Using MILP in Analysis of Feistel Structures

and Improving Type II GFS by Switching Mechanism

Mahdi Sajadieh and Mohammad Vaziri

Indocrypt 2018

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Background

- Feistel structures form a significant category of block ciphers.

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- Two methods for improving the immunity of Feistel structures:
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GFS Structures

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- Relation

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$$(X_0, X_1, ..., X_{l-1}) \rightarrow \pi(X_0, F_0(X_0) \oplus X_1, X_2, F_1(X_2) \oplus X_3, ..., F_{l/2-1}(X_{l-2}) \oplus X_{l-1})$$

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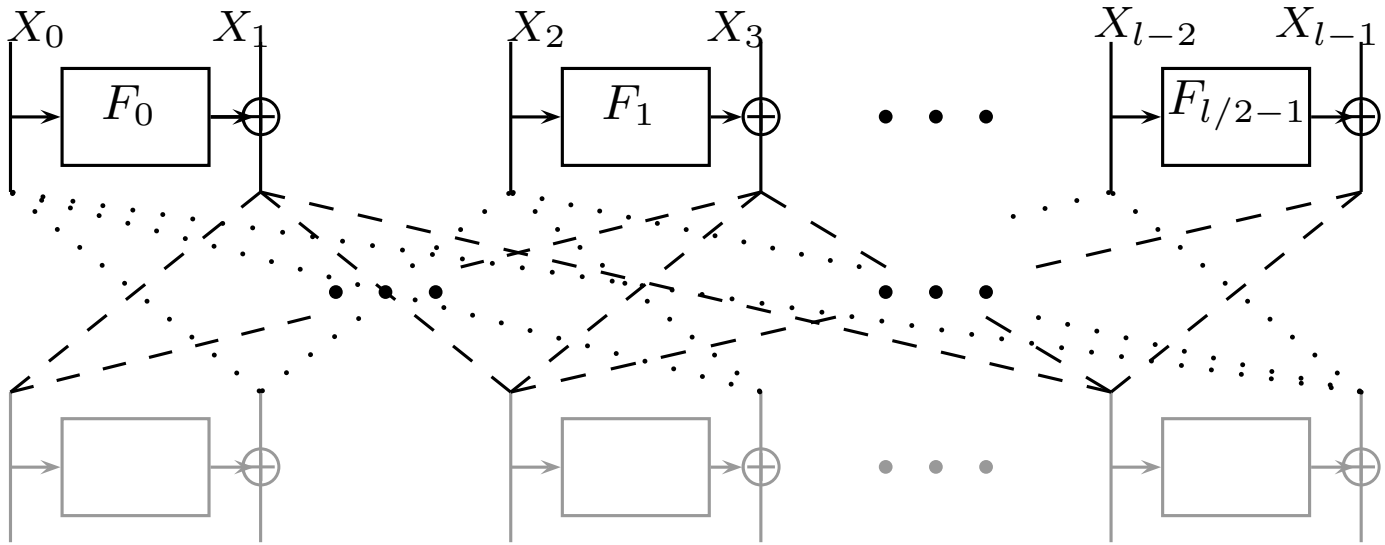

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$$(X_0, X_1, ..., X_{l-1}) \rightarrow \pi(X_0, F_0(X_0) \oplus X_1, X_2, F_1(X_2) \oplus X_3, ..., F_{l/2-1}(X_{l-2}) \oplus X_{l-1})$$

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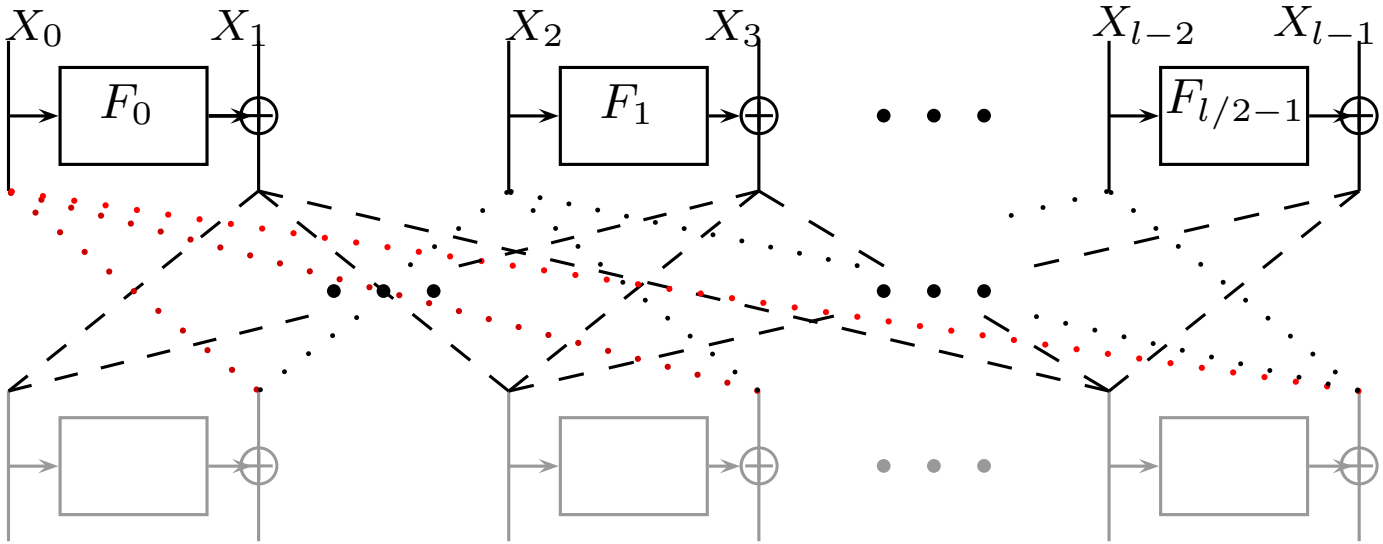

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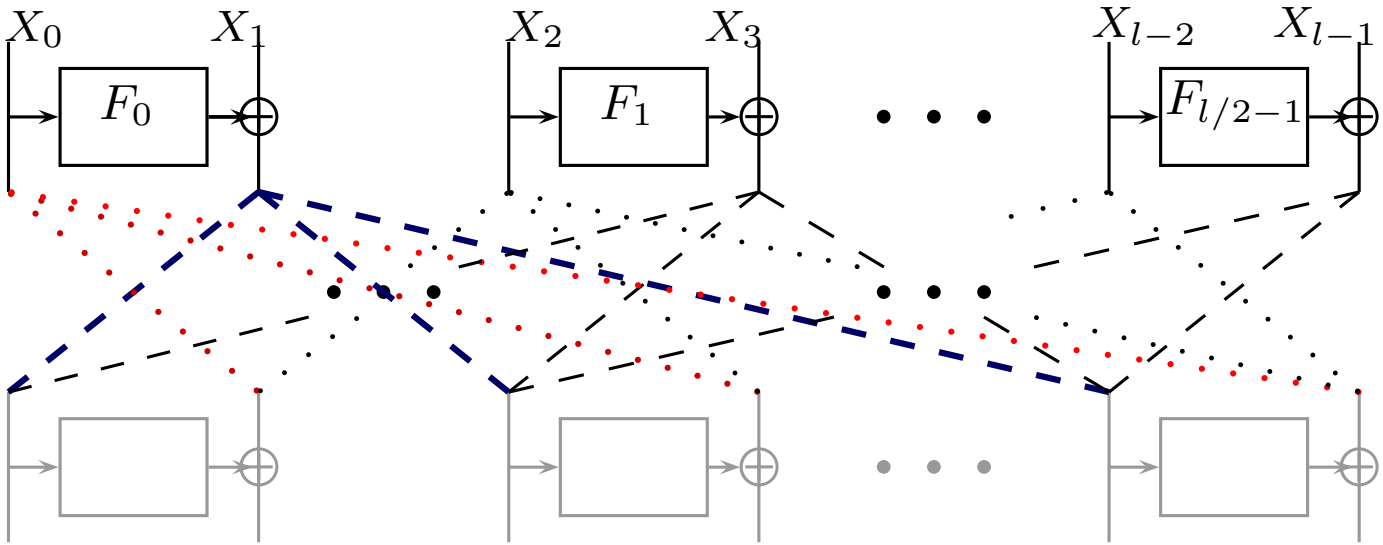
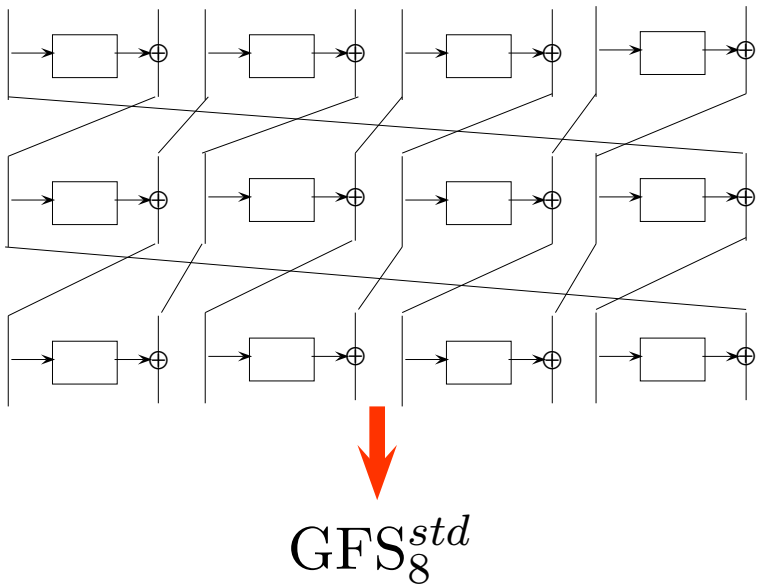
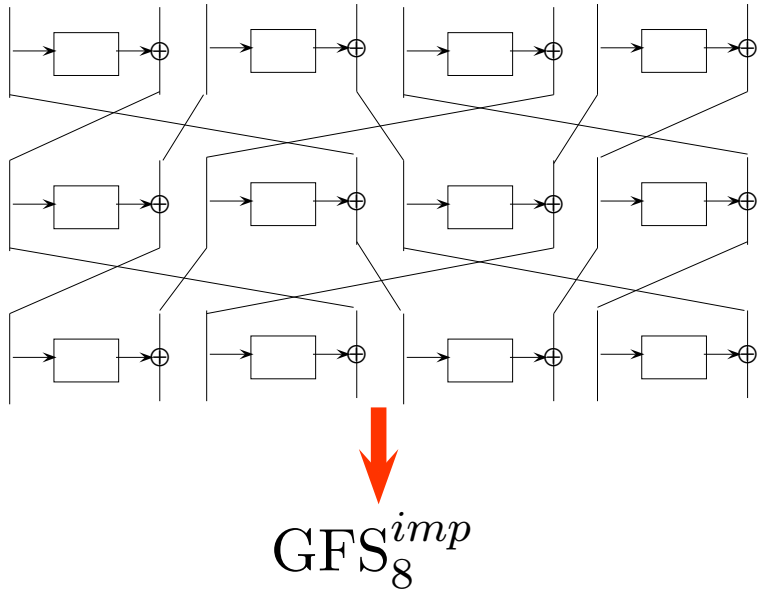


GFS Structures

- Figures of GFS_8^{std} and GFS_8^{imp}





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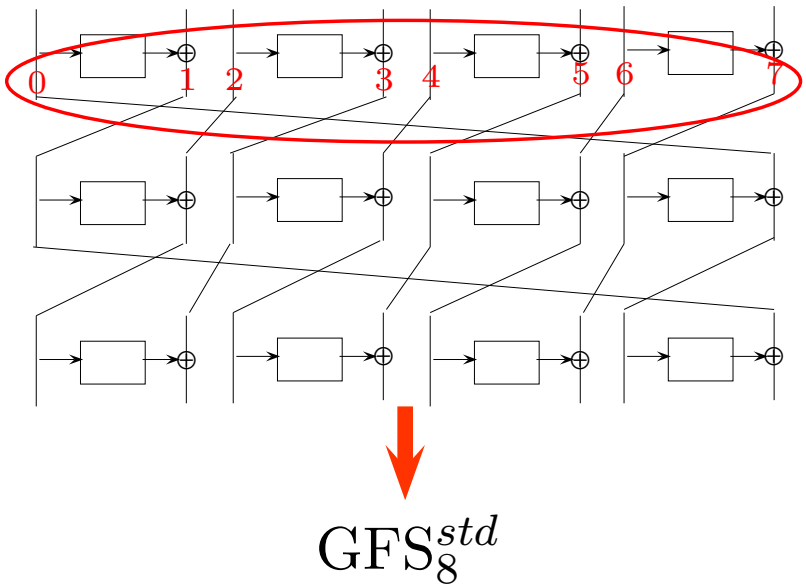
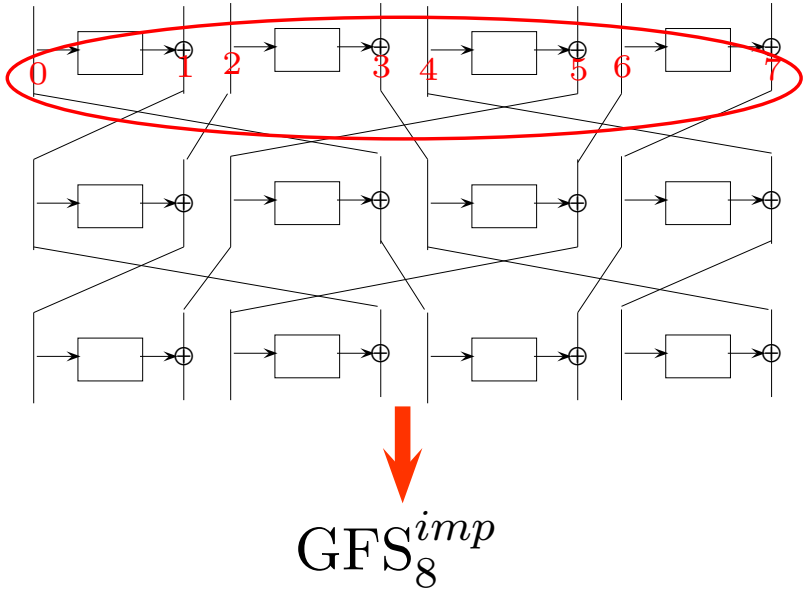


GFS_8^{std}

$\pi(0, 1, ..., 7) = (7, 0, 1, 2, 3, 4, 5, 6)$



GFS_8^{imp}

$\pi(0, 1, ..., 7) = (3, 0, 1, 4, 7, 2, 5, 6)$

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Summation Representation

$$\begin{pmatrix} 6 \\ 15 \\ 0 \\ 158 \end{pmatrix}$$

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$$\begin{pmatrix} 6 \\ 15 \\ 0 \\ 158 \end{pmatrix} \xrightarrow{truncated}$$

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$$\begin{pmatrix} 6 \\ 15 \\ 0 \\ 158 \end{pmatrix} \xrightarrow{truncated} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

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$$\begin{pmatrix} 6 \\ 15 \\ 0 \\ 158 \end{pmatrix} \xrightarrow{truncated} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \xrightarrow{summation}$$

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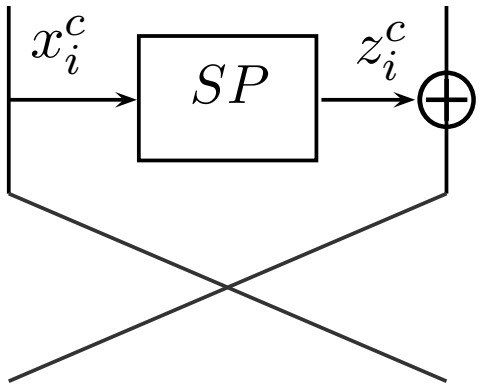
$$\begin{pmatrix} 6 \\ 15 \\ 0 \\ 158 \end{pmatrix} \xrightarrow{truncated} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \xrightarrow{summation} 3$$

Equations Describing the SP-Function



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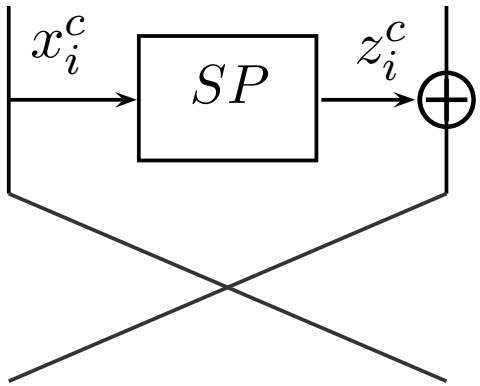

The branch number of an $n \times n$ matrix P is $\beta = n + 1$, since P is an MDS matrix.

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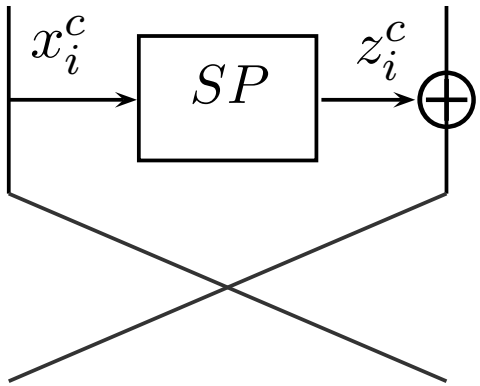

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$$\begin{cases} z_i^c = 0 & \text{if } x_i^c = 0 \\ x_i^c + z_i^c \geq n + 1 & \text{otherwise} \end{cases}$$

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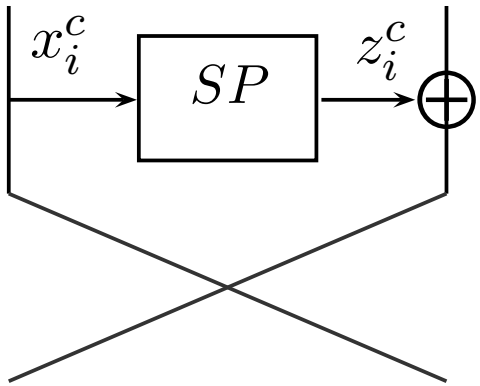

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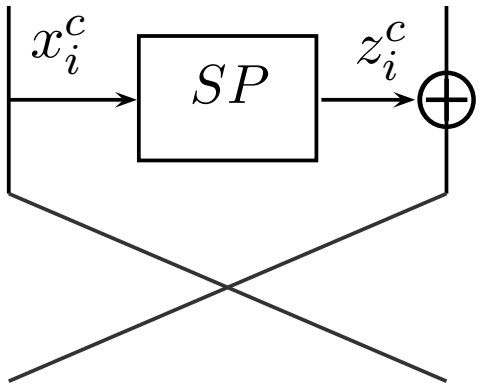

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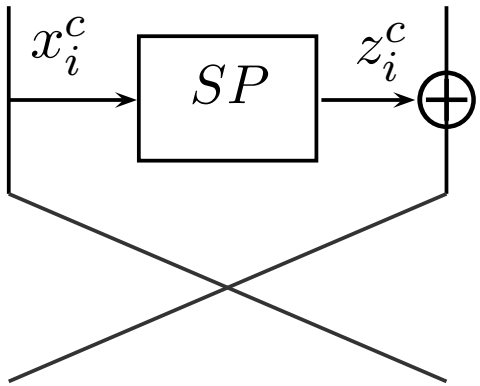

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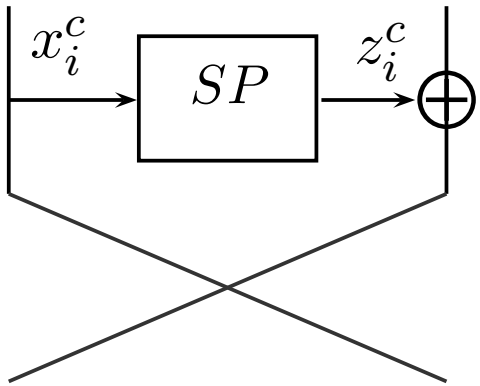

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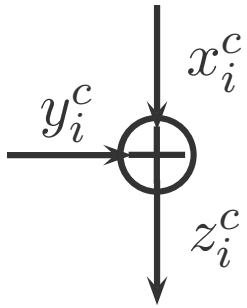
Equations Describing the XOR Operation



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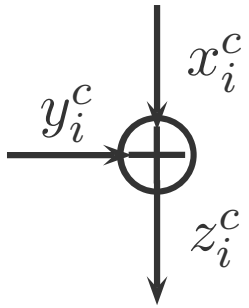

$$\begin{cases} x_i^c + z_i^c & \geq & y_i^c \\ \\ \|x_i^c - z_i^c\| & \leq & y_i^c \end{cases}$$

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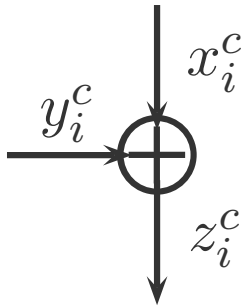

$$\begin{cases} x_i^c + z_i^c & \geq & y_i^c \\ \\ \|x_i^c - z_i^c\| & \leq & y_i^c \end{cases} \Longrightarrow$$

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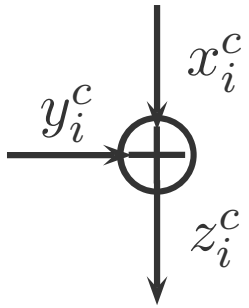

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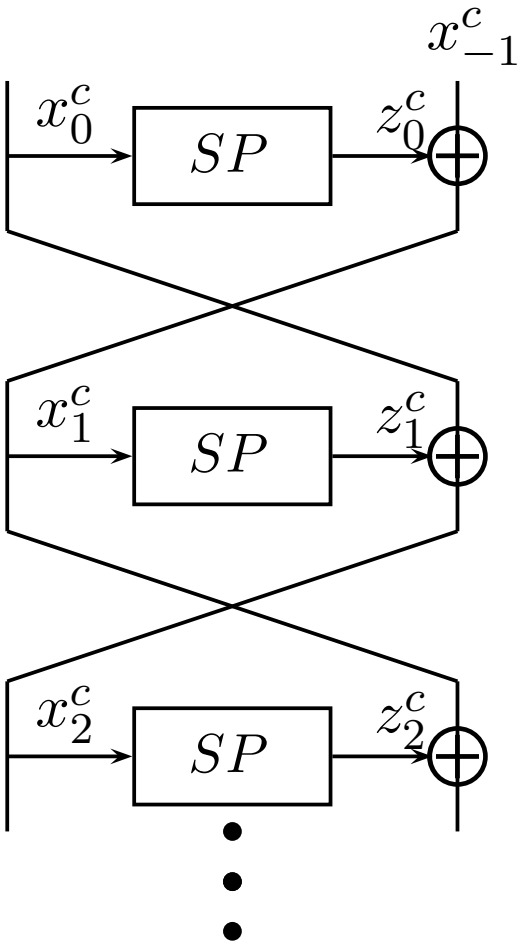
Evaluating Two Sub-Blocks Feistel Structure



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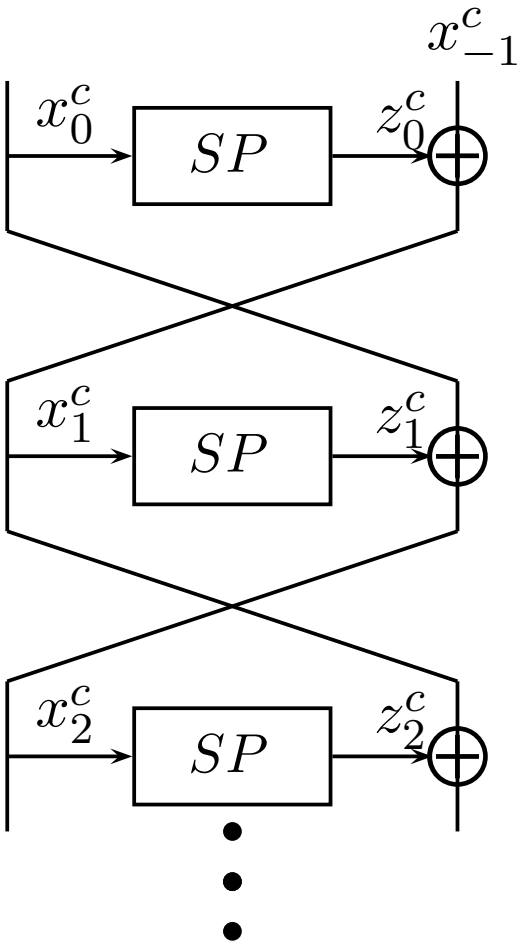

$$\begin{cases} 0 \leq x_i^c \leq n \\ 0 \leq z_i^c \leq n \\ x_i^c + z_i^c \geq (n+1)b_i \\ b_i \leq x_i^c \leq nb_i \\ z_i^c \leq nb_i \end{cases}$$

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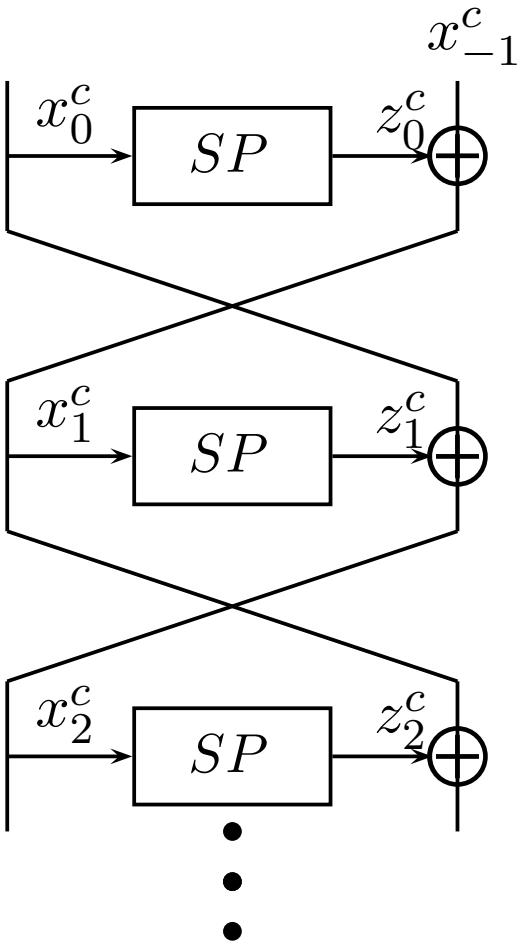

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$$\begin{cases} x_{i-2}^c + z_{i-1}^c \geq x_i^c \\ x_{i-2}^c - z_{i-1}^c \leq x_i^c \\ z_{i-1}^c - x_{i-2}^c \leq x_i^c \end{cases}$$

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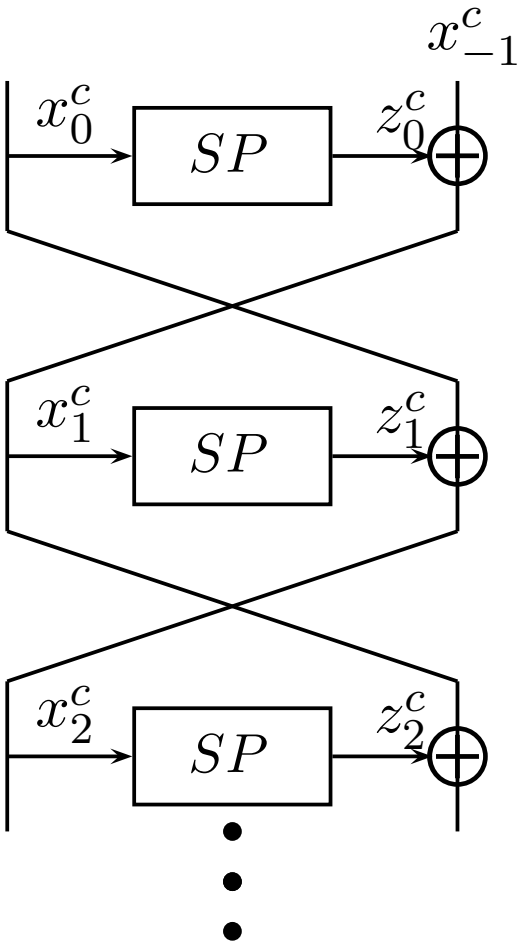

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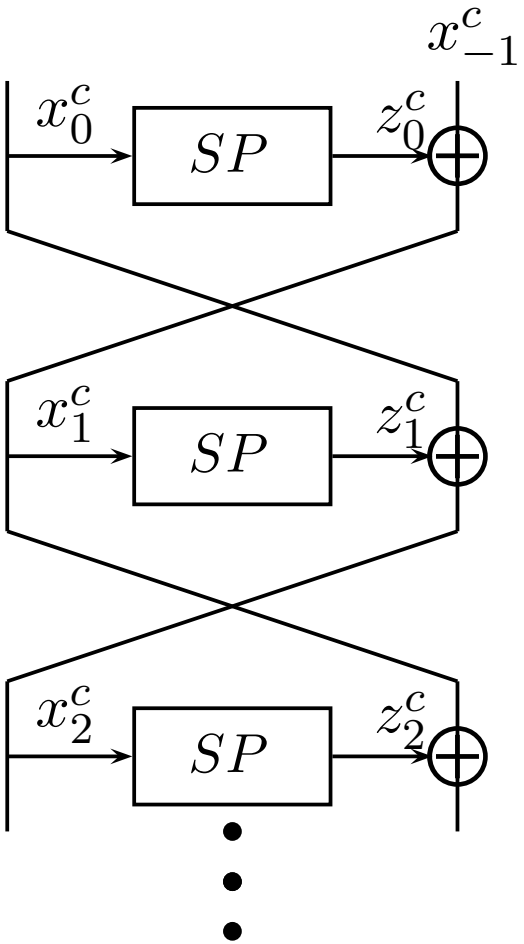
$$x_{-1}^c + x_0^c \geq 1$$

Evaluating Two Sub-Blocks Feistel Structure



$$\begin{cases} 0 \leq x_i^c \leq n \\ 0 \leq z_i^c \leq n \\ x_i^c + z_i^c \geq (n+1)b_i \\ b_i \leq x_i^c \leq nb_i \\ z_i^c \leq nb_i \end{cases}$$

$$\begin{cases} x_{i-2}^c + z_{i-1}^c \geq x_i^c \\ x_{i-2}^c - z_{i-1}^c \leq x_i^c \\ z_{i-1}^c - x_{i-2}^c \leq x_i^c \end{cases}$$

$$x_{-1}^c + x_0^c \geq 1$$

Objective function: $min(\sum_{j=0}^{n-1} x_j^c)$

Evaluating Two Sub-Blocks Feistel Structure

- **Table 1.** Minimum number of active S-boxes of two sub-blocks Feistel

Round	Feistel $(n = 4)$	Feistel $(n = 8)$
1	0	0
2	1	1
3	2	2
4	5	9
5	6	10
6	7	11
7	8	12
8	11	19
9	12	20
10	13	21
11	14	22
12	17	29
13	18	30
14	19	31
15	20	32
16	23	39
17	24	40
18	25	41

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Evaluating Generalized Feistel Structures

- **Table 2.** The Minimum Number of Active S-boxes in GFS_l^{std} with $n = 4$

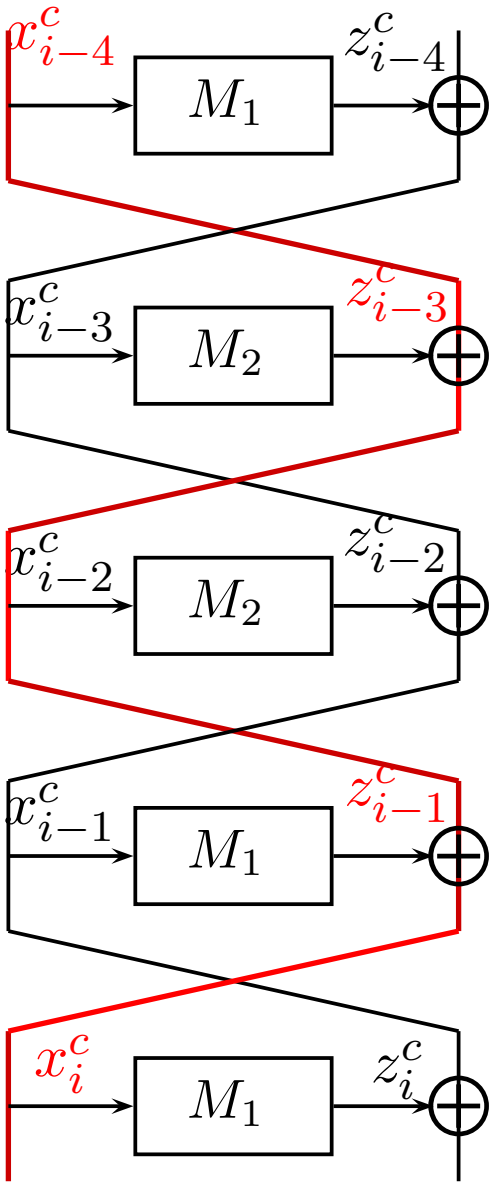
	$l=4$		$l=6$		$l=8$		$l=10$		$l=12$		$l=14$		$l=16$	
Round	*	[1]	*	[1]	*	[1]	*	[1]	*	[1]	*	[1]	*	[1]
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	2	2	2	2	2	2	2	2	2	2	2	2	2	2
4	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	8	8	8	8	8	8	8	8	8	8	8	8	8	8
6	12	12	12	12	12	12	12	12	12	12	12	12	12	12
7	12	12	14	14	14	14	14	14	14	14	14	14	14	14
8	13	13	18	18	18	18	18	18	18	18	18	18	18	18
9	14	14	21	21	21	21	21	21	21	21	21	21	21	21
10	18	18	25	25	25	25	25	25	25	25	25	25	25	25
11	20	20	27	27	28	28	28	28	28	28	28	28	28	28
12	24	24	30	30	36	36	36	36	36	36	36	36	36	36
13	24	24	31	31	36	36	39	39	39	39	39	39	39	39
14	25	25	35	35	37	37	43	43	43	43	43	43	43	43
15	26	26	37	37	38	38	47	47	47	47	47	47	47	47
16	30	30	41	41	42	42	54	54	54	54	54	54	54	54
17	32	32	43	43	44	44	58	58	58	58	58	58	52	52
18	36	36	47	47	48	48	**62**	58	62	62	62	62	62	62

Evaluating Generalized Feistel Structures

- **Table 3.** The Minimum Number of Active S-boxes in GFS_l^{imp} with $n = 4$

	$l = 6$		$l = 8$		$l = 10$		$l = 12$		$l = 14$		$l = 16$	
Round	*	[1]	*	[1]	*	[1]	*	[1]	*	[1]	*	[1]
1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1
3	2	2	2	2	2	2	2	2	2	2	2	2
4	6	6	6	6	6	6	6	6	6	6	6	6
5	8	8	8	8	8	8	8	8	8	8	8	8
6	12	12	12	12	12	12	12	12	12	12	12	12
7	14	14	14	14	14	14	14	14	14	14	14	14
8	**23**	22	23	23	**26**	23	18	18	**26**	23	**26**	23
9	24	24	26	26	29	29	21	21	29	29	31	31
10	26	26	29	29	**35**	34	29	29	37	37	**43**	40
11	28	28	32	32	36	36	32	32	40	40	48	48
12	32	32	39	39	**43**	45	**42**	39	**52**	49	**57**	54
13	**34**	33	**42**	40	44	44	45	45	54	54	60	60
14	38	38	**45**	44	48	48	**54**	53	**64**	60	**66**	63
15	40	40	46	46	50	50	57	57	**66**	63	**69**	70
16	**48**	46	50	50	54	54	**61**	60	**77**	71	76	76
17	48	48	52	52	56	56	64	64	**82**	76	78	78
18	50	50	56	56	**68**	65	**70**	68	**84**	83	87	87

Evaluating Switching Mechanism



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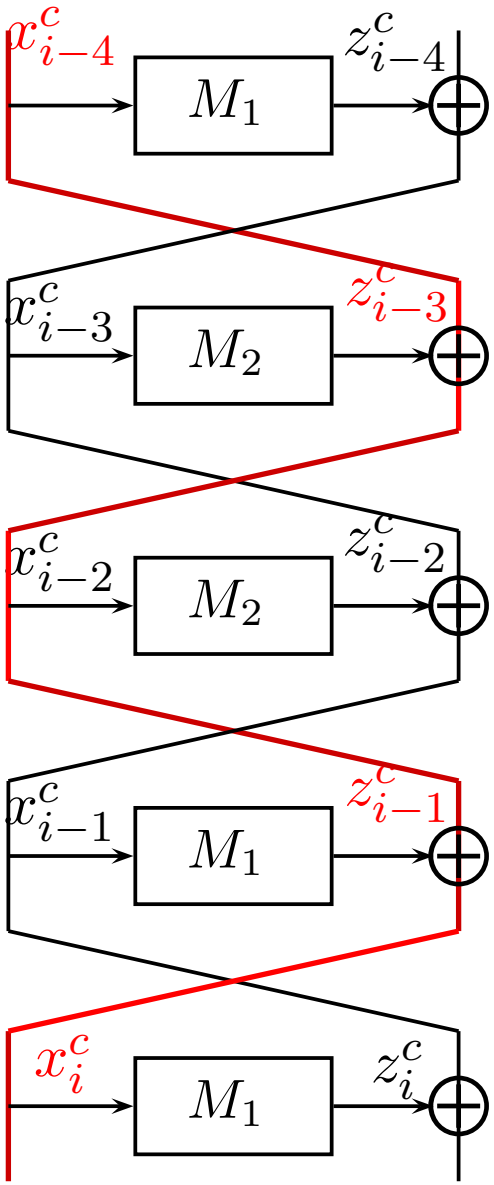
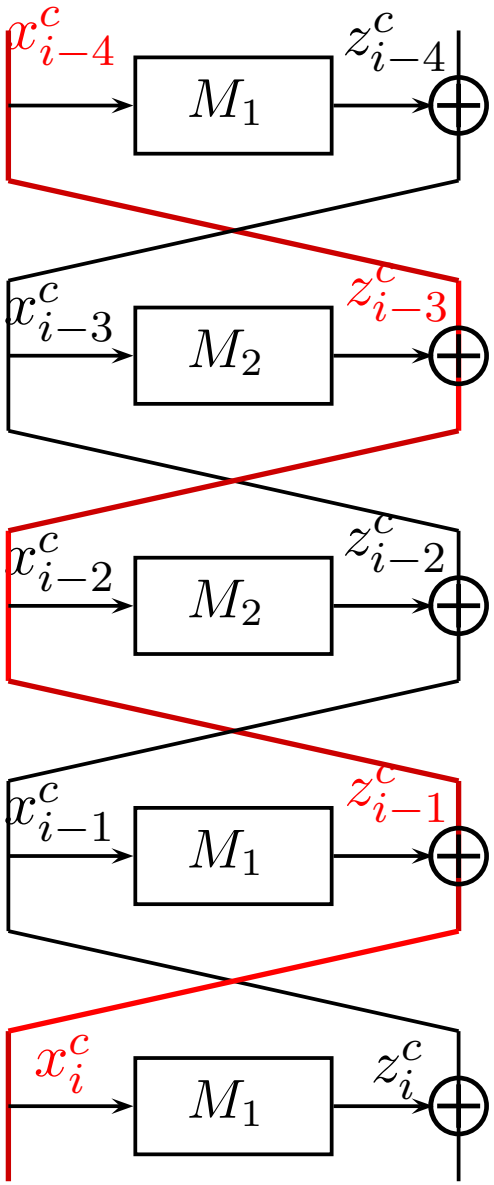


$$\mathbf{x_i} = \mathbf{z_{i-1}} \oplus \mathbf{z_{i-3}} \oplus \mathbf{x_{i-4}}$$

Evaluating Switching Mechanism



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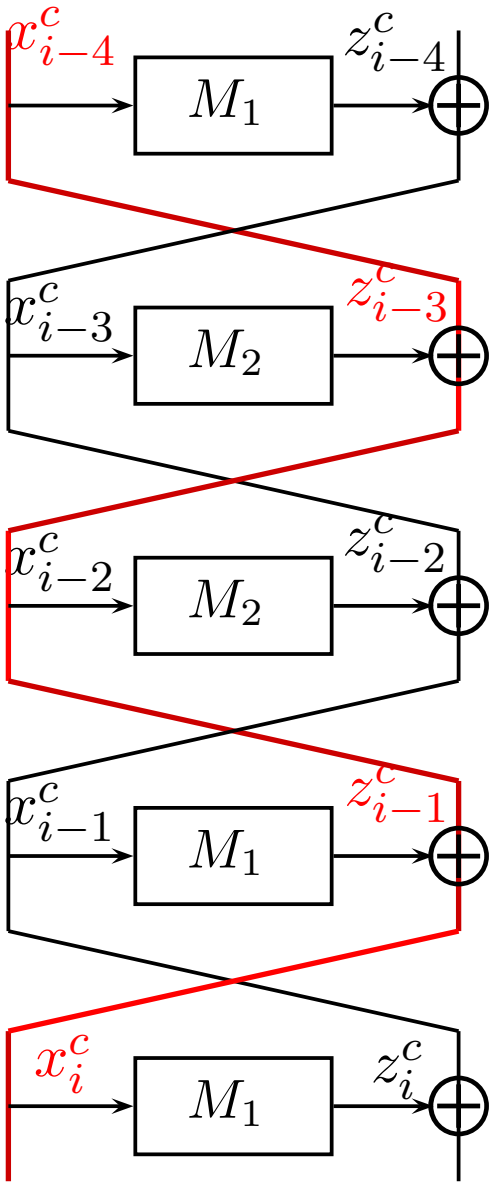

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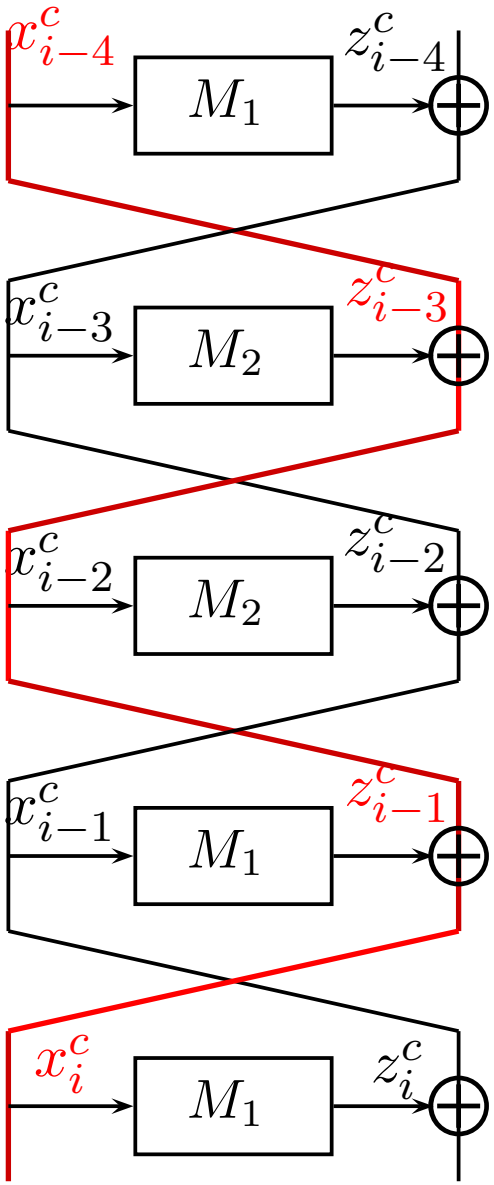

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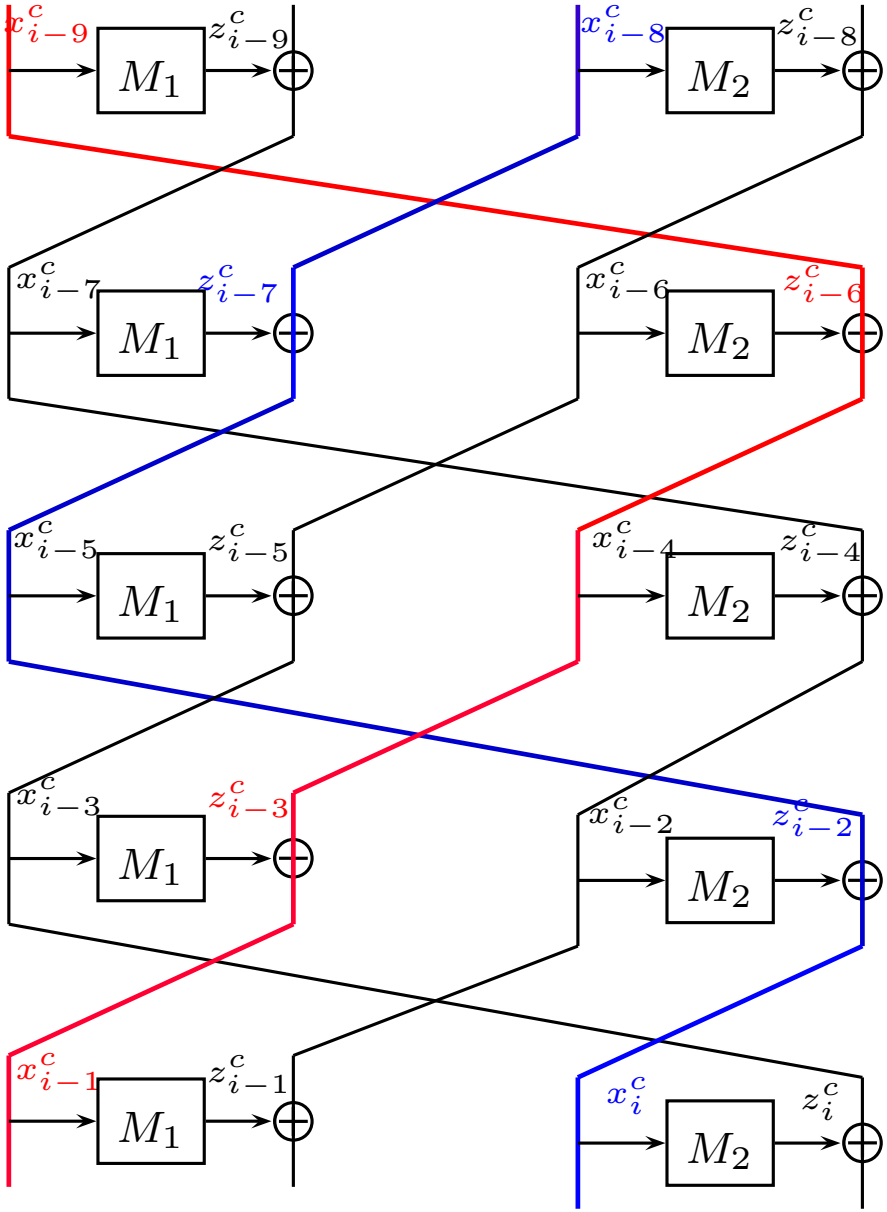
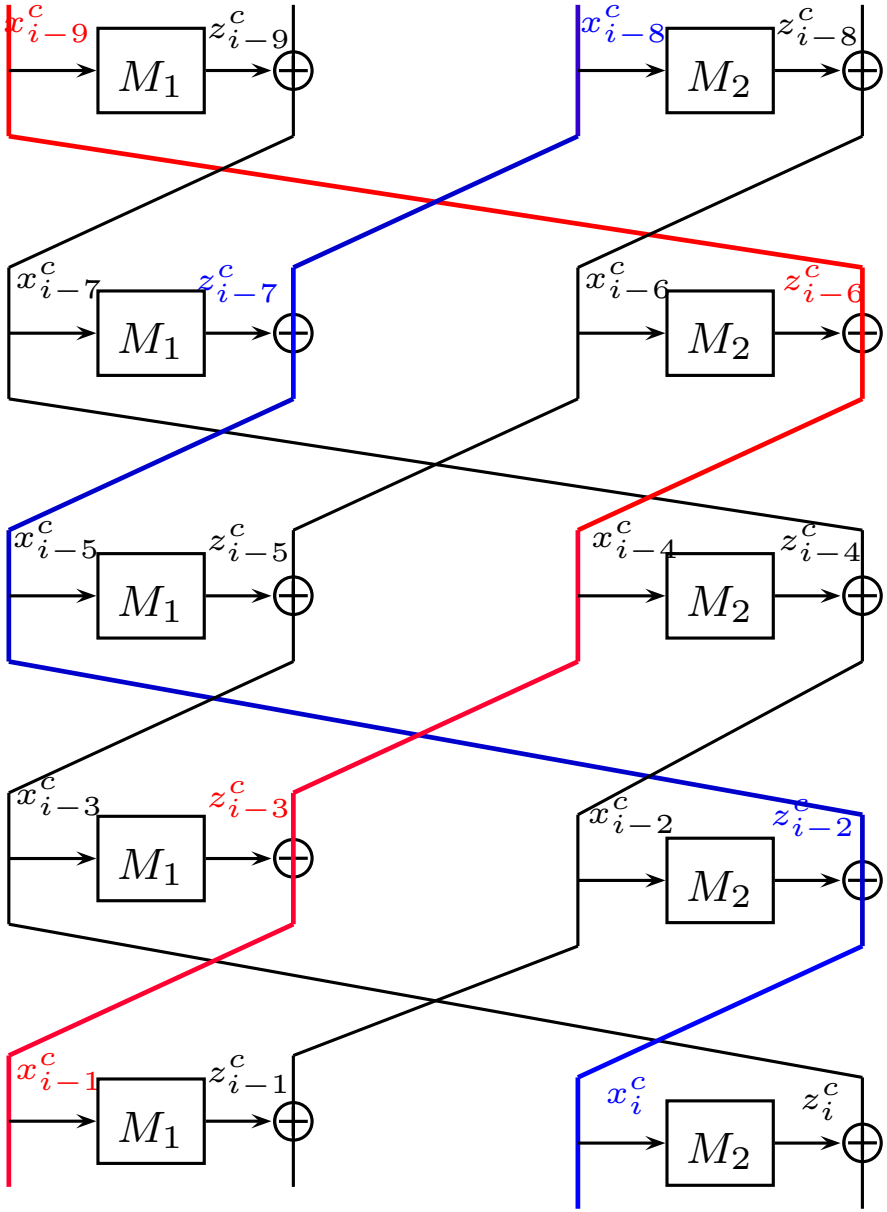


Evaluating Switching Mechanism



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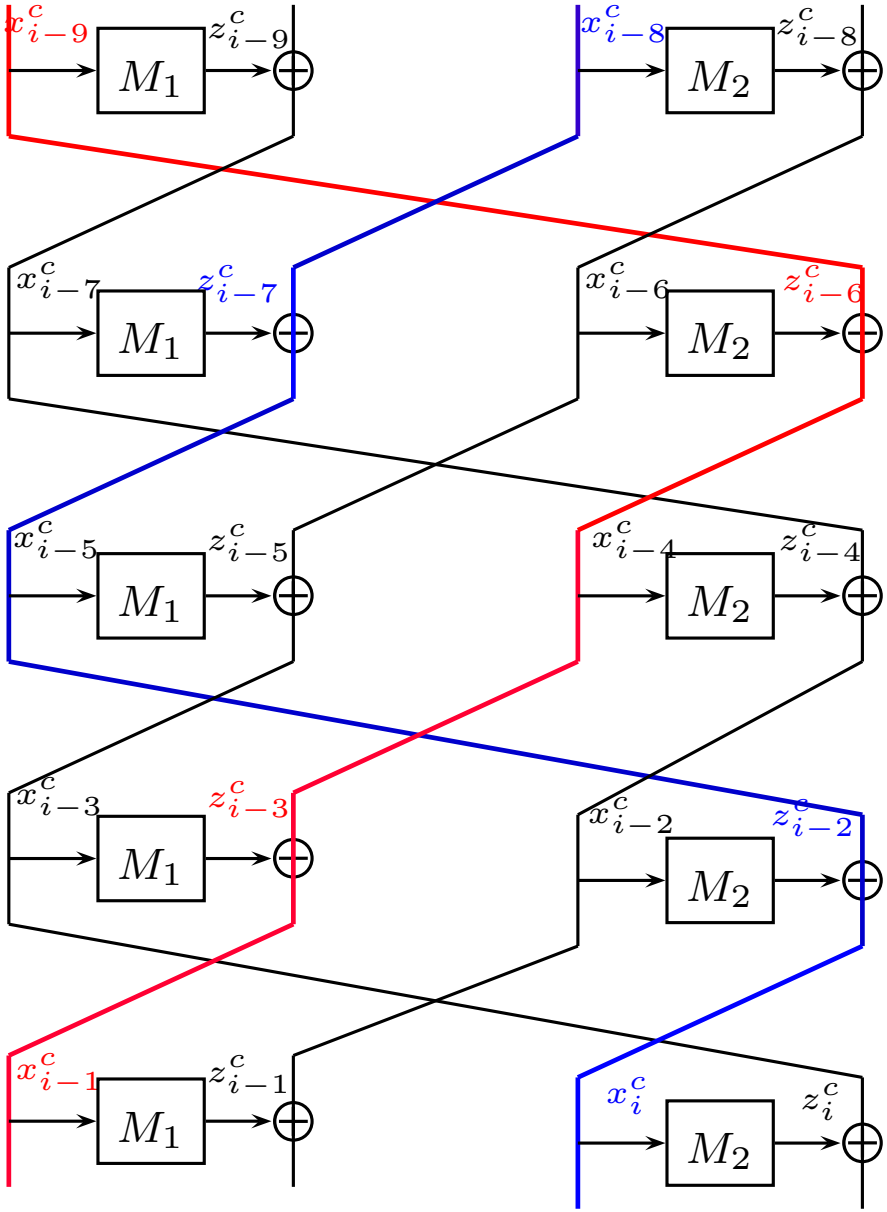

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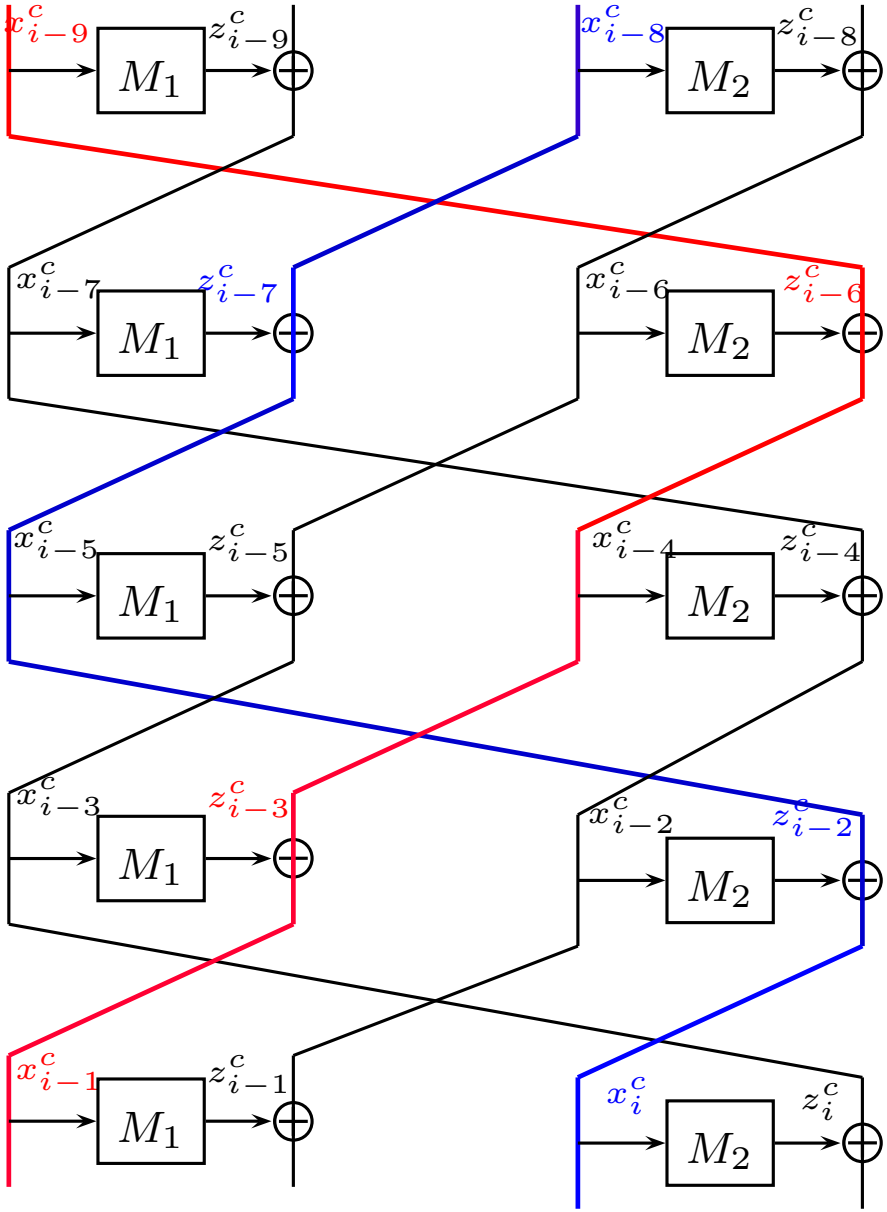

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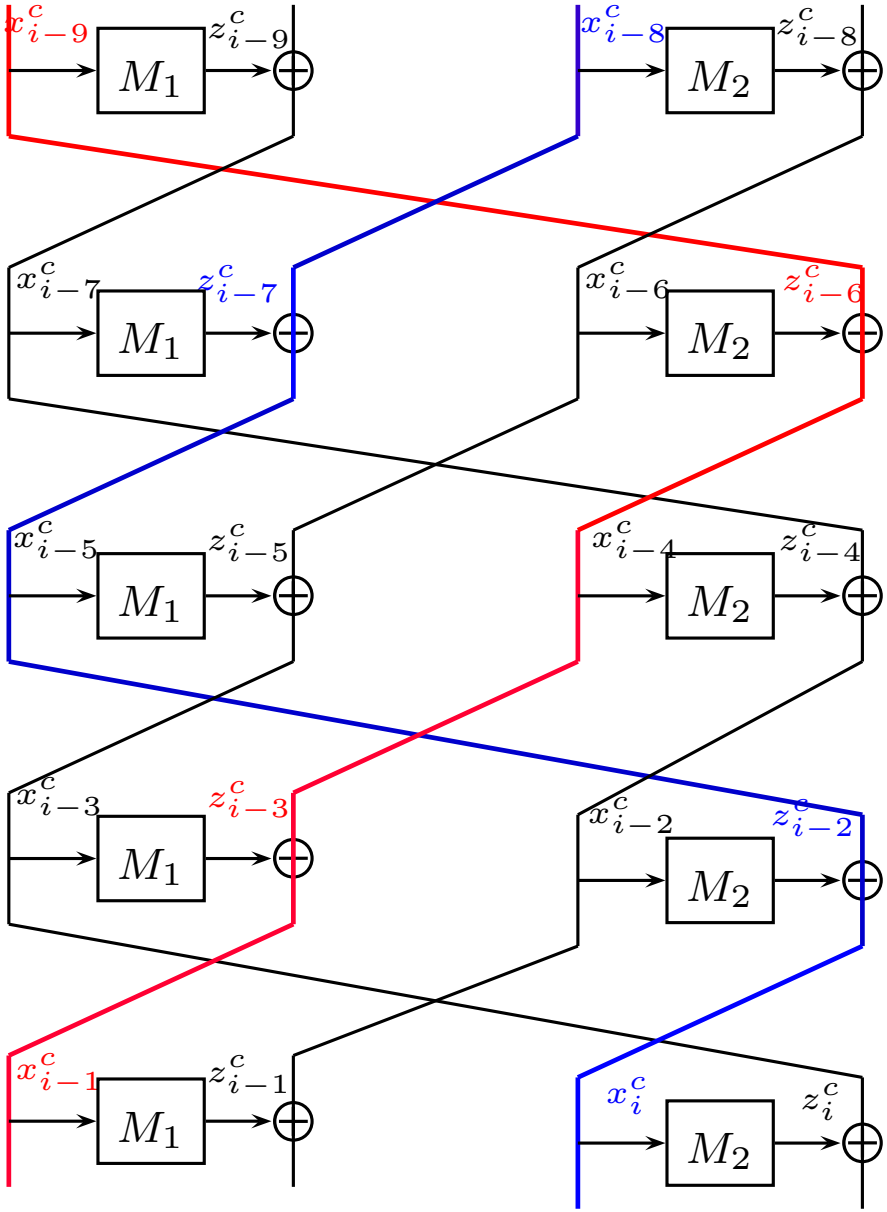

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Evaluating Switching Mechanism

- GFS_6^{std}

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$$\mathbf{x_{i-2}} = \mathbf{z_{i-5}} \oplus \mathbf{z_{i-9}} \oplus \mathbf{z_{i-16}} \oplus \mathbf{x_{i-20}}$$

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$$\begin{cases} x^c_{i-3} + x^c_{i-7} + x^c_{i-13} + x^c_{i-20} + x^c_{i-30} + x^c_{i-35} \geq (n+1)bb_{i-3} \\ bb_{i-3} \leq x^c_{i-7} + x^c_{i-13} + x^c_{i-20} + x^c_{i-30} \leq 4nbb_{i-3} \end{cases}$$

Evaluating Switching Mechanism

- **Table 3.** Minimum number of differentially active S-boxes of generalized Feistel structures imposed by switching properties with n=4

Evaluating Switching Mechanism

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	2 MDS matrices			3 MDS matrices			4 MDS matrices		5 MDS matrices		6 MDS matrices	
Round	Feistel	CLEFIA	GFS_8^{imp}	GFS_6^{std}	GFS_6^{imp}	GFS_{12}^{imp}	GFS_8^{std}	GFS_8^{imp}	GFS_{10}^{std}	GFS_{10}^{imp}	GFS_{12}^{std}	GFS_{12}^{imp}
1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1
3	2	2	2	2	2	2	2	2	2	2	2	2
4	5	6	6	6	6	6	6	6	6	6	6	6
5	6	8	8	8	8	8	8	8	8	8	8	8
6	10	12	12	12	12	12	12	12	12	12	12	12
7	10	14	14	14	14	14	14	14	14	14	14	14
8	11	18	26	18	25	26	18	23	18	26	18	26
9	12	20	29	21	27	29	21	26	21	29	21	29
10	15	22	34	25	31	41	26	32	25	37	25	37
11	16	24	37	28	33	44	31	36	28	40	28	42
12	20	28	42	34	36	56	36	44	36	49	36	50
13	20	30	43	37	38	61	41	46	39	51	39	54
14	21	34	47	38	43	67	48	49	47	54	45	65
15	22	36	49	42	46	69	50	52	51	56	50	72
16	25	38	53	44	52	72	53	56	58	62	56	77
17	26	40	55	48	55	75	56	60	64	66	62	79
18	30	44	64	50	59	79	59	67	68	70	67	85
19	30	46	67	54	61	81	62	69	72	78	75	88
20	31	50	72	57	64	92	66	77	74	83	85	..
21	32	52	74	61	67	..	69	80	78	88	88	..
22	35	55	83	64	72	..	73	86	80	95	92	..
23	36	56	84	69	76	..	76	88	84	97	94	..
24	40	59	88	73	81	..	80	91	87	..	..	..

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- Transforming differential vectors to linear masks

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    - $\mathbf{x}^c$ denotes the summation representation of vector $\mathbf{\Gamma}.\mathbf{x}$.

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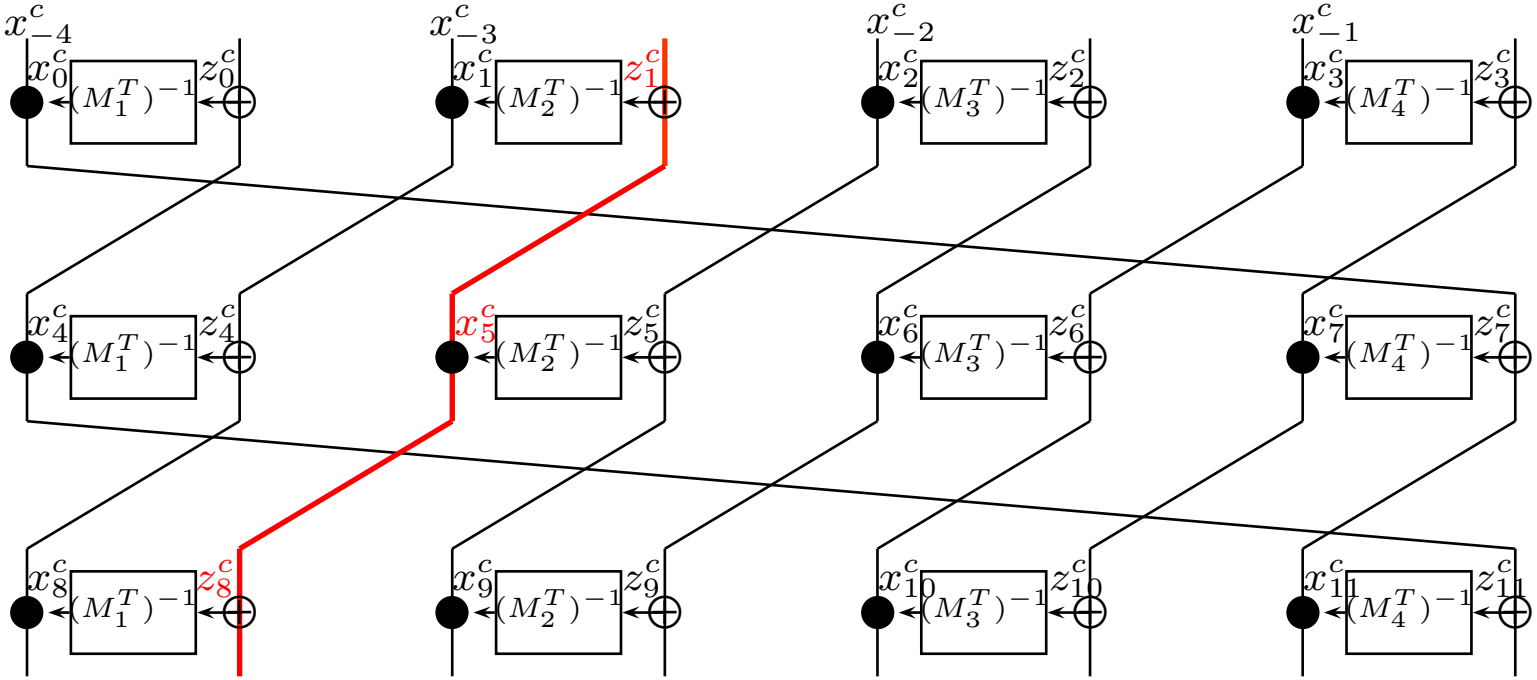

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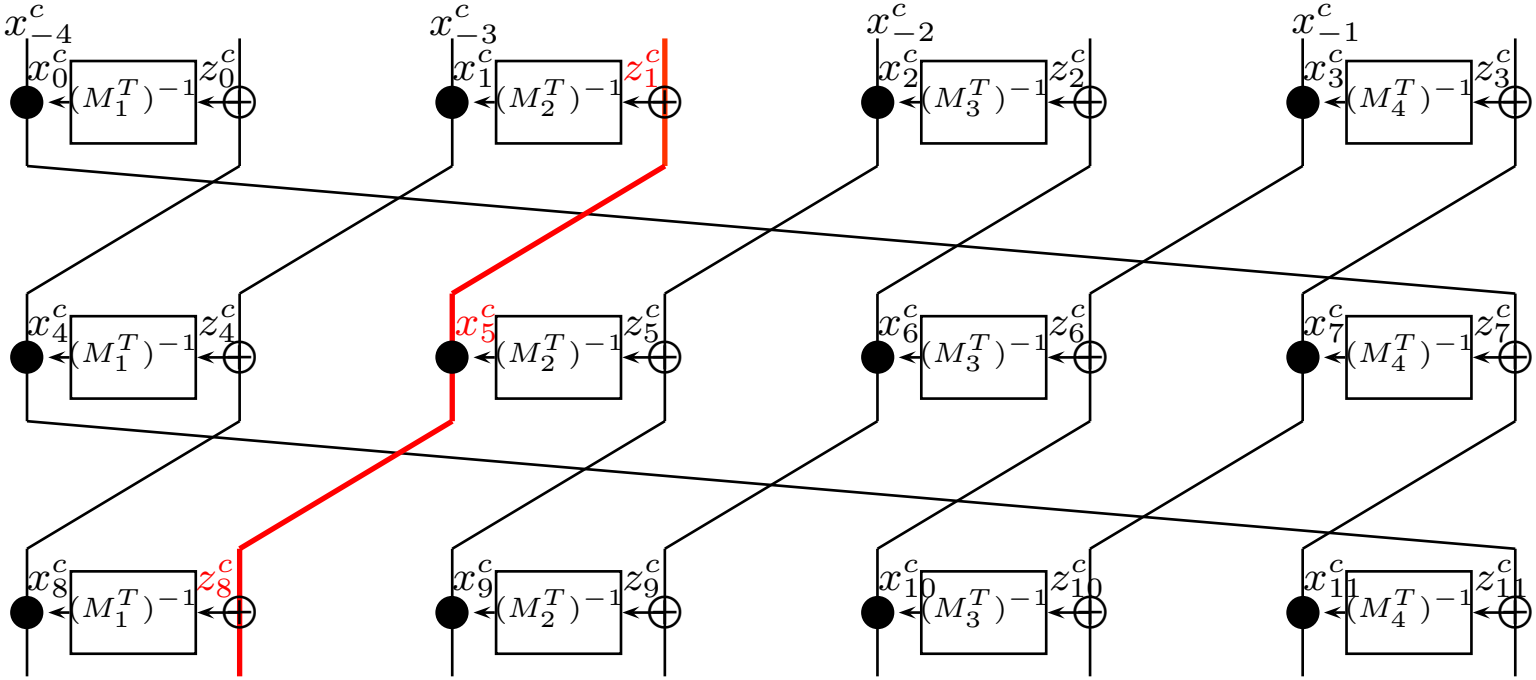

$$\mathbf{\Gamma.x_5 = \Gamma.x_1 \bullet \Gamma.x_8}$$

Counting the Linearly Active S-boxes

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$$\begin{cases} x_1^c + x_5^c + x_8^c \geq (n+1)bb_i \\ \\ bb_i \leq x_1^c + x_8^c \leq 2nbb_i \end{cases}$$

Counting the Linearly Active S-boxes

- **Table 7.** Minimum number of linearly active S-boxes of standard and improved generalized Feistel structures imposed by switching properties with $n = 4$

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- **Table 7.** Minimum number of linearly active S-boxes of standard and improved generalized Feistel structures imposed by switching properties with $n = 4$

round	Feistel	CLEFIA	GFS_6^{std}	GFS_6^{imp}	GFS_8^{std}	GFS_8^{imp}	GFS_{10}^{std}	GFS_{10}^{imp}	GFS_{12}^{std}	GFS_{12}^{imp}
1	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1
3	5	5	5	5	5	5	5	5	5	5
4	5	8	8	8	8	8	8	8	8	8
5	7	10	10	10	10	10	10	11	10	11
6	10	15	16	16	16	16	16	16	16	16
7	11	16	18	22	18	22	18	22	18	22
8	12	19	24	27	24	30	24	30	24	30
9	15	21	26	30	26	32	26	38	26	38
10	16	24	32	33	34	38	34	43	34	43
11	17	26	35	35	39	43	39	50	39	51
12	20	31	37	38	45	49	45	53	45	59
13	21	32	40	40	48	51	50	55	50	65
14	22	35	42	46	51	54	58	58	58	72
15	25	37	47	52	53	56	63	61	63	74
16	26	40	50	56	56	62	69	66	69	77
17	27	42	55	60	58	66	73	72	74	79
18	30	47	58	63	64	72	75	78	85	85
19	31	48	63	65	66	77	78	86	92	91
20	32	51	67	68	72	82	80	91	99	99
21	35	53	69	71	74	88	86	97	101	107
22	36	56	72	76	82	94	88	103	104	112
23	37	58	74	82	87	97	94	105	106	120
24	40	63	79	86	93	100	96	108	112	..

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- Employing Switching mechanism in GFS_8 enhance the number of active S-boxes almost $\%20$ for 18 rounds.
- Two different MDS matrices in GFS_8^{imp} leads to 3 lower differentially active S-boxes rather than 4 matrices after 24 rounds.

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Thank for your attention

Future Work

It is worth mentioning that, our approach can be generalized for other Feistel structures, and is usable in designing future block ciphers.