robust against the noise than that of the other methods here. Also, the limitation of the proposed method and room for improvement against each SNR can also be seen from the graph.

9.3. Qualitative validation

In this subsection, we report the qualitative results to validate the quantitative estimations. For a test image, we show one qualitative result among the 30 runs of the proposed Algorithm1. The qualitative result is the one which produced the minimum M.E among the 30 runs of the algorithm. The intermediate and final interval type-2 membership functions (after 10,15 and 20 generations), corresponding upper membership function, lower membership function, FOU(the black portion) and segmentation result for a sample image is shown in Figure 5. The figure shows how the BA gradually made the IT2FMF function. The FOU for each image represents the exact IT2FMF of the image with least entropy generated by our method at the corresponding generations. The results also show the successive improvement of the segmentation results. So, it can be said that the BA helped to reduce the uncertainties by generating the exact IT2FMF. Some more results of segmentation by the proposed method, the exact IT2FMF, histogram of the corresponding images and position of threshold point in the histogram are shown in Figure6. It is clear from the image histograms that the valleys were difficult to estimate. So it was difficult to locate the exact segmentation boundary, but the visual inspection shows that our proposed method located the boundary properly. The segmentation results by the proposed method on an another set of images are shown in Figure 7. The qualitative segmentation results of the NDT images by eight different methods (Kapur [7],Otsu [4], Cai [5], Ye [16],Huang [17], Pagola [25], Tizhoosh [24] and the proposed) are shown in the Figure 8. So, the results validated the powerful uncertainty handling capacity of the IT2FS. Moreover, the uncertainties arose due to the ambiguities present in a digital image and due to the opinion of different experts (i.e different membership functions) were reduced by the proposed method efficiently and adaptively. This shot of adaptation produced the proper threshold value which was also the segmentation boundary between object and background. In the next section, we tested the proposed method on the blurred images.

9.4. Result on blurred images

To verify the robustness of the proposed method, we have applied our method on the blurred images. The images from [47], [48] and some images collected from the

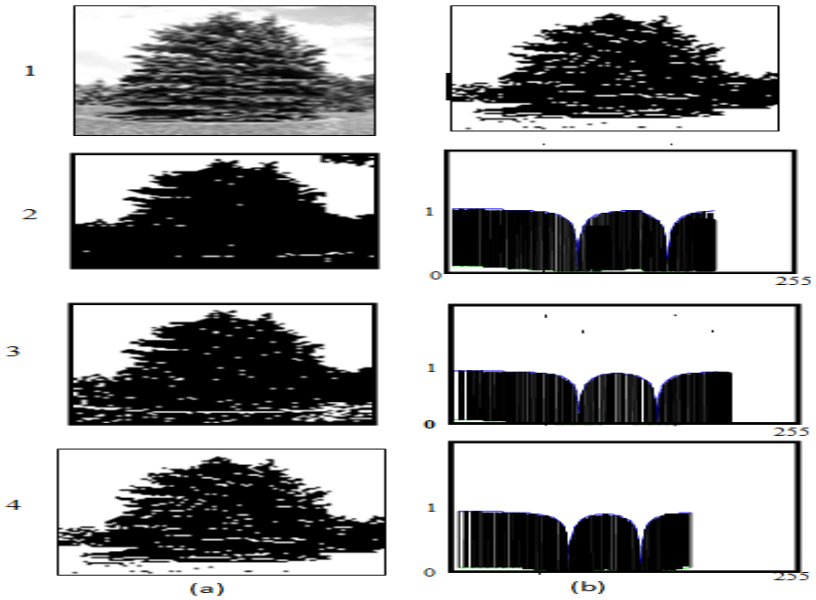

Figure 5: Row wise (1) Original test image(first column) from [46] and the corresponding ground truth (second column), (2),(3) and (4) are the segmentation results(first column) after 10, 15 and 20 generations and the corresponding IT2FMF (second column).

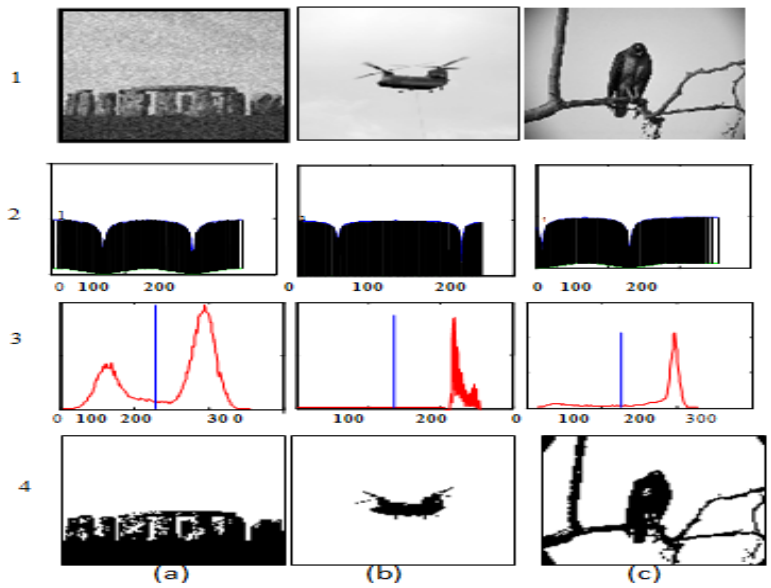

Figure 6: Row wise (1) Original test image (2)Exact IT2FMF of the segmented image (3)Histogram of the image with the position of threshold value.(4)Result using proposed method.

Figure 7: Some additional results by the proposed method. Row wise (1) Original test images (2) segmentation results by the proposed method

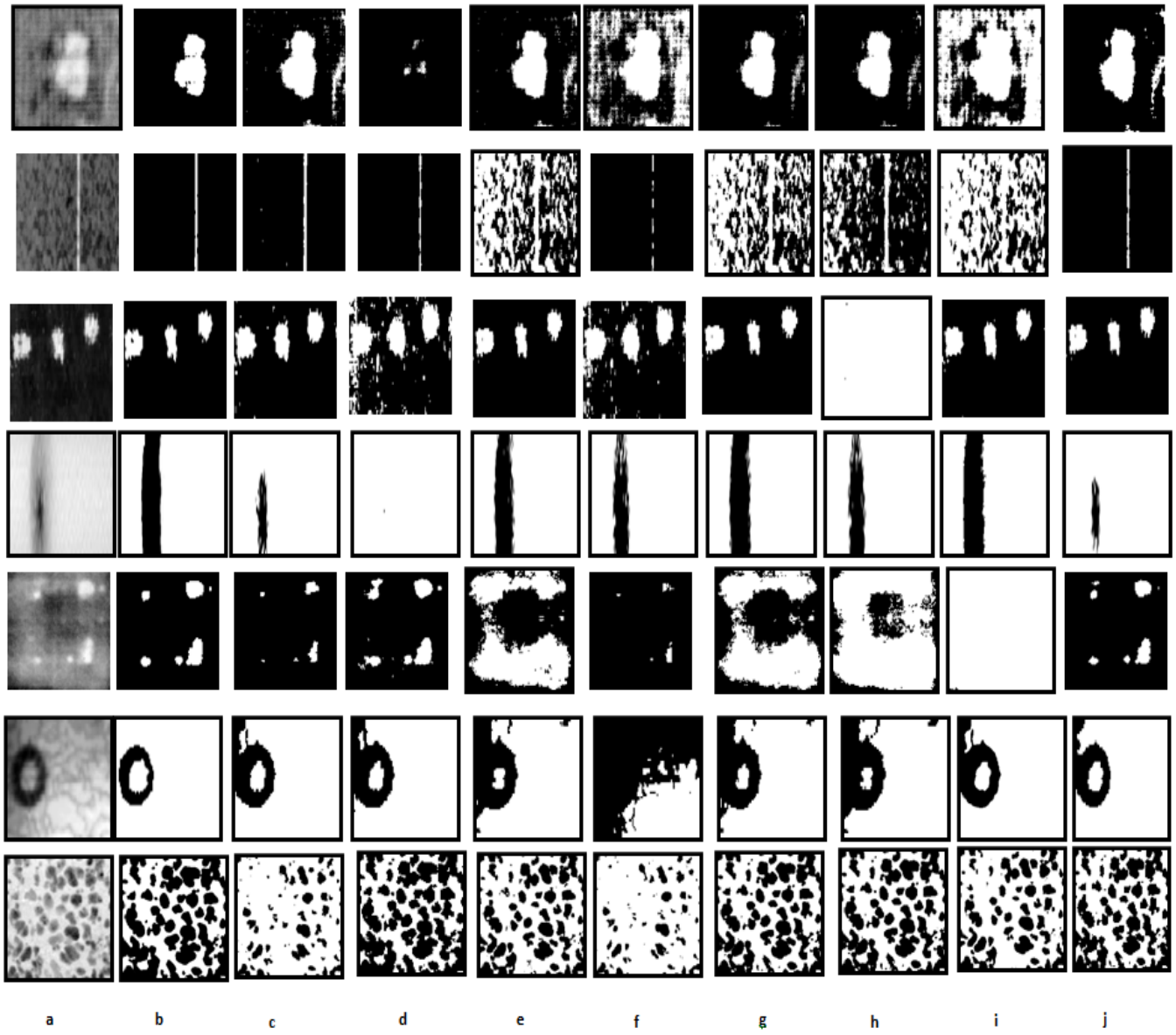

Figure 8: Column wise (a) Original NDT test image from [48] (b)Ground truth for segmentation result (c)Result using Kapur [7] (d) Result using Tizhoosh [24] (e) Results using Otsu [4] (f)Results using Huang [17] (g)Results using Ye [16] (h)Results using Pagola [25] (i)Results using Cai [5] (j)Results using proposed method

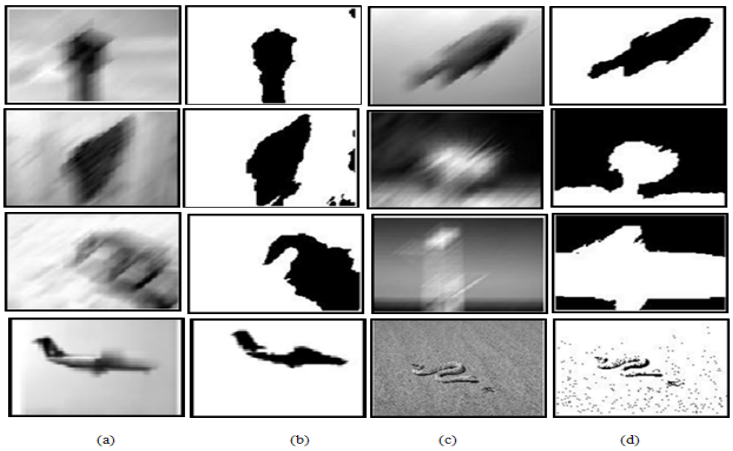

Figure 9: Segmentation results by the proposed method on the blurred images by Gaussian filter($\sigma = 4$). Column wise (a) Blurred test images (b) Segmentation by the proposed method. (c) Another set of blurred images (d) Segmentation by the proposed method

publicly available web sites were blurred using a Gaussian filter. The reason is that in a blurred image, the uncertainties increased. So, locating the segmentation boundary accurately became a difficult task. In Figure 9 some segmentation results on the blurred images are shown. The visual inspection shows that in almost all the blurred images, the segmentation boundary became exact. The reason is that, though uncertainties were increased in a blurred image, the adaptive uncertainty reduction with the changed statistics of the image helped to detect the boundaries properly.

9.5. Comparison of BA with Genetic algorithm and Particle Swam Optimization(PSO)

In this subsection we compared the BA with genetic algorithm(elitist model) [30] and an updated version of standard PSO called the MeanPSO [34]. To compare with the algorithms, we applied them on the same bag of type-1 membership functions to generate the IT2FMF of same image and calculated the threshold value. For genetic algorithms, we used the the mutation probability $p_m = 0.05$ and crossover probability of $p_c = 0.95$. For MeanPSO, we used the acceleration coefficients $c1 = c2 = 2$ and the inertia function $I = 1$. The quantitative result of each method was measured by taking the average of 30 runs. The quantitative results are shown in Table 2. From the result it

Table 2: Comparison with GA and MeanPSO

Methods	*Dataset*	*M.E*
$GA(Elitistmodel)$	*NDTimages*	0.0611
MeanPSO		0.0561
BA		0.0480
$GA(Elitistmodel)$	*Weizmann*	0.0600
MeanPSO		0.0543
BA		0.0434

can be seen that for generating the IT2FMF for segmentation, the BA is more efficient than that of GA and MeanPSO. The BA is superior to GA due to the fact that the BA takes into account the advantages of PSO and simulated annealing. Moreover, PSO is a special case of BA under appropriate simplifications.

10. Conclusions

In this paper, we have proposed an image thresholding method based on IT2FMF. The method generates the IT2FMF using a bag of type-1 membership functions depending on the statistics of the image at hand. The evolutionary Bat algorithm is used to judiciously find the optimal set of the membership functions from the bag and combine the different parameters of them to speed up the process. The method does not need any prior knowledge about the image and it does not require any external expert to select the membership functions. The IT2FMF generated in this way to achieve the object background segmentation result with higher accuracy in comparison to many existing and state-of-the-art techniques reported in the literature. It is to be noted the proposed method performed equally well for different types of images taken from standard datasets. The method also showed robustness under the moderate noise corruptions compared to other methods. It is verified by the modified Cramer-Rao bound for the MSE. Moreover, the method performed well on the blurred image with uncertain boundaries. With suitable modification in the current method, it can be extended for the multiclass problem, which is being currently investigated.

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Highlights

- **The method segments an image into object and background using interval type-2 fuzzy set.**
- **The interval type-2 fuzzy membership function is adaptively generated from a bag of type-1 membership function depending on the image statistics.**
- **Evolutionary Bat algorithm is used to increase the computational efficiency of the method.**

Graphical abstract