Reconsidering Generic Composition: the Tag-then-Encrypt case

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Objective

Obtain an Authenticated Encryption (AE) scheme composing:

- Encryption scheme
  - IV-based (ivE)
  - nonce-based (nE)
- PRF-MAC

Problem Distinguish secure from insecure combinations.

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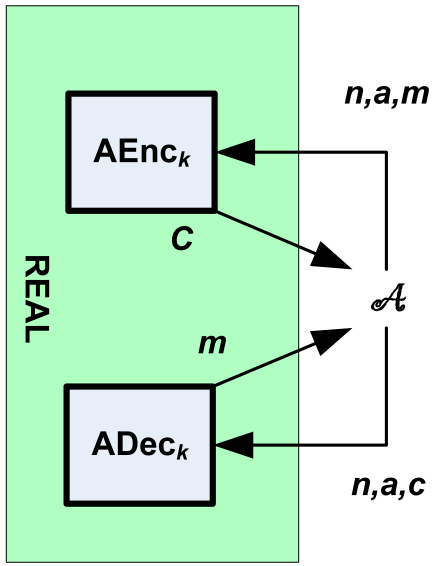
Problem Distinguish secure from insecure combinations.
NRS did most of the job (Eurocrypt 2014)
(160 [ivE] + 20 [nE] possible, 9 + 3 secure)
3+1 elusive in the Tag-then-Encrypt case

Outline

- Background
- Problem
- Results
- Attack on N4

Authenticated Encryption

Objective: Provide privacy and authenticity to a message and associated data (AD) with scheme $\Pi = (\mathcal{K}, \mathrm{AEnc}, \mathrm{ADec})$



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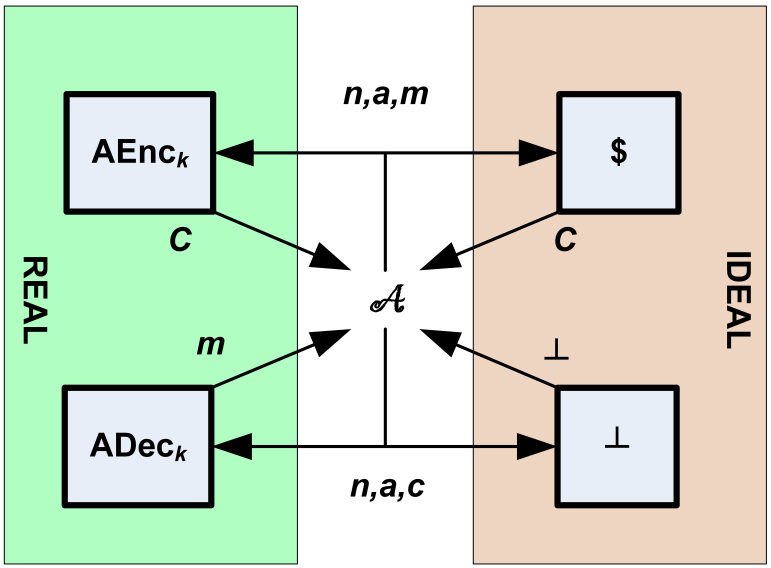

Real and Ideal should be indistinguishable. (nAE)
Not repeat n, if $c \leftarrow \text{AEnc}_k(n, a, m)$ not ask $\text{ADec}_k(n, a, c)$

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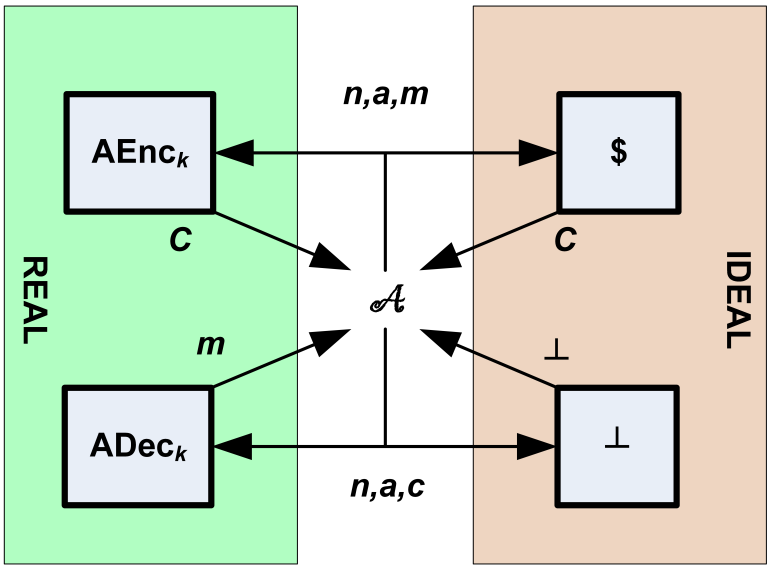

Correctness + Tidiness

$\mathrm{AEnc}_k^{n,a}(m) = c \Leftrightarrow \mathrm{ADec}_k^{n,a}(c) = m$ (Tidiness $\Leftarrow$)

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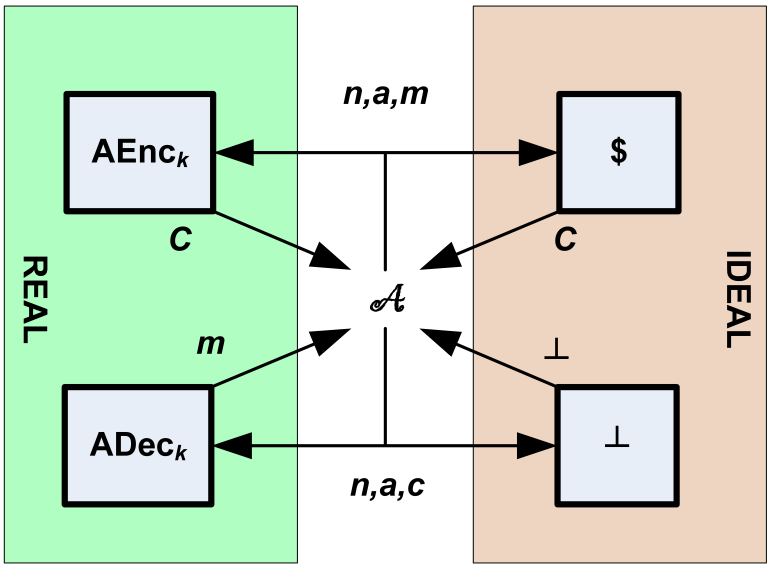

Privacy (nAE-E)
AEnc indistinguishable from \$.

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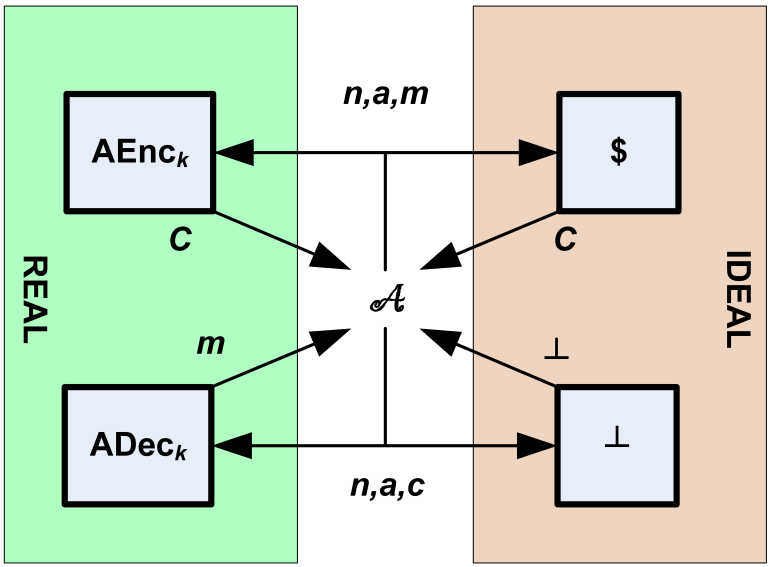

Authenticity

Difficult to provide a *valid and fresh* ciphertext c.

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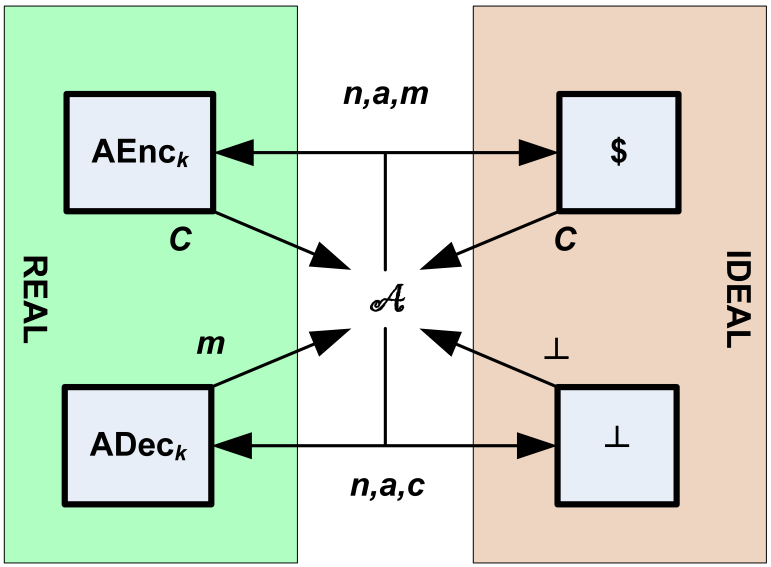
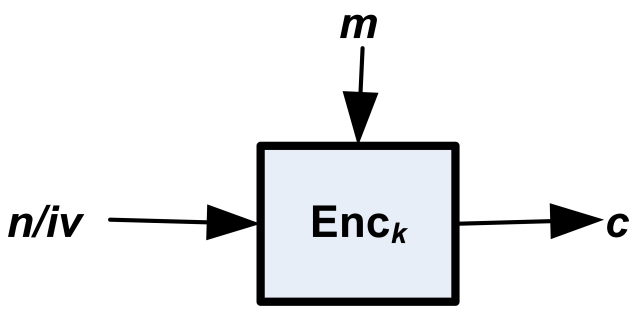


Privacy + **Authenticity** = nAE.

Encryption schemes

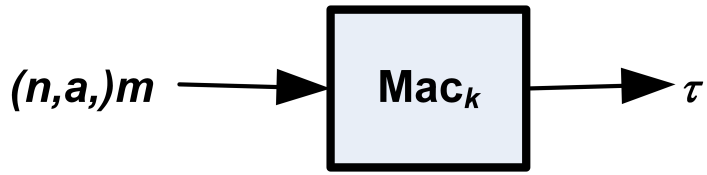
Scheme $\Pi = (\mathcal{K}_E, \mathrm{Enc}, \mathrm{Dec})$:



c should be **indistinguishable** from random

- if *n* is not repeated (nonce) [nE]
- if *iv* is picked randomly (iv) [ivE]

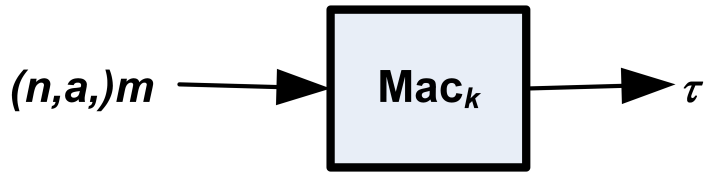
(PRF)-MAC



Security:

- τ should be *indistinguishable* from random (PRF-**security**)

(PRF)-MAC



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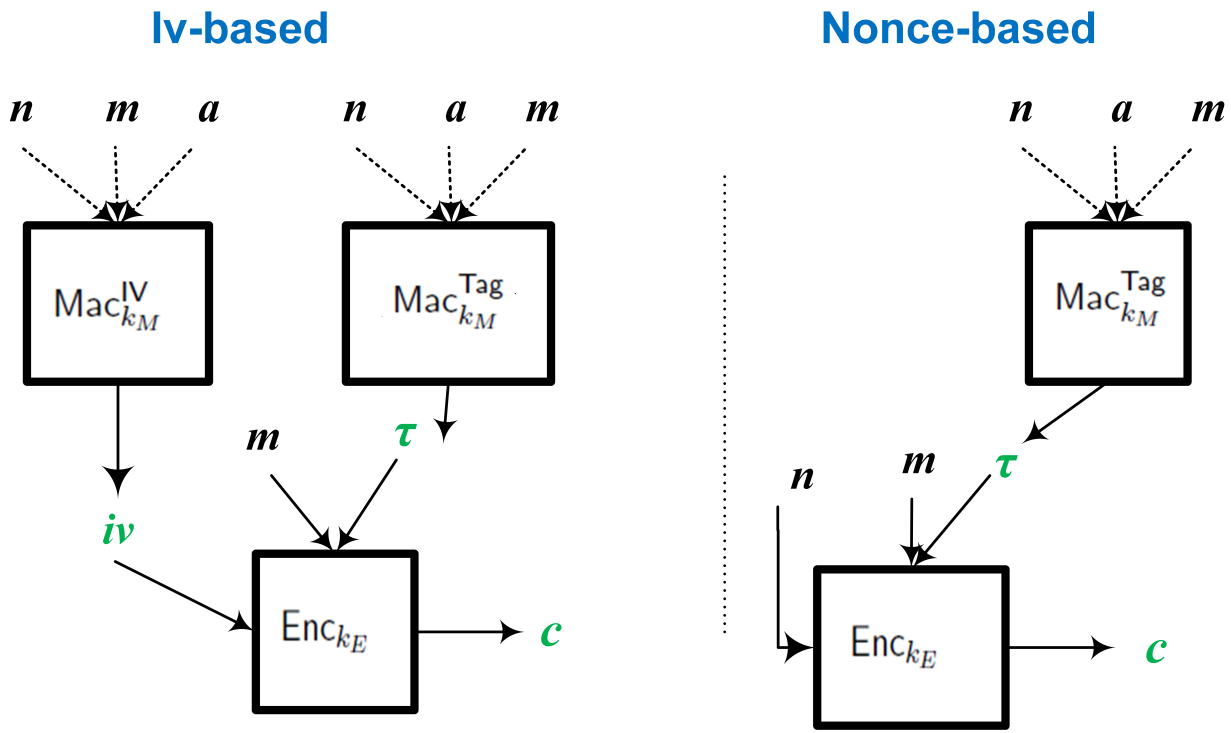
PRF-**security** $\Rightarrow$ (**unforgeability**)

hard to find a fresh valid tag τ

Composition

Many possible ways:
MAC-then-Enc (other Enc-then-MAC and Enc-and-MAC)



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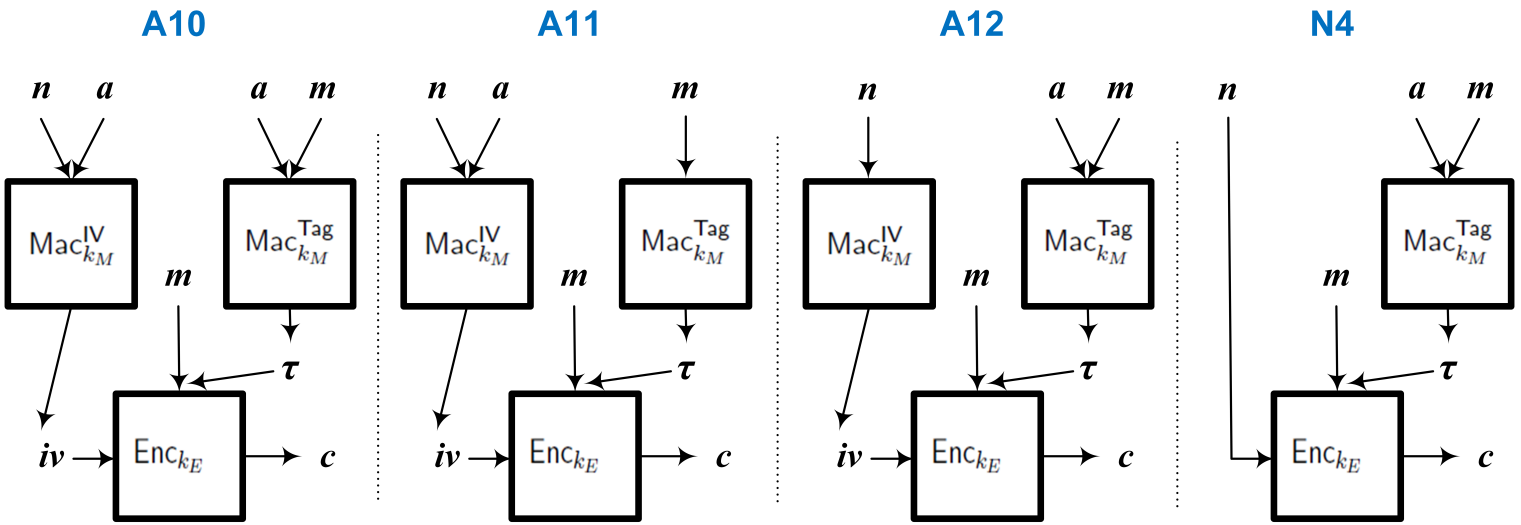
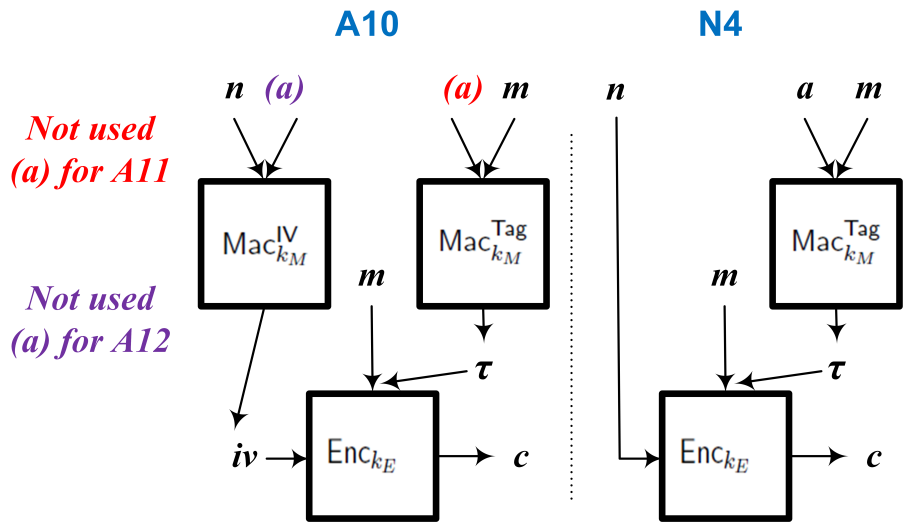
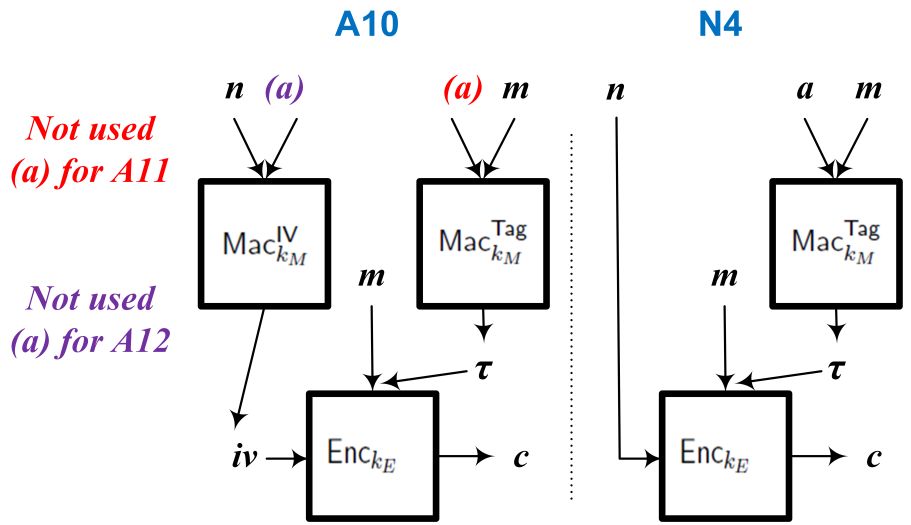




Privacy nAE-E is easy to prove.
Unforgeability Difficult to produce a *valid and fresh* (n, a, c)



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- c is (pseudo)random
- if the iv or the tag τ are fresh then it is unforgeable

Problem When iv and τ are not fresh, it may be used to forge a fresh valid c

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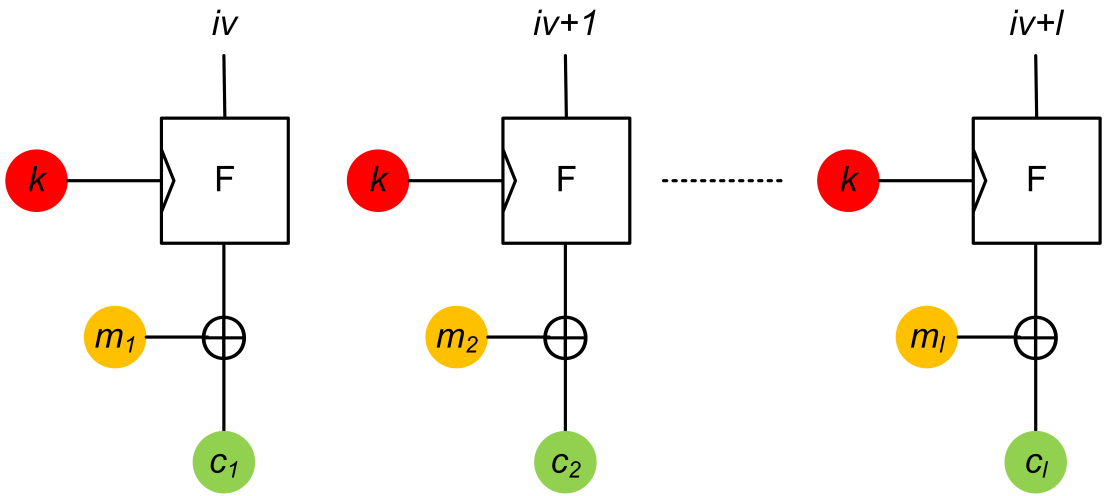
Results

- A10, A11, A12 are either all secure or insecure
- A10, A11, A12 are secure if
  - Enc is “message-malleable”
  - Enc is “misuse-resistant”
- A10, A11, A12 are insecure if
  - Enc is stateful
  - Enc is not tidy
- N4 is insecure

A10 with Message malleability

A10 is secure if ivE is message malleable. Example CTR



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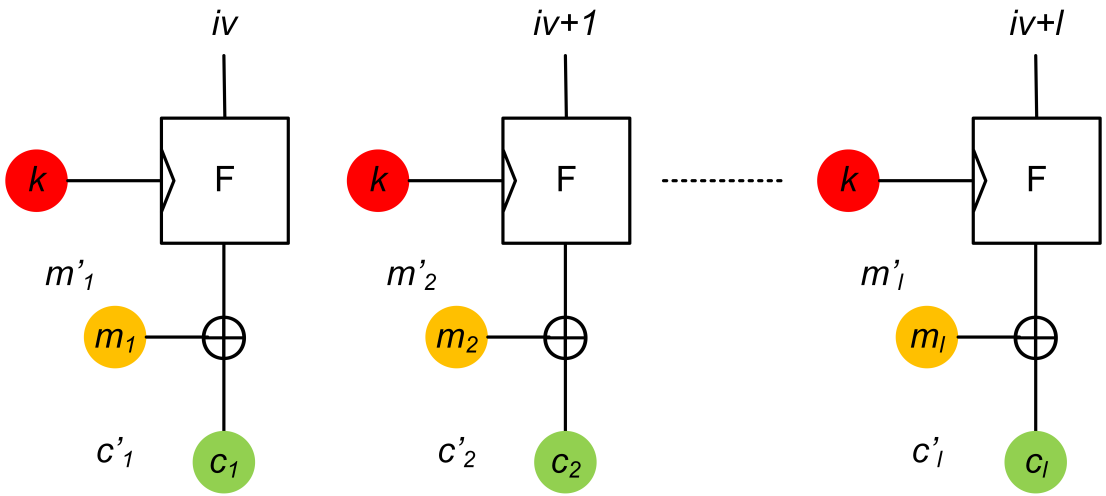

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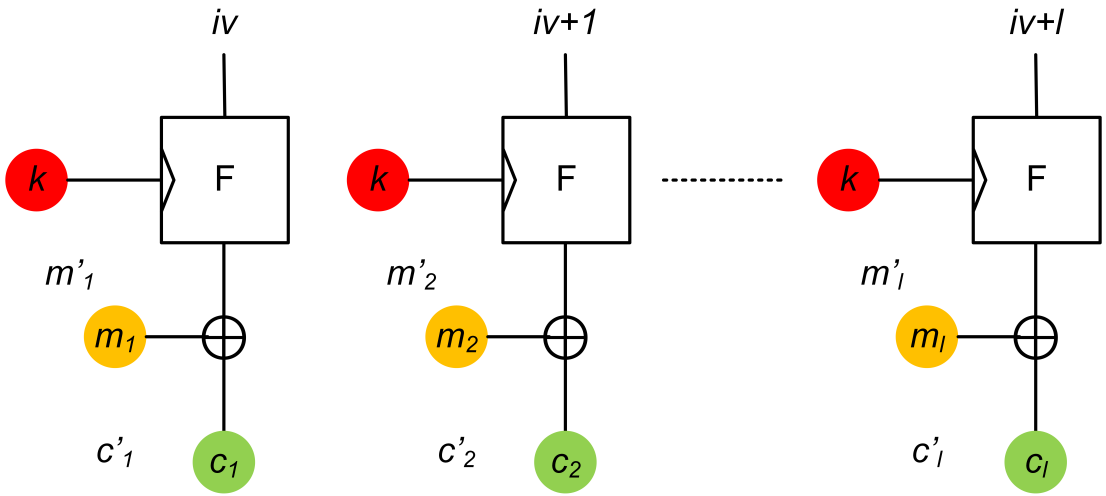

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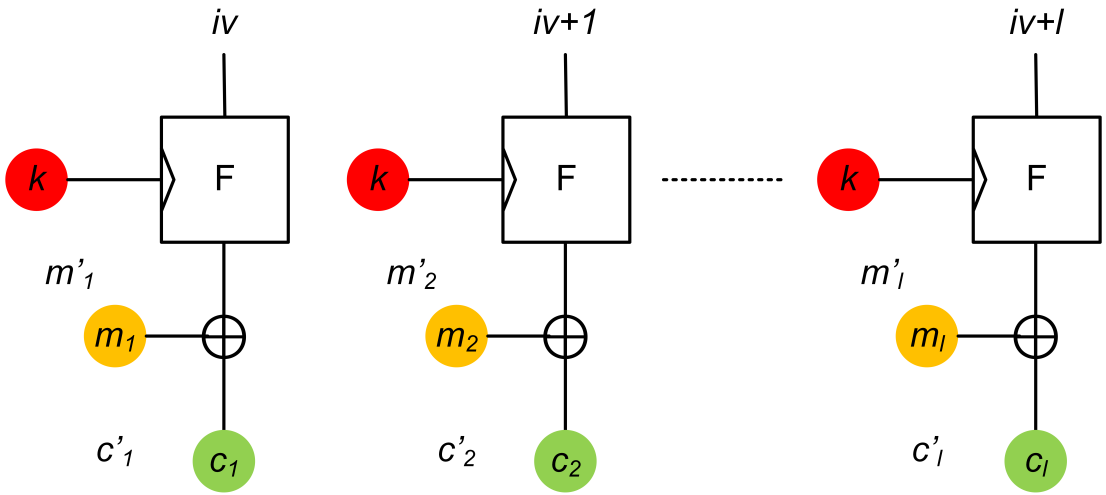

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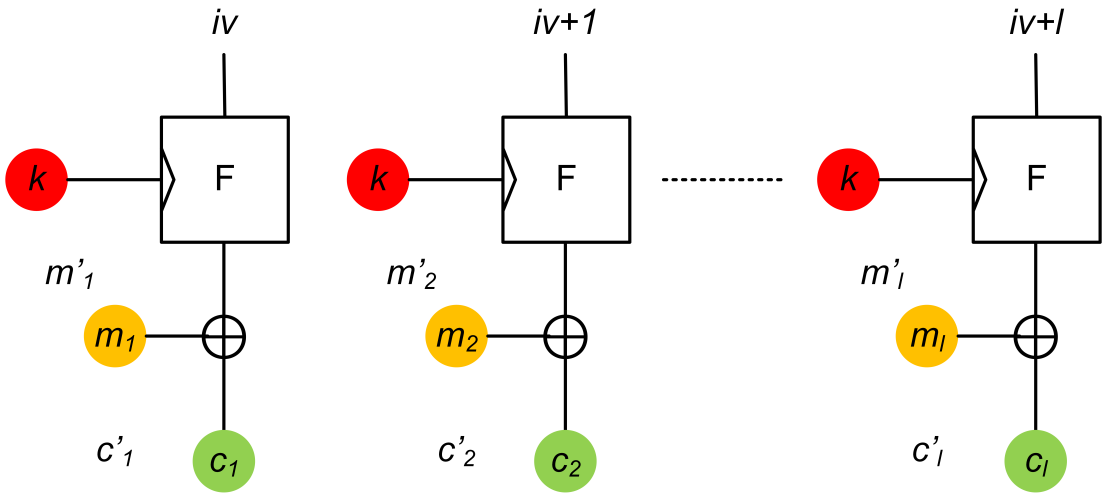

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Rember the only possible forgery is with iv and τ repeated.

Message-malleability

We can generalize:

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Definition

An ivE scheme is *message malleable* if given (iv, m, c) with $c \leftarrow \mathrm{Enc}^{iv}(m)$ then
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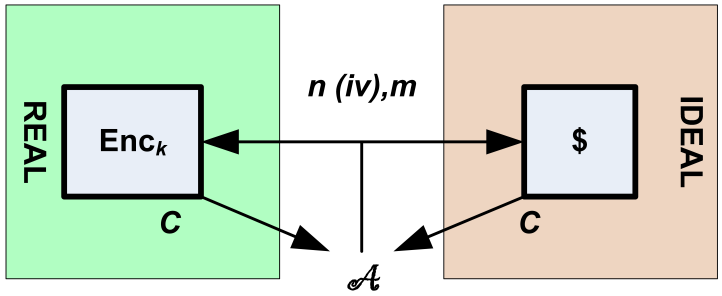
If ivE is message malleable then A10 is secure.

Misuse-resistant



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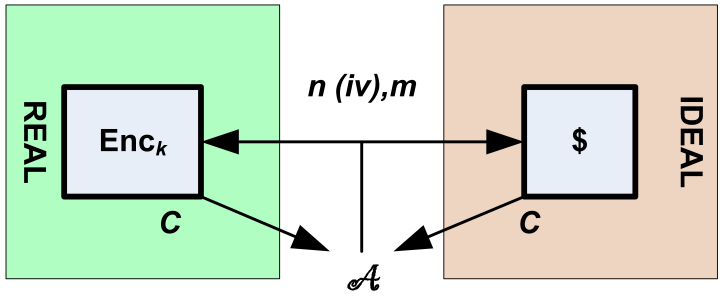

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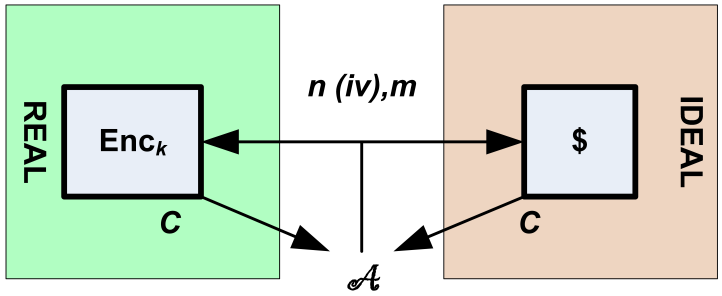

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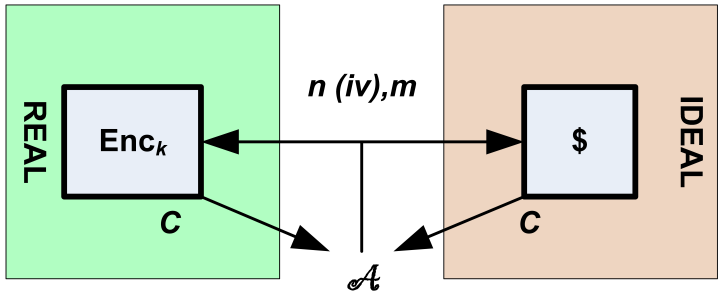

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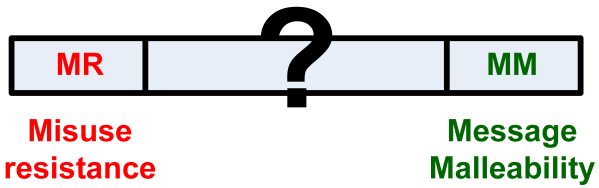

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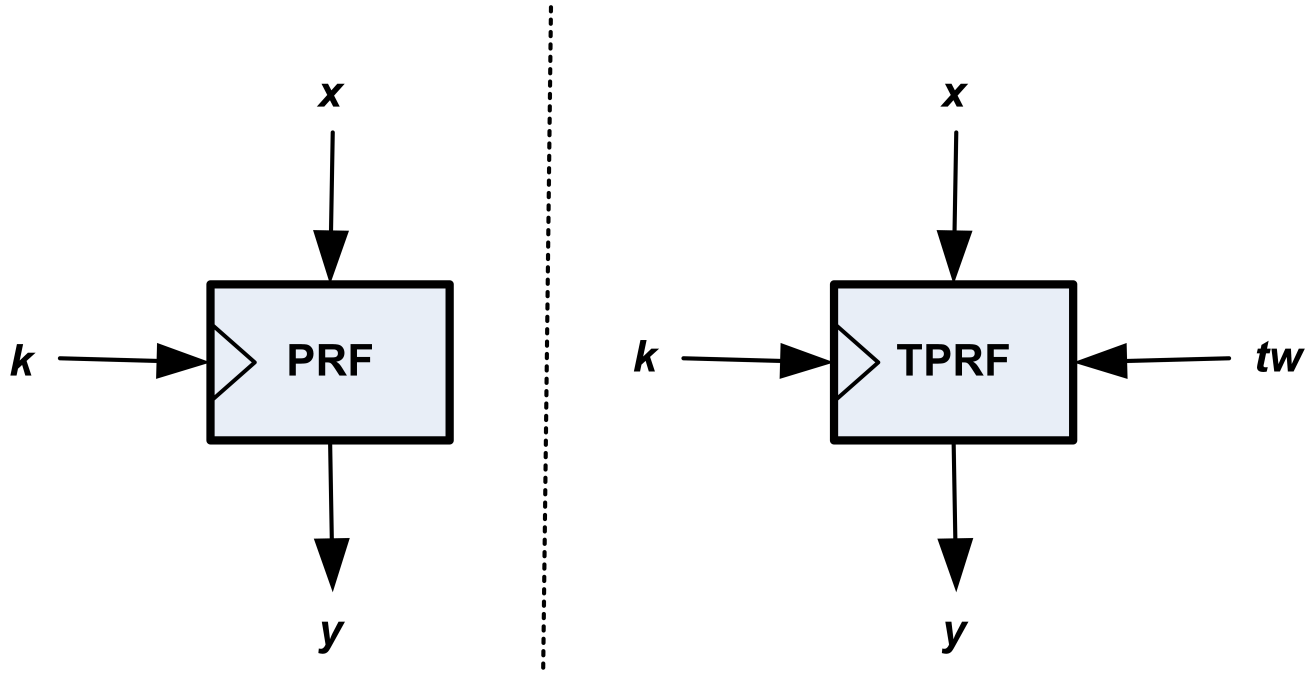
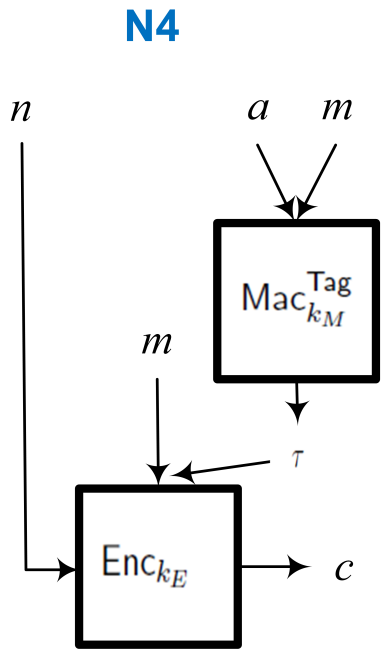
- *Attack vs N4*
  - PRF and TPRF
  - Attack.

PRF / TPRF



The output should be *indistinguishable* from random.

Attack N4 (1/2)



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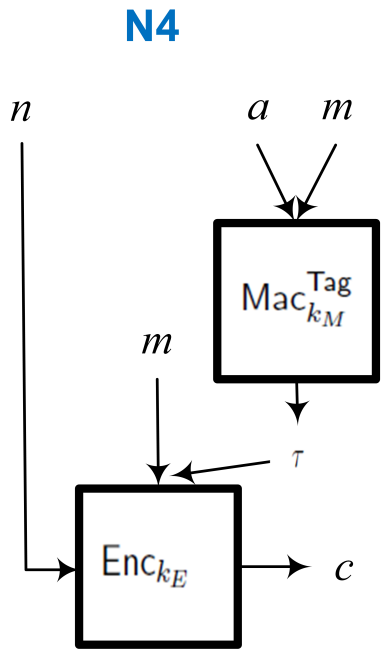

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Let $\mathcal{M} = \{0,1\}^l$ for AEnc, thus Enc $(m \| \tau)$.

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Forgery:

- $(1, a, m)$ obtaining $c^1 = (v^*, c^1_1, c^2_1)$ $[v^*, \mathsf{F}^{1,0}_k(1)]$
- $(2, a, v^*)$ obtaining $c^2 = (c^2_0, c^1_1, c^2_2)$ $[\mathsf{F}^{2,1}_k(v^*) \oplus \tau]$

Output $(1, a, c = [v^*, c^1_1 \oplus m \oplus v^*, c^2_2])$.

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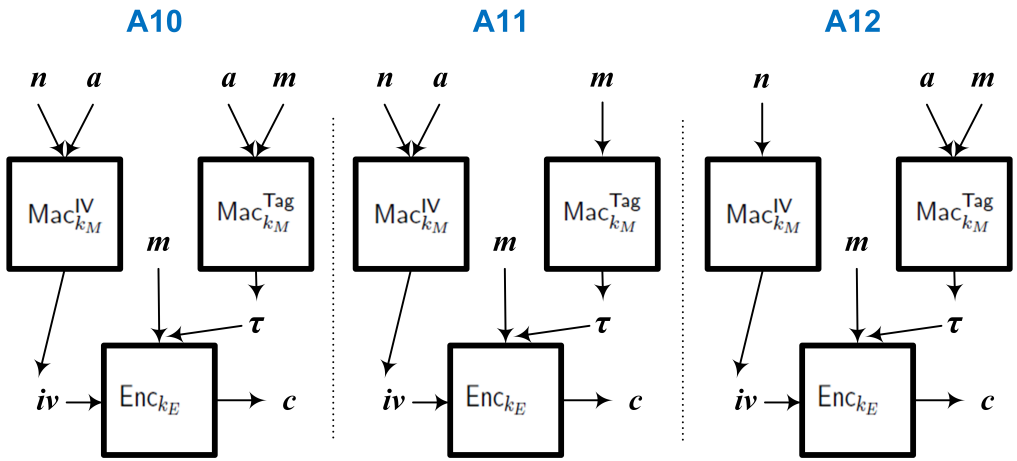
Output $(1, a, c = [\ v^*\ , \mathsf{F}^{1,0}_k(1) \oplus v^*, c^2_2 = \mathsf{F}^{2,1}_k(v^*) \oplus \tau\ ])$.

Observation of the attack

- N4 is secure if message malleable
- N4 is secure if nonce-misuse resistant
- The nE scheme just used is message malleable if the following does not happen:
  - $n = 1$ or 2 AND
  - $m = v^*$

Conclusions

Are A10, A11 and A12 secure?



Stateful generic composition

Questions?

Results

- A10, A11, A12 are either all secure or insecure
- A10, A11, A12 are secure if
  - Enc is “message-malleable”
  - Enc is “misuse-resistant”
- A10, A11, A12 are insecure if
  - Enc is stateful
  - Enc is not tidy
- N4 is insecure

Stateful

- **Stateful** The encryption algorithm uses a *state* which is
  - used only by Enc
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  - used only by Enc
  - kept in memory by Enc after executions
  - updated at every execution
- A10 is not secure if ivE is stateful
  - Attack similar to N4

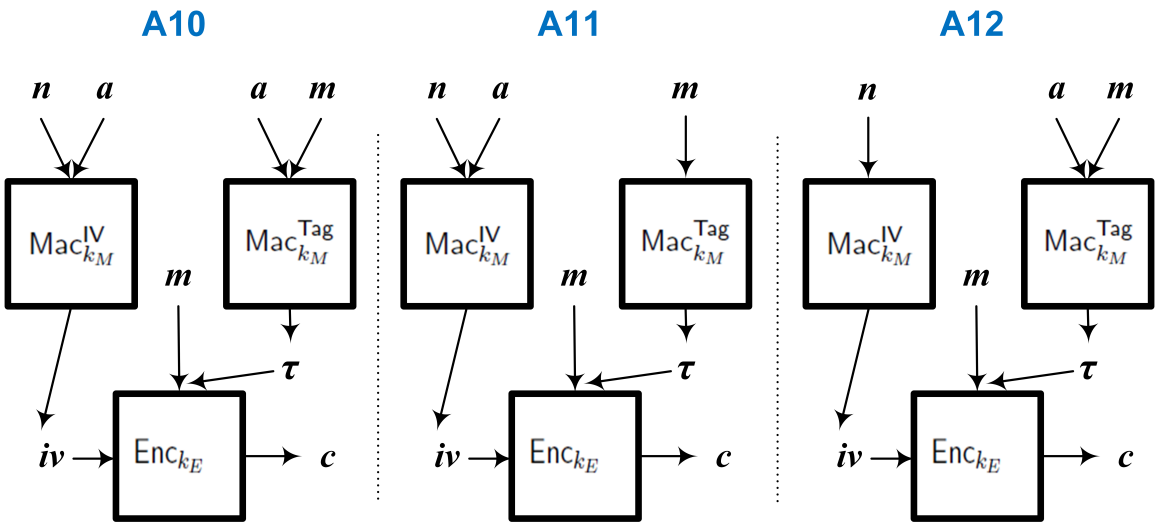
Unitidy

Untidy means

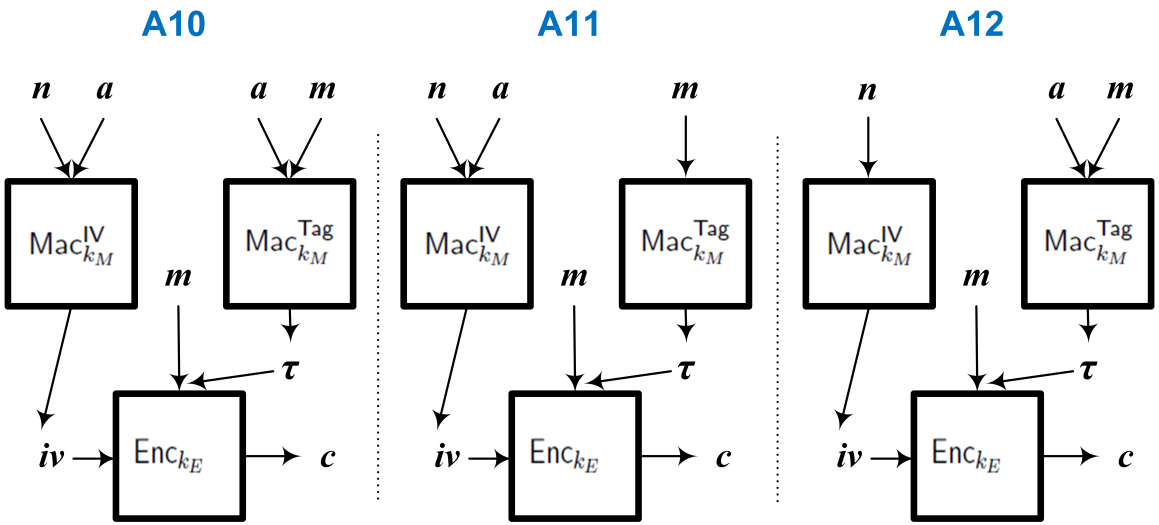
$$\mathsf{Dec}_k(iv, c) = m \nRightarrow \mathsf{Enc}_k(iv, m) = c$$

A10 in this case is not secure:

- $\mathsf{Enc}_k(iv, m) := (\mathsf{Enc}'_k(iv, m) \| \mathsf{Enc}'''_k(iv, m))$
- $\mathsf{Dec}_k(iv, c', c'') = \mathsf{Dec}_k(iv, c')$



We prove that: either all 3 are secure or all 3 are not secure.



We prove that: either all 3 are secure or all 3 are not secure.

Idea:

- Observe that $\mathrm{Mac}^{\mathrm{iv}}(n) = \mathrm{Mac}^{\mathrm{iv}}(n, a)$ since n not repeated.
- Replace n with $n' = n \| \mathrm{H}(a)$
- Replace m with $m' = \mathrm{H}(a) \| m$

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