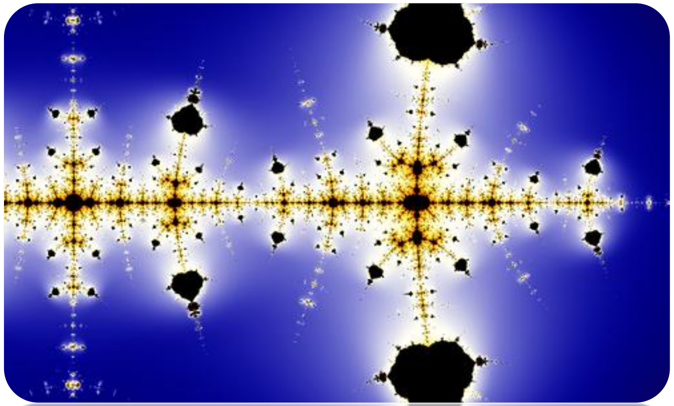
Collatz Conjecture in the light of Integral Value Transformations (IVTs)
&
Associated Applications of IVTs in Genomics



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Who is Collatz?

Lothar Collatz was a German mathematician. In 1937 he posed the famous **Collatz conjecture**, **during his PhD**.

Prof. Collatz *was convinced that mathematics and mathematicians had a responsibility to apply their results to, and be motivated by, real world phenomena. He never wearied of fighting for this conviction.*

(6th July 1910-26th Sept.1990)

Collatz Conjecture

Consider a function from $\mathbb{N}_0$ to $\mathbb{N}_0$, defined as follows:

$$\mathrm{T}(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ (3n+1)/2 & \text{if } n \text{ is odd} \end{cases}$$

Next define the iterate of T as usual:

$$\begin{cases} \mathrm{T}^{(0)}(n) = n \\ \mathrm{T}^{(i+1)}(n) = \mathrm{T}(\mathrm{T}^{i}(n)) \end{cases}$$

The question is now to show that for every $n \in \mathbb{N}_0$, there is a finite k, such that

$$\mathrm{T}^{(k)}(n) = 1.$$

This is the conjecture also known as the 3n + 1 conjecture

An illustration

A straightforward example: take $n = 7$, then we have the following sequence

$$
\begin{array}{cccccccccccc}
7 \rightarrow & 11 \rightarrow & 17 \rightarrow & 26 \rightarrow & 13 \rightarrow & 20 \rightarrow & 10 \rightarrow & 5 \rightarrow & 8 \rightarrow & 4 \rightarrow & 2 \rightarrow & 1 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11
\end{array}
$$

therefore $\mathrm{T}^{(11)}(7) = 1$ and $k = 11$.

Now we are free to play with some other natural numbers as we wish.

It is worth to be aware of the facts:

- The conjecture remains *Unanswered,* although it has been shown to be valid for all natural numbers up to $5.764 * 10^{18}$.
- Professor Paul ErdŐs once commented on the Collatz conjecture: "Mathematics is not yet ready for such problems”. He offered Rs. 2500000/- for a solution.

Generalizations and Analogue Conjectures

Conjecture in the domain of Rational Numbers

Let θ be the following function on $\mathbb{Q}_+$, the set of nonnegative rational numbers:

$$\theta(x) = \begin{cases} (x-1)/3 & x \geq 1 \\ 2x/(1-x) & x < 1 \end{cases}$$

We make the following conjecture:

Conjecture *For every $x \in \mathbb{Q}_+$, there exists $n \geq 1$ so that $\theta^n(x) = 0$.*

Further generalization

$$g(n) = \begin{cases} \frac{2}{3}n, & \text{if } n \equiv 0 \pmod 3, \\ \frac{4}{3}n - \frac{1}{3}, & \text{if } n \equiv 1 \pmod 3, \\ \frac{4}{3}n + \frac{1}{3}, & \text{if } n \equiv 2 \pmod 3, \end{cases}$$

Conjecture: Consider an iterative scheme $g^k(n) = g\left(g^{k-1}(n)\right) : k > 0$

There exists a positive integer k such that $g^k(n) = 1.$

This is how we can make the general conjecture in modulo n system.

A Collatz type Conjecture of Prof. Mridul K. Sen

$$g(n) = \frac{\varphi(n)}{2} : \text{n is even and} > 2$$

$$g(n) = 1 : n = 2$$

$$g(n) = \frac{3n+1}{2} : \text{n is odd}$$

Conjecture: Consider an iterative scheme $g^k(n) = g\left(g^{k-1}(n)\right) : k > 0$

There exists a positive integer k such that $g^k(n) = 1.$

So where is the challenge?

Challenge: To define an analogue Conjecture and to prove or disprove it!

This, in our view, will be one step forward towards the settlement of Collatz Conjecture in the sense that we may get some clue for settling the original conjecture.

Let us accept the Challenge.

Notion of Integral Value Transformations

Now let us define the IVT in $\mathbb{N}_0^K$ as the following:

$IVT^{p,k}{}_j : \mathbb{N}_0^K \rightarrow \mathbb{N}_0$

$IVT^{p,k}{}_j((n_1, n_2, \ldots . n_k) =$

$(f_j(a_0{}^{n_1}, a_0{}^{n_2}, \ldots, a_0{}^{n_k})\ f_j(a_1{}^{n_1}, a_1{}^{n_2}, \ldots, a_1{}^{n_k}) \ldots \ldots f_j(a_{l-1}{}^{n_1}, a_{l-1}{}^{n_2}, \ldots, a_{l-1}{}^{n_k}))_p = m$

where $n_1 = (a_0{}^{n_1} a_1{}^{n_1} \ldots . a_{l-1}{}^{n_1})_p$, $n_2 = (a_0{}^{n_2} a_1{}^{n_2} \ldots . a_{l-1}{}^{n_2})_p, \ldots \ldots n_k = (a_0{}^{n_k} a_1{}^{n_k} \ldots . a_{l-1}{}^{n_k})_p$

f_j: $\{0, 1, 2, \ldots, p-1\}^k \rightarrow \{0,1,2, \ldots, p-1\}$.

m is the decimal conversion from the p adic number.

Let us fix the domain of IVTs as $\mathbb{N}_0$ (k=1) and thus the above definition boils down to the following:

$$\mathrm{IVT}^{p,1}{}_j\ (x) = \left(f_j(x_n)\ f_j(x_{n-1}) \ldots \ldots \ldots f_j(x_1)\right)_p = m$$

where m is the decimal conversion from the p adic number, and $x = (x_n\ x_{n-1} \ldots \ldots x_1)_p$.

Let us denote the set of $\mathrm{IVT}^{p,1}{}_j$ as

$$T^{p,1} = \left\{ \mathrm{IVT}^{p,1}{}_j : \mathbb{N} \rightarrow \mathbb{N} \;\middle|\; \begin{array}{c} 0 \le j < p^p,\ \mathrm{IVT}^{p,1}{}_j\ (x) = \left(f_j(x_n)\ f_j(x_{n-1}) \ldots \ldots \ldots f_j(x_1)\right)_p = m \\ \text{where m is the decimal conversion from the p adic number} \\ \text{and } x = (x_n\ x_{n-1} \ldots \ldots x_1)_p \end{array} \right\}$$

$\mathrm{IVT}^{2,1}{}_{\#}$

Let us define the IVT in $\mathbb{N}_0$ in 2-adic number systems. There are 4 (2^{2^1}) one variable two state cellular automata rules. These are as follows:

Variable	$\mathbf{f_0}$	$\mathbf{f_1}$	$\mathbf{f_2}$	$\mathbf{f_3}$
0	**0**	**1**	**0**	**1**
1	**0**	**0**	**1**	**1**

$\mathrm{IVT}^{2,1}{}_{\#}$ mapping a non-negative integers to a non-negative integers.

$\mathrm{IVT}^{2,1}{}_0(\mathrm{a}) = \big((\mathrm{f}_0(\mathrm{a}_\mathrm{n})\mathrm{f}_0(\mathrm{a}_{\mathrm{n}-1}) \dots \mathrm{f}_0(\mathrm{a}_1)\big)_2 = b$

$\mathrm{IVT}^{2,1}{}_1(\mathrm{a}) = \big((\mathrm{f}_1(\mathrm{a}_\mathrm{n})\mathrm{f}_1(\mathrm{a}_{\mathrm{n}-1}) \dots \mathrm{f}_1(\mathrm{a}_1)\big)_2 = b$

$\mathrm{IVT}^{2,1}{}_2(\mathrm{a}) = \big((\mathrm{f}_2(\mathrm{a}_\mathrm{n})\mathrm{f}_2(\mathrm{a}_{\mathrm{n}-1}) \dots \mathrm{f}_2(\mathrm{a}_1)\big)_2 = b$

and $\mathrm{IVT}^{2,1}{}_3(\mathrm{a}) = \big((\mathrm{f}_3(\mathrm{a}_\mathrm{n})\mathrm{f}_3(\mathrm{a}_{\mathrm{n}-1}) \dots \mathrm{f}_3(\mathrm{a}_1)\big)_2 = b,$

where ‘a’ is a non-negative integer and $a = (a_n a_{n-1} \dots a_1)_2$ and ‘b’ is the decimal value corresponding to the binary number.

An Illustration : p=4, k=1 & j=120

$IVT^{4,1}_{\#}$ is mapping a non-negative integer to a non-negative integer.

$$IVT^{4,1}_{\#}(a) = \left((f_{\#}(a_n)f_{\#}(a_{n-1})\ldots f_{\#}(a_1)\right)_4 = b$$

Where 'a' is a non-negative integer and $a = (a_n a_{n-1} \ldots a_1)_4$ and 'b' is the decimal value corresponding to the 4-adic number.

For an example, let us consider $a = 225 = (3201)_4$ and $\# = 120$ so $f_{\#}(0) = 0;\ f_{\#}(1) = 2;\ f_{\#}(2) = 3\ and\ f_{\#}(3) = 1$

Therefore, $IVT^{4,1}_{120}(225) = (f_{120}(3)(f_{120}(2)(f_{120}(0)(f_{120}(1))_4 = (1302)_4 = 114$.

Consequently, $IVT^{4,1}_{120}(225) = 114$.

Algebraic relations of IVTs

$IVT^{2,1}{}_1(x) = (2^s - 1) - x$

$IVT^{2,1}{}_1(x) = (2^s - 1) - IVT^{2,1}{}_2(x)$

$IVT^{2,1}{}_2(x) = IVT^{2,1}{}_3(x) - IVT^{2,1}{}_1(x)$

i.e. $IVT^{2,1}{}_3(x) = IVT^{2,1}{}_1(x) + IVT^{2,1}{}_2(x)$ for all non-negative integers x.

Therefore, the relation becomes $IVT^{2,1}{}_3 = IVT^{2,1}{}_1 + IVT^{2,1}{}_2$

Collatz like Conjecture

Let us consider the iterative scheme $(IVT^{p,1}{}_j)^i(n) = IVT^{p,1}{}_j\left((IVT^{p,1}{}_j)^{i-1}(n)\right)$

Conjecture: there exists an integer i such that $(IVT^{p,1}{}_j)^i(n) = 0$

Let us see the above conjecture in case of $p = 2, j = 1$

Iterative convergence for few numbers

X_0	*Iterative sequences*
0	0
1	0
2	1, 0
3	0
4	3, 0
5	2, 1, 0
6	1, 0
7	0
8	7, 0
9	6, 1, 0

10	5, 2, 1, 0
11	4, 3, 0
12	3, 0
13	2, 1, 0
14	1, 0
15	0
16	15, 0
17	14, 1, 0
18	13, 2, 1, 0
19	12, 3, 0
20	11, 4, 3, 0

Main Result of the Collatz like Conjecture in IVT

The iterative scheme $\{X_n\}$ converges to 0 for any given X_0 where $X_{n+1} = IVT^{2,1}{}_1(X_n)$

Lemma:

(I) *For any non-negative integer of the form* $X_0 = 2^n + P$, $IVT^{2,1}{}_1(X_0) = 2^n - (P+1)$ *for some non-negative integer.*

(II) *For any non-negative integer of Merseene form* $X_0 = 2^n - 1$, $IVT^{2,1}{}_1(X_0) = 0$.

Main Proof is based on Mathematical Induction

The iterative scheme $\{X_n\}$ converges to 0 for any given X_0 where $X_{n+1} = IVT^{2,1}{}_1(X_n)$

Proof:
We use Strong Mathematical Induction (SMI) principle to prove the theorem. Let us consider a set T_n, a set of X_0s which is defined as $\{2^n + p: 0 \leq p \leq 2^n - 1\}$. Clearly for $n = 0, T_0 = \{1\}$, for $n = 1, T_1 = \{2, 3\}, for\ n = 2, T_2 = \{4, 5, 6, 7\}$ and so on…

In this way, all natural numbers along with zero could be captured by the said scheme. Let us define ${T_1}^n$ as a set of $IVT^{2,1}{}_1(X_0)$s corresponding to n for T.

For $n = 0, T_0 = \{1\}$ and so readily ${T_1}^0 = \{0\}$ i.e. X_0 converges to 0 by Lemma-2.2.1 (II).

For $n = 1$, by lemma-2.2.1 (I) and (II) $T_1 = \{2, 3\}$ becomes ${T_1}^1 = \{1, 0\} = T_0 \cup {T_1}^0$. Already, $T_0\ and\ {T_1}^0$ have converged to 0.

For $n = 2$, by the lemma 2.2.1 (I) and (II) in $T_2 = \{4, 5, 6, 7\}$ becomes ${T_1}^2 = \{3, 2, 1, 0\} = T_1 \cup {T_1}^1$. Previously, $T_1\ and\ {T_1}^1$ have converged to 0.

Let us hypothesize that the theorem be true for all $n = m$.

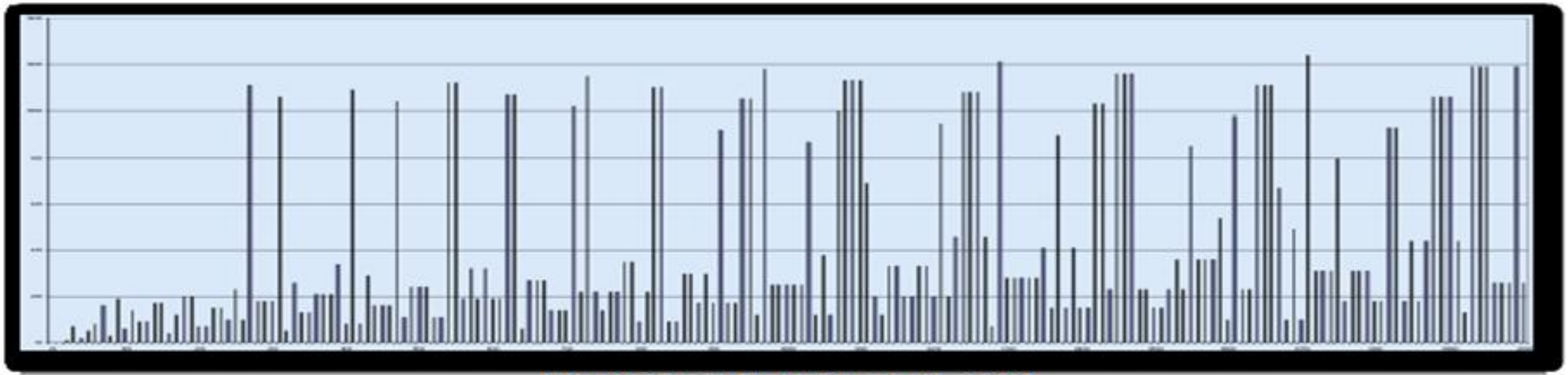
Let us try to establish the theorem is true for $n = m + 1$ also.

Now, ${T_1}^{m+1} = \{2^{m+1} + p: 0 \leq p \leq 2^{m+1} - 1\}$ then ${T_1}^{m+1} = \{p: 0 \leq p \leq 2^{m+1} - 1\} = T_m \cup {T_1}^m$. According to the SMI hypothesis we could say the iterative scheme is converging to 0.

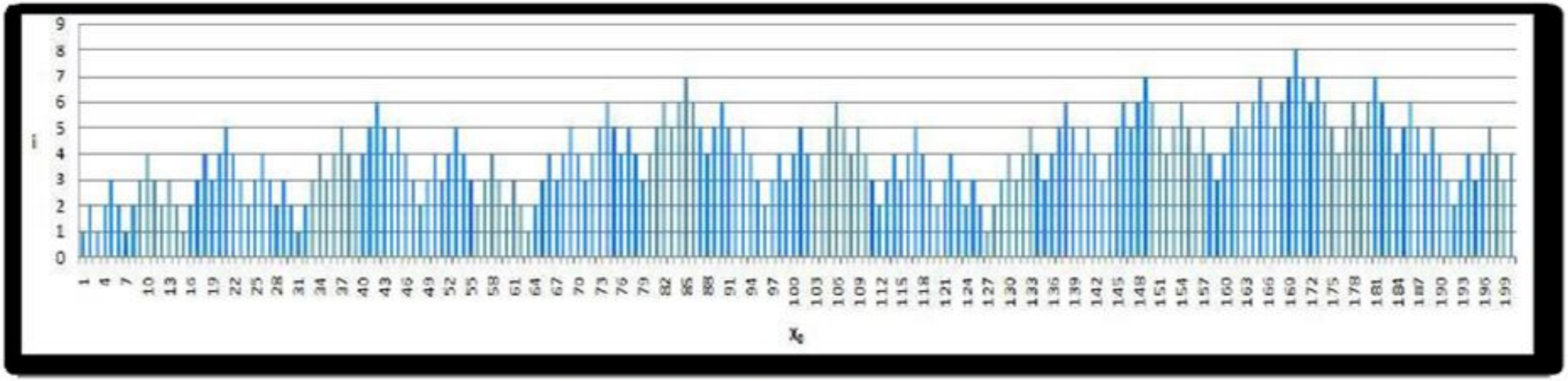
Therefore the required theorem is followed. (*Proved*).

In the subsequent section we would explore the convergence behavior of the iterative scheme corresponding to $IVT^{2,1}{}_1$ and Collatz function.

Convergence behavior of Collatz and IVT

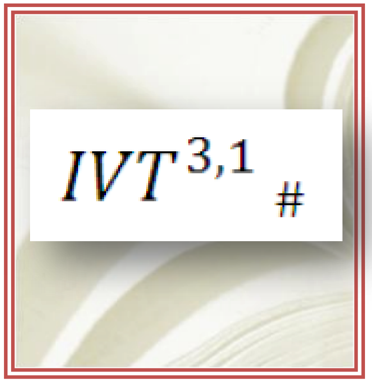


[Figure 1: Collatz Graph (1-200)]



[Figure 2: $IVT^{2,1}{}_1$ Graph (1-200)]

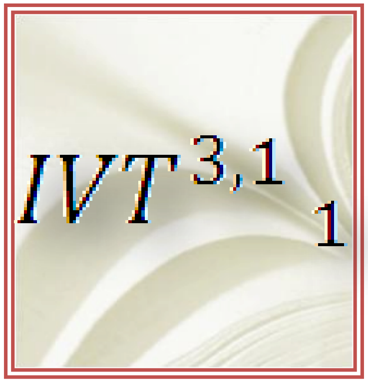
$$IVT^{3,1}{}_{\#}$$



Clearly there are $3^{3^1} = 27$ integral value transformations.

In this case $\{IVT^{3,1}{}_1, IVT^{3,1}{}_3, IVT^{3,1}{}_9\}$ is basis for the $IVT^{3,1}{}_{\#}$ system.

Algebraic form of IVTs



$\mathbf{IVT^{3,1}}_1$

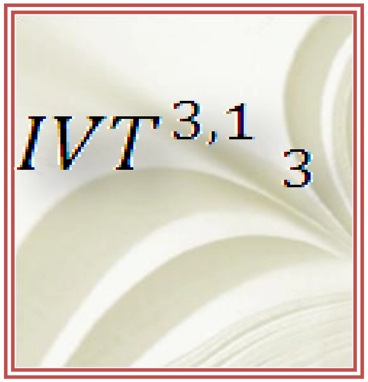
$$\mathrm{IVT^{3,1}}_1(\mathrm{n}) = \frac{3^p - 1}{2} - \left(\sum_{i=0}^{p-1} 3^i\right) : n = \sum_{i=0}^{p} 3^i$$

$$= \frac{3^p - 1}{2} - \left(\sum_{i=0}^{p-1} 3^i\right) : n = 2.\left(\sum_{i=0}^{p} 3^i\right)$$

$$= \frac{3^h - 1}{2} - \left[\left(\sum_{i=0}^{h-1} 3^i\right) + \left(\sum_{j=0}^{k} 3^i\right)\right] : n = \left(\sum_{i=0}^{h} 3^i\right) + 2.\left(\sum_{i=0}^{k} 3^i\right), h > k$$

$$= \frac{3^p - 1}{2} - \left[\left(\sum_{i=0}^{h} 3^i\right) + \left(\sum_{j=0}^{p-1} 3^i\right)\right] : n = \left(\sum_{i=0}^{h} 3^i\right) + 2.\left(\sum_{i=0}^{p} 3^i\right), h < k$$

Contd…



$\mathbf{IVT^{3,1}{}_{3}}$

$$\mathrm{IVT}^{3,1}{}_{3}(n) = \left(\sum_{j=0}^{p} 3^{i}\right) \; when \; n = \left(\sum_{j=0}^{p} 3^{i}\right)$$

$$= 0 \; when \; n = 2.\left(\sum_{j=0}^{p} 3^{i}\right)$$

$$= \left(\sum_{j=0}^{p} 3^{i}\right) \; when \; n = \left(\sum_{j=0}^{p} 3^{i}\right) + 2.\left(\sum_{k=0}^{q} 3^{i}\right)$$

Contd…

The algebraic formulations of the remaining IVTs can be defined as a linear combination of the above three IVTs. Each $IVT^{3,1}_{\#}$ satisfies the following linear combination.

$$IVT^{3,1}_{\#} = a.\ IVT^{3,1}_{1} + b.\ IVT^{3,1}_{3} + c.\ IVT^{3,1}_{9}$$

$$where\ a, b, c \in \{0,1,2\}$$

satisfies the following relation:

$$\# = a + 3b + 9c$$

Collatz behavioral IVTs in $T^{3,1}$

$IVT^{3,1}{}_{0}$

$IVT^{3,1}{}_{1}$

$IVT^{3,1}{}_{2}$

$IVT^{3,1}{}_{6}$

$IVT^{3,1}{}_{7}$

$IVT^{3,1}{}_{8}$

$IVT^{3,1}{}_{9}$

$IVT^{3,1}{}_{10}$

$IVT^{3,1}{}_{11}$

As a general remark,there are $p^{p-1}-1$ number of Collatz like functions in $T^{p,1}$

So where is the clue?

If we can express the original Collatz function in terms of IVTs in some manner (some hybridization of IVTs), then we may come across some clue since we know the behaviour of all IVTs in the Collatz context.

So far, we have explored the mathematical beauty of Integral Value Transformations in deciphering Collatz Conjecture.

Let us see in next few slides, how IVTs play a crucial role in deciphering *Proteomic Sequence Evolution*.

Protein Sequence Evolution Through IVTs

We took a protein sequence (of length 312) of OR1D2 (Human Olfactory Receptor), which is shown below:

MDGGNQSEGSEFLLLGMSESPEQQRILFWMFLSMYLVTVVGNVLIILAIS
SDSRLHTPVYFFLANLSFTDLFFVTNTIPKMLVNLQSHNKAISYAGCLTQ
LYFLVSLVALDNLILAVMAYDRYVAICCPLHYTTAMSPKLCILLLSLCWV
LSVLYGLIHTLLMTRVTFCGSRKIHYIFCEMYVLLRMACSNIQINHTVLI
ATGCFIFLIPFGFVIISYVLIIRAILRIPSVSKKYKAFSTCASHLGAVSL
FYGTLCMVYLKPLHTYSVKDSVATVMYAVVTPMMNPFIYSLRNKDMHGAL
GRLLDKHFKRLT

Method

- First we have segmented the whole sequence into multiple blocks of length 50. The last block may contain less than 50 amino acids.
- We then applied IVTs (as shown in next slide) into each block. Consequently, the blocks are updated and finally we got a sequence of same length as it was initially.
- We follow the same process as long as we wish to iterate to have new sequences.

Map between amino acids and natural numbers

$A \rightarrow 0; C \rightarrow 1; D \rightarrow 2; E \rightarrow 3; F \rightarrow 4; G \rightarrow 5; H \rightarrow 6; I \rightarrow 7; K \rightarrow 8; L \rightarrow 9; Q \rightarrow 10; N \rightarrow 11;$

$P \rightarrow 12; Q \rightarrow 13; R \rightarrow 14; S \rightarrow 15; T \rightarrow 16; V \rightarrow 17; W \rightarrow 18; Y \rightarrow 19;$

IVTs applied to each block

We choose the IVTs which are bijective in nature as well as which restrict the image set as domain. Here domain of act is {0,1,2,3,…., 19}

BLOCK	Sequence-1 in 2 adic IVT	Sequence-1 in 3 adic IVT	Sequence-1 in 4 adic IVT
Block-1	$IVT^{2,1}{}_{1}$	$IVT^{3,1}{}_{5}$	$IVT^{4,1}{}_{99}$
Block-2	$IVT^{2,1}{}_{1}$	$IVT^{3,1}{}_{5}$	$IVT^{4,1}{}_{114}$
Block-3	$IVT^{2,1}{}_{2}$	$IVT^{3,1}{}_{11}$	$IVT^{4,1}{}_{147}$
Block-4	$IVT^{2,1}{}_{1}$	$IVT^{3,1}{}_{11}$	$IVT^{4,1}{}_{177}$
Block-5	$IVT^{2,1}{}_{2}$	$IVT^{3,1}{}_{21}$	$IVT^{4,1}{}_{180}$
Block-6	$IVT^{2,1}{}_{2}$	$IVT^{3,1}{}_{21}$	$IVT^{4,1}{}_{210}$
Block-7	$IVT^{2,1}{}_{2}$	$IVT^{3,1}{}_{21}$	$IVT^{4,1}{}_{225}$

The following sequences are generated:

Sequence-1 generated from 2-adic IVTs:

GCDDFDAADAAEHHHDGAAAEADDCAHEQGEHAGPHRSRRDFRHAAHCAA
ACACHCSERPEEHCFHAESCHEERSFSAEIGHRFHDACFICAAPCDAHSD
LYFLVSLVALDNLILAVMAYDRYVAICCPLHYTTAMSPKLCILLLSLCWV
HARHPDHACSHHGSCRSEADACIACPAEAAGPRHHCGCAAFADAFCSRHA
ATGCFIFLIPFGFVIISYVLIIRAILRIPSVSKKYKAFSTCASHLGAVSL
FYGTLCMVYLKPLHTYSVKDSVATVMYAVVTPMMNPFIYSLRNKDMHGAL
GRLLDKHFKRLT

Sequence-1 generated from 3-adic IVTs:

TAEESQNGENGFVVVETNGNRGQQPCVFKTFVNTIVLMLLESLVCCVDCN
NANPVDMRLIFFVDSVNFMAVFFLMSMCRATVLSVQNDSADCNIDECVMQ
KSAKFGKFDKCIKEKDFHDSCCSFDEAADKGSEEDHGDFKAEKKKGKAVF
KGFKSCKEGEKKHECFEAACGCFEGSEAADHSFKKCHDAGIEAEIGEFKE
ATGCFIFLIPFGFVIISYVLIIRAILRIPSVSKKYKAFSTCASHLGAVSL
FYGTLCMVYLKPLHTYSVKDSVATVMYAVVTPMMNPFIYSLRNKDMHGAL
GRLLDKHFKRLT

Sequence-1 generated from 4-adic IVTs:

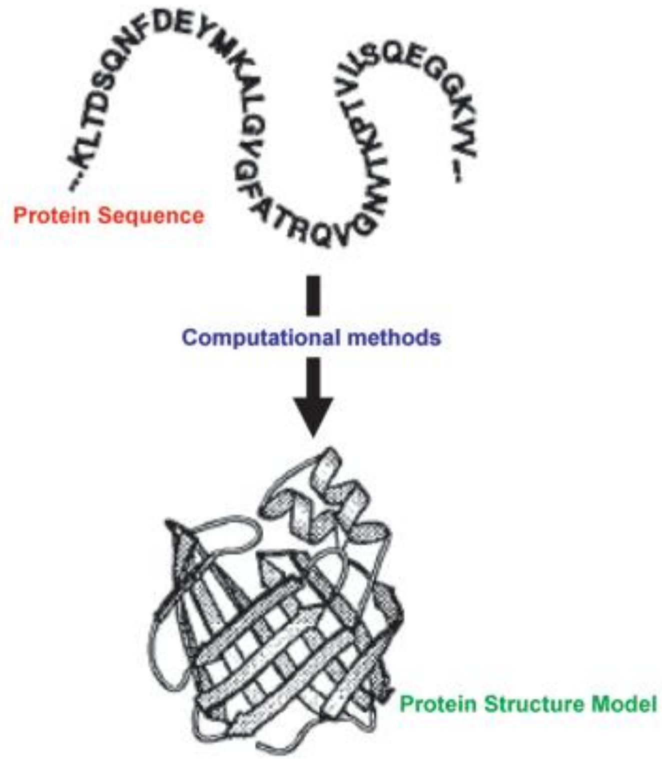
MDAALFGCAGCEKKKAMGCGICFFHCKERMEKGMQKPSPPALPKCCKECG
GEGIPEMHKLDDPDQPGDMEPDDKMQMCHRSPKQPFGEQRDCGLDAAPMF
FREFPMFPEFCHFDFEPGERCLRPEDAANFCRSSEGMNIFADFFFMFAQP
PMFPHAPDEGPPSGNFGCAAMNQDEHDCADSHFPPNSCAMRDKDREGFPD
ATGCFHFQHKFGFVHHMWVQHHNAHQNHKMVMPPWPAFMTCAMIQGAVMQ
DNAMFAGKNFHRFCMNSKHCSKDMKGNDKKMRGGIRDENSFQIHCGCADF
ARKKDLDCLRKG

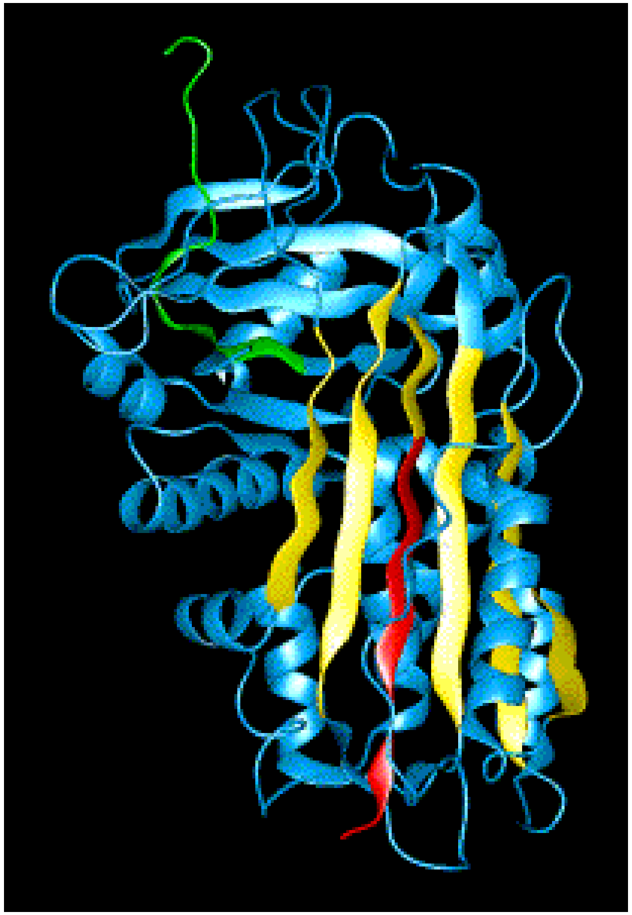
The sequences map to the following ORs of different species (NCBI blast)

adic system	Sequence	Maps to
2	1	G protein-coupled receptor [Homo sapiens]
3	1	G protein-coupled receptor [Homo sapiens]
4	1	hypothetical protein VITISV_006535 [Vitis vinifera]

Protein Structure

Beta strand and Alpha helix





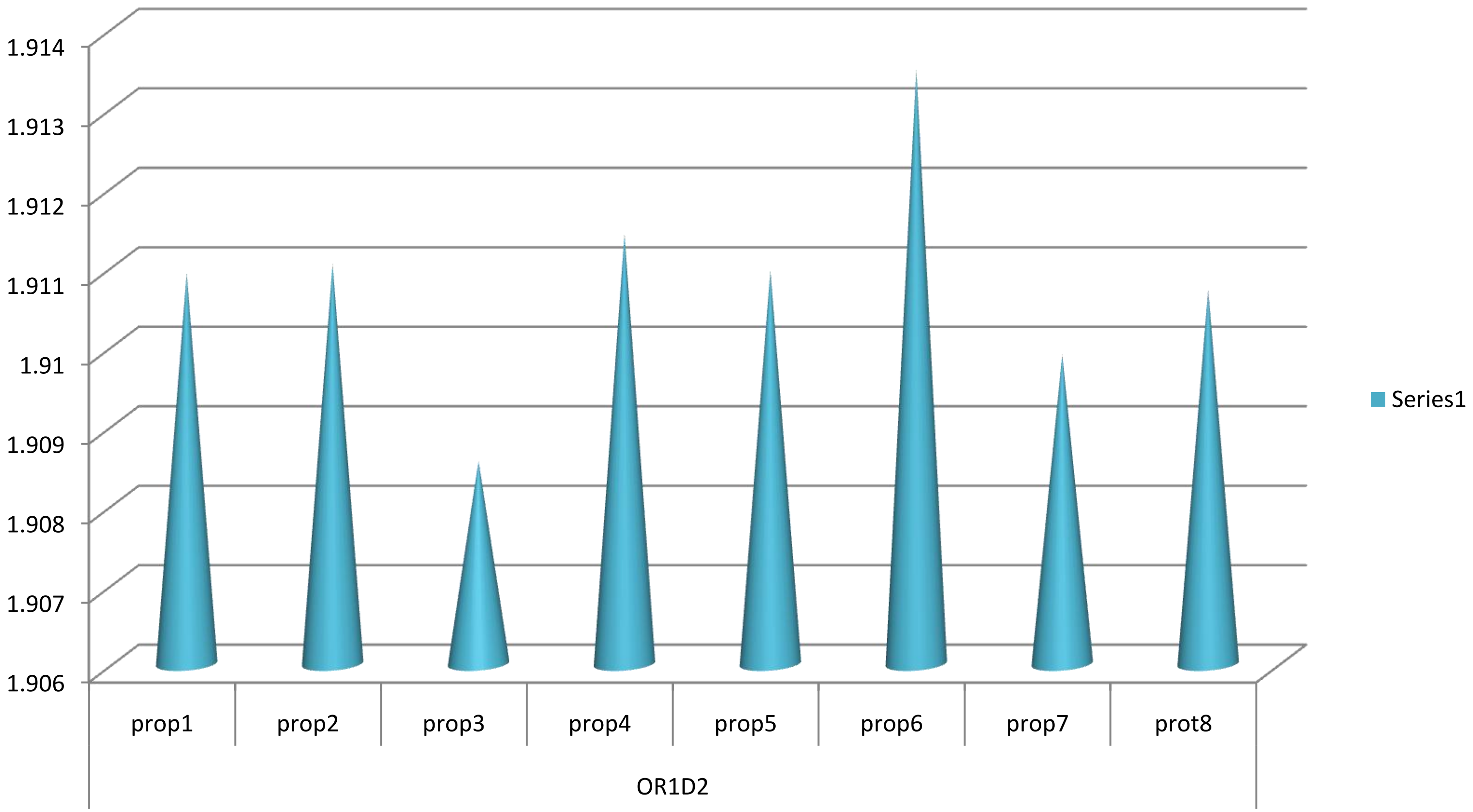
Protein properties, we have considered:

- Accessible residues (%)
- Alpha helix (Chou & Fasman)
- Amino acid composition (%)
- Beta sheet (Chou & Fasman)
- Beta turn (Chou & Fasman)
- Coil (Deleage & Roux)
- Hydrophobicity (Aboderin)
- Total beta strand

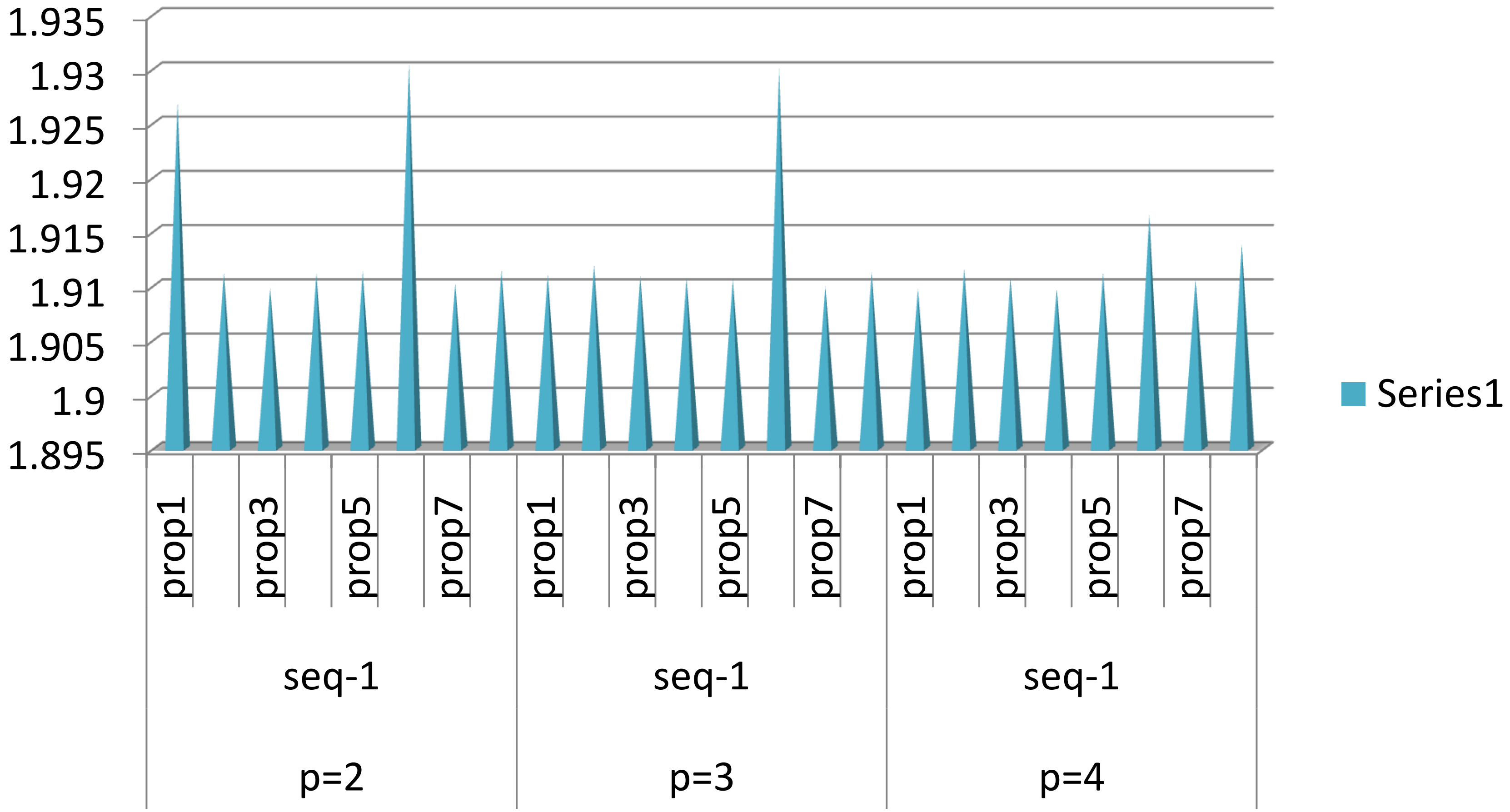
Data Representation and Quantification through Fractal Dimension (Box-counting method)

We got different graphs corresponding to each of those 8 protein properties and then we perform Box counting method (using BENOIT software) to have Fractal dimension for each of those property-graph.

Protein properties of OR1D2



Protein properties of generated sequences



Observations

- The generated sequences have mapped to Olfactory Receptors of *Homo sapiens* and *Vitis vinifera.*
- The generated sequences preserved the protein properties as in the protein sequence of OR1D2 of *Homo sapiens*.

Inference Drawn

We see that IVTs steer a given OR sequence of a species to another of the same or different (most likely) species, preserving the protein properties of the original sequence.

Concluding remarks

- What is the underlying Biological methodology that governs the entire process?
- IVTs may also be thought of as a platform to comprehend the morphological connections among the various species.
- A naïve question to the Biologists:

Suppose, we are given an olfactory receptor *or1* of a species *s1*which help it to identify the odors *x1, x2,...*

Now, we apply the proposed methodology to *or1* and obtain a new olfactory receptor *or2* (supposedly) of species *s2*.So, does *or2* help *s2* in identifying the same odors x1,x2,….?

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Thank You !!!!