DWCDM: A Single-Keyed Nonce Based BBB Secure MAC

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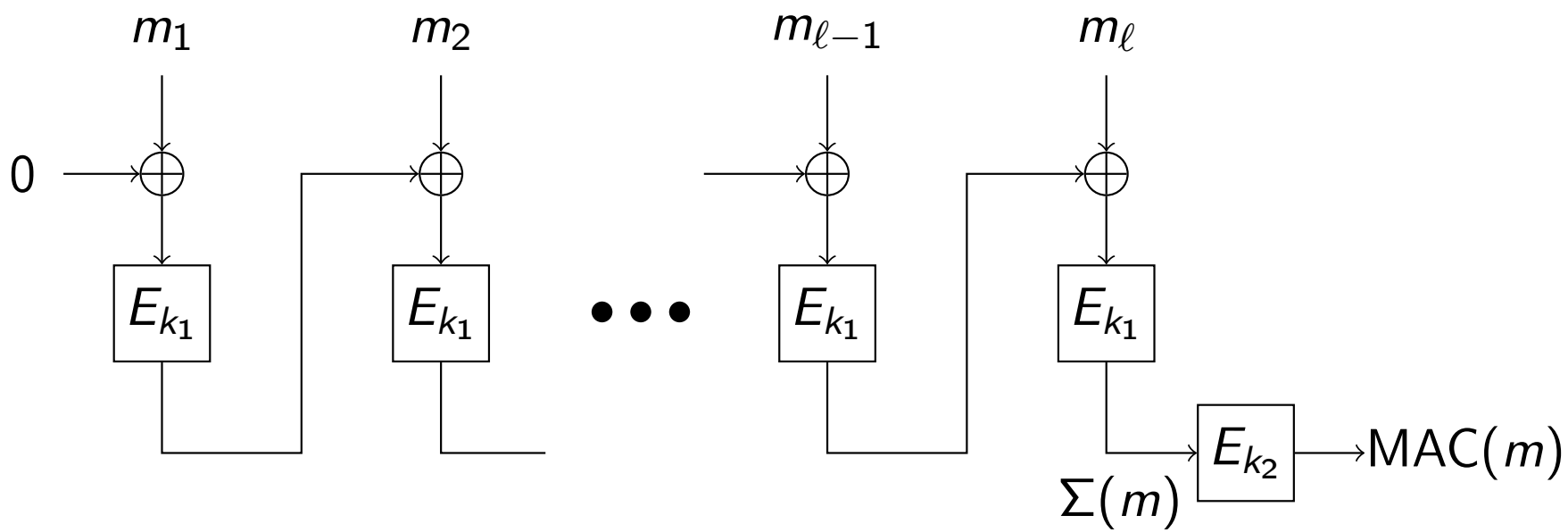
- Encrypt or Decrypt? To Make a Single-Key Beyond Birthday Secure Nonce-Based MAC – Nilanjan Datta, Avijit Dutta, Mridul Nandi and Kan Yasuda, CRYPTO 2018.




Introduction

- **Symmetric cryptography:** Alice and Bob share the same key.
- **Active attacker:** Eve might intercept and manipulate Alice's messages.
- **Authentication:** Alice computes and appends a keyed MAC or tag T... Bob recomputes tag and matches with T.

An Example of MAC: ECBC-MAC

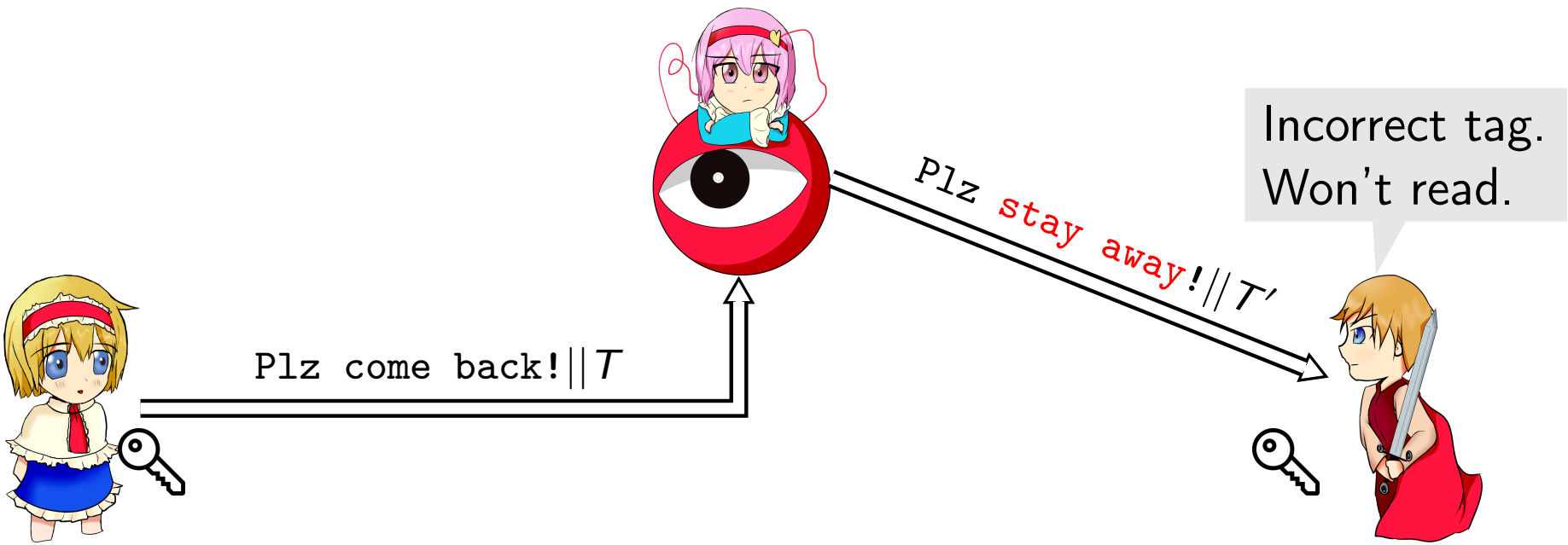


– $m_1||\cdots||m_\ell$: "`Plz come back || pad`", Tag : $T = \mathrm{MAC}(m)$.

– Alice sends T along with m to guarantee authenticity.

Introduction

- **Verifying:** Bob verifies the tag with the shared key and only reads the message if tags match.
- **Forgery:** Eve cannot modify the message without forging a new and correct tag.



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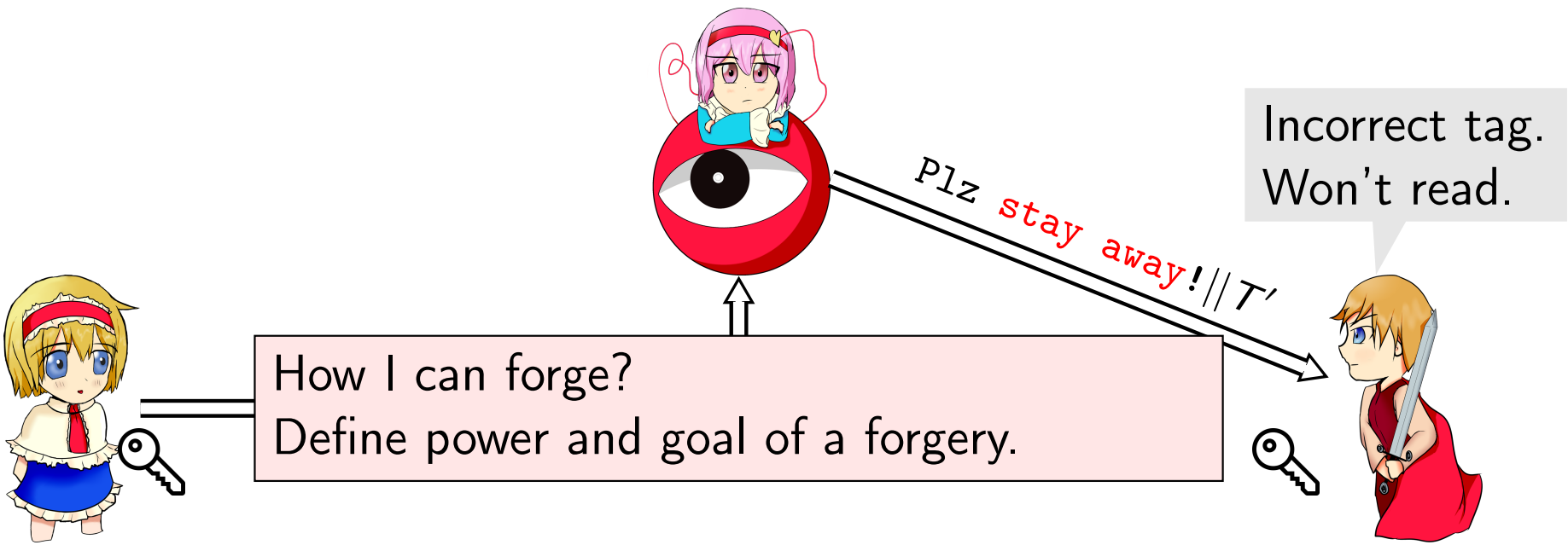

Forgery security game

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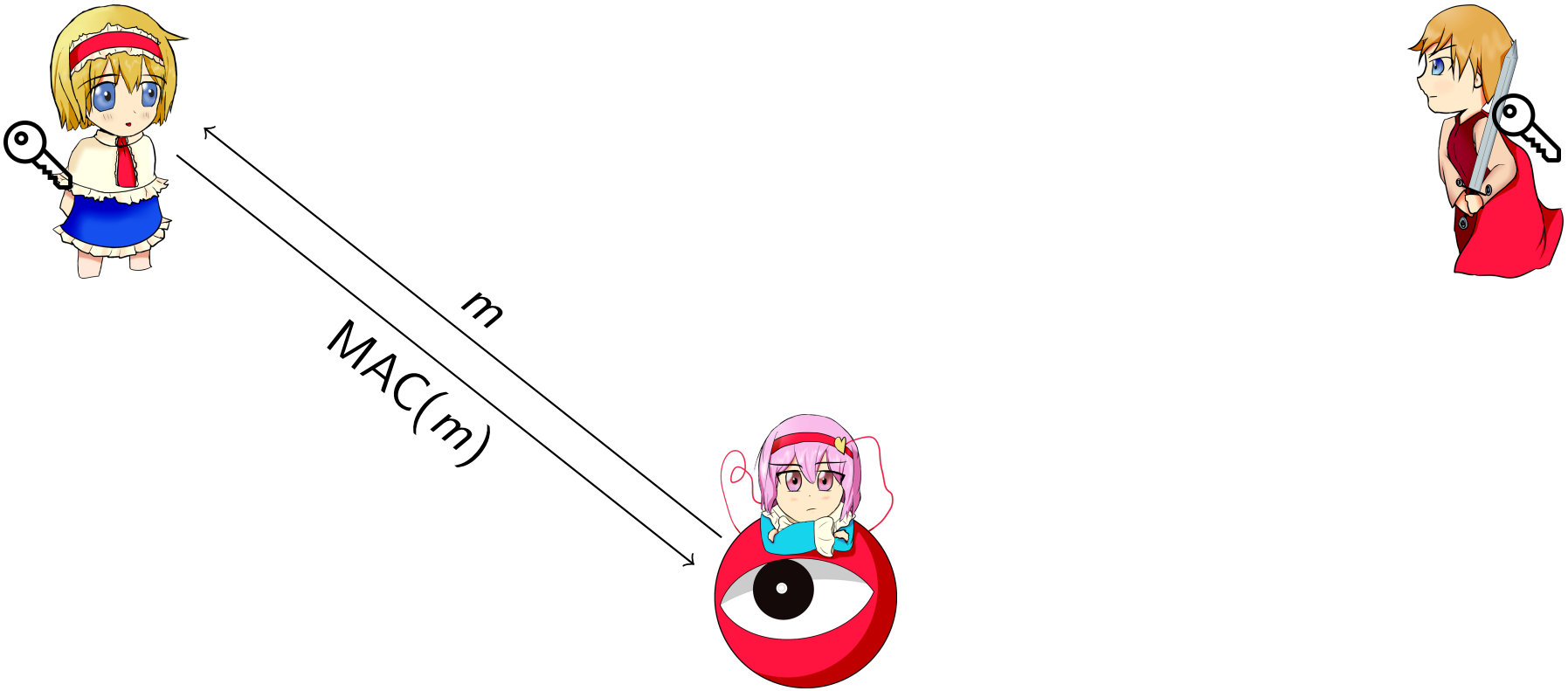

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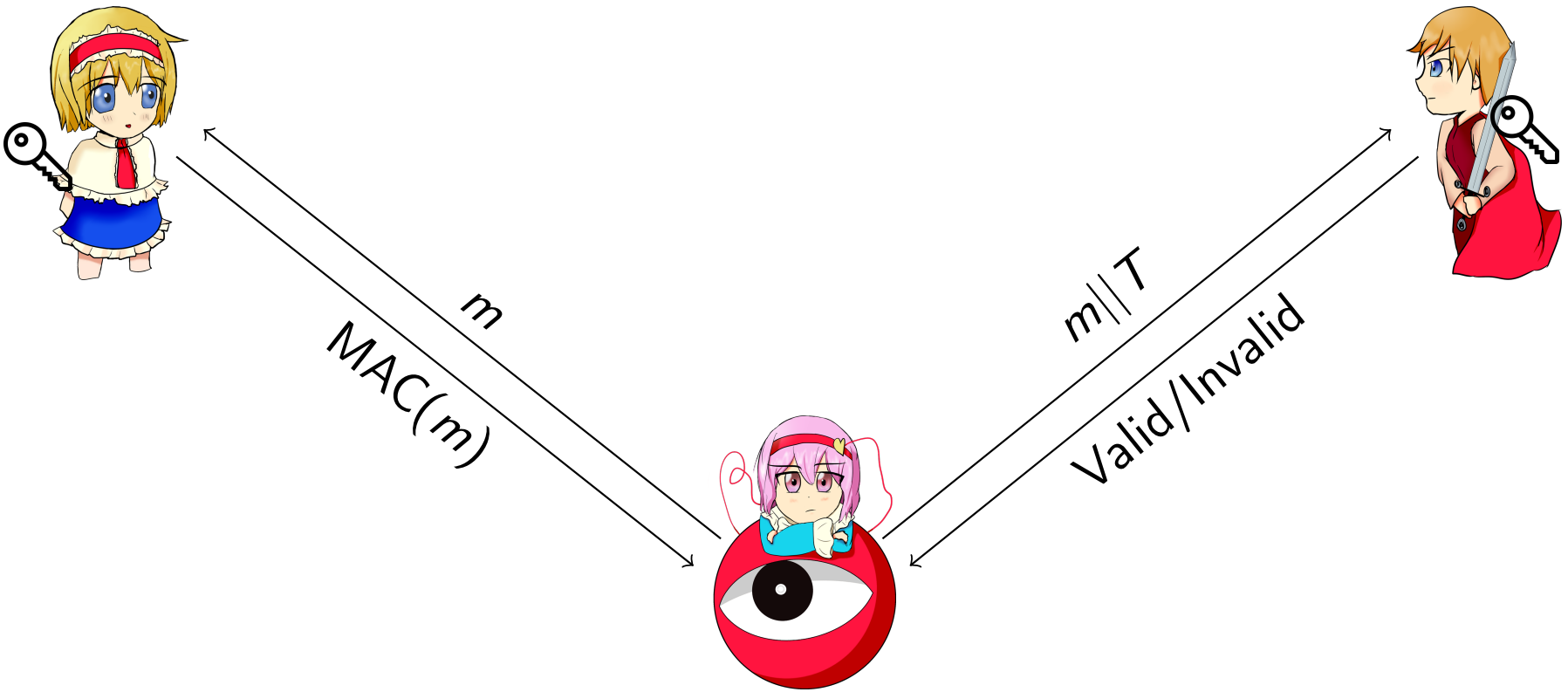

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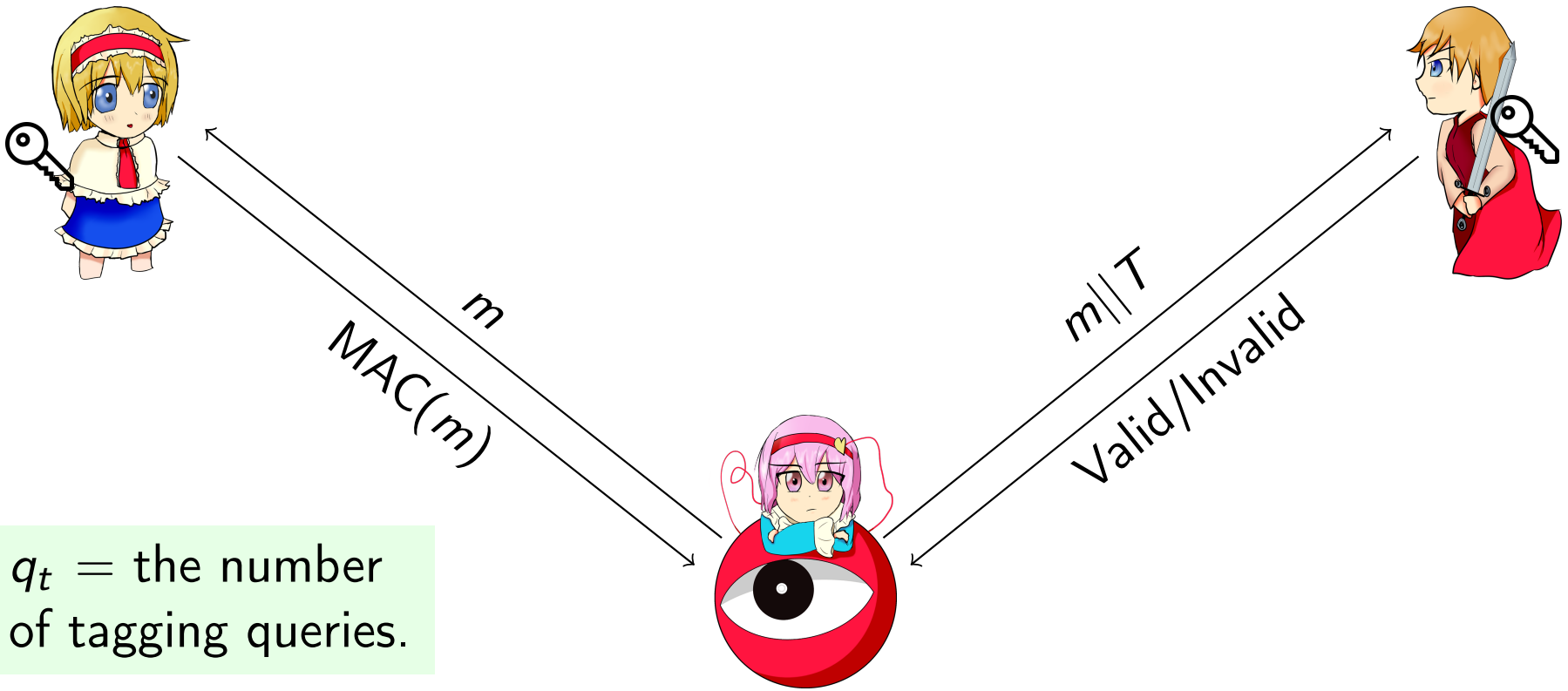

q_t = the number of tagging queries.

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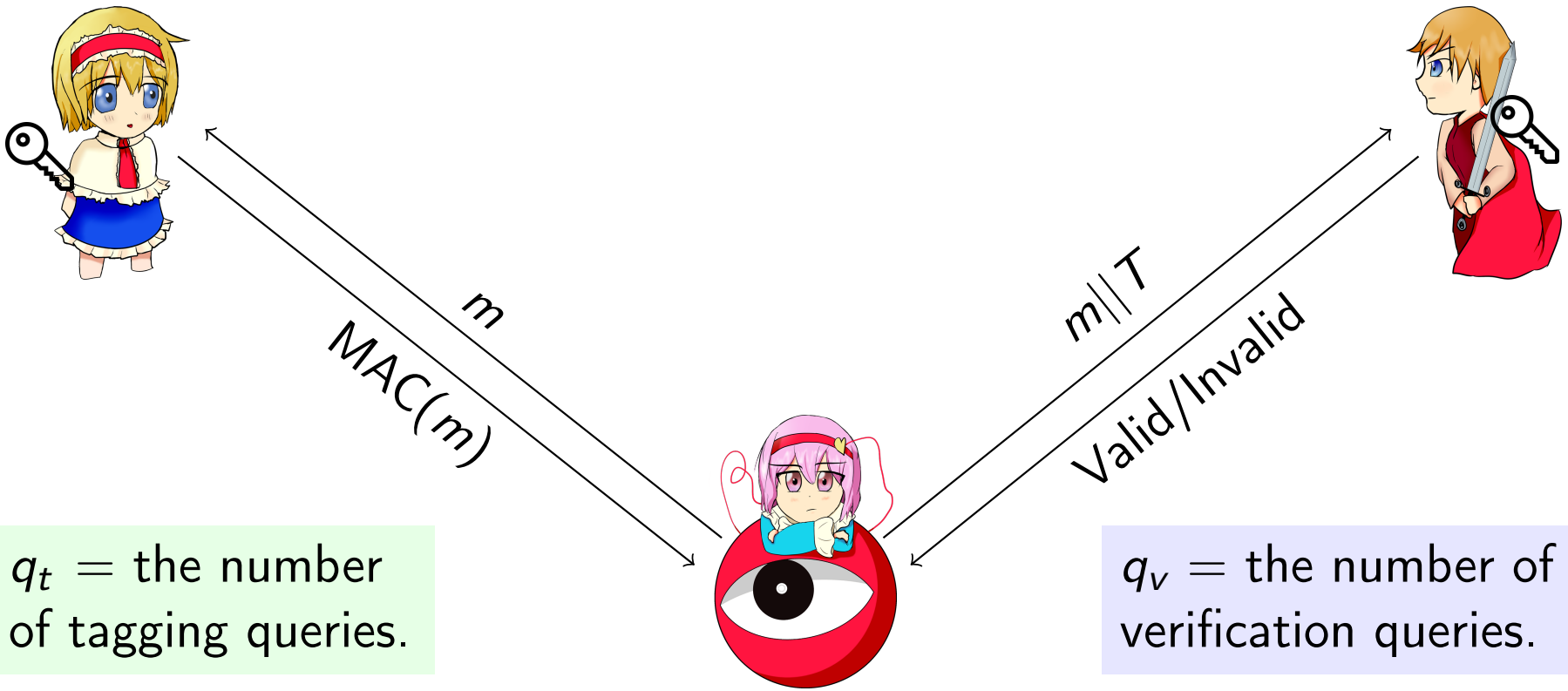

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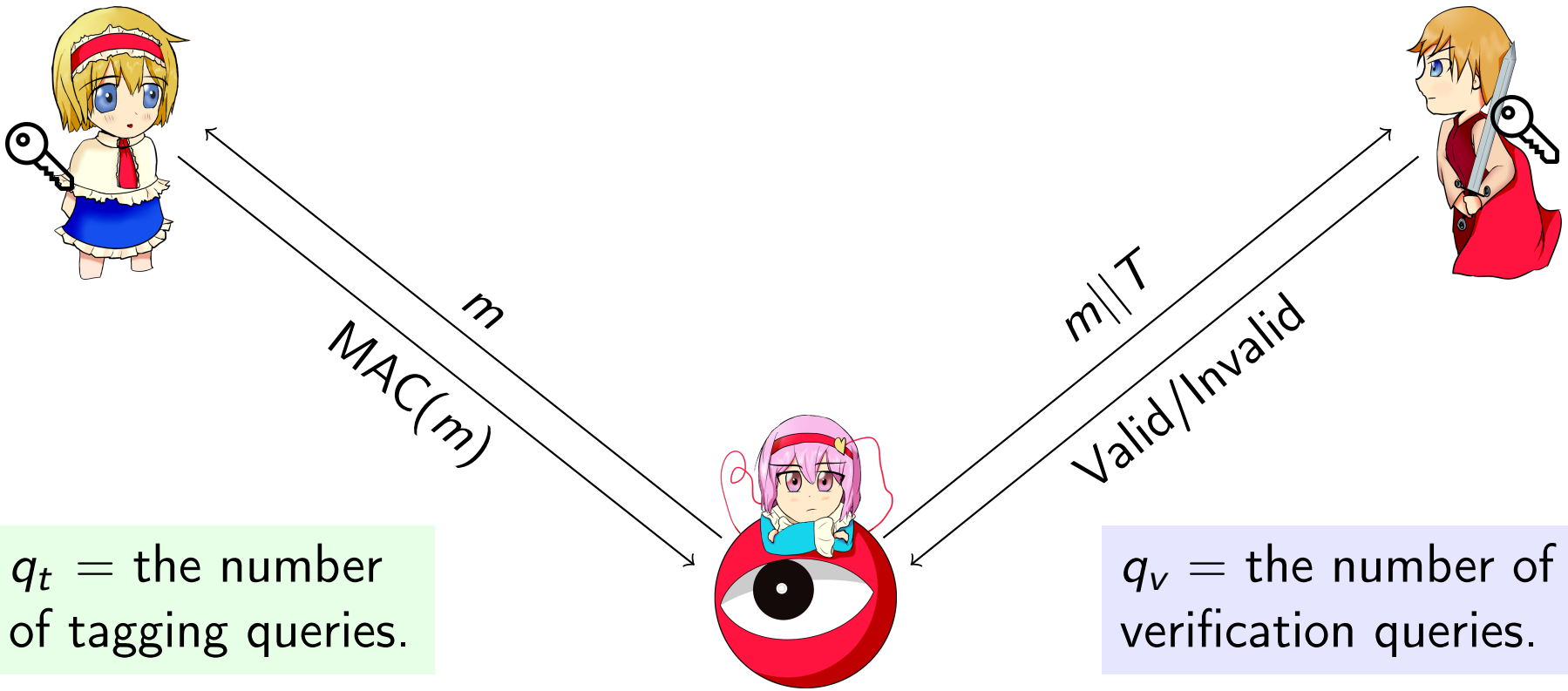


q_t = the number of tagging queries.

q_v = the number of verification queries.

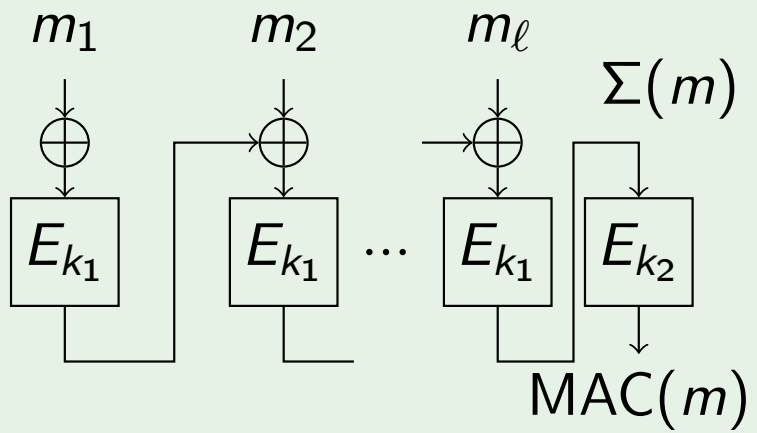
Can Eve forge a valid tag for a message that Alice never saw?

Case of ECBC

Properties of ECBC for all messages m, m', c:

$$
\begin{aligned}
& \mathrm{MAC}(m) = \mathrm{MAC}(m') \\
\iff \quad & E_{k_2}\big(\Sigma(m)\big) = E_{k_2}\big(\Sigma(m')\big) \\
\iff \quad & \Sigma(m) = \Sigma(m') \\
\iff \quad & \Sigma(m||c) = \Sigma(m'||c) \\
\iff \quad & \mathrm{MAC}(m||c) = \mathrm{MAC}(m'||c)
\end{aligned}
$$

ECBC mode



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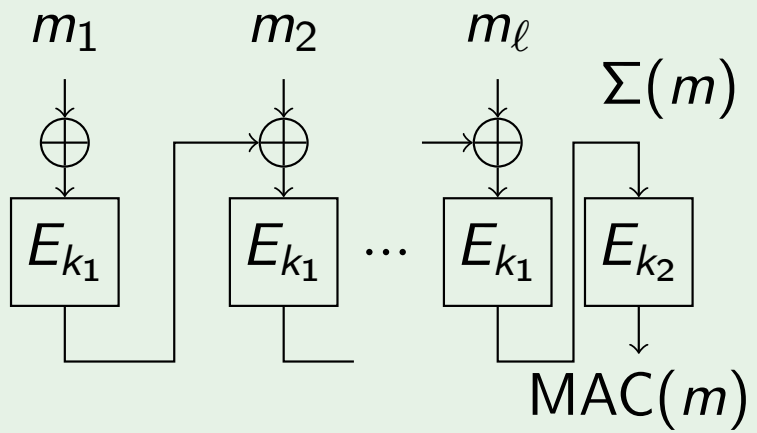
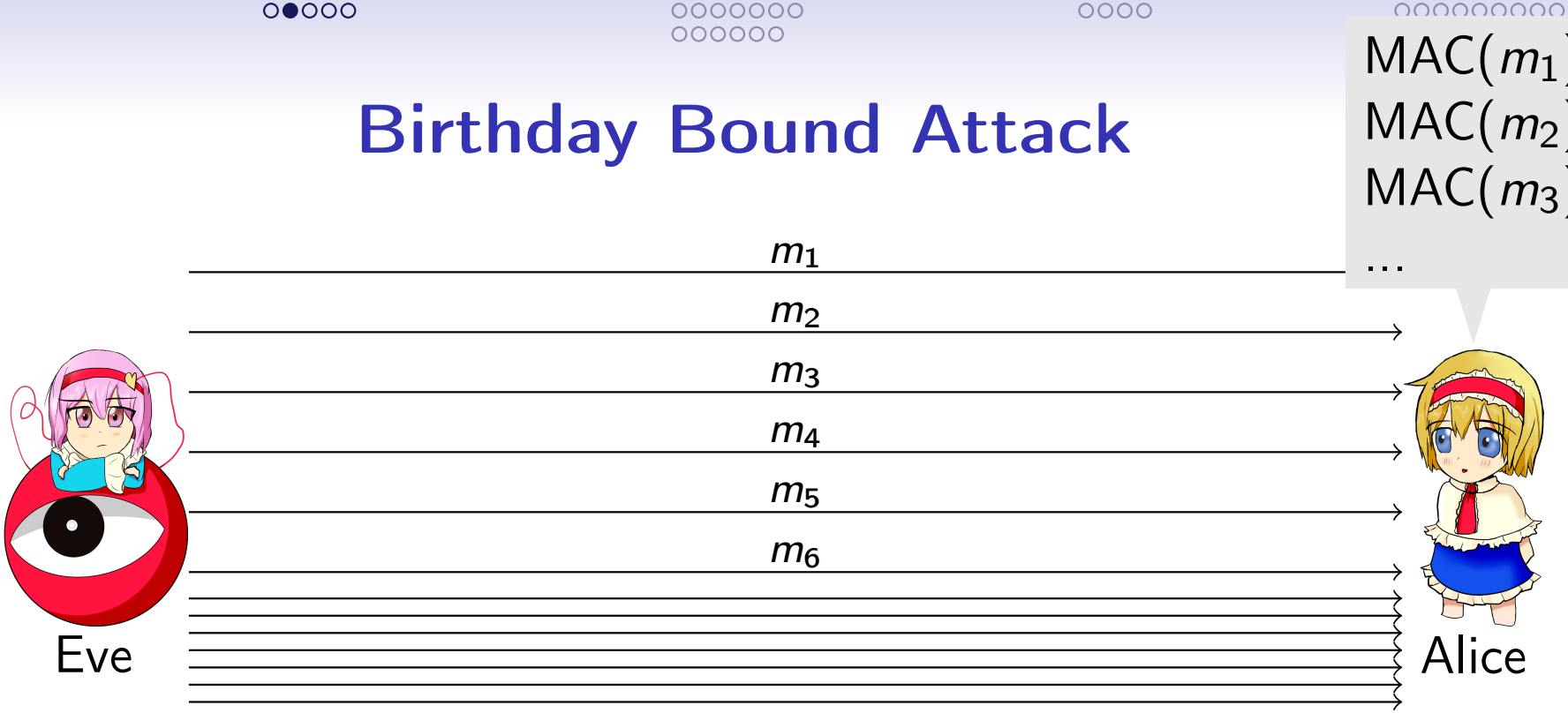


Expansion Property

Look for a pair of messages m, m' such that $\mathrm{MAC}(m) = \mathrm{MAC}(m')$. Then, for all c,

$$\mathrm{MAC}(m||c) = \mathrm{MAC}(m'||c)$$

Birthday Bound Attack



Looking for collisions

Eve looks for $\mathrm{MAC}(m_i) = \mathrm{MAC}(m_j)$ for some $i \neq j$.
She has $\simeq q_t^2$ pairs for an n-bit relationship so chances grow as:

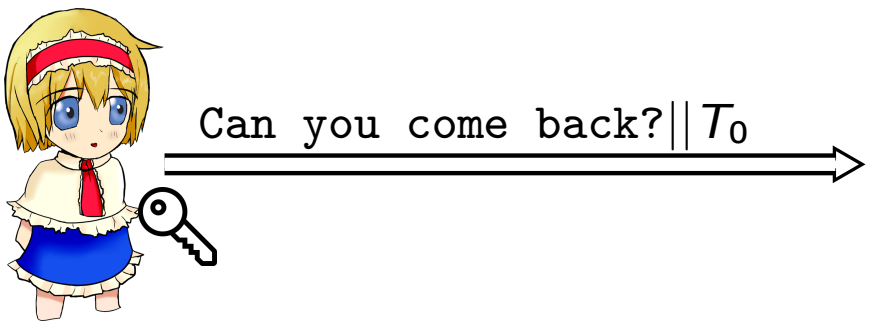
$$\mathbf{Adv}(\mathcal{A}) \simeq \frac{q_t^2}{2^n}$$

Forgery from collisions

Expansion property

$$\mathrm{MAC}(m) = \mathrm{MAC}(m') \implies \mathrm{MAC}(m||c) = \mathrm{MAC}(m'||c)\ \forall c$$

Collision found:
$\mathrm{MAC}(\texttt{You must}) =$
$\mathrm{MAC}(\texttt{No, don't})$



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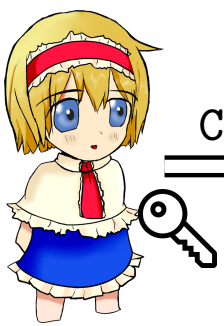
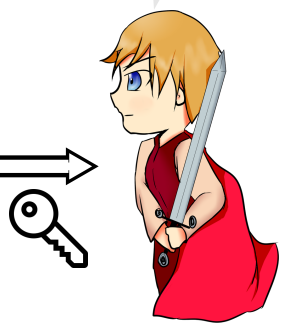
Correct tag.
Will read.



$\texttt{Can you come back?}||T_0$



Forgery from collisions

Expansion property

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Tell Bob **he must** come back!

Collision found:
$\mathrm{MAC}(\texttt{You must}) =$
$\mathrm{MAC}(\texttt{No, don't})$

Oh you are right!

Forgery from collisions

Expansion property

$$\mathrm{MAC}(m) = \mathrm{MAC}(m') \implies \mathrm{MAC}(m||c) = \mathrm{MAC}(m'||c)\ \forall c$$

Collision found:
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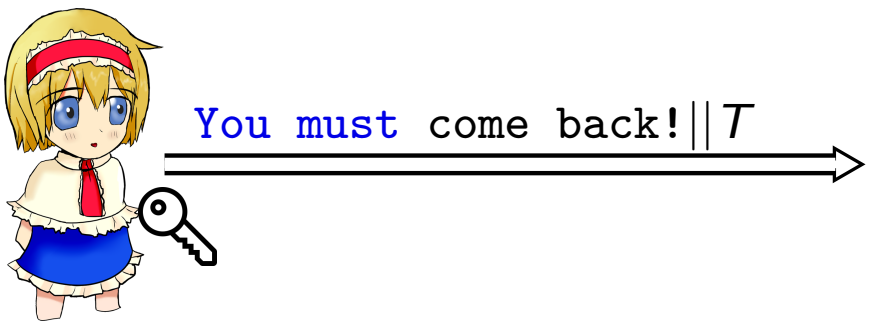

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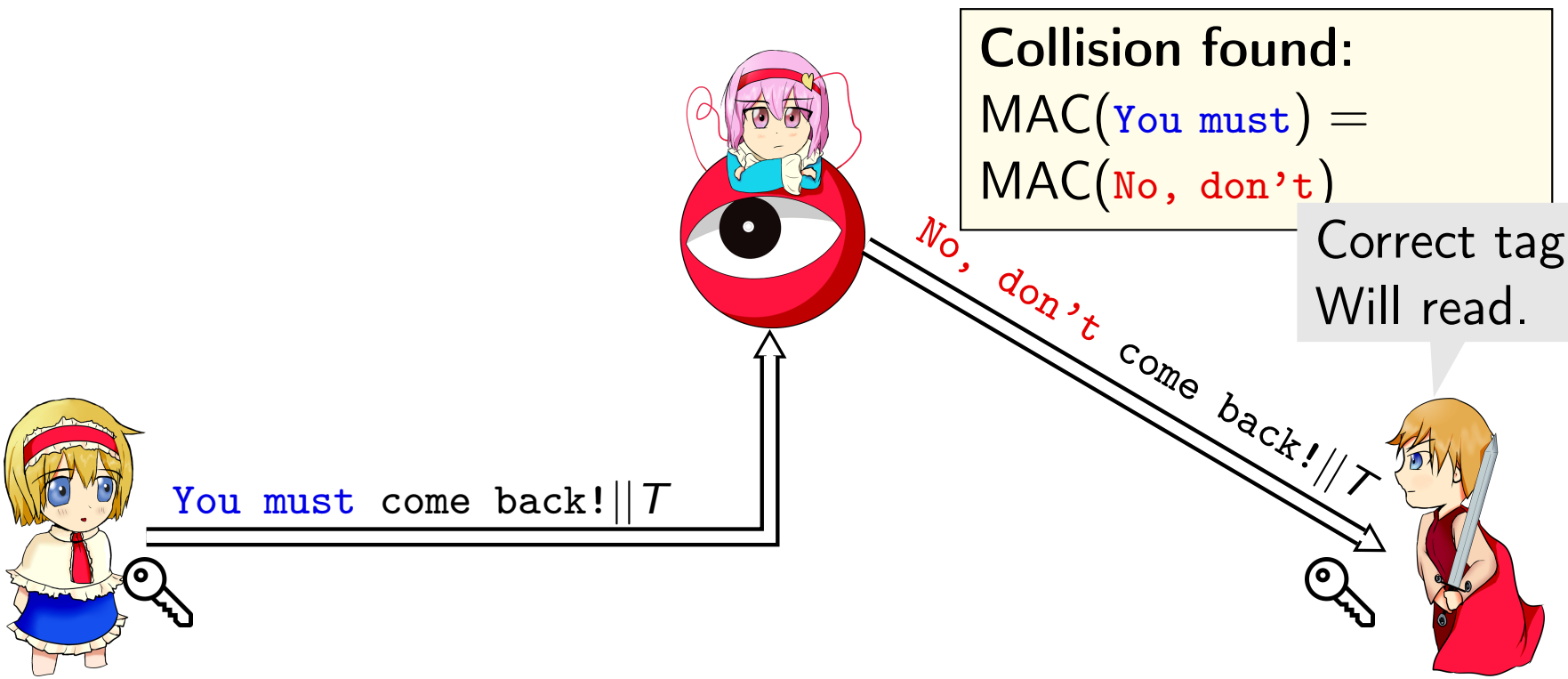

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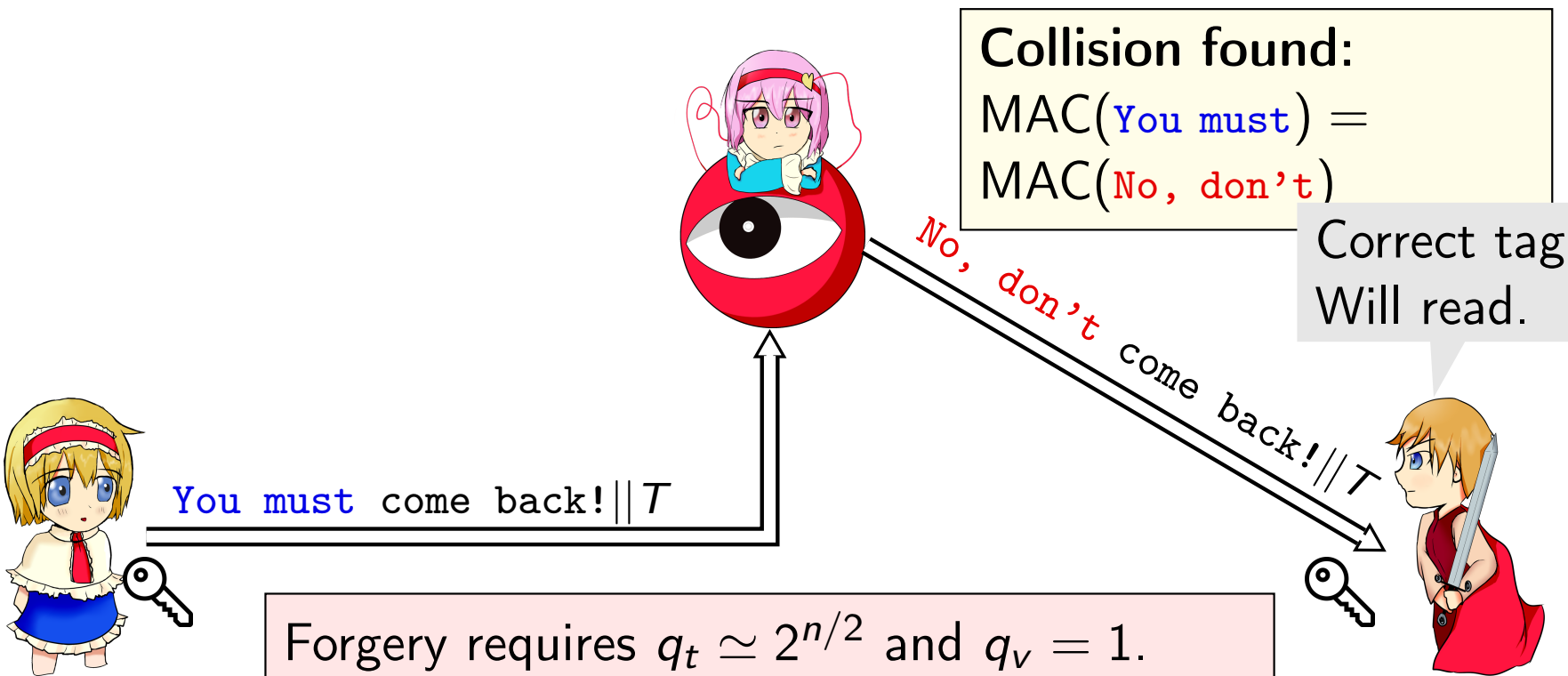


Forgery requires $q_t \simeq 2^{n/2}$ and $q_v = 1$.
Not secure beyond birthday bound ($2^{n/2}$)

Why Beyond Birthday Security?

- BBB security is useful in lightweight cryptography
- Consider the security advantage $\epsilon = 2^{-10}$, $n = 64$ and $\ell = 2^{16}$ blocks.

Construction	Security	# of queries
ECBC	$16q_t^2/2^n$	$\approx 2^{25}$
PMAC	$5\ell q_t^2/2^n$	$\approx 2^{18}$

Table: Data limit of constructions achieving birthday bound security.

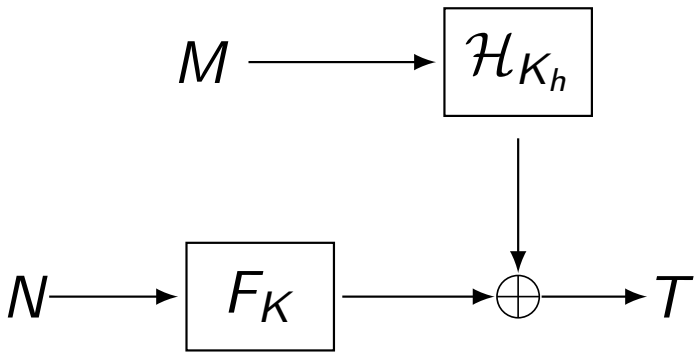
§ BBB security can allow to process larger number of blocks per session key.

Summary So Far

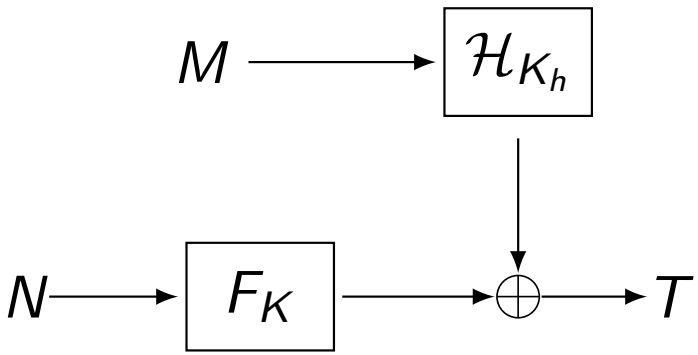
- Forgery Game of Message Authentication Code (MAC)
- Birthday Bound Forgery for ECBC MAC.
- Birthday Bound is not suitable for small block cipher based MAC

§ Coming up: How to get BBB secure MAC constructions.

Wegman Carter MAC [JCSS, 1981]

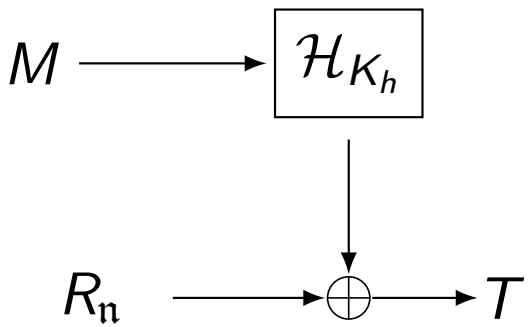


Wegman Carter MAC [JCSS, 1981]



- Nonce based authenticator.
- Proposed by Wegman and Carter in 1981
  - based on a Code of Gilbert, MacWilliams and Sloane [GMS74] – a fresh key of size as large as message

Wegman Carter MAC



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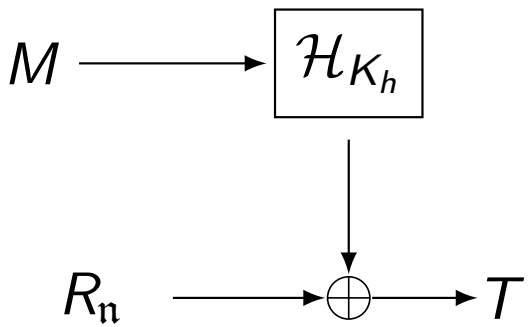


- universal$_2$ is relaxed to a weaker hash AXU in [Kra94/Rog95]
  - $\Pr[H_k(M) \oplus H_k(M') = \delta]$ is small
- polynomial hashing over n bits:
  $\mathrm{Poly}_k(M) := m_d k \oplus \ldots \oplus m_1 k^d$ is $d/2^n$-AXU hash.

Getting rid of one time masking

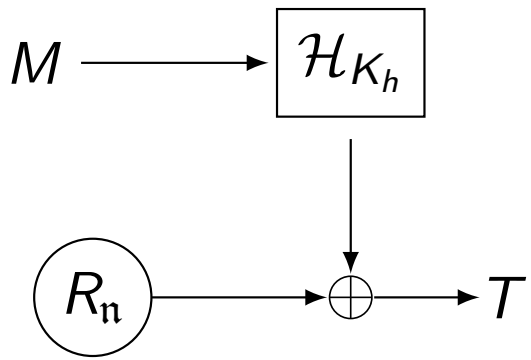
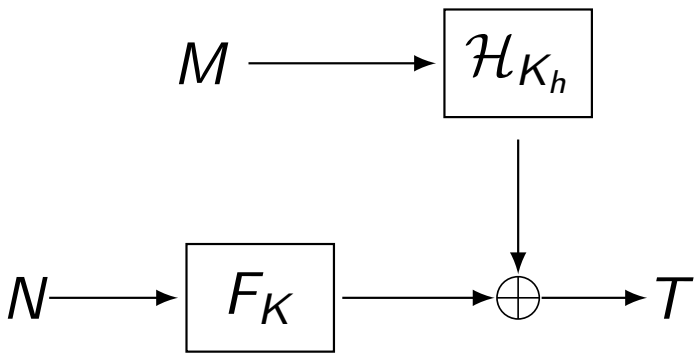


Figure: We can compute $R_{\mathfrak{n}}$ directly from $\mathfrak{n}$ and a secret key.

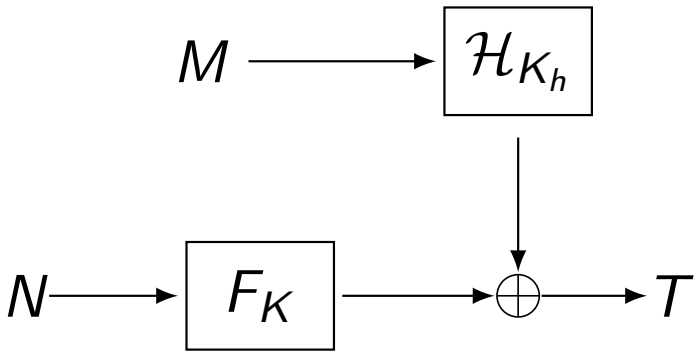
- Use PRBG (Brassard [Bra83]).
- Sequential in nature
  - Direct efficient computation of $R_{\mathfrak{n}}$ (Blum-Blum-Shub PRBG)
  - also modeled as pseudorandom function.

Getting rid of one time masking



- Use pseudorandom function on a unique value nonce.

Wegman Carter MAC [JCSS, 1981]

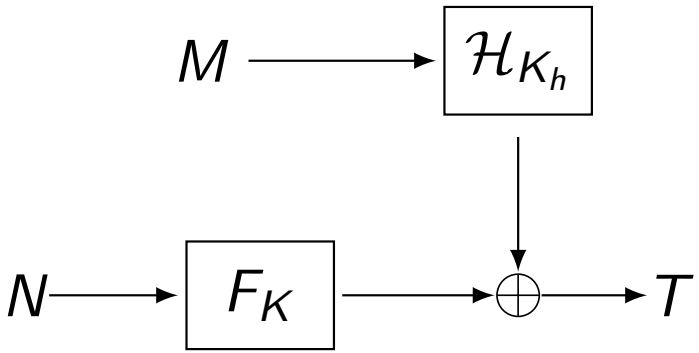


Construction	Security	# of queries
ECBC	$16q_t^2/2^n$	$\approx 2^{25}$
WC (Nonce Respecting)	$q_v\ell/2^n$	$\approx 2^{54}$

Table: The security advantage $\epsilon = 2^{-10}$, $n = 64$ and $\ell = 2^{16}$ blocks.

§ Security of WC relies on a crucial assumption: Nonce cannot repeat.

Nonce Misuse Attack of Wegman Carter MAC



- A makes two queries (N, M) and (N, M') and obtains reply T and T' respectively.
- A makes another query (N', M) and obtains reply T''.
- A forges with $(N', M', T \oplus T' \oplus T'')$.

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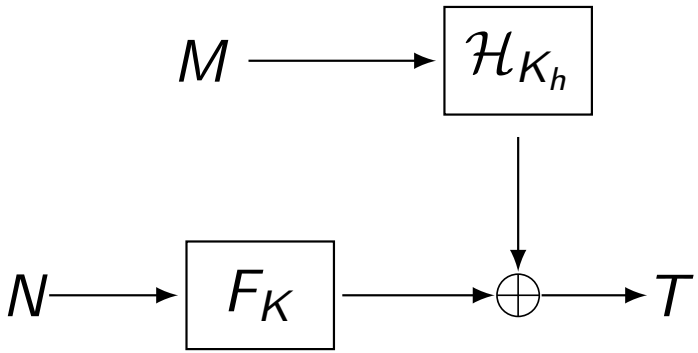

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When $\mathcal{H}$ is Polyhash, then one can recover the hash key and mounts universal forgery.

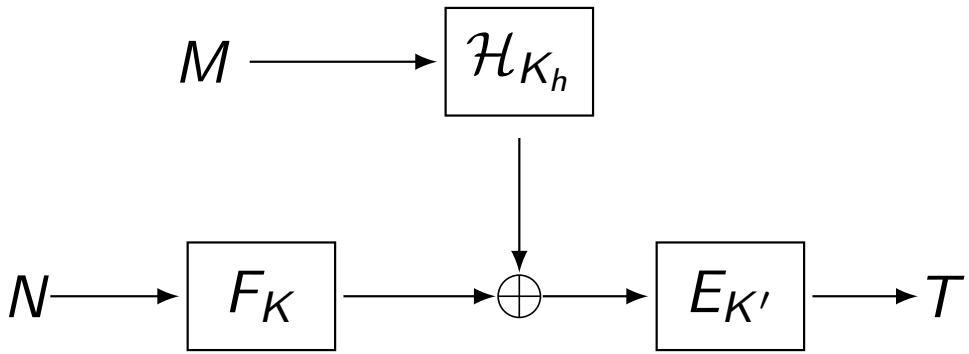
What We Want to Achieve ?

- Maintaining the uniqueness of nonce is difficult in certain applications

What We Want to Achieve ?

- Maintaining the uniqueness of nonce is difficult in certain applications
- Our Goal of this talk:
  § Constructions that give some security (birthday bound) in NM model but gives the beyond birthday bound security in NR model.

Encrypted Wegman Carter MAC [Cogliati and Seurin, CRYPTO 2016]



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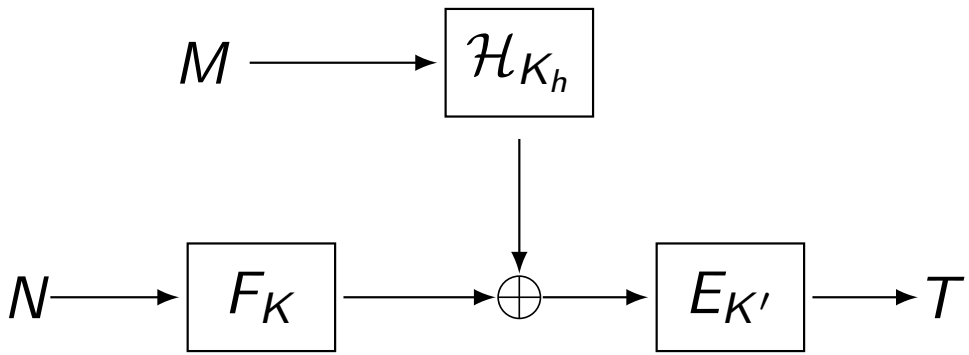
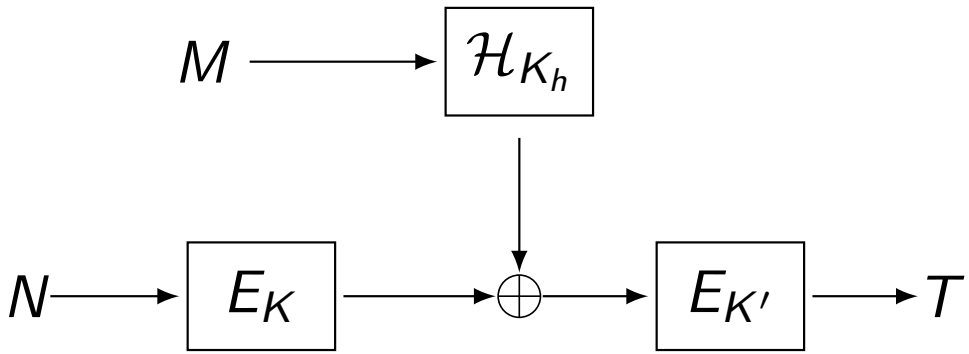


- Nonce Respecting (NR): Same security as Wegman Carter MAC in NR case
- Nonce Misuse (NM): Birthday Bound security

Instantiation of F_K

1. Encrypted Wegman-Carter-Shoup : $F_K \to E_K$



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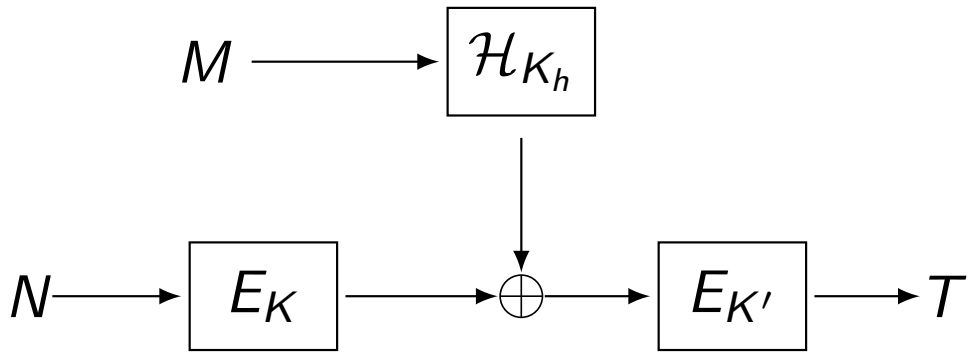
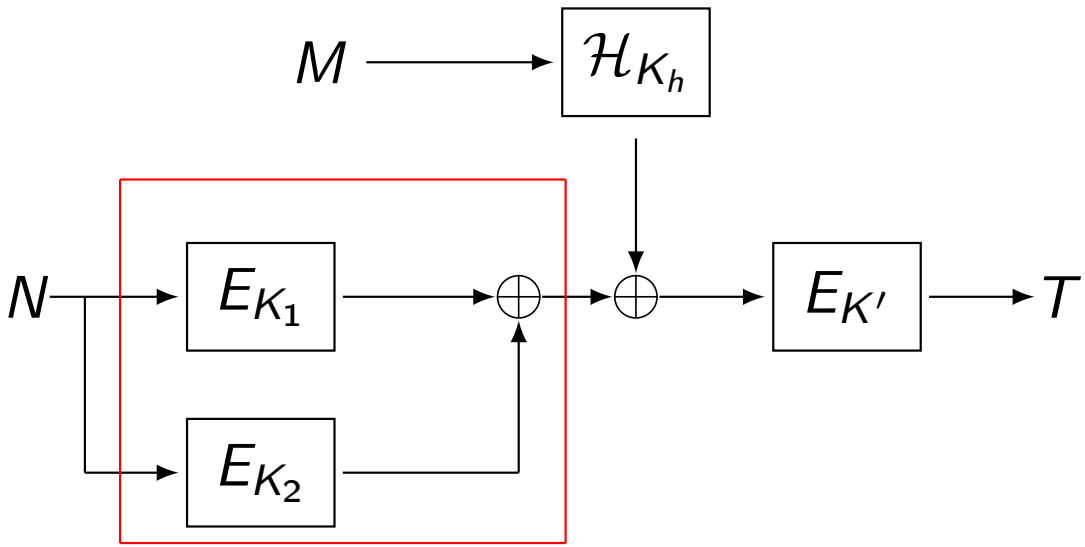


- Nonce Respecting (NR): Birthday Bound Security (distinguishing attack)
- Nonce Misuse (NM): Birthday Bound security
- Block cipher is more popular (can we use block cipher E_K as F_K?).

Instantiation of F_K

2. Encrypted Wegman-Carter with Sum of Permutation :
$F_K(N) \to E_{K_1}(N) + E_{K_2}(N)$



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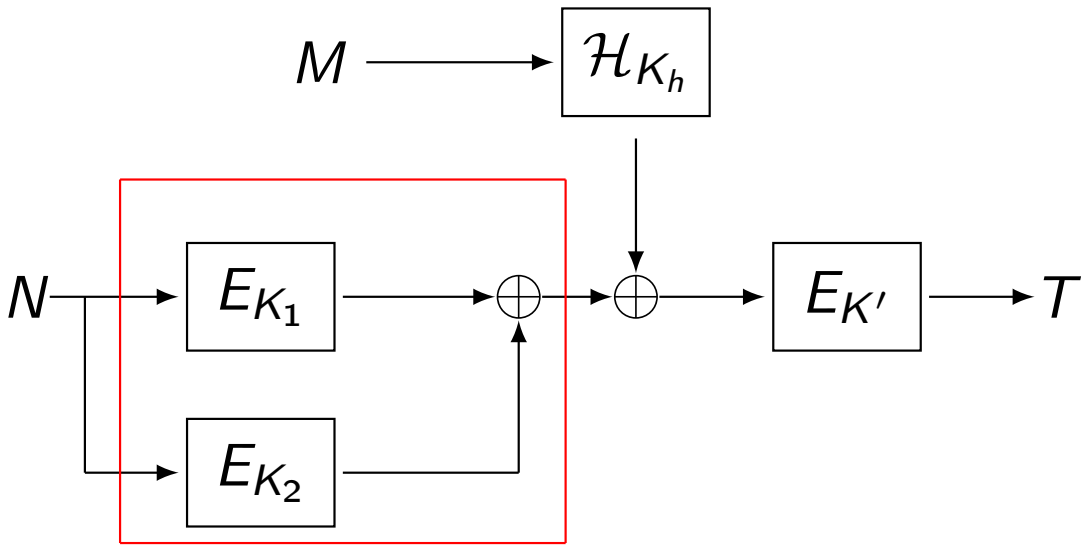

- Nonce Respecting (NR) : Beyond Birthday Bound Security.
- Nonce Misuse (NM) : Birthday Bound Security.

Instantiation of F_K

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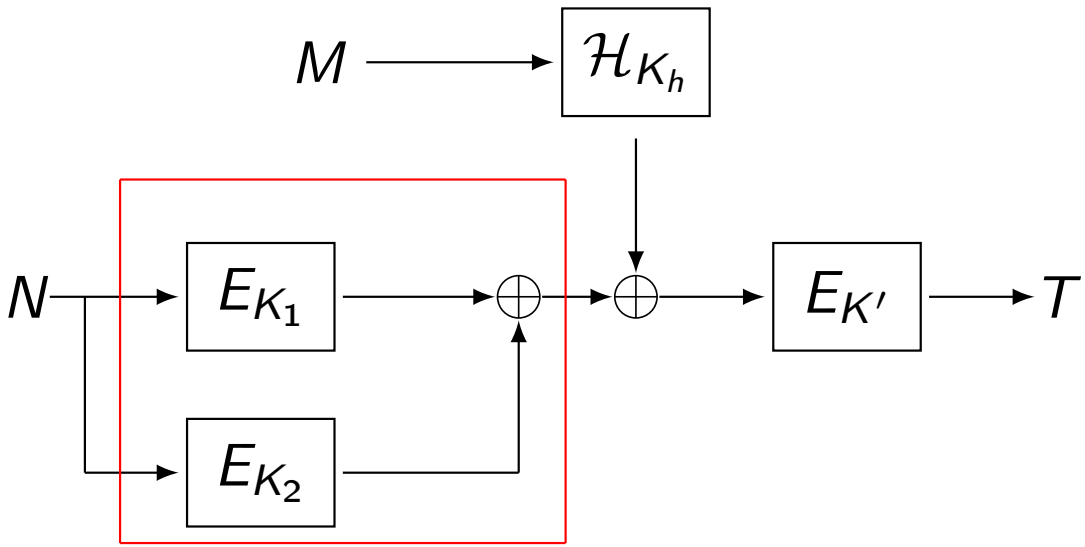
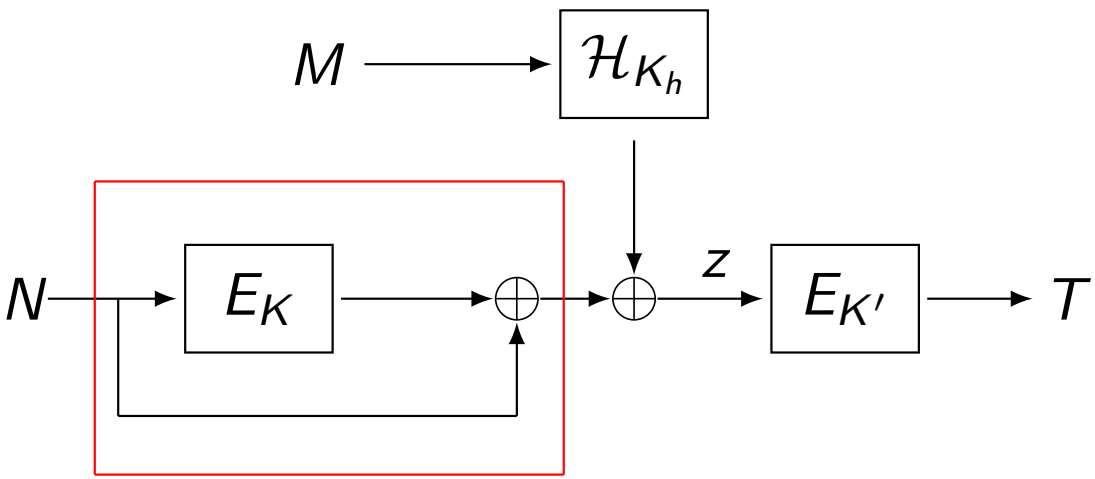


- Nonce Respecting (NR) : Beyond Birthday Bound Security.
- Nonce Misuse (NM) : Birthday Bound Security.

Can we reduce the number of BC calls?

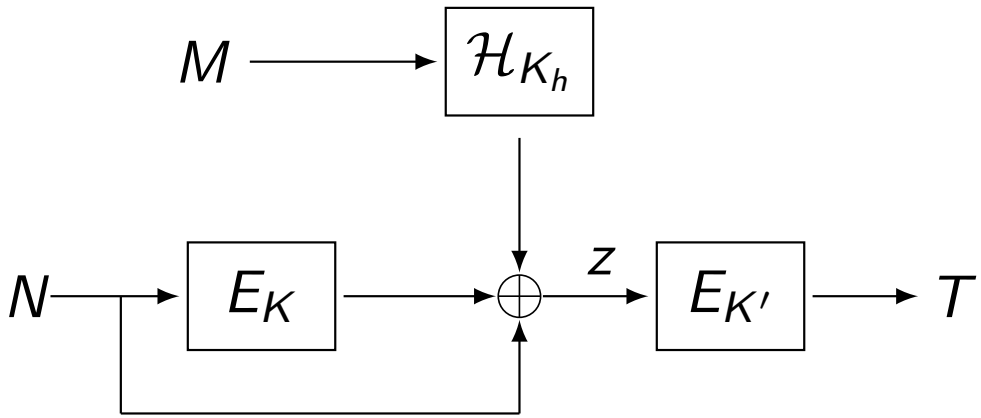
Instantiation of F_K

3. Encrypted Wegman-Carter with Davies-Meyer :
$F_K(N) \to E_K(N) + N$



Instantiation of F_K by Keyed Davies-Meyer Construction

EWCDM [Cogliati and Seurin, CRYPTO 2016]



Conjecture of Cogliati and Seurin

Single keyed EWCDM (i.e $K = K'$) is BBB Secure against NR adversaries.

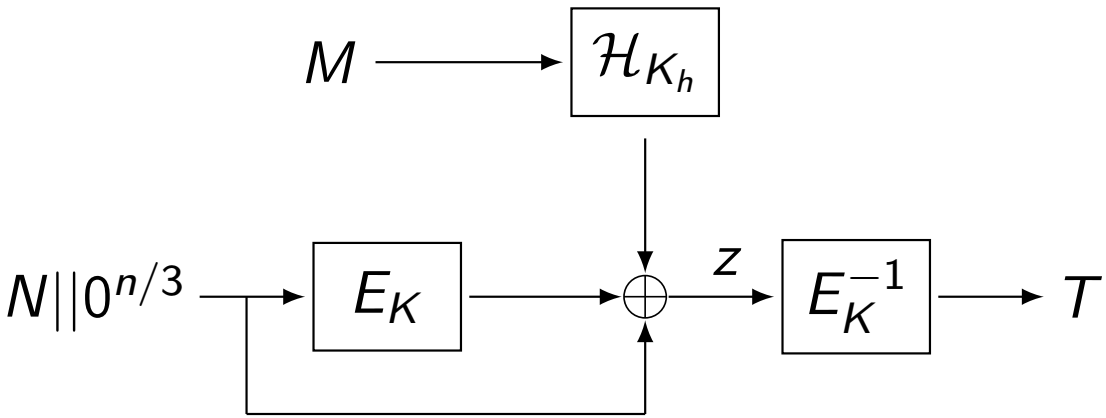
Summary So Far

- Birthday Bound Forgery for ECBC MAC.
- WC is completely broken for a single repetition of nonce.
- EWCDM has the following features:
  1. Based on block cipher (pseudorandom permutation)
  2. BBB security against NR adversaries
  3. Birthday bound security against NM adversaries
  4. But, it has three keys!!

§ Coming up: How to get Single Keyed BBB secure MAC.

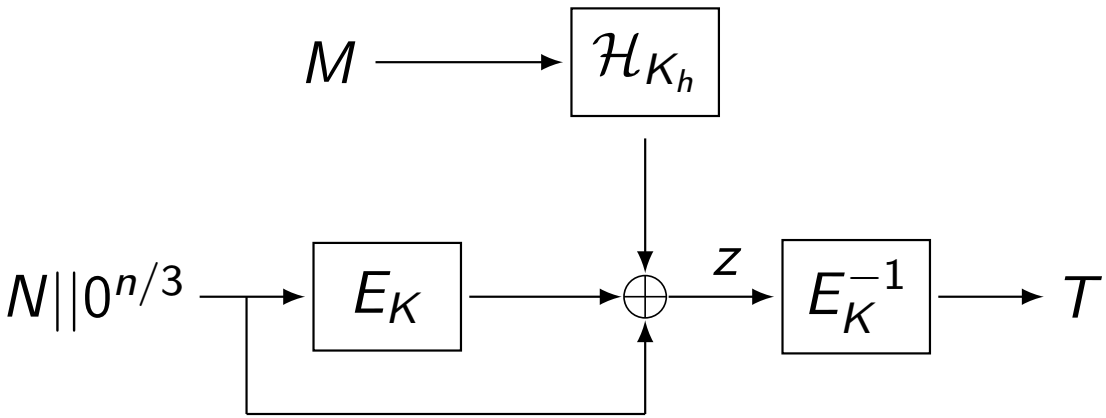
1. Decrypted Wegman-Carter with Davies-Meyer (DWCDM)

DWCDM



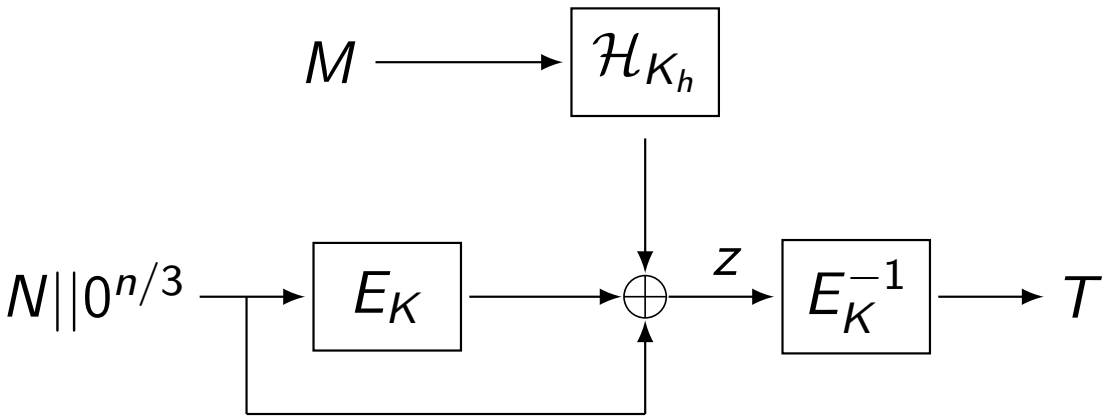
- Single Keyed Nonce Based MAC (Nonce Space: $2n/3$ bits)

DWCDM



- Single Keyed Nonce Based MAC (Nonce Space: $2n/3$ bits)
  why $2n/3$ bits ?

DWCDM

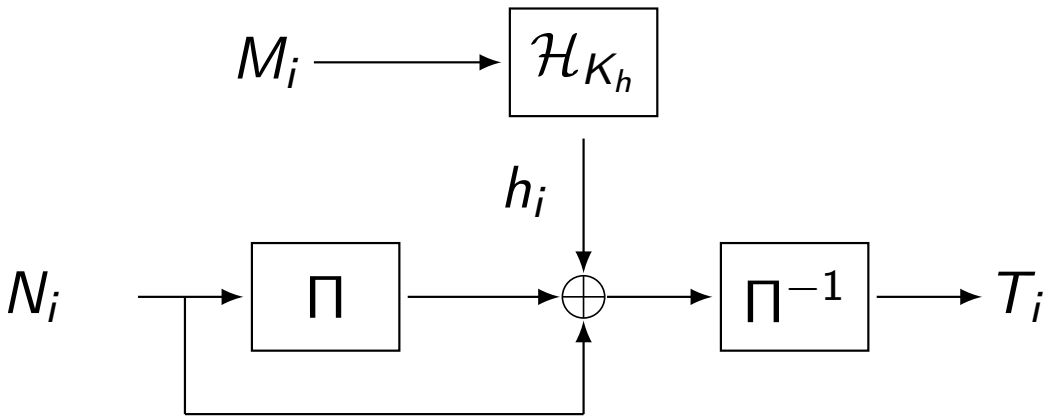


- Single Keyed Nonce Based MAC (Nonce Space: $2n/3$ bits) why $2n/3$ bits ?
- MAC security: $2n/3$-bit (NR setting), $n/2$-bit (NM setting)

A Glimpse of Pure Single Keyed DWCDM

- Derive the hash key as $K_h = \mathrm{E}_K(0^{n-1}1)$
- Polynomial hash applied on message
- Provides same level of security of DWCDM

Basic Idea of Proving Security



1. $\Pi(N_i) \oplus \Pi(T_i) = N_i \oplus h_i := \lambda_i$ for all i.
2. Sufficient to show high probability of above event for any fixed T_i and h_i (by fixing the hash key).
3. Want to estimate probability of $\Pi(N_i) \oplus \Pi(T_i) = \lambda_i$.

Analyzing the security of a MAC

1. The probability of $\Pi(n_i) \oplus \Pi(t_i) = n_i \oplus h_i := \lambda_i$ for all i is at least a value close to that of a random function.
2. In other words, want to find number of solutions of $P_{n_i} \oplus P_{t_i} = \lambda_i$ for all i, where $P_{n_i} = \Pi(n_i)$ and $P_{t_i} = Pi(t_i)$.
3. Group all n_i and t_i values depending on equalities. Let $x_1, \ldots, x_r$ denotes the distinct values.
4. Mirror Theory : Need to find number of solutions (P_x) of system of linear equations such that P_x's are distinct.

2. Mirror Theory

Mirror Theory

A system of q equations over $\mathcal{P} = \{P_1, \ldots, P_r\}$ variables.

$$
\begin{aligned}
P_{n_1} \oplus P_{t_1} &= \lambda_1 \\
P_{n_2} \oplus P_{t_2} &= \lambda_2 \\
&\vdots \\
P_{n_q} \oplus P_{t_q} &= \lambda_q
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where n_i's and t_i's represent indices from 1 to r not necessarily distinct.

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where n_i's and t_i's represent indices from 1 to r not necessarily distinct.

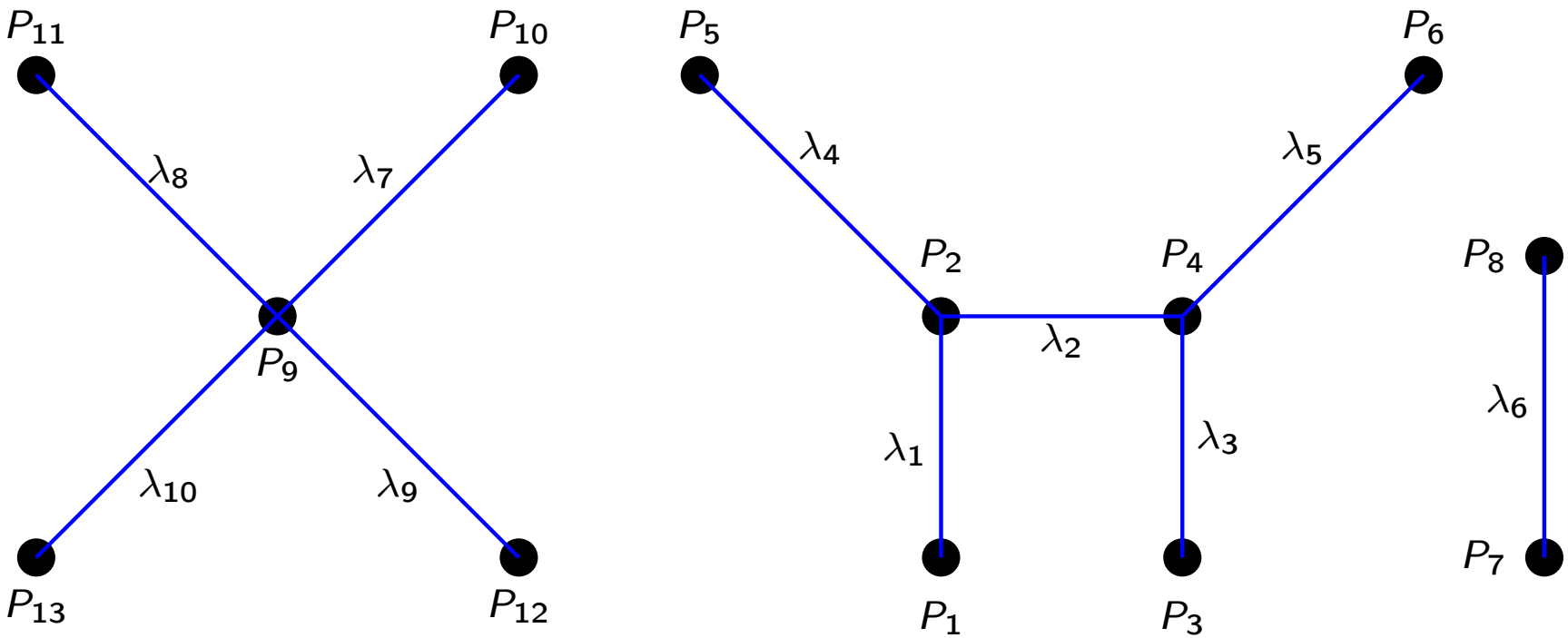
Goal of Mirror Theory

Count the number of solutions of $\mathcal{P}$ such that $P_a \neq P_b$ for $a \neq b \in \{1, \ldots, r\}$.

Current Status of Mirror Theory

- Optimal bound proved by Patarin.
- But the proof is not clear to the community.
- However, the proof is verifiable up to a certain security limit (e.g $3n/4$ in some cases).

Graphical view of Equations

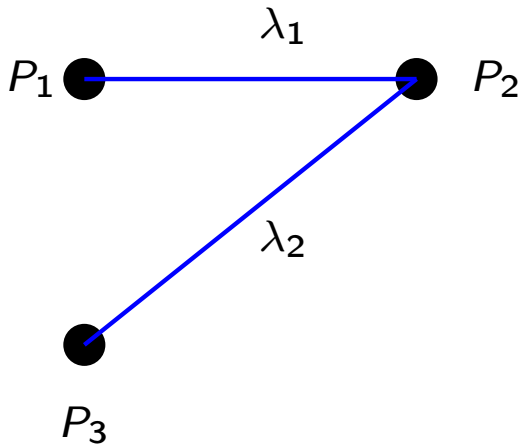


Cycle leads to dependency. So we assume acyclic graph.

Mirror Theory Example 1

$$P_1 \oplus P_2 = \lambda_1$$

$$P_2 \oplus P_3 = \lambda_2$$



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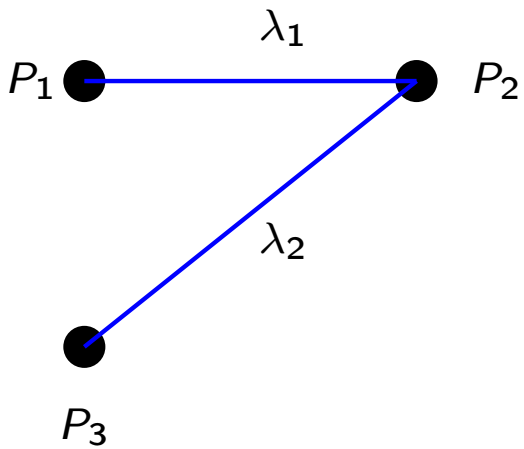

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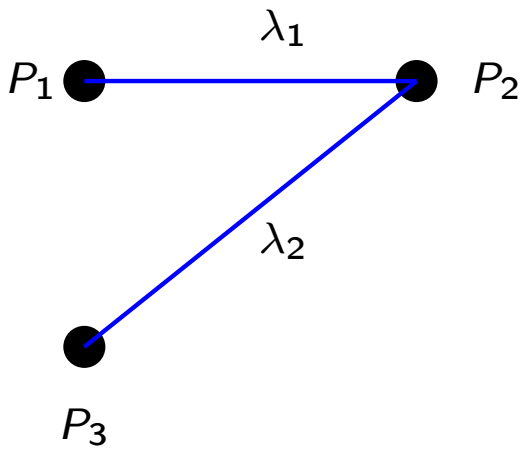

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- if $\lambda_1 \neq 0, \lambda_2 \neq 0, \lambda_1 \neq \lambda_2$, then the # of soln. is 2^n.

Mirror Theory Example 2

$$P_1 \oplus P_2 = \lambda_1$$
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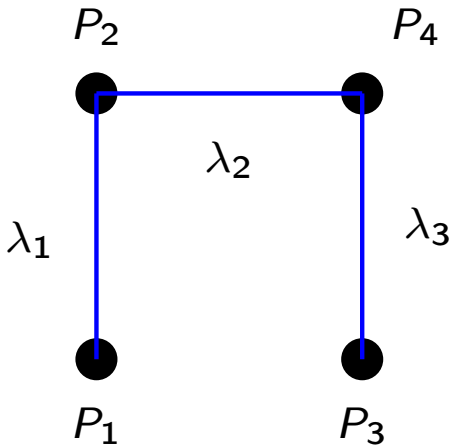

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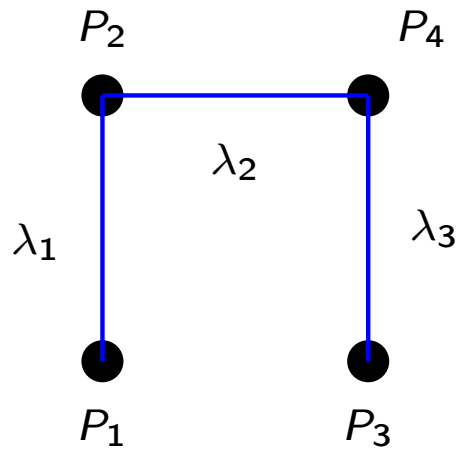

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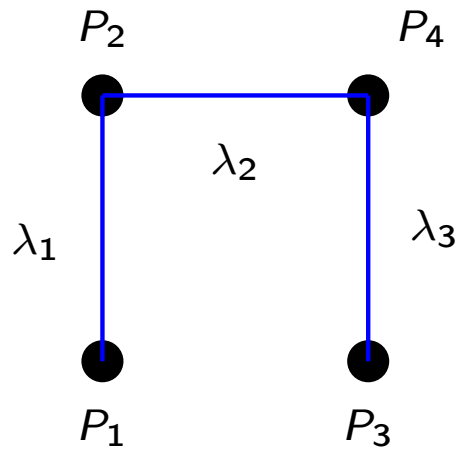

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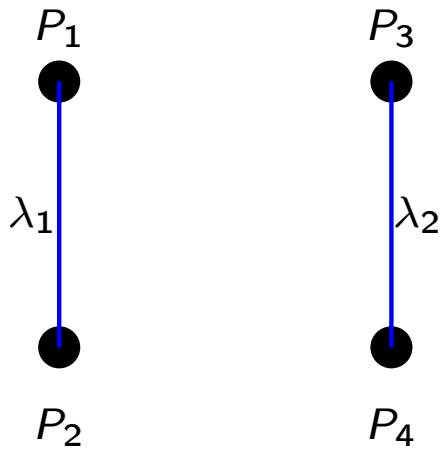
- if no contradiction, then the # of solution is 2^n.

Degeneracy

1. To have a solution the sum of all labels of any path (can be an edge) should be nonzero.
2. If so, the number of solutions for a single component is 2^n.

Mirror Theory Example 3

$$P_1 \oplus P_2 = \lambda_1$$
$$P_3 \oplus P_4 = \lambda_2$$



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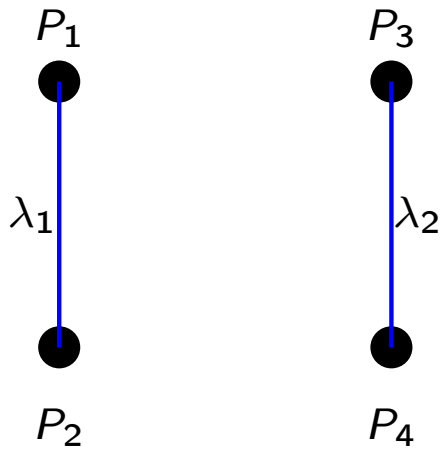

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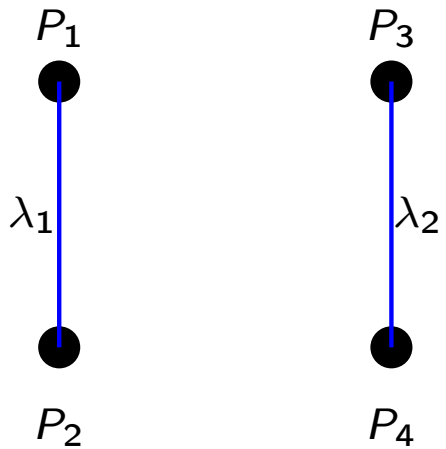

- Contradiction: $\lambda_1 = 0$ or $\lambda_2 = 0$. (Degeneracy)
- if $\lambda_1 \neq 0, \lambda_2 \neq 0$, then the # of soln. is at least $2^n(2^n - 4)$ (why?).

Mirror Theory

In summary, Graph G is bad if

- (P.1) G contains a cycle
- (P.2) There is a path P in G, the sum of label of P is zero.

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Main result (Mirror Theory by Patarin)

If $G = (\mathcal{V}, \mathcal{E})$ is a good graph, where $|\mathcal{V}| = r, |\mathcal{E}| = q$, then the distinct number of solutions is at least

$$\frac{(2^n)_r}{2^{nq}},$$

provided $(\xi_{\max} - 1)^2 \cdot r \leq 2^n/67$.

Mirror Theory

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Main result (Verifiable)

If $G = (\mathcal{V}, \mathcal{E})$ is a good graph, where $|\mathcal{V}| = r, |\mathcal{E}| = q$, then the distinct number of solutions is at least

$$\frac{(2^n)_r}{2^{nq}}(1 - \frac{q^3}{2^{2n}}).$$

MAC and Verification Eqn

MAC Eqn.

$$\Pi(N_1) \oplus \Pi(T_1) = \lambda_1$$
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$$\vdots$$
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Verification Non-Eqn

$$\Pi(N'_1) \oplus \Pi(T'_1) \neq \lambda'_1$$
$$\Pi(N'_2) \oplus \Pi(T'_2) \neq \lambda'_2$$
$$\vdots$$
$$\Pi(N'_{q_v}) \oplus \Pi(T'_{q_v}) \neq \lambda'_{q_v}$$

$$\boxed{\lambda_i = N_i \oplus H_k(M_i), \quad \lambda'_i = N'_i \oplus H_k(M'_i)}$$

Mirror theory can be extended for non-equations also.

Summary

1. Classical constructions are birthday bound secure.
2. Wegman-Carter MAC and its different variants can provide BBB security in NR model.
3. DWCDM: A simple variant of EWCDM (Decryption instead of encryption) is single-keyed.
4. Security analysis relies on Mirror Theory.
5. Better understanding of Mirror Theory and its variants is important for many PRF and MAC Constructions.

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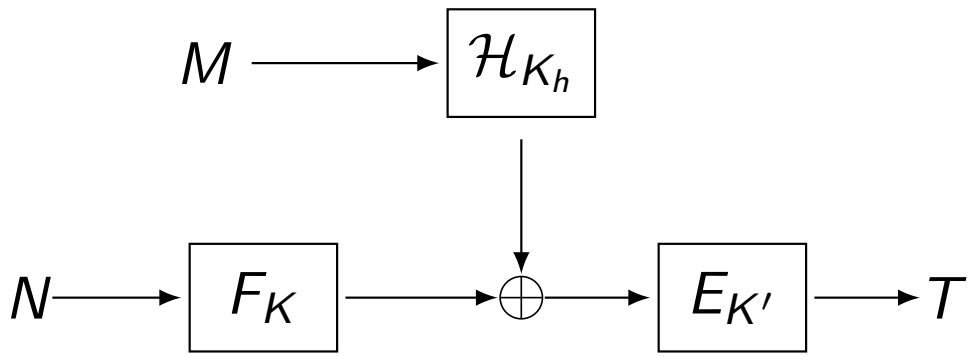
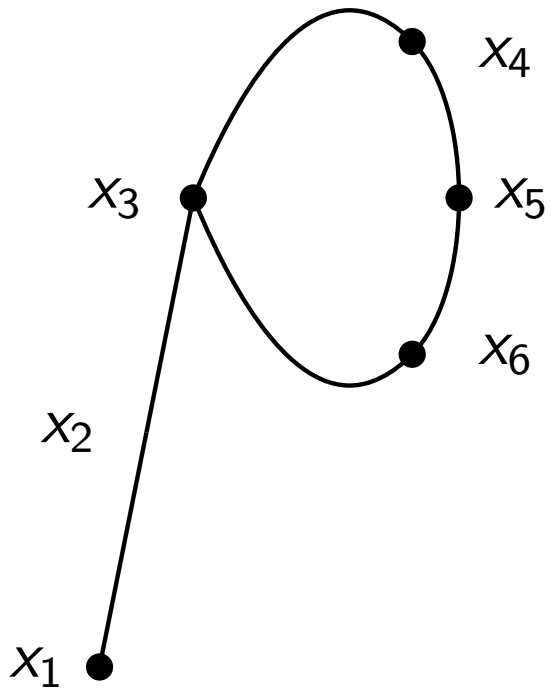
Thank You For Your Attention.

Nonce Misuse Attack on Encrypted Wegman Carter MAC



- A makes $2^{n/2}$ queries (N_i, M) with distinct nonces N_i.
- W.h.p A finds a collision in the tag $(T_i = T_j)$.
- A makes another queries (N_i, M') and obtains reply T
- A forges with (N_j, M', T).

What if Nonce has n bits? (Nonce feedback attack)



$$
\begin{aligned}
\Pi(x_1) \oplus \Pi(x_2) &= H_k(m) + x_1 \\
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$$x_3 + x_4 + x_5 + x_6 = 0$$

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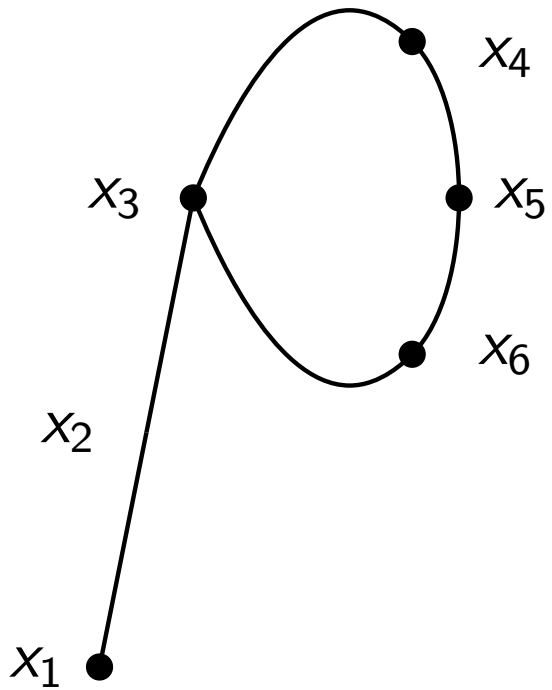

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$$x_3 + x_4 + x_5 + x_6 = 0$$

Forging Event

$(x_i + x_{i+1} + \cdots + x_j = 0) \Rightarrow (x_j, m, x_i)$ is a valid forgery.