

>>> Tweakable HCTR: A BBB Secure Tweakable Enciphering Scheme

Name: Avijit Dutta and Mridul Nandi (Indian Statistical Institute, Kolkata, India.)

Date: December 10, 2018

INDOCRYPT, 2018

>>> Outline

- * Introduction
- * HCTR Construction
- * Tweakable HCTR
- * Security Proof

>>> Tweakable Enciphering Scheme

TES is a triplet of three algorithms: $\mathcal{I} = (\text{KGen}, \text{Enc}, \text{Dec})$.

>>> Tweakable Enciphering Scheme

TES is a triplet of three algorithms: $\mathcal{I} = (\text{KGen}, \text{Enc}, \text{Dec})$.

- * Alice and Bob shares a secret key $K \leftarrow \text{KGen}(1^n)$.
- * Alice generates the ciphertext $C \leftarrow \text{Enc}(K, M, T)$ and sends C to Bob.
- * Bob decrypts the ciphertext C to $M \leftarrow \text{Dec}(K, C, T)$.

>>> Tweakable Enciphering Scheme

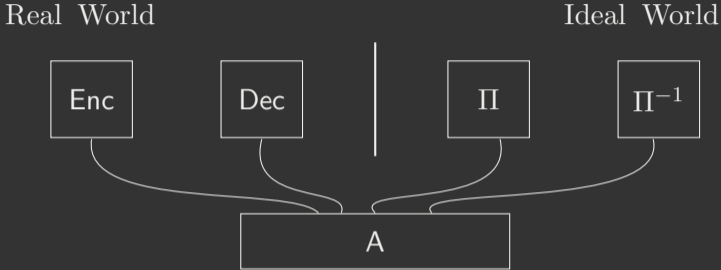
TES is a triplet of three algorithms: $\mathcal{I} = (\text{KGen}, \text{Enc}, \text{Dec})$.

- * Alice and Bob shares a secret key $K \leftarrow \text{KGen}(1^n)$.
- * Alice generates the ciphertext $C \leftarrow \text{Enc}(K, M, T)$ and sends C to Bob.
- * Bob decrypts the ciphertext C to $M \leftarrow \text{Dec}(K, C, T)$.

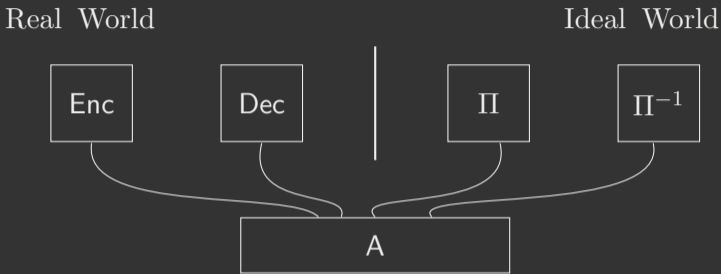
Correctness Condition

$$\forall K, M, T : \text{Dec}(K, \text{Enc}(K, M, T), T) = M.$$

>>> Security Game of TES

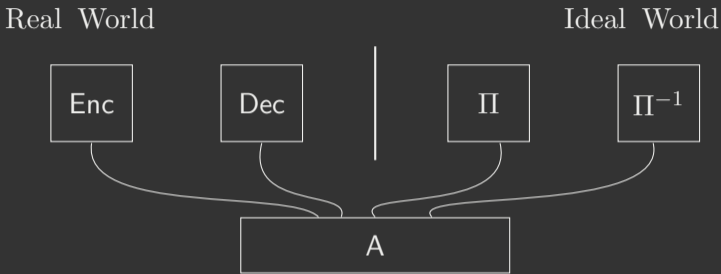


>>> Security Game of TES



$$\text{Adv}_{\mathcal{I}}^{\text{tes}}(A) = | \Pr[A^{\mathcal{I}, \text{Enc}, \mathcal{I}, \text{Dec}} = 1] - \Pr[A^{\Pi, \Pi^{-1}} = 1] |.$$

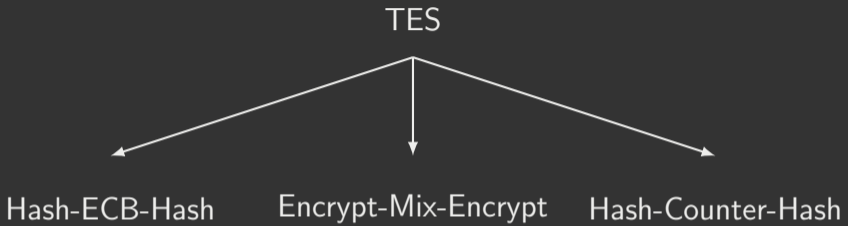
>>> Security Game of TES



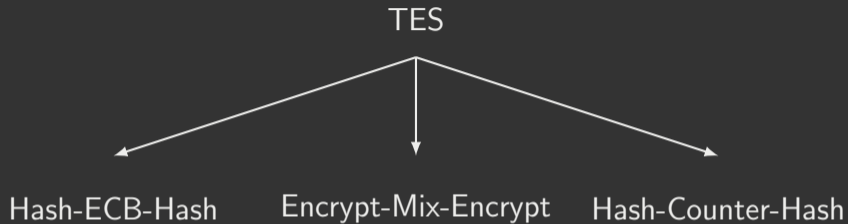
$$\text{Adv}_{\mathcal{I}}^{\text{tes}}(A) = | \Pr[A^{\mathcal{I}.Enc, \mathcal{I}.Dec} = 1] - \Pr[A^{\Pi, \Pi^{-1}} = 1] |.$$

\mathcal{I} is secure against all such computationally bounded adversary A , if the advantage is small.

>>> Types of TES

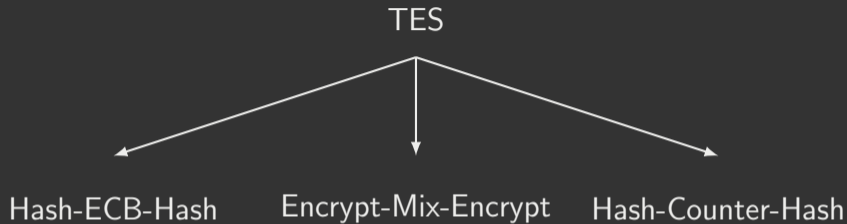


>>> Types of TES



- * I. Hash-ECB-Hash: PEP, TET, HEH.

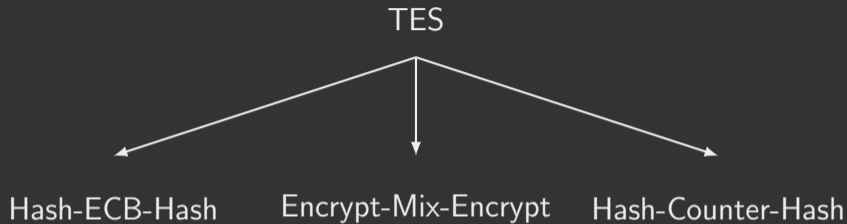
>>> Types of TES



* I. Hash-ECB-Hash: PEP, TET, HEH.

* II. Encrypt-Mix-Encrypt: CMC, EME, EME*, FMix.

>>> Types of TES



- * I. Hash-ECB-Hash: PEP, TET, HEH.
- * II. Encrypt-Mix-Encrypt: CMC, EME, EME*, FMix.
- * III. Hash-Counter-Hash: XCB, HCTR, HCH, FAST.

>>> Comparison Chart for various TES

| Type | Constructions | Prim. Calls | Mult. Calls | Bound |
|------|---------------|------------------|-------------|-----------------|
| I | PEP | $(l + 5)$ | $(4l - 6)$ | $\sigma^2/2^n$ |
| | TET | $(l + 2)$ | $(l - 1)$ | $\sigma^2/2^n$ |
| | HEH | $(l + 2)$ | $2(l - 1)$ | $\sigma^2/2^n$ |
| II | CMC | $(2l + 1)$ | 0 | $\sigma^2/2^n$ |
| | EME | $(2l + 2)$ | 0 | $\sigma^2/2^n$ |
| | EME* | $(2l + l/n + 1)$ | 0 | $\sigma^2/2^n$ |
| | FMix | $(2l + 1)$ | 0 | $q^2 + l^2/2^n$ |
| III | XCB | $(l + 6)$ | $2(l + 1)$ | - |
| | HCTR | l | $2(l - 1)$ | $\sigma^2/2^n$ |
| | HCH | $(l + 3)$ | $2(l - 2)$ | $\sigma^2/2^n$ |
| | FAST (★) | l | $2(l - 3)$ | $\sigma^2/2^n$ |

>>> Comparison Chart for various TES

| Type | Constructions | Prim. Calls | Mult. Calls | Bound |
|------|---------------|------------------------|---------------|--------------------|
| I | PEP | $(\ell + 5)$ | $(4\ell - 6)$ | $\sigma^2/2^n$ |
| | TET | $(\ell + 2)$ | $(\ell - 1)$ | $\sigma^2/2^n$ |
| | HEH | $(\ell + 2)$ | $2(\ell - 1)$ | $\sigma^2/2^n$ |
| II | CMC | $(2\ell + 1)$ | 0 | $\sigma^2/2^n$ |
| | EME | $(2\ell + 2)$ | 0 | $\sigma^2/2^n$ |
| | EME* | $(2\ell + \ell/n + 1)$ | 0 | $\sigma^2/2^n$ |
| | FMix | $(2\ell + 1)$ | 0 | $q^2 + \ell^2/2^n$ |
| III | XCB | $(\ell + 6)$ | $2(\ell + 1)$ | - |
| | HCTR | ℓ | $2(\ell - 1)$ | $\sigma^2/2^n$ |
| | HCH | $(\ell + 3)$ | $2(\ell - 2)$ | $\sigma^2/2^n$ |
| | FAST (*) | ℓ | $2(\ell - 3)$ | $\sigma^2/2^n$ |

* FAST is a PRF based TES.

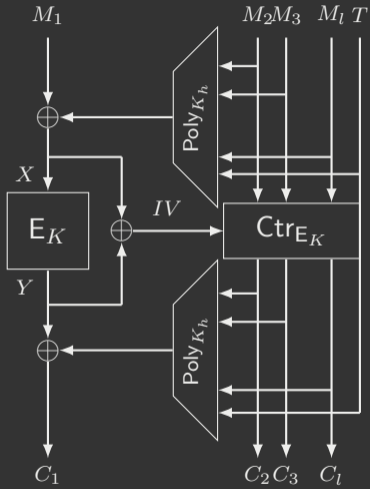
>>> Key Points

- * Only CMC, EME and EME* are based only on block ciphers.
- * Field Multiplication for other constructions.
- * HCTR is the efficient one ¹.
- * Secured upto the birthday bound.

¹Lopez et al., INDOCRYPT 07.

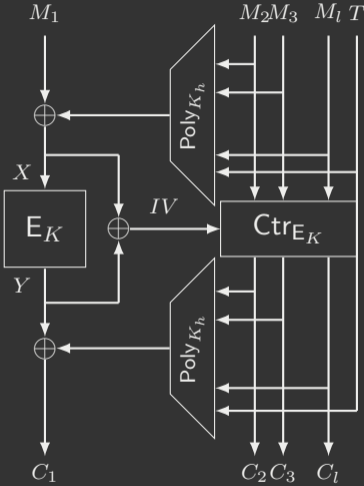
>>> HCTR [Wang et al., CISC 05]

* Most efficient candidate.



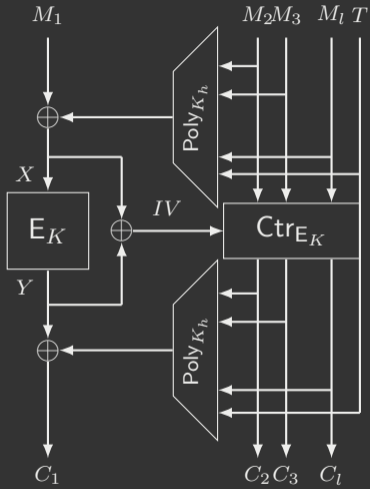
>>> HCTR [Wang et al., CISC 05]

- * Most efficient candidate.
- * Sec. bound $\sigma^3/2^n$ [Wang et al.].



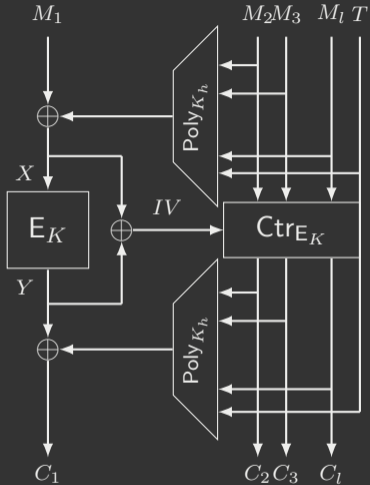
>>> HCTR [Wang et al., CISC 05]

- * Most efficient candidate.
- * Sec. bound $\sigma^3/2^n$ [Wang et al.].
- * Improved bound $\sigma^2/2^n$ [Chakraborty and Nandi, FSE 08].



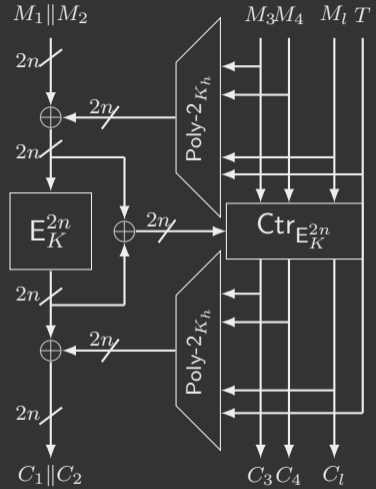
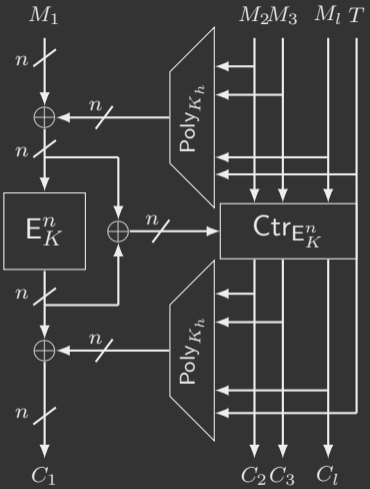
>>> HCTR [Wang et al., CISC 05]

- * Most efficient candidate.
- * Sec. bound $\sigma^3/2^n$ [Wang et al.].
- * Improved bound $\sigma^2/2^n$ [Chakraborty and Nandi, FSE 08].
- * The bound is tight (due to hash collision).



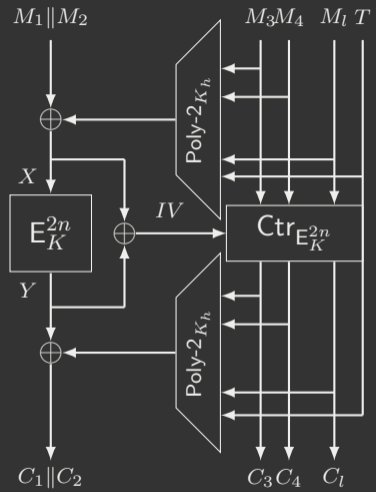
Can we construct a BBB secure variant of HCTR ?

>>> Naive Approach to construct BBB Secure variant of HCTR



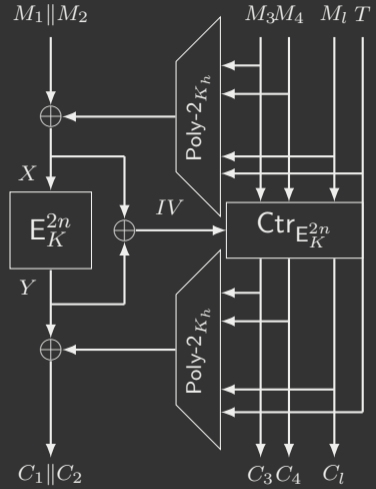
>>> Drawback of the Scheme

* 2 * field mutiplications.



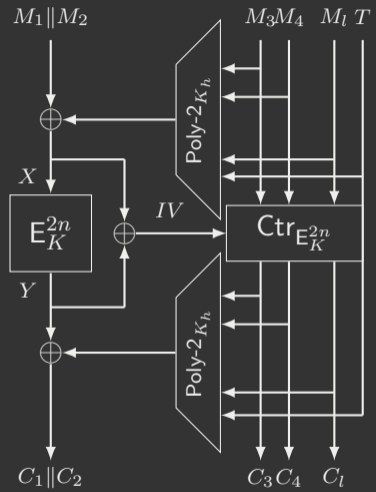
>>> Drawback of the Scheme

- * 2 * field mutiplications.
- * 2 * state size.



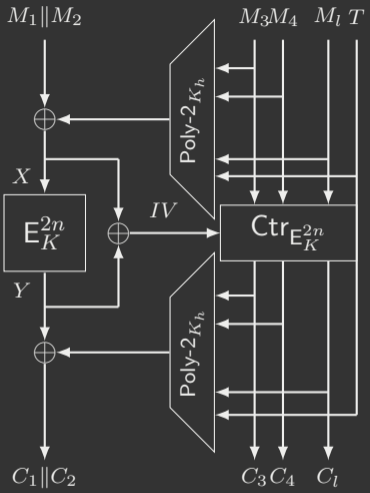
>>> Drawback of the Scheme

- * 2 * field mutiplications.
- * 2 * state size.
- * Optimally secure SPRP requires at least 6 BC calls.



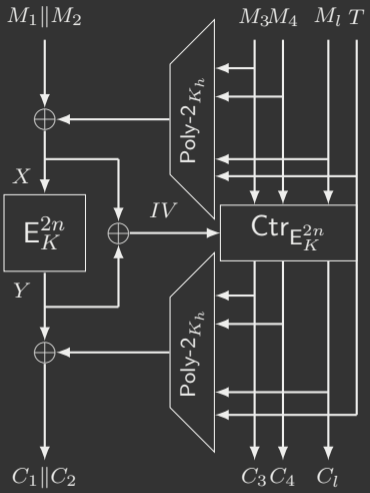
>>> Drawback of the Scheme

- * 2 * field mutiplications.
- * 2 * state size.
- * Optimally secure SPRP requires at least 6 BC calls.
- * Ctr mode requires 2 BC calls per block of message.



>>> Drawback of the Scheme

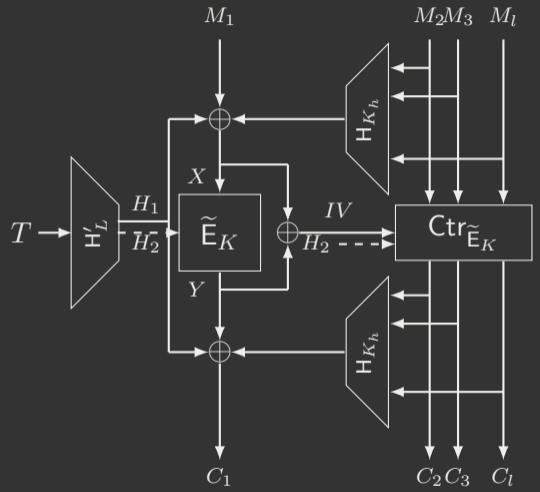
- * 2 * field mutiplications.
- * 2 * state size.
- * Optimally secure SPRP requires at least 6 BC calls.
- * Ctr mode requires 2 BC calls per block of message.



Can we make the scheme BBB secure without increasing state size?

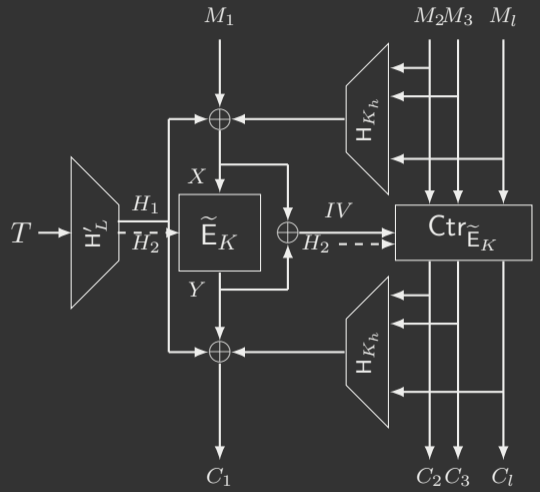
>>> Tweakable HCTR

* BC \rightarrow m-bit TBC.



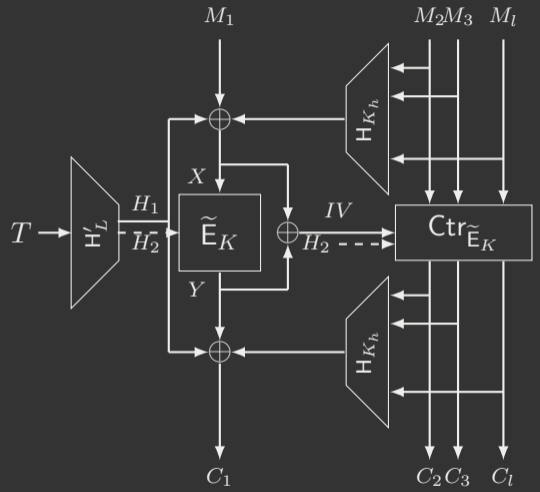
>>> Tweakable HCTR

- * BC \rightarrow m-bit TBC.
- * $H'_L(T)$ for processing variable length tweak.



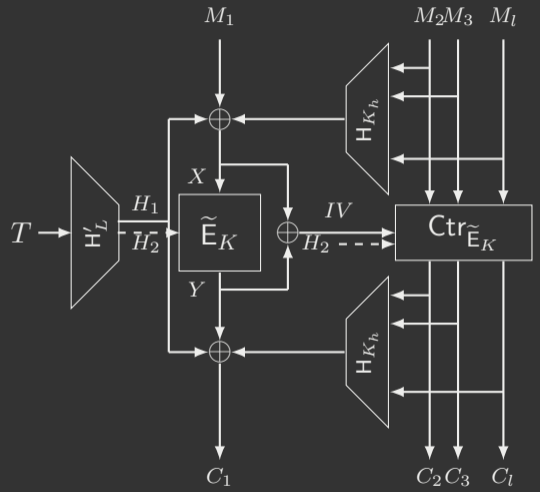
>>> Tweakable HCTR

- * BC \rightarrow m-bit TBC.
- * $H'_L(T)$ for processing variable length tweak.
- * First n bit of $H'_L(T)$ is xor-ed with the state.



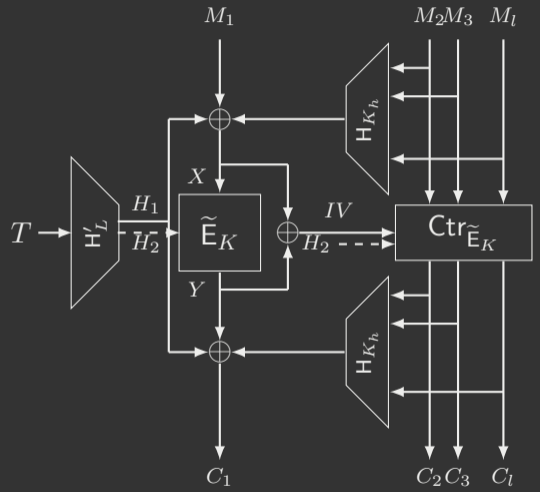
>>> Tweakable HCTR

- * BC \rightarrow m-bit TBC.
- * $H'_L(T)$ for processing variable length tweak.
- * First n bit of $H'_L(T)$ is xor-ed with the state.
- * Remaining m bit of $H'_L(T)$ is the tweak to the underlying TBC.



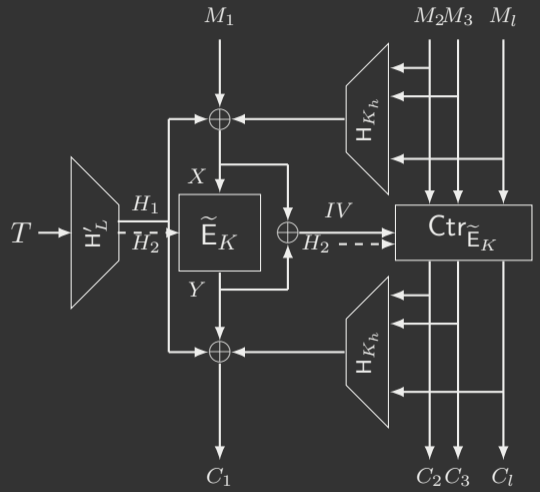
>>> Tweakable HCTR

- * BC \rightarrow m-bit TBC.
- * $H'_L(T)$ for processing variable length tweak.
- * First n bit of $H'_L(T)$ is xor-ed with the state.
- * Remaining m bit of $H'_L(T)$ is the tweak to the underlying TBC.
- * n bit state.



>>> Tweakable HCTR

- * BC \rightarrow m-bit TBC.
- * $H'_L(T)$ for processing variable length tweak.
- * First n bit of $H'_L(T)$ is xor-ed with the state.
- * Remaining m bit of $H'_L(T)$ is the tweak to the underlying TBC.
- * n bit state.
- * Provides graceful security degradation.

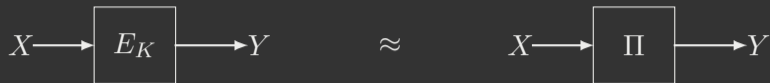


>>> Block Cipher vs Tweakable Block Cipher

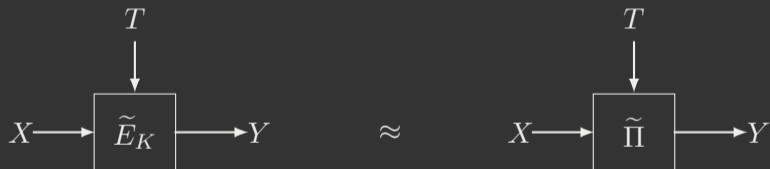


Indistinguishable from random permutation.

>>> Block Cipher vs Tweakable Block Cipher



Indistinguishable from random permutation.



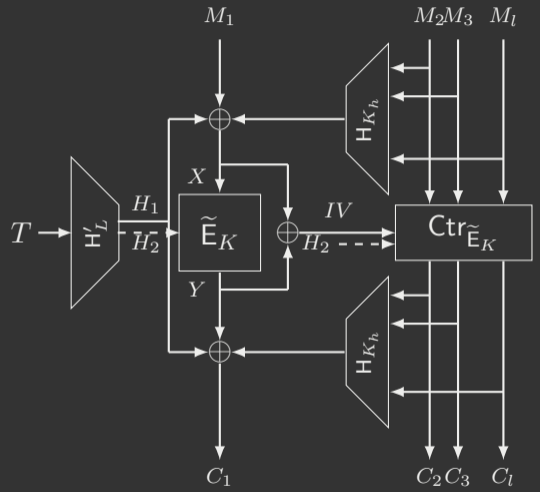
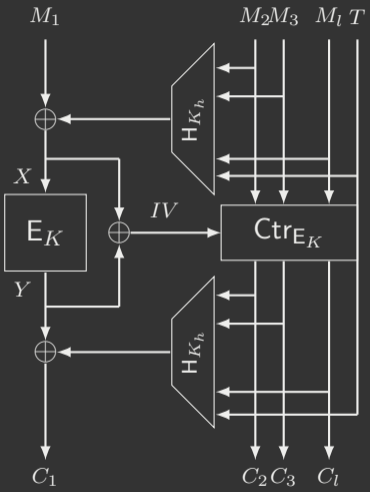
Indistinguishable from tweakable random permutation.

>>> Application of Tweakable Block Cipher

| Author | Construction | Bound | Publication |
|-------------------|---|---------------------------------|---------------|
| Rogaway | PMAC1 | $n/2$ | ASIACRYPT, 04 |
| Naito | PMAC_TBC3k, PMAC_TBC1k | n | Provsec, 15 |
| Peyrin and Seurin | SCT | Graceful BBB | CRYPTO, 16 |
| List and Nandi | PMAC _x , PMAC2 _x , SIV _x | n | CT-RSA, 17 |
| Iwata et al. | ZMAC, ZAE | $\min\{n, (n+t)/2\}$ | CRYPTO, 17 |
| List and Nandi | ZMAC+ | $q(q+\sigma)/2^{n+\min\{n,t\}}$ | ToSC, 17 |
| Chen et al. | LDT | $n/2$ (★) | FSE, 18 |
| Chen et al. | LDT | $2n/3$ | ASIACRYPT, 18 |

(★) Beyond the birthday bound for domain $[n, 3n/2)$.

>>> HCTR vs Tweakable HCTR



How H'_L is different from H_{K_h} ?

>>> How H'_L is different from H_{K_h} ?

* H_{K_h} requires to be n bit ϵ -AXU and ϵ -almost regular hash function.

>>> How \mathbf{H}'_L is different from \mathbf{H}_{K_h} ?

* \mathbf{H}_{K_h} requires to be n bit ϵ -AXU and ϵ -almost regular hash function.

* AXU:

$$\forall M \neq M', \forall \delta, \Pr[\mathbf{H}_{K_h}(M) \oplus \mathbf{H}_{K_h}(M') = \delta] \leq \epsilon.$$

* Almost Regular:

$$\forall M, \forall \delta, \Pr[\mathbf{H}_{K_h}(M) = \delta] \leq \epsilon.$$

>>> How H'_L is different from H_{K_h} ?

* H_{K_h} requires to be n bit ϵ -AXU and ϵ -almost regular hash function.

* AXU:

$$\forall M \neq M', \forall \delta, \Pr[H_{K_h}(M) \oplus H_{K_h}(M') = \delta] \leq \epsilon.$$

* Almost Regular:

$$\forall M, \forall \delta, \Pr[H_{K_h}(M) = \delta] \leq \epsilon.$$

* H'_L requires to be $(n+m)$ bit ϵ -partial AXU and $H'_L[2]$ requires to be ϵ -almost universal hash function.

>>> How H'_L is different from H_{K_h} ?

* H_{K_h} requires to be n bit ϵ -AXU and ϵ -almost regular hash function.

* AXU:

$$\forall M \neq M', \forall \delta, \Pr[H_{K_h}(M) \oplus H_{K_h}(M') = \delta] \leq \epsilon.$$

* Almost Regular:

$$\forall M, \forall \delta, \Pr[H_{K_h}(M) = \delta] \leq \epsilon.$$

* H'_L requires to be $(n+m)$ bit ϵ -partial AXU and $H'_L[2]$ requires to be ϵ -almost universal hash function.

* partial AXU:

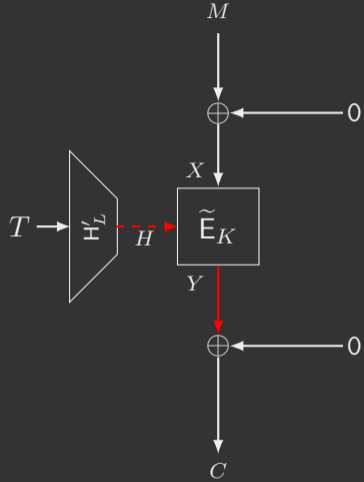
$$\forall M \neq M', \forall \delta, \Pr[H_{K_h}(M) \oplus H_{K_h}(M') = (\delta, 0)] \leq \epsilon.$$

* Almost Universal:

$$\forall M \neq M', \Pr[H'_L[2](M) = H'_L[2](M')] \leq \epsilon.$$

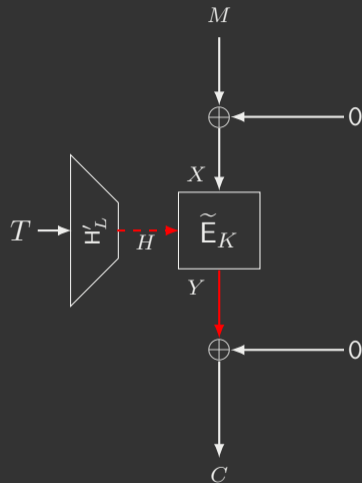
>>> Processing of tweak with m bit Almost Universal Hash

* A makes $2^{n/2}$ single block message query with distinct tweaks, i.e., $(M, T_1), \dots, (M, T_{2^{n/2}})$.

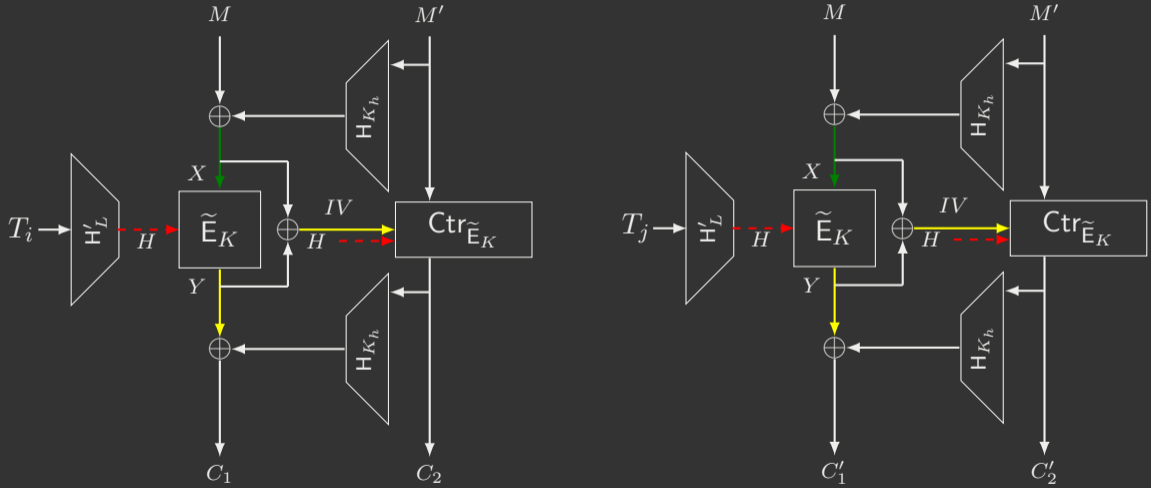


>>> Processing of tweak with m bit Almost Universal Hash

- * A makes $2^{n/2}$ single block message query with distinct tweaks, i.e., $(M, T_1), \dots, (M, T_{2^{n/2}})$.
- * Let $C_i = C_j$ (w.h.p due to m -bit hash collision).



>>> Processing of tweak with m bit Almost Universal Hash



Is $C_1 || C_2 = C'_1 || C'_2$?

>>> Security Result of Tweakable HCTR

Theorem

If H_{K_h} is ϵ -AXU and ϵ_1 -almost regular hash function and H'_L be an (n, m, δ) -partial AXU hash function and $H'[2]$ is a δ_{au} -almost universal hash function. Then,

$$\text{Adv}_{\text{HCTR}}^{\text{tsprp}} \leq 2(\mu - 1)(q\epsilon + \sigma/2^n) + 2q\sigma\delta_{\text{au}}/2^n + q^2\delta \\ + 2 \max\{q\ell(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n, \sigma\epsilon_1\}.$$

$\mu := \#$ of repetition of tweaks.

>>> Security Result of Tweakable HCTR

Theorem

If H_{K_h} is ϵ -AXU and ϵ_1 -almost regular hash function and H'_L be an (n, m, δ) -partial AXU hash function and $H'[2]$ is a δ_{au} -almost universal hash function. Then,

$$\text{Adv}_{\text{HCTR}}^{\text{tsprp}} \leq 2(\mu - 1)(q\epsilon + \sigma/2^n) + 2q\sigma\delta_{\text{au}}/2^n + q^2\delta \\ + 2 \max\{q\ell(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n, \sigma\epsilon_1\}.$$

$\mu := \#$ of repetition of tweaks.

Corollary

- * Assuming $\epsilon, \epsilon_1 \approx 2^{-n}$, $\delta_{\text{au}} \approx 2^{-m}$, $\delta \approx 2^{-(n+m)}$ and $m > n$, security $\approx 2^n/\mu\ell$ queries.

>>> Security Result of Tweakable HCTR

Theorem

If H_{K_h} is ϵ -AXU and ϵ_1 -almost regular hash function and H'_L be an (n, m, δ) -partial AXU hash function and $H'[2]$ is a δ_{au} -almost universal hash function. Then,

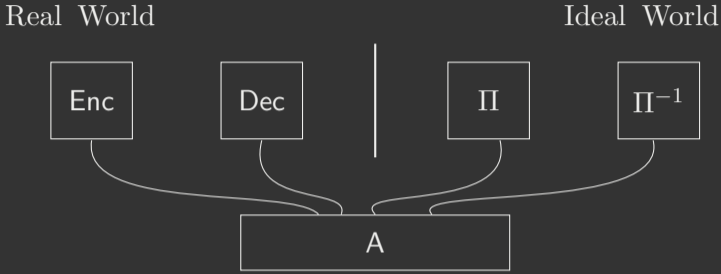
$$\text{Adv}_{\text{HCTR}}^{\text{tsprp}} \leq 2(\mu - 1)(q\epsilon + \sigma/2^n) + 2q\sigma\delta_{\text{au}}/2^n + q^2\delta \\ + 2 \max\{q\ell(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n, \sigma\epsilon_1\}.$$

$\mu := \#$ of repetition of tweaks.

Corollary

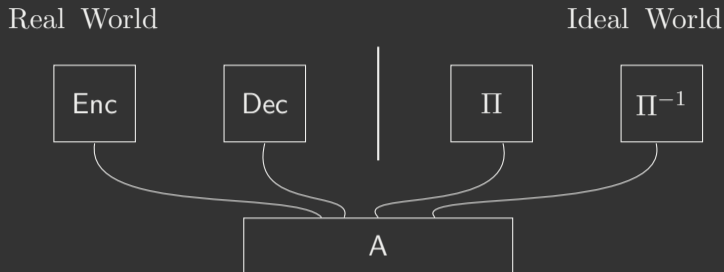
- * Assuming $\epsilon, \epsilon_1 \approx 2^{-n}$, $\delta_{\text{au}} \approx 2^{-m}$, $\delta \approx 2^{-(n+m)}$ and $m > n$, security $\approx 2^n/\mu\ell$ queries.
- * Moreover, when $\mu = 1$, security $\approx 2^n$ many message blocks.

>>> H-Coefficient Technique



$$\text{Adv}_{\text{ideal}}^{\text{real}}(A) = | \Pr[A^{\mathcal{I}.\text{Enc}, \mathcal{I}.\text{Dec}_K} = 1] - \Pr[A^{\Pi, \Pi^{-1}} = 1] |.$$

>>> H-Coefficient Technique



$$\mathbf{Adv}_{\text{ideal}}^{\text{real}}(\mathbf{A}) = | \Pr[\mathbf{A}^{\mathcal{I}.\text{Enc}, \mathcal{I}.\text{Dec}_K} = 1] - \Pr[\mathbf{A}^{\Pi, \Pi^{-1}} = 1] |.$$

- * Transcript: $\tau = (M_1, T_1, C_1), \dots, (M_q, T_q, C_q)$.
- * $X_{\text{re/id}}$:= probability distribution of transcript in real / ideal world.
- * $\mathcal{V} = \text{GoodT} \sqcup \text{BadT}$.

>>> H-Coefficient Technique

Main Theorem (H-Coefficient Technique)

If there exists $\epsilon_{\text{ratio}}, \epsilon_{\text{bad}} \geq 0$ such that

- (i) for all $\tau \in \text{GoodT}$, $\frac{\Pr[X_{\text{re}}=\tau]}{\Pr[X_{\text{id}}=\tau]} \geq 1 - \epsilon_{\text{ratio}}$ and
- (ii) $\Pr[X_{\text{id}} \in \text{BadT}] \leq \epsilon_{\text{bad}}$,

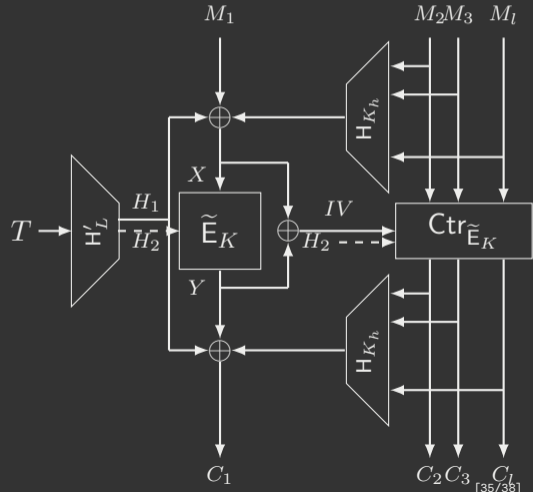
then

$$\text{Adv}_{\text{ideal}}^{\text{real}}(\mathbf{A}) \leq \epsilon_{\text{ratio}} + \epsilon_{\text{bad}}.$$

>>> Identifying Bad Event

Input or output collision for same tweak is bad.

- * 1. $H_{2,i} = H_{2,j}, X_i = X_j$ or (b) $H_{2,i} = H_{2,j}, Y_i = Y_j$.
- * 2. $H_{2,i} = H_{2,j}, IV_a^i = IV_b^j$.
- * 3. $H_{2,i} = H_{2,j}, M_a^i \oplus C_a^i = M_b^j \oplus C_b^j$.
- * 4. $H_{2,i} = H_{2,j}, X_i = IV_a^j$.
- * 5. $H_{2,i} = H_{2,j}, Y_i = M_a^j \oplus C_a^j$.



>>> Analysing Bad Event

Analysing Bad Event I:

$$\mathbf{B} := H_{2,i} = H_{2,j}, X_i = X_j \Leftrightarrow H_{2,i} = H_{2,j}, \mathbf{H}_{K_h}(\widehat{M}_i) \oplus H_{1,i} \oplus M_i = \mathbf{H}_{K_h}(\widehat{M}_j) \oplus H_{1,j} \oplus M_j$$

where $\widehat{M}_i := M_2^i \parallel \dots \parallel M_{l_i}^i, \widehat{M}_j := M_2^j \parallel \dots \parallel M_{l_i}^j$.

>>> Analysing Bad Event

Analysing Bad Event I:

$$\mathbf{B} := H_{2,i} = H_{2,j}, X_i = X_j \Leftrightarrow H_{2,i} = H_{2,j}, \mathbf{H}_{K_h}(\widehat{M}_i) \oplus H_{1,i} \oplus M_i = \mathbf{H}_{K_h}(\widehat{M}_j) \oplus H_{1,j} \oplus M_j$$

where $\widehat{M}_i := M_2^i \parallel \dots \parallel M_{l_i}^i$, $\widehat{M}_j := M_2^j \parallel \dots \parallel M_{l_i}^j$.

* Case a: $T_i = T_j \Leftrightarrow (H_{1,i}, H_{2,i}) = (H_{1,j}, H_{2,j})$.

$$\Pr[\mathbf{B}] \leq q(\mu - 1)\epsilon, \quad \# \text{ of } (i, j) = (q, (\mu - 1)).$$

>>> Analysing Bad Event

Analysing Bad Event I:

$B := H_{2,i} = H_{2,j}, X_i = X_j \Leftrightarrow H_{2,i} = H_{2,j}, \mathbf{H}_{K_h}(\widehat{M}_i) \oplus H_{1,i} \oplus M_i = \mathbf{H}_{K_h}(\widehat{M}_j) \oplus H_{1,j} \oplus M_j$
where $\widehat{M}_i := M_2^i \parallel \dots \parallel M_{l_i}^i, \widehat{M}_j := M_2^j \parallel \dots \parallel M_{l_j}^j$.

* Case a: $T_i = T_j \Leftrightarrow (H_{1,i}, H_{2,i}) = (H_{1,j}, H_{2,j})$.

$$\Pr[B] \leq q(\mu - 1)\epsilon, \quad \# \text{ of } (i, j) = (q, (\mu - 1)).$$

* Case (b): $T_i \neq T_j$, then

$$\Pr[B] \leq \binom{q}{2}\delta, \quad \# \text{ of } (i, j) = \binom{q}{2}.$$

Analysing Bad Event I:

$B := H_{2,i} = H_{2,j}, X_i = X_j \Leftrightarrow H_{2,i} = H_{2,j}, \mathbf{H}_{K_h}(\widehat{M}_i) \oplus H_{1,i} \oplus M_i = \mathbf{H}_{K_h}(\widehat{M}_j) \oplus H_{1,j} \oplus M_j$
where $\widehat{M}_i := M_2^i \parallel \dots \parallel M_{l_i}^i, \widehat{M}_j := M_2^j \parallel \dots \parallel M_{l_i}^j$.

* Case a: $T_i = T_j \Leftrightarrow (H_{1,i}, H_{2,i}) = (H_{1,j}, H_{2,j})$.

$$\Pr[B] \leq q(\mu - 1)\epsilon, \quad \# \text{ of } (i, j) = (q, (\mu - 1)).$$

* Case (b): $T_i \neq T_j$, then

$$\Pr[B] \leq \binom{q}{2}\delta, \quad \# \text{ of } (i, j) = \binom{q}{2}.$$

Similarly, we can bound the other bad events too.

>>> Summarizing the proof

Summary of the Bad Events Bound

| Bad Event | Bound |
|-----------|--|
| B.1 | $2(\mu - 1)q\epsilon + q^2\delta$ |
| B.2 | $\sigma(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n$ |
| B.3 | $\sigma(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n$ |
| B.4 | $\max\{q\ell(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n, \sigma\epsilon_1\}$ |
| B.5 | $\max\{q\ell(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n, \sigma\epsilon_1\}$ |

>>> Summarizing the proof

Summary of the Bad Events Bound

| Bad Event | Bound |
|-----------|---|
| B.1 | $2(\mu - 1)q\epsilon + q^2\delta$ |
| B.2 | $\sigma(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n$ |
| B.3 | $\sigma(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n$ |
| B.4 | $\max\{ql(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n, \sigma\epsilon_1\}$ |
| B.5 | $\max\{ql(\mu - 1)/2^n + q\sigma\delta_{\text{au}}/2^n, \sigma\epsilon_1\}$ |

If Bad Events do not happen then for two same tweak to the BC, either its input or output is fresh. Therefore,

$$\frac{\Pr[X_{\text{re}} = \tau]}{\Pr[X_{\text{id}} = \tau]} \geq 1.$$

>>> Conclusion & Future Work

- * Tweakable Enciphering Scheme with Tweakable Block Cipher
- * Mode with n bit state.
- * Provides BBB security with graceful degradation

BBB Secure TES using BC (other than trivial option) is still open.

>>> Conclusion & Future Work

- * Tweakable Enciphering Scheme with Tweakable Block Cipher
- * Mode with n bit state.
- * Provides BBB security with graceful degradation

BBB Secure TES using BC (other than trivial option) is still open.



Thank You