# Comparison of Loss Ratios of Different Scheduling Algorithms

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# Comparison of loss ratios of different scheduling algorithms

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**Abstract**

It is well known that in a firm real time system with a renewal arrival process, exponential service times and stochastic deadlines till the end of service of a job, the earliest deadline first (EDF) scheduling policy has smaller loss ratio (expected fraction of jobs that are not completed) than any other service time independent scheduling policy, including the first come first served (FCFS) policy. Various modifications to the EDF and FCFS policies have been proposed in the literature, with a view to improving performance. In this article, we compare the loss ratios of these two policies along with some of the said modifications, as well as their counterparts with deterministic deadlines. The results include some formal inequalities and some counter-examples to establish non-existence of an order. A few relations involving loss ratios are posed as conjectures, and simulation results in support of these are reported. These results lead to a complete picture of dominance and non-dominance relations between pairs of scheduling policies, in terms of loss ratios.

*Key words:* Firm real time system, Earliest Deadline First, First Come First Served, loss ratio comparison

## 1. Introduction

In real time systems consisting of aperiodic jobs, such as web server, network router or real time database; it is typically not known when a job

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will arrive or what its service time and deadline will be. If too many jobs arrive simultaneously, the system becomes overloaded and the jobs begin to miss their deadlines. The service requirements for the jobs are often not known beforehand, and hence are specified in probabilistic terms. So a fundamental problem in such systems is to schedule a set of jobs such as to allow the maximum possible number of jobs to meet their respective deadlines. A common measure of performance of a scheduling algorithm is the loss ratio, that is the expected fraction of jobs that are not completed by their respective deadlines.

In this article, we consider various scheduling algorithms for firm real time systems (i.e., systems where a job must leave the queue latest by its deadline [4]) with a single processor and an aperiodic workload, under their commonly used model as a $G/G/1+G$ queue with an infinite buffer [6]. We assume that the deadline of a job is till the end of its service, that its service time is known at its arrival epoch and it can be preempted at any time. We also assume that the number of priority levels is unlimited, and the context switch overhead is negligibly small.

In the above set up, the simplest scheduling policy is the First Come First Served (FCFS) policy, which stipulates that jobs be serviced in the order of their arrival. A more complex scheduling policy that has some attractive optimality properties is the Earliest Deadline First (EDF) policy [7]. According to this policy, jobs that have arrived and await execution are kept in a ready queue, sorted in ascending order by their absolute deadlines. When the processor finishes a job, the first job in the queue is selected for execution. When a job arrives, it is inserted in the proper position of the queue (breaking ties arbitrarily). A variant of the EDF policy provides for preemption of the currently running job by a newly arrived job, if the absolute deadline of this job is earlier than that of the currently running job. If it is assumed that a job can always be preempted, and that there is no cost of preemption, then it can be shown that preemptive EDF is the optimal policy within the class of non-idling service-time independent preemptive policies [5], in the sense that if a set of jobs with with arbitrary release times and deadlines on a single processor can be scheduled in such away that no job misses its deadline, then the EDF scheduler would necessarily produce such a schedule. Also, it has been shown that EDF stochastically minimizes the loss ratio in both preemptive and non-preemptive models [9, 10].

There have been attempts to reduce the loss ratio by controlling admission of newly arriving jobs in the queue, through a scheduling test. Prominent examples of this innovation are utilization based admission controller [1] and the exact admission controller [7]. The Utilization based admission-controller for aperiodic jobs is pessimistic in the sense that it sometimes denies admission to a job even if that job can be scheduled at that instant. It can be shown that a utilization based admission-controller also passes some jobs that would not be completed before their respective deadlines. The exact admission controller (EAC) seeks to remove these shortcomings at the cost of increased computational complexity ($O(\log n)$ for EAC as opposed to $O(1)$ for the utilization based admission controller) [2].

While an admission controller takes into account the history of jobs already in the queue, a particular decision regarding admission may appear to be unduly conservative in the light of events that follow that decision. If the decision to serve a job is deferred till the epoch of it being served, then that decision can be made on the basis of additional information. Here, we consider a simple modification to scheduling protocols, called the early job discarding (EDT) technique. The EDT does not check the scheduling feasibility of a job on its arrival, but rather admits each incoming job into the system, inserts the job in an appropriate place of the queue according to the protocol being used and lets the system evolve. It discards a job at the epoch of its getting the server from the head of the queue, irrespective of it being a fresh job or a previously preempted job requesting the server again, if it is clear at that moment that the job cannot be completed before the deadline. The name *early* job discarding technique reflects the fact that it discards a job *before* its deadline epoch. It should be noted that this common sense belt-tightening step in improving the performance of a scheduling policy may not be feasible in applications that demand guaranteed completion of jobs once they are admitted to the queue. Even where it is feasible, the value of EDT has never been formally studied.[1] We show that this step can be more effective than admission controllers in reducing the loss ratio.

In this article, we undertake a comprehensive and comparative study of the loss ratios for the FCFS and the EDF scheduling policies along with their variations. We show that, under a purely random environment, the inclusion of EDT in the FCFS and the EDF scheduling policy reduces the loss ratio.

[1] The only relevant work that we could access in this connection is a simulation study in [1], where it was found that EDT works marginally better than the utilization based admission controller in a particular situation.

We also show that the inclusion of EAC reduces the loss ratio for the FCFS scheduling policy, while the same is not true for the EDF scheduling policy under general conditions. We also prove that EDF along with EDT has smaller loss ratio than all other scheduling algorithms considered here, for an $M/M/1+G$ system.

This article is organized as follows. In Section 2, possible dominance relations of the scheduling strategies in terms of loss ratio are discussed. Special attention to systems with deterministic job deadlines is given in Section 3. Some concluding remarks are provided in Section 4.

## 2. Comparing loss ratios for various scheduling policies

In this section, we undertake a comprehensive and comparative study of the loss ratios for the FCFS and the EDF scheduling policies with their variations. We use the notations $\lambda$ for arrival rate (reciprocal of the mean inter-renewal time), $\mu$ for the service rate (reciprocal of mean service time), $\theta$ for mean relative deadline. Further, we use the notation $\alpha_{sp}^{H}$ to denote the loss ratio of a system under the scheduling policy *sp* and with relative deadline distribution $H(\cdot)$.

### *2.1. Performance enhancement through EAC*

Generally, EAC is used to provide a guarantee of service completion of a job once it is admitted to the queue. It can be expected that inclusion of EAC also reduces the loss ratio of the system by removing the unproductive utilization of the server. However, there has so far been no study to verify whether this is indeed the case. In this section, we will investigate whether the inclusion of EAC reduces the loss ratio of firm real-time systems operated under the FCFS or the EDF schedulers.

#### *2.1.1. FCFS scheduling policy*

Here, we formally prove that EAC reduces the loss ratio of an FCFS scheduler under a more general set-up.

*Proposition 2.1.* In a $G/G/1+G$ queue, the loss ratio under the FCFS scheduling policy can only be reduced when Exact Admission Control is used, i.e., $\alpha_{FCFS\text{-}EAC}^{H} \leq \alpha_{FCFS}^{H}$.

*Proof.* Let $A_i$, $Y_i$ and $D_i$ be the arrival epoch, the service time and the relative deadline of the $i^{th}$ job. If $V_i$ is the workload upon arrival of the $i^{th}$

job under the FCFS scheduling policy, then

$$\alpha^H_{FCFS} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} E[I_{\{V_i+S_i>D_i\}}]. \tag{1}$$

Likewise, if $V_i^e$ is the workload upon arrival of the $i^{th}$ job under the FCFS-EAC scheduling policy, then

$$\alpha^H_{FCFS-EAC} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} E[I_{\{(V_i^e+S_i>D_i\}}]. \tag{2}$$

Note that the workloads in the two cases follow the recursions

$$\begin{aligned} V_i &= \Big[(V_{i-1}+Y_{i-1})I_{\{V_{i-1}+Y_{i-1}\le D_{i-1}\}} \\ &\quad +(V_{i-1}\vee D_{i-1})I_{\{V_{i-1}+Y_{i-1}>D_{i-1}\}} - (A_i - A_{i-1})\Big] \vee 0 \qquad (3)\\ V_i^e &= \Bigg[(V_{i-1}^e+Y_{i-1})I_{\{V_{i-1}^e+Y_{i-1}\le D_{i-1}\}} \qquad (4)\\ &\quad +V_{i-1}^e I_{\{V_{i-1}^e+Y_{i-1}>D_{i-1}\}} - (A_i - A_{i-1})\Bigg] \vee 0. \end{aligned}$$

We prove by induction that $V_i \ge V_i^e$ for all $i$. The inequality holds trivially for $i=1$. Assuming that it holds for all indices up to $i-1$, we consider three cases: $V_{i-1} \le D_{i-1}-Y_{i-1}$, $V_{i-1}^e \le D_{i-1}-Y_{i-1} < V_{i-1}$ and $D_{i-1}-Y_{i-1} < V_{i-1}^e$.

In the first case,

$$\begin{aligned} V_i &= [(V_{i-1}+Y_{i-1}) - (A_i - A_{i-1})] \vee 0 \\ V_i^e &= \big[(V_{i-1}^e+Y_{i-1}) - (A_i - A_{i-1})\big] \vee 0. \end{aligned}$$

In the second case,

$$\begin{aligned} V_i &= [(V_{i-1}\vee D_{i-1}) - (A_i - A_{i-1})] \vee 0 \\ V_i^e &= \big[(V_{i-1}^e+Y_{i-1}) - (A_i - A_{i-1})\big] \vee 0. \end{aligned}$$

In the third case,

$$\begin{aligned} V_i &= [(V_{i-1}\vee D_{i-1}) - (A_i - A_{i-1})] \vee 0 \\ V_i^e &= \big[V_{i-1}^e - (A_i - A_{i-1})\big] \vee 0. \end{aligned}$$

In all the cases, we have $V_i \ge V_i^e$, which concludes the induction argument. The stated result follows from (1) and (2). □

*2.1.2. EDF scheduling policy*

In the previous section, we have proved that EAC not only provides guaranteed service but it also reduces the loss ratio of a system run with the FCFS scheduling policy. This is due to the fact that, along with the FCFS scheduling policy, EAC screens only those jobs whose deadlines are not sufficiently large for their own successful completions. Thus, under FCFS, all the unproductive server usages are removed through EAC. As a result, loss ratio of the system is reduced. Similarly, under EDF, EAC denies services to all the jobs that would not have been successfully served due to their smaller deadline. However, under EDF, EAC also denies services to a job due to the server's commitment to previously admitted jobs. The latter cause of service denial is detrimental to the reduction of the loss ratio, because service commitment to a previously admitted job with large service time can trigger service denial to several small jobs that could otherwise have been admitted. Therefore, inclusion of EAC may not in general reduce the loss ratio of a real-time $G/G/1+G$ queue, operated under the EDF scheduling policy. We demonstrate this through an example.

Consider a real-time $G/G/1+G$ queue under EDF and EDF-EAC scheduling policies, where the service times have the two-point distribution with probabilities 0.8 and 0.2 assigned to the points 1 and 19.5, respectively, the relative deadline distribution is also two-point with equal probabilities assigned to the points 2 and 20, respectively and the inter-arrival times are uniformly distributed with support $[\mu/\lambda - 1, \mu/\lambda + 1]$. The normalized arrival rate $(\lambda/\mu)$ can vary from 0 to 4. The loss ratios of the system under the two policies are plotted in Figure 1 as a function of the normalized arrival rate. The values of the loss ratios are computed on the basis of simulations consisting of three different runs of the process, each with about ten million arrivals. We can see from Figure 1 that neither of the loss ratios under the two policies uniformly dominates the other.

We have found that for general real-time queues, EDF sometimes performs better than EDF-EAC in terms of loss ratio. However, the loss ratio under EDF-EAC may be less than that under EDF scheduling policy, in the special case of the $M/M/1+G$ queue. Even though we could not prove this dominance relation, it appears to supported by the simulation results shown in Figure 2, for a number of deadline distributions. We ran the simulations for a wide range of normalized arrival rates (with $\lambda/\mu$ varying from 0 to 4), and four types of relative deadline distributions, namely exponential, uni-

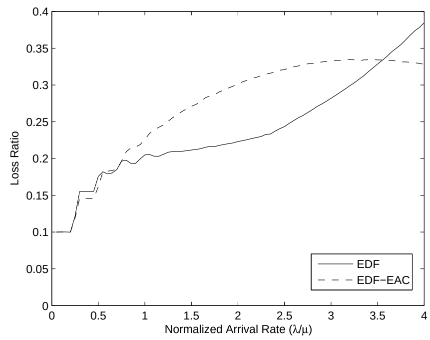


Figure 1: Loss ratios of the EDF-EAC and EDF scheduling policies for two-point relative deadline distribution and various normalized arrival rates ($\lambda/\mu$).

form, log-normal and two-point. The mean ($\theta$) of the deadline distribution was varied from $\frac{1}{\mu}$ to $\frac{16}{\mu}$. We plot the loss ratios for these deadline distributions under the EDF-EAC scheduling policy, normalized by the loss ratio under the EDF scheduling policy, for various values of normalized arrival rate ($\lambda/\mu$) and normalized mean relative deadline ($\mu\theta$).

The exponential deadline distribution is completely characterized by its mean. For the uniform deadline distribution, we have chosen the support as $[0, 2\theta]$. In the case of the log-normal distribution of the normalized deadline, we have chosen the coefficient of variation was as 1 for all values of $\theta$. In the case of the two-point distribution, the probabilities 0.9 and 0.1 were assigned to the points $\frac{5}{9}\theta$ and $5\theta$, respectively, for all values of $\theta$. The values of the loss ratios were computed on the basis of three independent simulation runs, each consisting of about one million arrivals.

It is seen that the normalized loss ratio over the entire range of normalized arrival rates and normalized mean relative deadlines for all the deadline distributions is less than one. On the basis of these findings, we make the following conjecture.

*Conjecture 2.1.* The loss ratio of an $M/M/1 + G$ queue under the EDF-EAC scheduling policy is less than that under the EDF scheduling policy, i.e., $\alpha^H_{EDF\text{-}EAC} \leq \alpha^H_{EDF}$.

We also observe from Figure 2 that the normalized loss ratio approaches the value 1 when the normalized arrival rate goes to zero, as expected. The loss ratio under EDF-EAC becomes gradually smaller than that un-

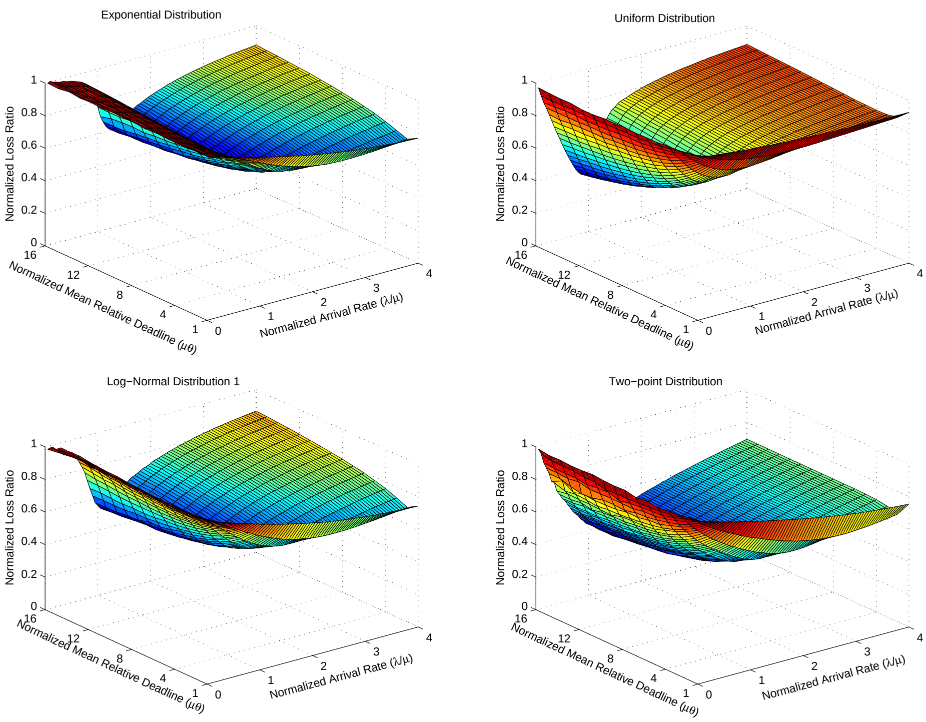


Figure 2: Loss ratios for various deadline distributions under the EDF-EAC scheduling policy normalized by loss ratio under the EDF scheduling policy, for various values of normalized arrival rate $(\lambda/\mu)$ and normalized mean relative deadline $(\mu\theta)$.

der EDF as the normalized arrival rate increases to 1. There is a subsequent turnaround, provided the mean relative deadline is large. The turnaround point depends on the mean of the relative deadline and, to some extent, on its distribution. Another interesting fact is that, irrespective of the normalized arrival rate, the normalized loss ratio is a non-monotone function of the mean relative deadline, with very large and very small values of the latter producing high values of the normalized loss ratio.

*2.2. Performance enhancement through EDT*

While the exact admission controller is meant for assuring guaranteed service, the counter-example given in the forgoing section shows that it is not necessarily effective in reducing the loss ratio. This fact gives rise to the question: Can reduction of loss ratio of a general real-time queue be achieved through EDT, which aims directly at performance enhancement? We now show that the inclusion of EDT in FCFS and EDF schedulers indeed reduces the loss ratio of firm real-time systems.

*2.2.1. FCFS scheduling policy*

We first consider the impact of EDT on a $G/G/1+G$ queue, operated under the FCFS scheduling policy.

*Proposition 2.2.* The loss ratio of a $G/G/1+G$ queue, operated under the FCFS scheduling policy, can only reduce when EDT is used, i.e., $\alpha^H_{FCFS\text{-}EDT} \leq \alpha^H_{FCFS}$.

*Proof.* The stated result follows from Proposition 2.1 and the fact that the loss ratios of FCFS-EDT and FCFS-EAC are identical, as proved in Proposition 2.4 below. However, here we give a direct argument.

Consider a finite number of job arrivals, and arrange all the jobs in order of their arrival. A job that is discarded under EDT would have missed the deadline in any case. On the other hand, the act of discarding a particular job can only reduce the waiting times of the subsequent jobs (arranged as above). Consequently, the act of discarding that job can only reduce the number of subsequent jobs missing their respective deadlines. This argument holds for every single event of discarding of jobs under EDT. Thus, for any given configuration of a finite number of jobs, the proportion of jobs missing deadline under FCFS-EDT is less than or equal to that under FCFS. It follows that the expected proportion of jobs (out of the first $n$ arrivals for any fixed

$n$) is less for FCFS-EDT than for FCFS. The stated result is obtained by taking the limit of the expected proportions as $n$ goes to infinity. $\square$

*2.2.2. EDF scheduling policy*

We now consider the impact of EDT on a $G/G/1+G$ queue, operated under the EDF scheduling policy.

*Proposition 2.3.* The loss ratio of a $G/G/1+G$ queue, operated under the EDF scheduling policy, can only reduce when EDT is used, i.e., $\alpha^H_{EDF\text{-}EDT} \leq \alpha^H_{EDF}$.

*Proof.* We prove this result by using a path-wise argument, where successive acts of discarding jobs, that can not be completed are shown to improve the action of completed jobs under the EDF scheduling policy. Let, for $i = 1, 2, \ldots, N$, $A_i$, $Y_i$ and $D_i$ be the arrival epoch, service time and relative deadline of the job $J_i$. The jobs are indexed in the order of arrival. Let $y_{i,h}$ be the remaining service time of job $J_h$ at time $A_i$, $1 \leq h \leq i \leq N$. In case $J_h$ has been fully served before $A_i$, or $A_h + D_h \leq A_h$, $y_{i,h} = 0$. Further, let $c_i$ be the aggregate number of jobs completed successfully till time $A_i$.

As per the EDF scheduling policy, the server is obliged to serve even those jobs that have no chance of being completed. Let $t_0$ be a point of time when the server serves the first job of this kind, say $J_k$. Suppose $J_k$ is discarded at $t_0$, and subsequently server engagement of all the remaining jobs are rescheduled as per the EDF policy. This rescheduling could alter $y_{i,h}$ and $c_i$ for $1 \leq h \leq i \leq N$. Let the (possibly) modified versions of these quantities be denoted by $y^{(1)}_{i,h}$ and $c^{(1)}_i$. We shall show by induction that the set of inequalities

$$y^{(1)}_{i,h} \leq y_{i,h},\ c^{(1)}_i \geq c_i \quad 1 \leq h \leq i,\ h \neq k, \tag{5}$$

hold for $i = 1, 2, \ldots, N$. Let $i_0$ be such that $A_{i_0} \leq t_0 < A_{i_0+1}$. It is easy to see that (5) holds with equality for $1 \leq i \leq i_0$. Further, as far as the time interval $[A_{i_0}, A_{i_0+1})$ is concerned, the act of discarding $J_k$ at $t_0$ does not alter the priority list of the remaining jobs; it merely allows some jobs to be served longer. As a result, (5) holds for $i = i_0 + 1$.

Now suppose (5) holds for some $i > i_0 + 1$. The priority lists of the jobs in the original and the altered queue are identical, both being in the order of increasing values of $A_h + D_h$, $1 \leq h \leq i$, $h \neq k$. However, in the altered queue, the remaining service times of some jobs are smaller. Therefore,(5) holds for $i+1$. This concludes the induction, and we can infer that $c_N^{(1)} \geq c_N$.

We can also consider the effects of subsequent actions of discarding of jobs under the EDF-EDT policy. In particular, if $c_N^{(j)}$ is the number completed jobs following the first $j$ actions of discarding of jobs, then an adaptation of the above argument would show that $c_N^{(j)} \geq c_N^{(j-1)}$.

It follows by repeated application of this logic that every successive discarding of jobs under the EDF-EDT policy increases the number of successfully completed jobs. In particular, the number of successfully completed jobs under the EDF-EDT policy, say $c_N^*$, is greater than $c_N$. Therefore,

$$1 - \lim_{N\to\infty} E\left(\frac{c_N^*}{N}\right) \leq 1 - \lim_{N\to\infty} E\left(\frac{c_N}{N}\right).$$

This completes the proof. □

*2.3. Comparison of EAC with EDT*

In Section 2.1 and 2.2 we examined whether the inclusion of admission or exit control reduce the loss ratios of FCFS and EDF scheduling policies. A natural question that arises from these investigations is: How do the loss ratios of systems operated under EDT or EAC compare with one another? We explore answers to this question in this section.

*2.3.1. FCFS scheduling policy*

We first compare the impacts of EAC and EDT on the loss ratio in the case of an FCFS scheduler.

*Proposition 2.4.* In a $G/G/1+G$ queue, the loss ratios under the FCFS-EDT and FCFS-EAC scheduling policies are identical, i.e., $\alpha^H_{FCFS\text{-}EAC} = \alpha^H_{FCFS\text{-}EDT}$.

*Proof.* Consider the implementation of the FCFS-EAC scheduling policy, where a job that does not satisfy the admission criterion of EAC is not discarded at the time of admission, but is merely tagged for eventual rejection at the epoch of its getting the server. Note that this modification does not change the completion status of any job. On the other hand, under the modified policy, the order of the untagged jobs getting the server becomes the same as that under FCFS. The fact that the tagged job would have been denied admission under the FCFS-EAC procedure indicates that this job, if served, would have missed its own deadline. Thus, this job would also be discarded under FCFS-EDT. It can be seen that, out of the first $n$ arrivals, the set of jobs that would be discarded under FCFS-EDT is precisely the set of jobs tagged as above. It follows that, for any given configuration of $n$ job arrivals, the proportion of jobs missing deadline is the same under FCFS-EAC and FCFS-EDT. The result follows by taking expectation of this proportion and allowing $n$ to go to infinity. □

*2.3.2. EDF scheduling policy*

For a general $M/G/1+G$ queue, there is no dominance relationship between the loss ratios under the EDF-EAC and the EDF-EDT scheduling policies. Consider a real-time queue, where arrivals follow a Poisson process with rate 0.5, the service times have the two-point distributions over the values 3 and 3.5 with probabilities 0.9 and 0.1, respectively, and the relative deadlines also have the two-point distribution over the values 3.75 and 6 with probabilities 0.9 and 0.1, respectively. In this case, simulation results show that the loss ratio of a real-time queue operated under the EDF-EDT scheduling policy is 0.5329, while that under the EDF-EAC scheduling policy is 0.5302. The values of the loss ratios are computed on the basis of simulations consisting of ten different runs of the process, each with about one million arrivals. The 95% confidence intervals of the loss ratios under EDF-EDT and under EDF-EAC are [0.5326, 0.5332] and [0.5299, 0.5306], respectively. Since the intervals are nonoverlaping, we can conclude that the loss ratio under EDF-EDT is sigificantly larger than that under EDF-EAC.

The reverse order exists when the service time distribution is chosen as exponential with mean as before, the loss ratios under the EDF-EDT and the EDF-EAC policies being 0.3555 and 0.3570, respectively. In this case, 95% confidence intervals of the loss ratios under EDF-EDT and under EDF-EAC are [0.3552, 0.3558] and [0.3567, 0.3573], respectively. The nonoverlap of the intervals implies that the loss ratio under EDF-EDT is sigificantly smaller than that under EDF-EAC.

However a definite order between the loss ratios under these two scheduling polices appear to emerge in the case of an $M/M/1+G$ queue. Figure 3 indicates that the loss ratio in the case of EDF-EDT is smaller, when the deadlines distribution is any one of the four considered there. On the basis of these findings, we make the following conjecture.

*Conjecture 2.2.* The loss ratio of an $M/M/1+G$ queue under the EDF-

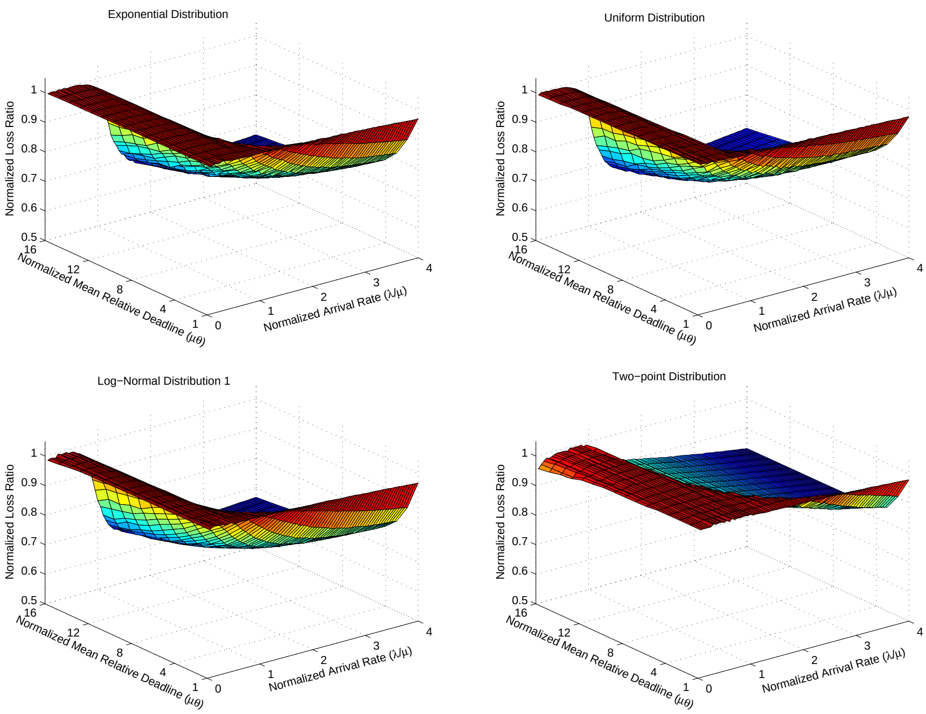


Figure 3: Loss ratios for various deadline distributions under the EDF-EDT scheduling policy normalized by loss ratio under the EDF-EAC scheduling policy, for various values of normalized arrival rate ($\lambda/\mu$) and normalized mean relative deadline ($\mu\theta$).

EDT scheduling policy is less than that under the EDF-EAC scheduling policy, i.e., $\alpha^H_{EDF\text{-}EDT} \leq \alpha^H_{EDF\text{-}EAC}$.

It also transpires from Figure 3 that the loss ratios of an $M/M/1+G$ system under the EDF-EDT and the EDF-EAC scheduling policies are close to one another in two contrasting situations: when the mean normalized relative deadline is small (i.e., both the loss ratios are large), and when the normalized arrival rate is small (i.e., both the loss ratios are small). EDF-EDT produces significantly smaller loss ratio when both the mean normalized relative deadline and the normalized arrival rate are large.

*2.4. Superiority of EDF with admission/exit control*

We now compare the loss ratios under the FCFS and the EDF scheduling policies under various circumstances, viz. with or without EDT or EAC.

Optimality of the EDF scheduling policy within the class of service time independent policies is well known [9]. The following proposition follows from Theorem 1 of Towsley and Panwar [9].

*Proposition 2.5.* In an $G/M/1+G$ queue, the loss ratio under the EDF scheduling policy is smaller than that under any other service time independent scheduling policy with deadline till the end of service. In particular, EDF produces smaller loss ratio than FCFS, i.e., $\alpha^H_{EDF} \leq \alpha^H_{FCFS}$.

The result of Towsley and Panwar [9] on the optimality of the EDF scheduling policy among the class of all service-time independent policies is not applicable in the presence of EDT or EAC, which make the scheduling policy dependent on service time. This fact gives rise to the question of possible optimality, in terms of loss ratio, of EDF among the modified class of scheduling policies that accommodate EDT or EAC. This question appears to be a particularly difficult one. Therefore, we turn to the simpler question of possible superiority of EDF over a specific policy such as FCFS, in terms of loss ratio, in the presence of EDT or EAC.

*2.4.1. EDF with EDT*

In general, EDF-EDT may not have smaller loss ratio than that of FCFS-EDT. However, for exponential service time distribution, we will show that even after the inclusion of either EDT, EDF indeed has smaller loss ratio than FCFS on an average.

Let $\nu_t(n)$ be the expected count of completed jobs in a $G/M/1+G$ queue of size $n$ under the EDF-EDT policy subject to an initial server commitment of $t$ units of time.

*Lemma 2.1.* In a finite length $G/M/1+G$ queue operated under the EDF-EDT scheduling policy and satisfying the condition $\nu_{t_1}(n) \geq \nu_{t_2}(n)$ for all $t_1 \leq t_2$ and all $n > 1$, the inequality $E[\nu_{Y_1}(n)] \geq E[\nu_{Y_2}(n)]$ holds whenever $Y_1$ is stochastically smaller than $Y_2$.

*Proof.* Let $F_{Y_i}(t) = P(Y_i \leq t)$ for $i = 1, 2$. We have

$$E[\nu_{Y_i}(n)] = \int_0^\infty \nu_t(n) dF_{Y_i}(t) = -\int_0^\infty F_{Y_i}(t) d\nu_t(n), \quad i = 1, 2.$$

The result follows from the fact that $F_{Y_1}(t) \geq F_{Y_2}(t)$. □

*Proposition 2.6.* In a finite length $G/M/1+G$ queue satisfying the condition $\nu_{t_1}(n) \leq \nu_{t_2}(n)$ for all $t_1 \leq t_2$ and all $n > 1$, the loss ratio under the EDF-EDT scheduling policy is less than that of the FCFS-EDT scheduling policy, i.e., $\alpha^H_{EDF\text{-}EDT} \leq \alpha^H_{FCFS\text{-}EDT}$.

*Proof.* In order to compare the EDF-EDT and FCFS-EDT policies, we consider a sequence of scheduling policies signifying transition from the former policy to the latter.

Let $P_n$ denote the scheduling policy, where the jobs are scheduled according to the FCFS-EDT policy for the first $n$ arrivals, and there is a switch to the EDF-EDT policy before the arrival of the next job. In particular, up to the first $(n-1)$ arrivals, jobs are placed according to the order of arrivals. All the subsequent jobs, starting from the $n^{th}$ arrived job (labeled as $J_n$), are placed in the queue in the order of their absolute deadlines, without altering the positions of the jobs arrived before the switch-over. Note that, as far as the first $N$ arrivals are concerned, $P_1$ corresponds to the EDF-EDT policy, while $P_N$ corresponds to the FCFS-EDT policy.

We first establish the superiority of the EDF-EDT scheduling policy over the FCFS-EDT policy, for $N$ arrivals, and then let $N$ go to infinity. We aim at showing that the expected count of completed jobs (out of the total of $N$ jobs) for $P_n$ is a decreasing function of $n$.

In order to facilitate comparison between $P_n$ and $P_{n+1}$ in terms of loss ratio, we introduce another scheduling policy, $P'_n$. Consider the situation where the job $J_n$ is successfully serviceable under $P_{n+1}$. Let $J_r$ be the first job that departs unsuccessfully under $P_{n+1}$ but is completed under $P_n$. If such a job does not exist, then the completion status of all the jobs in the two queues are identical. If there is a job labeled as above, then the absolute deadline of $J_r$ must be smaller than that of $J_n$. (Else, $J_r$ would be placed below in $P_n$, and the completion status of all the jobs departing before $J_r$ in the two queues would be identical, making it impossible for $J_r$ to have different completion status under the two queues.) We define $P'_n$ as a modification of $P_n$, in which the job $J_n$ is routinely discarded whenever the following conditions hold: (a) $J_n$ is successfully serviceable under $P_{n+1}$ and (b) there is a job $J_r$ that can be labeled as above. The discarding occurs at the epoch of successful departure of $J_r$.

We compare $P_n$, $P'_n$ and $P_{n+1}$ in two cases, depending on the status of the job $J_n$ under $P_{n+1}$.

Case 1. Let the job $J_n$ not be successfully serviceable under $P_{n+1}$. It follows that $J_n$ is not successfully serviceable under $P_n$ also. Hence, the completion status of all the jobs under the three policies are identical.

Case 2. Let the job $J_n$ be successfully serviceable under $P_{n+1}$. Up to and including the job $J_r$, we find that exactly the same number of jobs meet their deadlines under $P_n$, $P'_n$ and $P_{n+1}$. However, the workloads for the subsequent jobs in the queue under $P'_n$ and $P_{n+1}$ are different, even though the two queues consist of the same set of jobs arranged in identical order. We will show that the workload on the subsequent jobs is stochastically larger under $P_{n+1}$ than under $P'_n$.

Let us take the epoch of the server commencing service to $J_n$ as the time 0 (reference time). Let $Y_r$ and $Y_n$ be the service times of jobs $J_r$ and $J_n$, respectively. Let $\tau$ be the aggregated service times of the jobs arriving after $J_n$ and having absolute deadlines earlier than that of $J_r$. Let $d_r$ and $d_n$ $(= d_r + d')$ be the absolute deadlines of $J_r$ and $J_n$, respectively.

We will now show that, given these circumstances, $Y_r$ is stochastically smaller than $Y_n$, i.e., the workload on jobs placed in the queue after $J_r$ is stochastically larger under $P_{n+1}$ than under $P'_n$, for every combination of values of $\tau$, $d_r$, and $d'$.

For any set of fixed and positive values of $\tau$, $d_r$ and $d'$ satisfying $\tau < d_r$, the following simultaneous conditions fully characterize this case.

1. Job $J_r$ meets its deadline under $P'_n$, so that $\tau + Y_r \leq d_r$.
2. Job $J_n$ meets its deadline under $P_{n+1}$, so that $Y_n \leq d_r + d'$.

3. Job $J_r$ misses its deadline under $P_{n+1}$, so that $\tau + Y_r + Y_n > d_r$.

We refer to the simultaneous occurrence of these three conditions as the event $E$.

$$E = \{\tau + Y_r \leq d_r,\ Y_n \leq d_r + d',\ \tau + Y_r + Y_n > d_r\}.$$

We will first obtain the conditional probability distribution functions of $Y_r$ and $Y_n$ given by $E$, denoted by $F_{Y_r|E}(x)$ and $F_{Y_n|E}(x)$. These are obtained from their joint conditional probability density function $f_{Y_r,Y_n|E}(x, x')$, which we now deduce. If the service rate is $\mu$, the joint density of $Y_r$ and $Y_n$ given $E$ is their unconditional density (product of iid $\exp(\mu)$) truncated to the set $E$, i.e.,

$$f_{Y_r,Y_n|E}(x, x') = \frac{\mu^2 e^{-\mu(x+x')}}{P(E)}, \quad x' \leq a,\ x \leq b < x + x',$$

where $a$ and $b$ are obtained by considering the conditions on $Y_r$ and $Y_n$ that must be satisfied for event $E$ to be true. The range of validity of $Y_r$ is obtained from conditions 1 and 3, while that of $Y_n$ is obtained from conditions 2 and 3. Thus, we have $a = \{d_r + d'\}$ and $b = d_r - \tau$ (with $b \leq a$).
Also $P(E)$ is the unconditional probability

$$\iint_{x' \leq a,\ x \leq b < x+x'} \mu^2 e^{-\mu(x+x')}\, dx\, dx'.$$

Now, the conditional density of $Y_r$ given $E$ is

$$\begin{aligned} f_{Y_r|E}(x) &= \int_0^\infty f_{Y_r,Y_s|E}(x, x')\, dx' \\ &= \int_{b-x}^{a} \frac{\mu^2 e^{-\mu(x+x')}}{P(E)}\, dx' \\ &= \frac{\mu}{P(E)} \left[e^{-\mu b} - e^{-\mu(x+a)}\right], \quad 0 \leq x \leq b, \end{aligned}$$

and the corresponding distribution function is

$$F_{Y_r|E}(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ \dfrac{1}{P(E)} \left[\mu x e^{-\mu b} - e^{-\mu a}\left(1 - e^{-\mu x}\right)\right] & \text{if } 0 < x \leq b, \\ 1 & \text{if } x > b. \end{cases}$$

On the other hand, the conditional density of $Y_n$ given $E$ is

$$\begin{aligned} f_{Y_n|E}(x') &= \int_0^\infty f_{Y_r,Y_n|E}(x,x')\,dx \\ &= \int_{b-x'}^{b} \frac{\mu^2 e^{-\mu(x+x')}}{P(E)}\,dx \\ &= \frac{\mu}{P(E)} e^{-\mu b}\left(1-e^{-\mu x'}\right), \quad 0 \le x' \le a, \end{aligned}$$

and the corresponding distribution function is

$$F_{Y_n|E}(x') = \begin{cases} 0 & \text{if } x' \le 0, \\ \dfrac{e^{-\mu b}}{P(E)}\left[\mu x' - \left(1-e^{-\mu x'}\right)\right] & \text{if } 0 < x' \le a, \\ 1 & \text{if } x' > a. \end{cases}$$

By comparing the conditional distribution functions of $Y_r$ and $Y_n$, we observe that the inequality $F_{Y_r|E}(t) \ge F_{Y_n|E}(t)$ holds $\forall\ t$, since $b \le a$. This proves that $Y_n$ is stochastically larger than $Y_r$.

The total count of completed jobs up to the disposing of $J_r$ is the same under $P_n'$ and $P_{n+1}$. Note that, in a time scale that starts from $J_n$ getting the server, this disposal time is either $\tau+Y_r$ or $\tau+Y_n$, depending on whether $P_n'$ or $P_{n+1}$ is used. Therefore, conditional on $\tau$, the workload on a job subsequent to $J_r$ is stochastically smaller under $P_n'$ than under $P_{n+1}$. The result would then hold unconditionally also.

We now show a similar order between $P_n$ and $P_n'$ can be established. Whenever the queues produced by these two policies are different, their only difference is that the queue under $P_n$ contains the job $J_n$, – in addition to the jobs contained in $P_n'$. This difference disappears if $J_n$ is not found to be serviceable under $P_n$. In case $J_n$ is successfully serviceable under $P_n$, it may cause a subsequent job (say, $J_s$) to miss its deadline, which would be successfully served under $P_n'$. In such a case, $J_s$ must have absolute deadline larger than $d_n$. By an argument similar to the one used in comparing $J_n$ and $J_r$, it is seen that $J_s$ has stochastically larger service time than $J_n$. Therefore, the queue of $P_n$ produces stochastically smaller workload on the jobs subsequent to $J_s$ than the corresponding workload produced by $P_n'$.

By applying Lemma 1, we find that the expected count of completed jobs under $P_n'$ is larger than that under $P_{n+1}$, and the expected count of completed jobs under $P_n$ is larger than that under $P_n'$. □

*2.4.2. EDF with EAC*

The following example shows that FCFS-EAC can have smaller loss ratio than EDF-EAC for some configuration of arrival times, service times and relative deadlines. Let there be five jobs, $J_1$, $J_2$, $J_3$, $J_4$ and $J_5$ in the system with the profile given in Table 1.

| Job | Arrival time | Service time | Relative deadline | Absolute deadline |
|---|---|---|---|---|
| $J_1$ | 0 | 5 | 10 | 10 |
| $J_2$ | 1 | 1 | 5 | 6 |
| $J_3$ | 1.5 | 3.5 | 5.5 | 7 |
| $J_4$ | 1.6 | 3 | 7.4 | 9 |
| $J_5$ | 6.1 | 1.5 | 4.5 | 10.6 |

Table 1: Job profile that ensures superiority of FCFS-EAC over EDF-EAC

So if the queue is operated under EDF-EAC, then jobs $J_1$, $J_2$ and $J_3$ are completed. However, under FCFS-EAC, jobs $J_1$, $J_2$, $J_4$ and $J_5$ have successful completion. Hence the number of successful jobs under FCFS-EAC is one more than that under EDF-EAC. However, this finding does not rule out the possible superiority of EDF-EAC over FCFS-EAC for an $M/M/1+G$ queue in terms of loss ratio. In fact, this dominance relation is supported by extensive simulations for a number of relative deadline distributions. We considered Poisson arrival process with a range of normalized arrival rates (with $\lambda/\mu$ varying from 0 to 4), and four types of relative deadline distributions, described above. The values of the loss ratio were computed on the basis of three independent runs of the queue, each consisting of about one million arrivals. The results, summarized in Figure 4, lead us to the following.

*Conjecture 2.3.* In an $M/M/1+G$ queue, the loss ratio under the EDF-EAC scheduling policy is less than that of FCFS-EAC scheduling policy, i.e., $\alpha^H_{EDF\text{-}EAC} \leq \alpha^H_{FCFS\text{-}EAC}$.

It transpires from Figure 4 that the normalized loss ratio generally has a non-monotone relation with the arrival rate and the mean relative deadline. However, the normalized loss ratio assumes the smallest value when the normalized arrival rate is about 1 and the mean relative deadline is large.

*2.5. Comparison of loss ratios of various scheduling policies*

By combining all the results discussed so far, we can build a graph of dominance relations between various pairs of scheduling policies. The policies

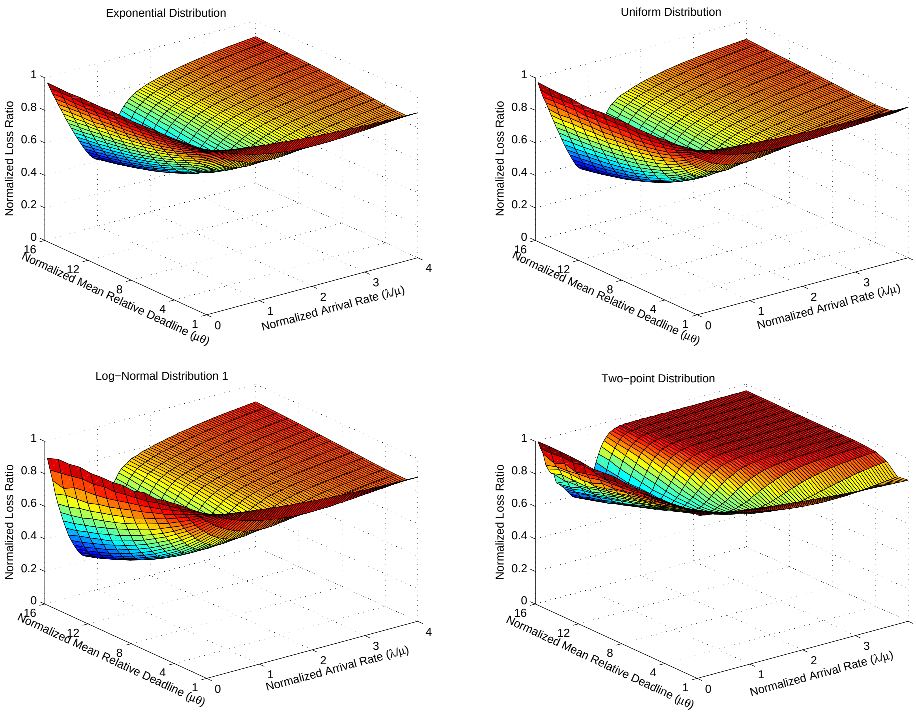


Figure 4: Loss ratios for various deadline distributions under the EDF-EAC scheduling policy normalized by loss ratio under the FCFS-EAC scheduling policy, for various values of normalized arrival rate ($\lambda/\mu$) and normalized mean relative deadline ($\mu\theta$).

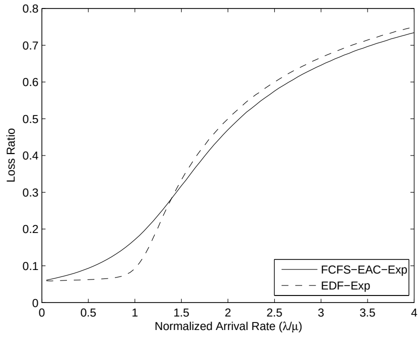


Figure 5: Loss ratios of the FCFS-EAC and EDF scheduling algorithms for exponential relative deadline with $\theta = \frac{16}{\mu}$ and various normalized arrival rates ($\lambda/\mu$).

under consideration include FCFS, EDF and their respective modifications through EAC and EDT. For the sake of completeness, we present a non-dominance relation before presenting the graph.

The following counter-example shows that there is no dominance relation between the loss ratios of the EDF and FCFS-EAC (or FCFS-EDT) scheduling policies, i.e., neither of $\alpha^{Exp}_{FCFS\text{-}EAC}$ and $\alpha^{Exp}_{EDF}$ dominates the other in general.

*Counter-example 2.1.* Consider the $M/M/1$ queue with deadline till the end of the service, where the relative deadline has the exponential distribution with mean equal to 16 times the mean service time ($\theta = \frac{16}{\mu}$). The loss ratios, plotted in Figure 5 as a function of the normalized arrival rate ($\lambda/\mu$), show that the inequality $\alpha^{Exp}_{EDF} \leq \alpha^{Exp}_{FCFS\text{-}EAC}$ holds for small arrival rates, while the inequality $\alpha^{Exp}_{FCFS\text{-}EAC} \leq \alpha^{Exp}_{EDF}$ holds for large arrival rates. The values of the loss ratios are computed on the basis of simulations of about one million arrivals. Thus, neither of $\alpha^{exp}_{EDF}$ and $\alpha^{exp}_{FCFS\text{-}EAC}$ uniformly dominates the other.

Figure 6 shows the graph of dominance relations (in terms of loss ratio of an $M/M/1+G$ system) between various scheduling algorithms. In this figure, an arrow extending from the scheduling policy $sp_1$ to the policy $sp_2$ indicates that $\alpha^H_{sp_1} \leq \alpha^H_{sp_2}$, a double headed arrow indicates equality of the loss ratios, while a pair of arrows facing each other indicates that there is no dominance relation. The dashed arrow represents a conjectured relation, based on simulation studies.

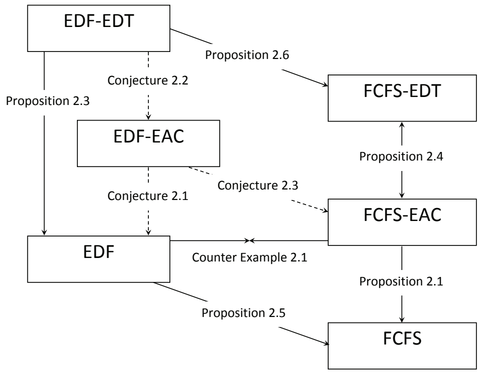


Figure 6: Relationship between various scheduling algorithms in terms of order of loss ratios of an $M/M/1$ system, for stochastic relative deadlines till the end of service.

## 3. Simplifications for deterministic deadlines

In this section we study how the loss ratio of a firm real time system, operating under any of the schedulers considered in Section 2, changes when the deadline distributions becomes degenerate, i.e., the deadline is deterministic. When the deadline distribution is degenerate, one can observe the followings equivalence in terms of loss ratio.

1. The FCFS and EDF scheduling policies are equivalent.
2. The FCFS-EDT and EDF-EDT scheduling policies are equivalent.
3. The FCFS-EDT and FCFS-EAC scheduling policies are equivalent.
4. The EDF-EDT and EDF-EAC scheduling policies are equivalent.

These equivalences follow from simple path-wise analyses of the pairs of policies. In view of the above facts, the relations depicted in Figure 6 for an M/M/1 system simplify to those given in Figure 7. In fact, these results hold in general for a G/G/1 system, since the smallness of the loss ratio for the FCFS-EAC scheduler in comparison to that of the FCFS scheduler had been proved through a path-wise argument (see Proposition 2.1.).

### *3.1. Degenerate and stochastic deadlines: Dominance relations*

Movaghar [8] showed that the loss ratio for the FCFS scheduling policy is bounded from below by the corresponding loss ratio for the case where the deadline is degenerate. In particular, the following proposition follows from Lemma 5.1.3 of Movaghar [8].

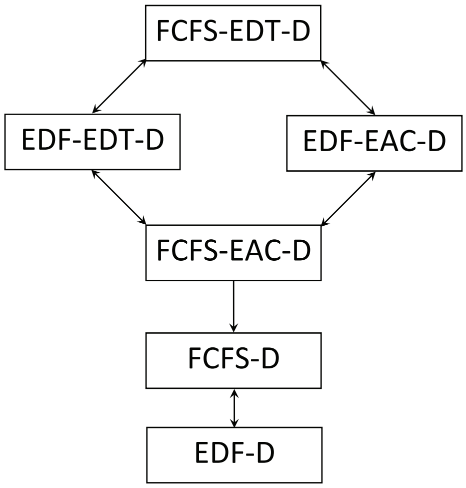


Figure 7: Relationship between various scheduling algorithms in terms of order of loss ratios of a $G/G/1$ system, for degenerate relative deadlines till the end of service.

*Proposition 3.1.* In an $M/M/1+G$ queue with a specified mean deadline till the end of service, the loss ratio under the FCFS scheduling policy happens to be the minimum when the deadline distribution is degenerate, i.e., $\alpha_{FCFS}^{Deg} \leq \alpha_{FCFS}^{H}$.

The above result gives rise to the question as to whether the loss ratio for a DES system under other scheduling policies also attains a minimum value when the relative deadline distribution is degenerate. One can look for an answer to this question for the FCFS scheduling policy with EAC, by using the explicit expression of the loss ratio give in Proposition 4 of [3]. While we could not prove this optimality, numerical computations indicate that the result may hold in the case of an $M/M/1$ queue.

We considered a range of normalized arrival rates (with $\frac{\lambda}{\mu}$ varying from 0 to 4), and four types of relative deadline distributions, as mentioned before. The mean ($\theta$) of the deadline distribution was varied from $\frac{1}{\mu}$ to $\frac{16}{\mu}$. The other parameters of the deadline distributions were chosen as in the case of the simulations reported in the previous sections. The values of the loss ratios were computed from Proposition 4 of [3]. The results are summarized in Figure 8. The loss ratio is found to have a common pattern of dependence on the arrival rate and mean deadline for different deadline distributions. On the basis of these findings, we make the following conjecture.

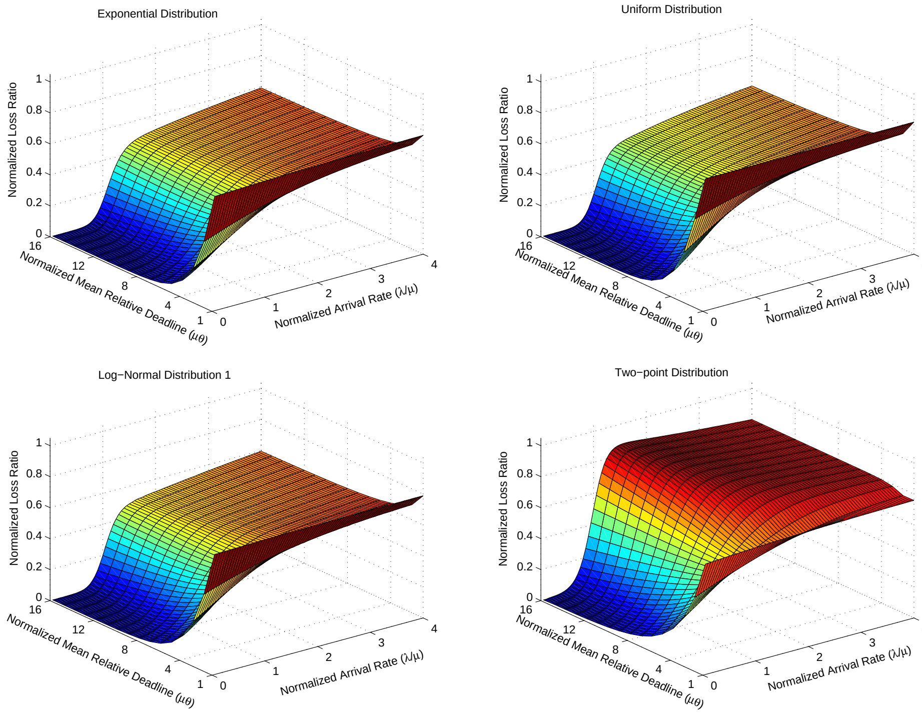


Figure 8: Loss ratio for degenerate deadline normalized by loss ratios for various deadline distributions under the FCFS-EAC scheduling policy, for various values of normalized arrival rate ($\lambda/\mu$) and normalized mean relative deadline ($\mu\theta$).

*Conjecture 3.1.* In an $M/M/1+G$ queue with a specified mean deadline till the end of service, the loss ratio under the FCFS-EAC scheduling policy happens to be the minimum when the deadline distribution is degenerate, i.e., $\alpha^{Deg}_{FCFS-EAC} \le \alpha^{H}_{FCFS-EAC}$.

A result similar to Proposition 3.1 for the EDF scheduling policy was conjectured in [6], and we state it below.

*Conjecture 3.2.* In an $M/M/1+G$ queue with a specified mean deadline till the end of service, the loss ratio under the EDF scheduling policy happens to be the minimum when the deadline distribution is degenerate, i.e., $\alpha^{Deg}_{EDF} \le \alpha^{H}_{EDF}$.

We were unable to find either a proof of the above conjecture or a counter-example to disprove it. However, we conducted simulations for a number of relative deadline distributions. The conditions of these simulations were identical to those used for Figure 8. The values of the loss ratios were computed on the basis of simulations consisting of three different runs of the process, each with about one million arrivals. The results, summarized in Figure 9, support the above conjecture.

We looked for a similar result for the EDF-EDT scheduling policy, but were unable to find either a proof or a counter-example. We state it in the form of a conjecture, which is supported by the simulation results summarized in Figure 10. The conditions for this simulation experiment were the same as before.

*Conjecture 3.3.* In an $M/M/1+G$ queue with a specified mean deadline till the end of service, the loss ratio under the EDF-EDT scheduling policy happens to be the minimum when the deadline distribution is degenerate, i.e., $\alpha^{Deg}_{EDF-EDT} \le \alpha^{H}_{EDF-EDT}$.

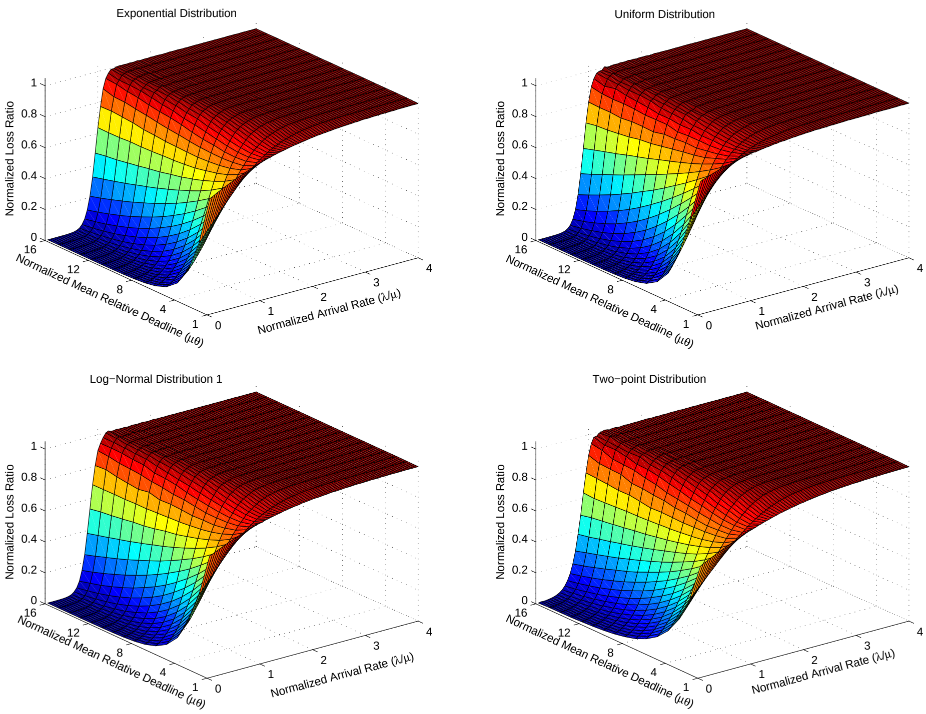


Figure 9: Loss ratio for degenerate deadline normalized by loss ratios for various deadline distributions under the EDF scheduling policy, for various values of normalized arrival rate $(\lambda/\mu)$ and normalized mean relative deadline $(\mu\theta)$.

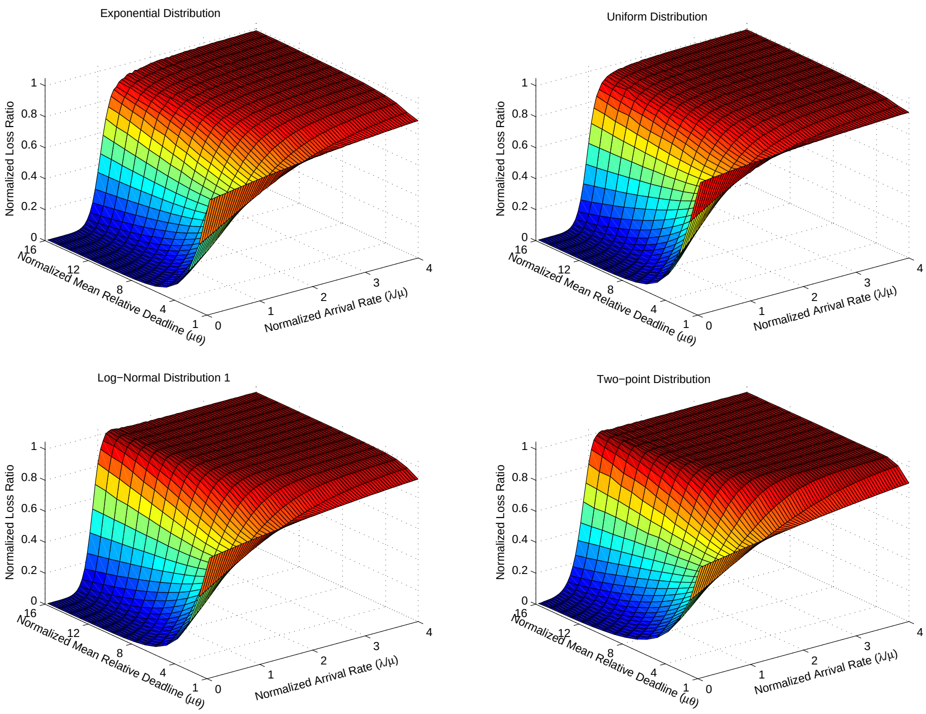


Figure 10: Loss ratio for degenerate deadline normalized by loss ratios for various deadline distributions under the EDF-EDT scheduling policy, for various values of normalized arrival rate $(\lambda/\mu)$ and normalized mean relative deadline $(\mu\theta)$.

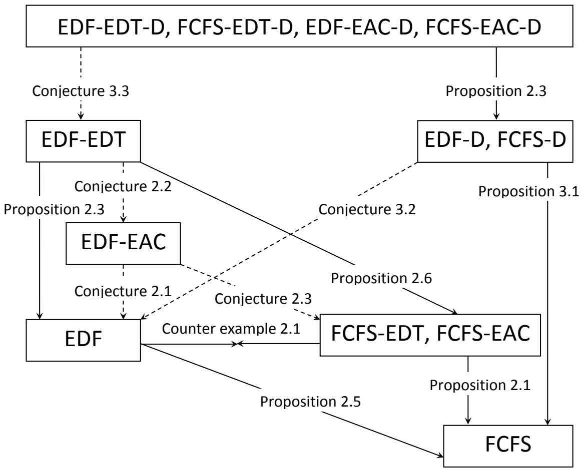


Figure 11: Relationship between various scheduling algorithms in terms of order of loss ratios, for stochastic and degenerate relative deadlines.

The propositions and conjectures presented in this section link the nodes of the graph shown in Figure 6 with corresponding nodes in Figure 7. The combined graph representing the order of loss ratios is shown in Figure 11.

*3.2. Degenerate and stochastic deadlines: Non-dominance relations*

Figure 11 has some unconnected pairs of nodes. The following three counter-examples complete the set of connections between these pairs.

*Counter-example 3.1.* Consider the $M/M/1$ queue with DES, where the relative deadline is either degenerate with value $2/\mu$ or exponentially distributed with mean $2/\mu$. Loss ratios plotted in Figure 12 as a function of the normalized arrival rate ($\lambda/\mu$), computed on the basis of simulations of about one million arrivals, show that the inequality $\alpha^{Deg}_{FCFS} \leq \alpha^{Exp}_{FCFS\text{-}EDT}$ holds for small arrival rates, while the inequality $\alpha^{Exp}_{FCFS\text{-}EDT} \leq \alpha^{Deg}_{FCFS}$ holds for large arrival rates. Thus, neither of $\alpha^{Deg}_{FCFS}$ and $\alpha^{H}_{FCFS\text{-}EDT}$ uniformly dominates the other.

*Counter-example 3.2.* Consider the $M/M/1$ queue with DES, where the relative deadline is either degenerate with value $16/\mu$ or exponentially distributed with mean $16/\mu$. The loss ratios plotted in Figure 13 as a function of the normalized arrival rate ($\lambda/\mu$), computed on the basis of simulations of about one million arrivals, show that the inequality $\alpha^{Deg}_{FCFS} \leq \alpha^{Exp}_{EDF\text{-}EDT}$

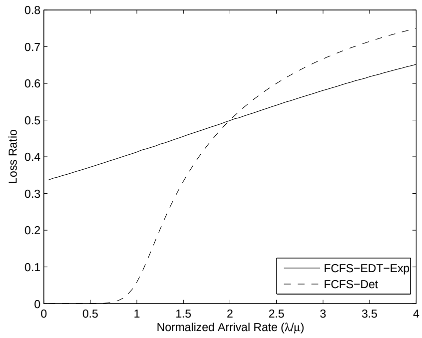


Figure 12: Loss ratios of the *FCFS-EDT,Exp* and *FCFS,Deg* scheduling algorithms for mean relative deadline $\theta = 2/\mu$ and various normalized arrival rates $(\lambda/\mu)$.

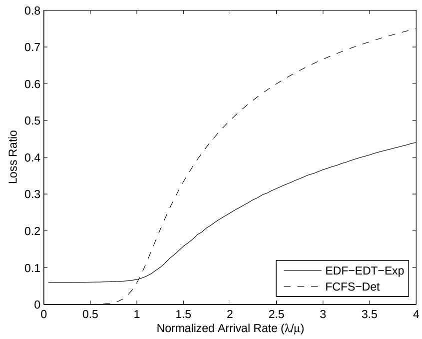


Figure 13: Loss ratios of the *EDF-EDT,Exp* and *FCFS,Deg* scheduling algorithms for mean relative deadline $\theta = 16/\mu$ and various normalized arrival rates $(\lambda/\mu)$.

holds for small arrival rates, while the inequality $\alpha^{Exp}_{EDF\text{-}EDT} \leq \alpha^{Deg}_{FCFS}$ holds for large arrival rates. Thus, neither of $\alpha^{Deg}_{FCFS}$ and $\alpha^{H}_{EDF\text{-}EDT}$ uniformly dominates the other.

*Counter-example 3.3.* Consider the $M/M/1$ queue with DES, where the relative deadline is either degenerate with value $16/\mu$ or exponentially distributed with mean $16/\mu$. The loss ratios plotted in Figure 14 as a function of the normalized arrival rate $(\lambda/\mu)$, computed on the basis of simulations of about one million arrivals, show that the inequality $\alpha^{Deg}_{FCFS} \leq \alpha^{Exp}_{EDF\text{-}EAC}$ holds for small arrival rates, while the inequality $\alpha^{Exp}_{EDF\text{-}EAC} \leq \alpha^{Deg}_{FCFS}$ holds for large arrival rates. Thus, neither of $\alpha^{Deg}_{FCFS}$ and $\alpha^{H}_{EDF\text{-}EAC}$ uniformly

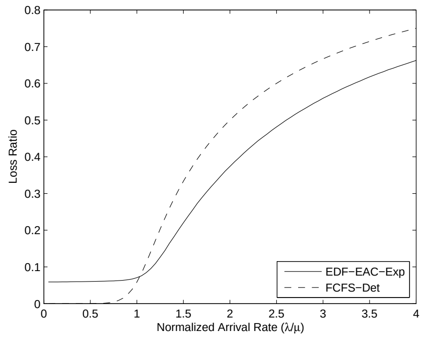


Figure 14: Loss ratios of the *EDF-EAC,Exp* and *FCFS,Deg* scheduling algorithms for mean relative deadline $\theta = 16/\mu$ and various normalized arrival rates $(\lambda/\mu)$.

dominates the other.

## 4. Summary and concluding remarks

In this paper, we have proved some dominance relations between various scheduling algorithms in terms of their respective loss ratios. We have also proved, through counter-examples, the non-existence of a dominance relation between some pairs of scheduling algorithms. A few possible dominance relations are left as conjectures, supported by extensive simulations. These relations help one construct a comprehensive dominance structure of scheduling algorithms in terms of loss ratios, parts of which were given in Figures 6 and 7. The combined structure is shown in Figure 15.

We were unable to establish a clear order between loss ratios of a $G/G/1+G$ system operating under the EDF scheduler with admission control (EAC) on the one hand and exit control (EDT) on the other. However, we have proved that EDT definitely reduces the loss ratio, while EAC may not reduce it. On the other hand, for FCFS schedulers, there is no difference between the improvement in the loss ratio resulting from the adoption of EAC and EDT. From these considerations, it may be concluded that EDT should be preferred over EAC as far as loss ratio is concerned. Of course, this comparison is relevant only for those systems that do not require guaranteed completion of a job once admitted.

The result of Towsley and Panwar [9] on the optimality of the EDF scheduling policy among the class of all service-time independent policies does not hold in the presence of EDT, which makes the scheduling policy

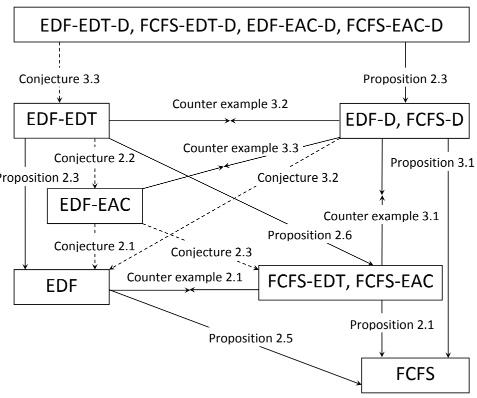


Figure 15: Relationship between various scheduling algorithms in terms of order of loss ratios, for stochastic and deterministic relative deadlines.

dependent on service time. In this paper, we have considered the possible optimality of EDF among the modified class of scheduling policies that accommodate EDT. The result stated in Proposition 2.6 establishes the superiority of EDF over FCFS, and keeps open the possibility of overall optimality. This issue may be taken up for research in future. The conjectures presented in this paper also provide opportunity for further research.

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