New Yoyo Tricks with AES-based Permutations

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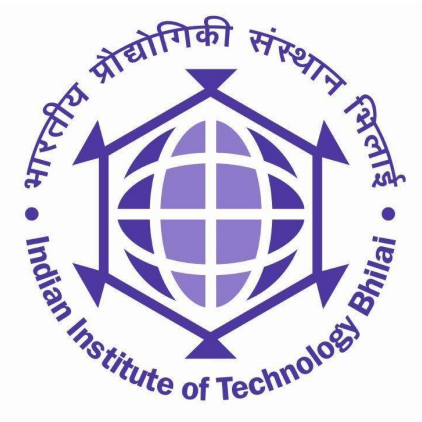


ASK 2018
Kolkata, INDIA

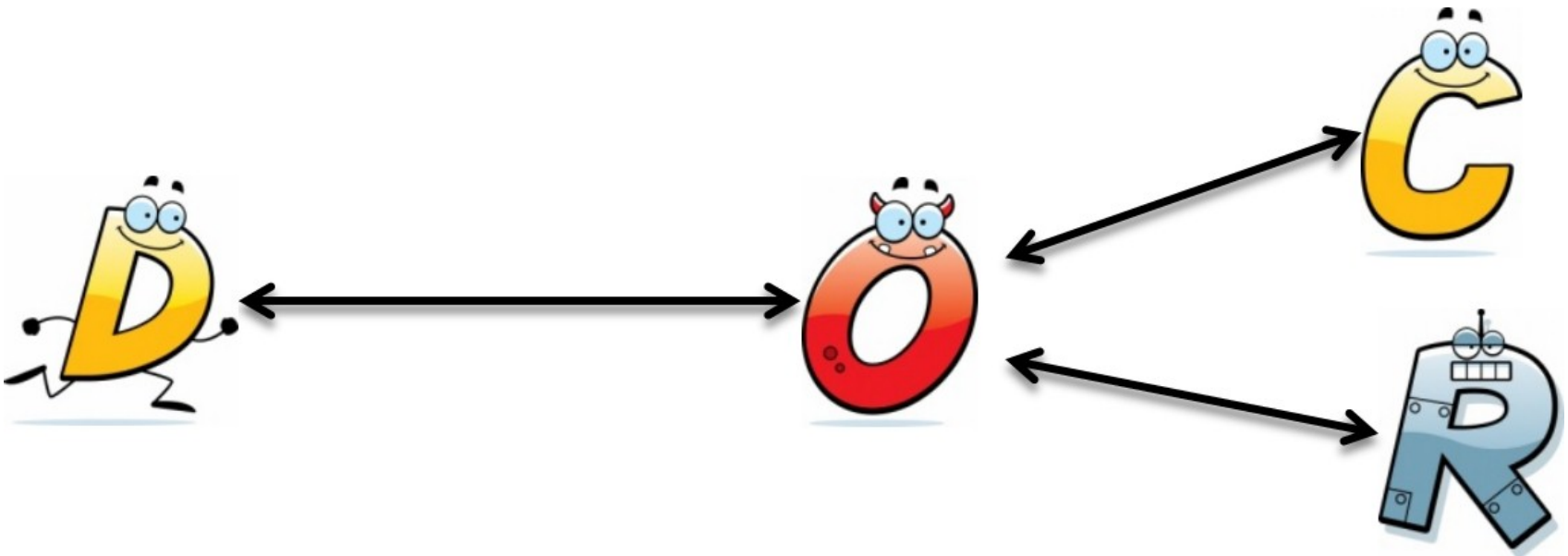
The Problem: Devising Distinguishers

Distinguish between **what** and **why**?

Exhibiting Non-random Behavior

The Distinguishing Setting

1. D tries to distinguish between C and R
2. Can make queries to O
3. O behaves as either C or R
4. At the end D has to guess who is O impersonating
5. D wins if its guess is right



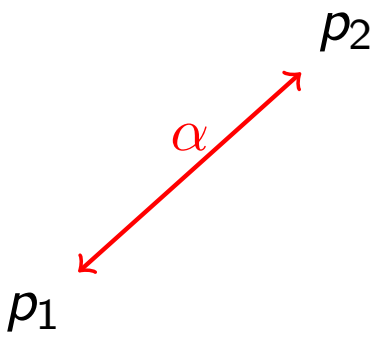
Lets play a Game

Setting: Adaptive Chosen Plaintext/Ciphertext

Will look similar to Boomerang Attack

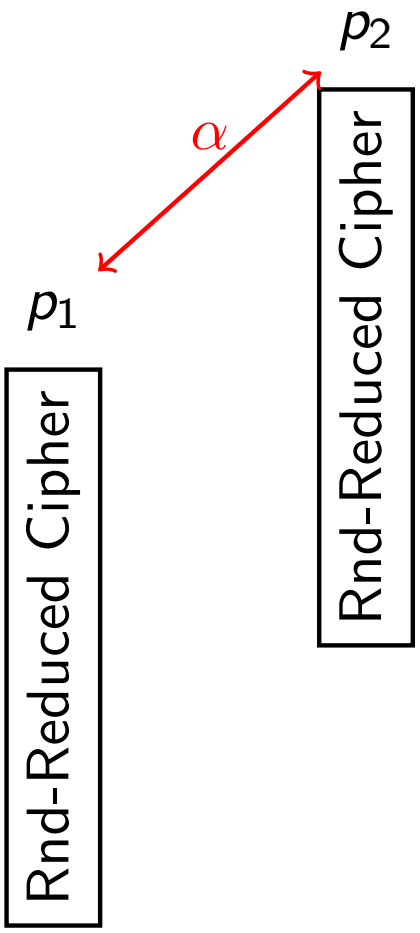
Select messages p_1, p_2 with $p_1 \oplus p_2 = \alpha$

Is there a special way to choose α?



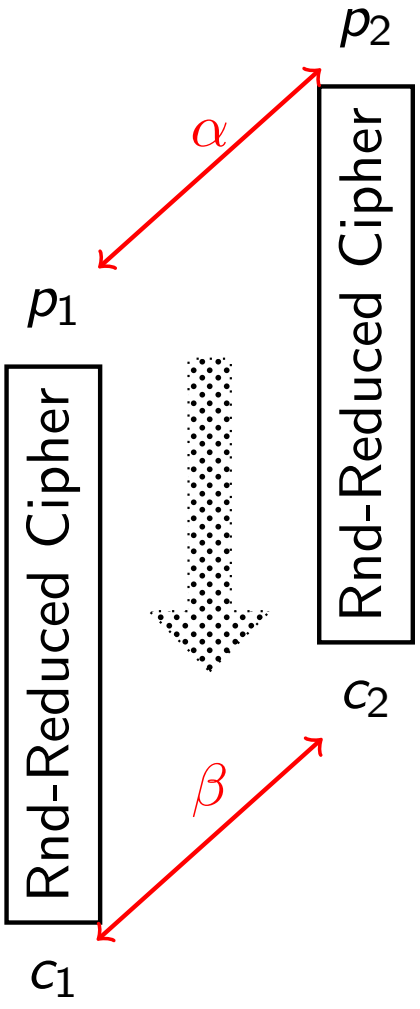
Apply some rounds of some cipher

How many rounds? What type of cipher?



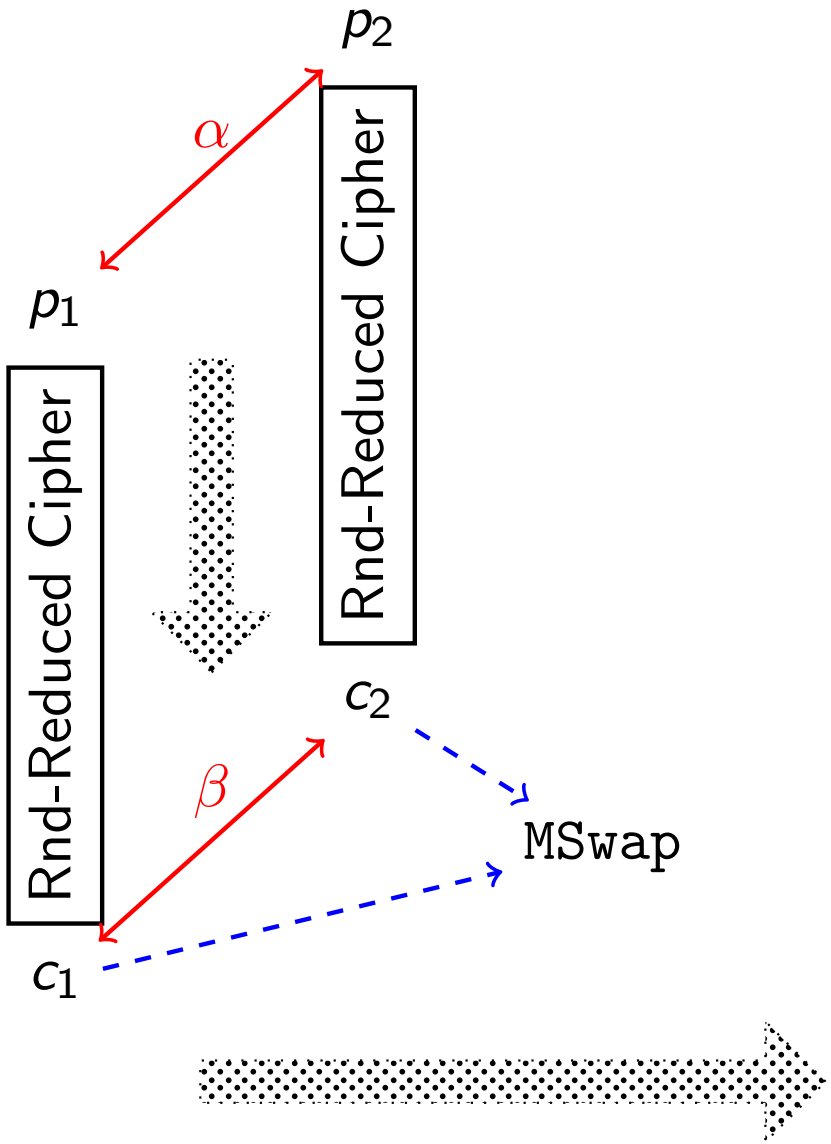
Get c_1, c_2 with $c_1 \oplus c_2 = \beta$

β is the ciphertext difference



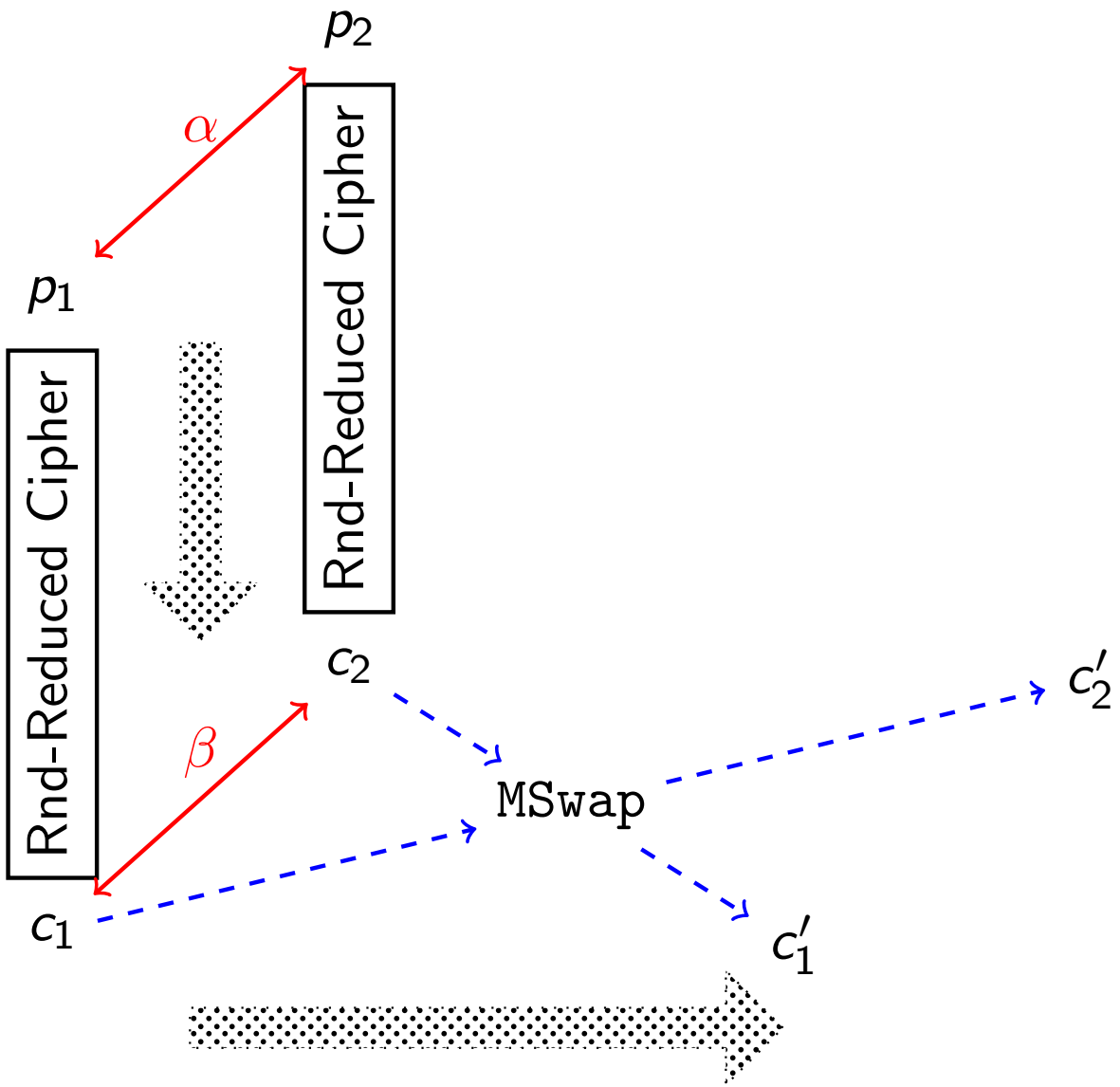
Use MSwap to swap bytes/words of c_1, c_2

How does this swap work?



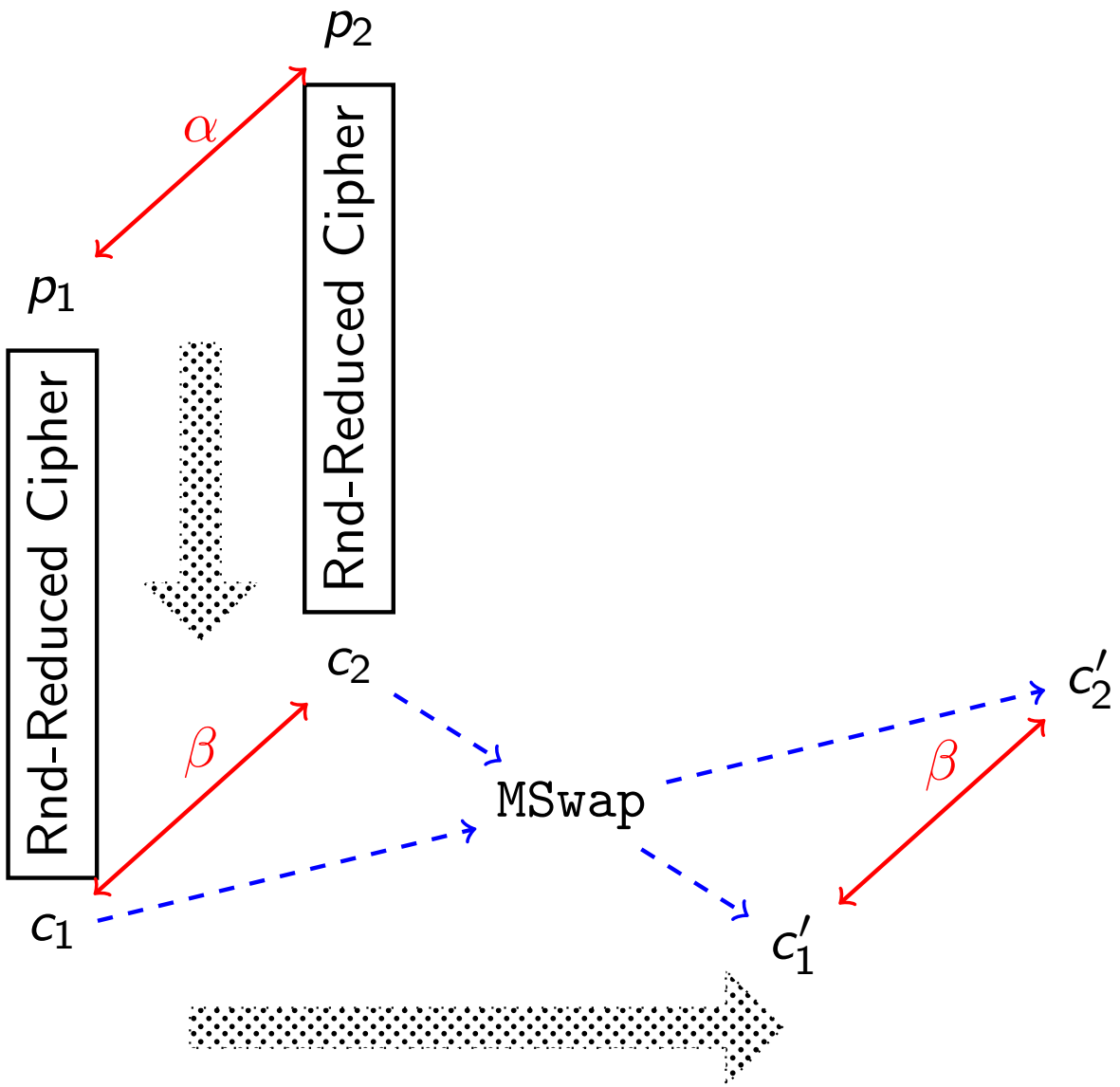
Generate new ciphertext pair c_1', c_2'

What is the relation between c_1', c_2'?



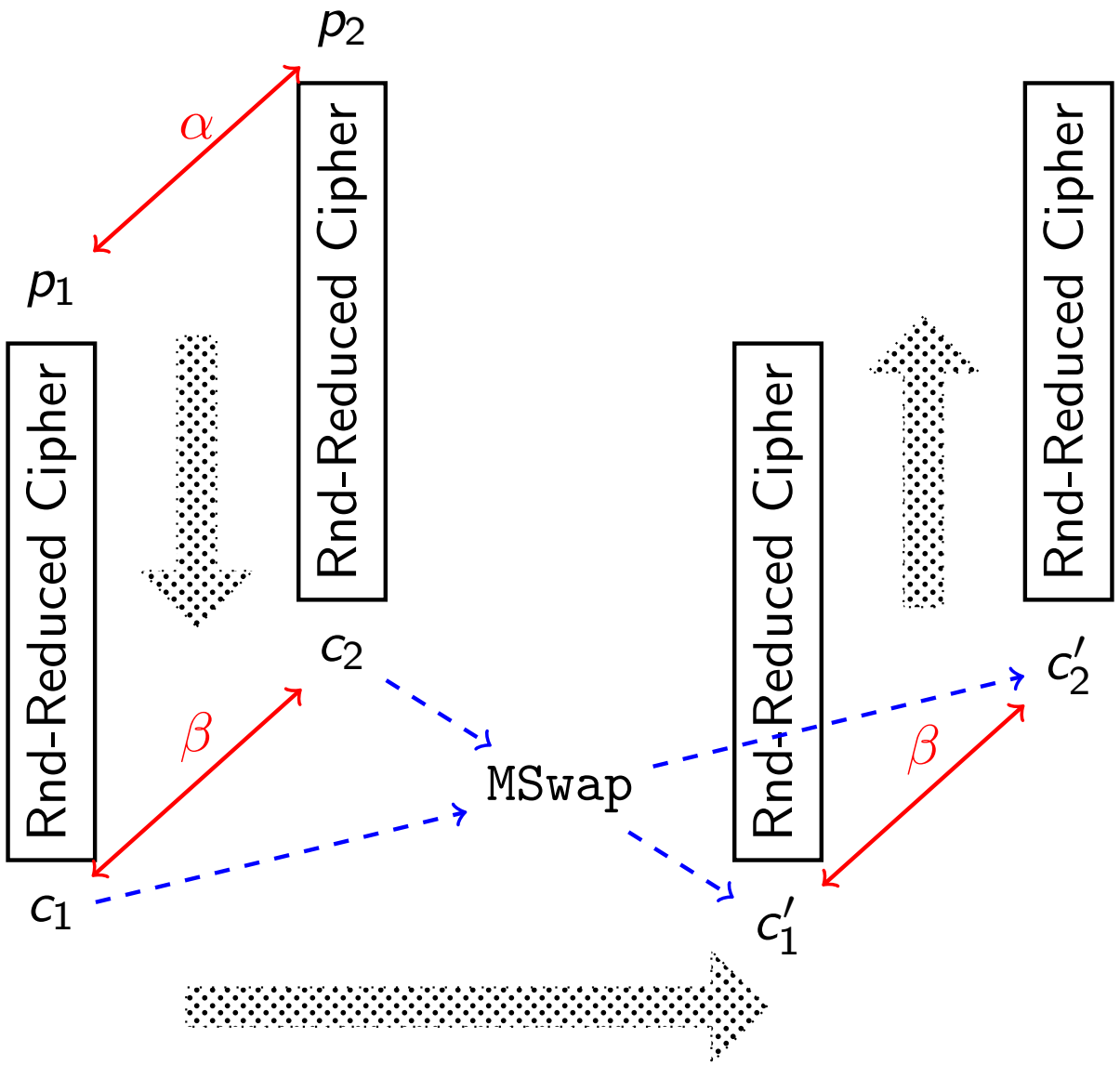
Invariant: $c_1' \oplus c_2' = \beta$

How does this part differ from the **Boomerang** Attack?



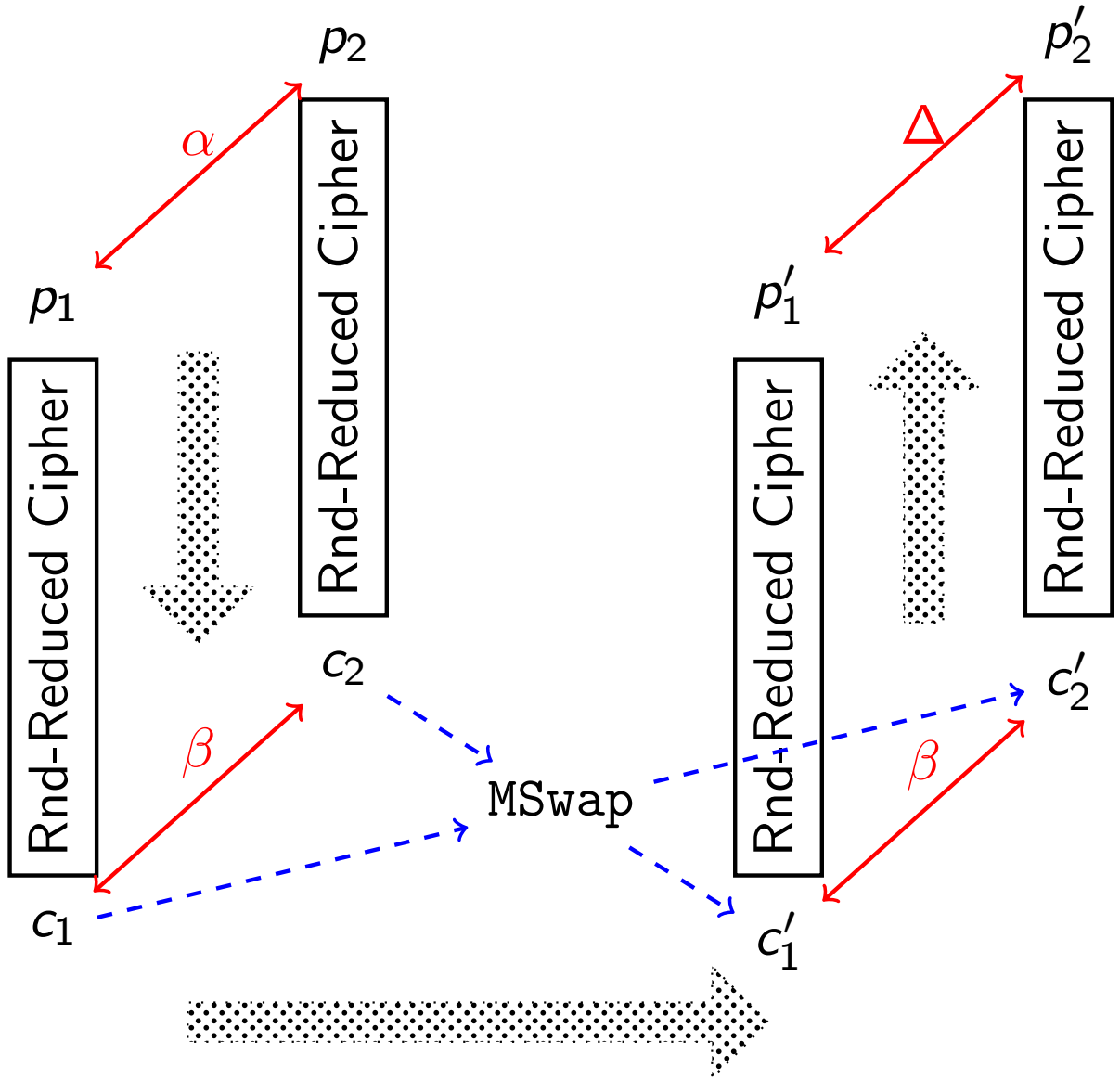
Invert same number of rounds

Decrypting new ciphertext pair



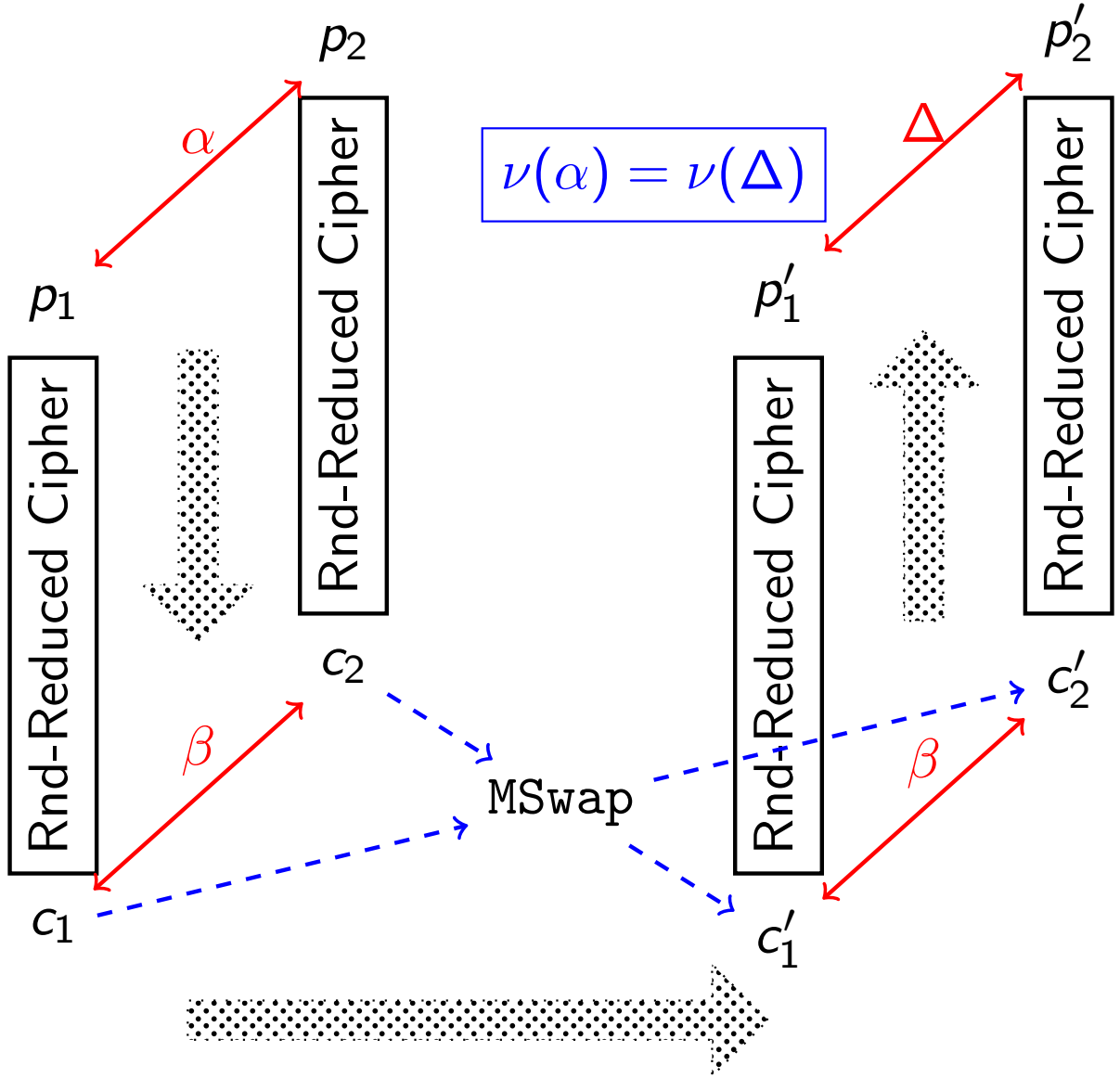
Get p'_1, p'_2 with $p'_1 \oplus p'_2 = \Delta$

Does Δ have a special property?



Hypothesis: Property ν induced in α is preserved by Δ

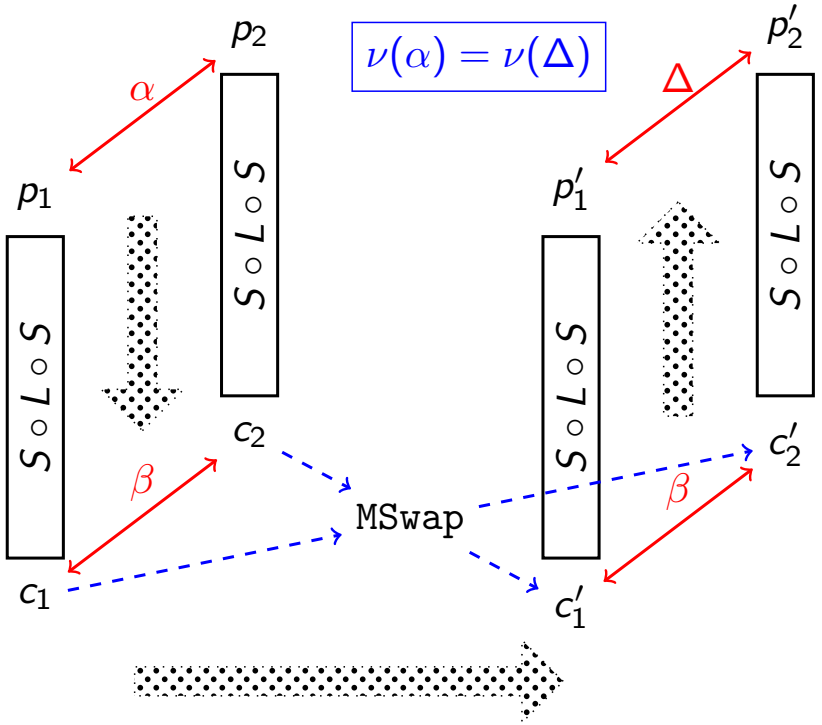
What is this **property** ν?



- Is there a special way to choose α?
  - Zero Difference Pattern (ZDP). Explained later.
- How many rounds? What type of cipher?
  - **2-Rnd Generic SPN**
- How does the swap work?
  - Swap based on non-linear layer. Explained later.
- Does Δ have a special property?
  - Same as α
- What is this **property** ν?
  - Zero Difference Pattern (ZDP)

$G'_2 = L \circ S \circ L \circ S$ Two full generic Rounds

$G_2 = S \circ L \circ S$ ← Dropping final linear layer (to simplify)



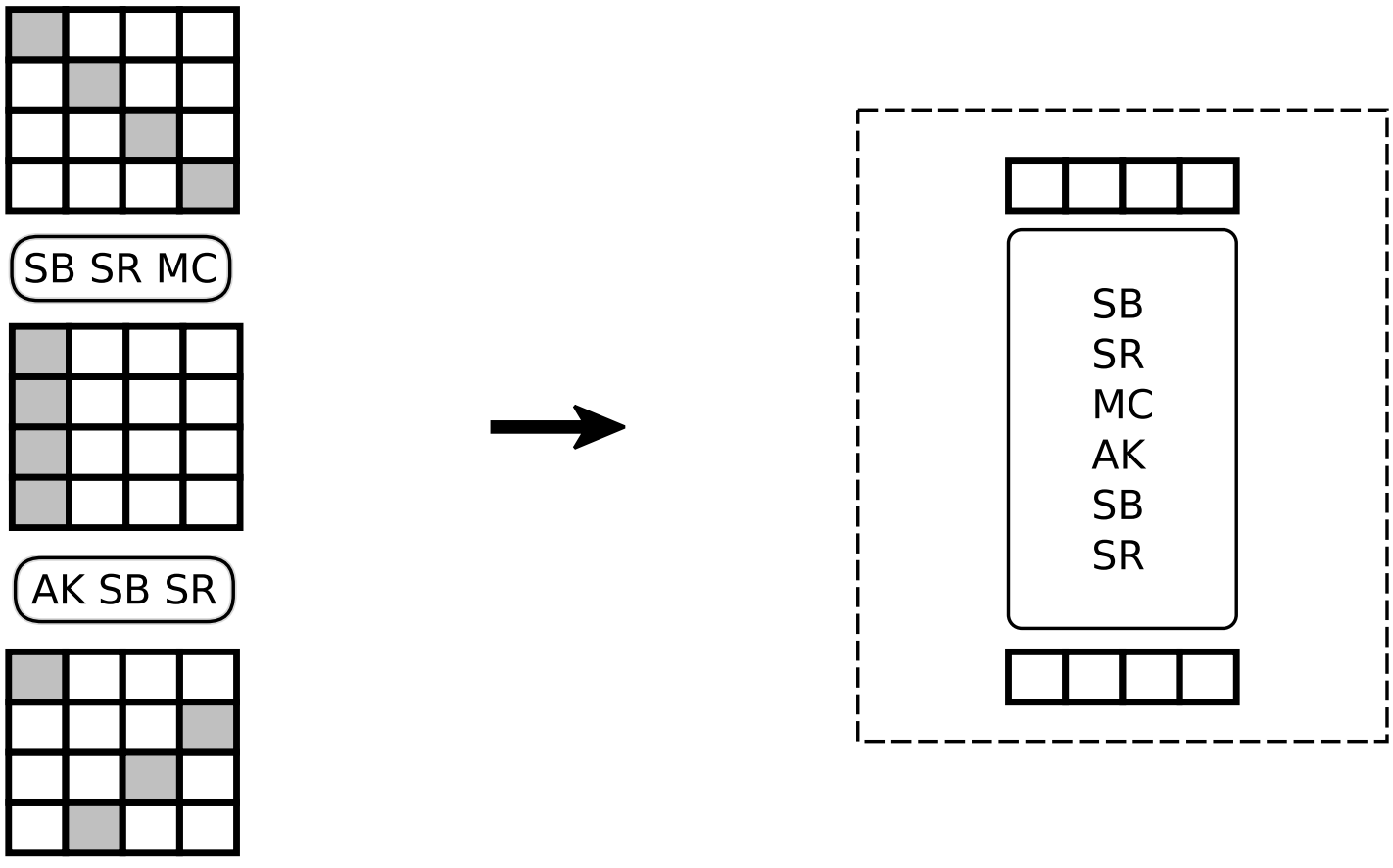
- ZDP of α is preserved by Δ

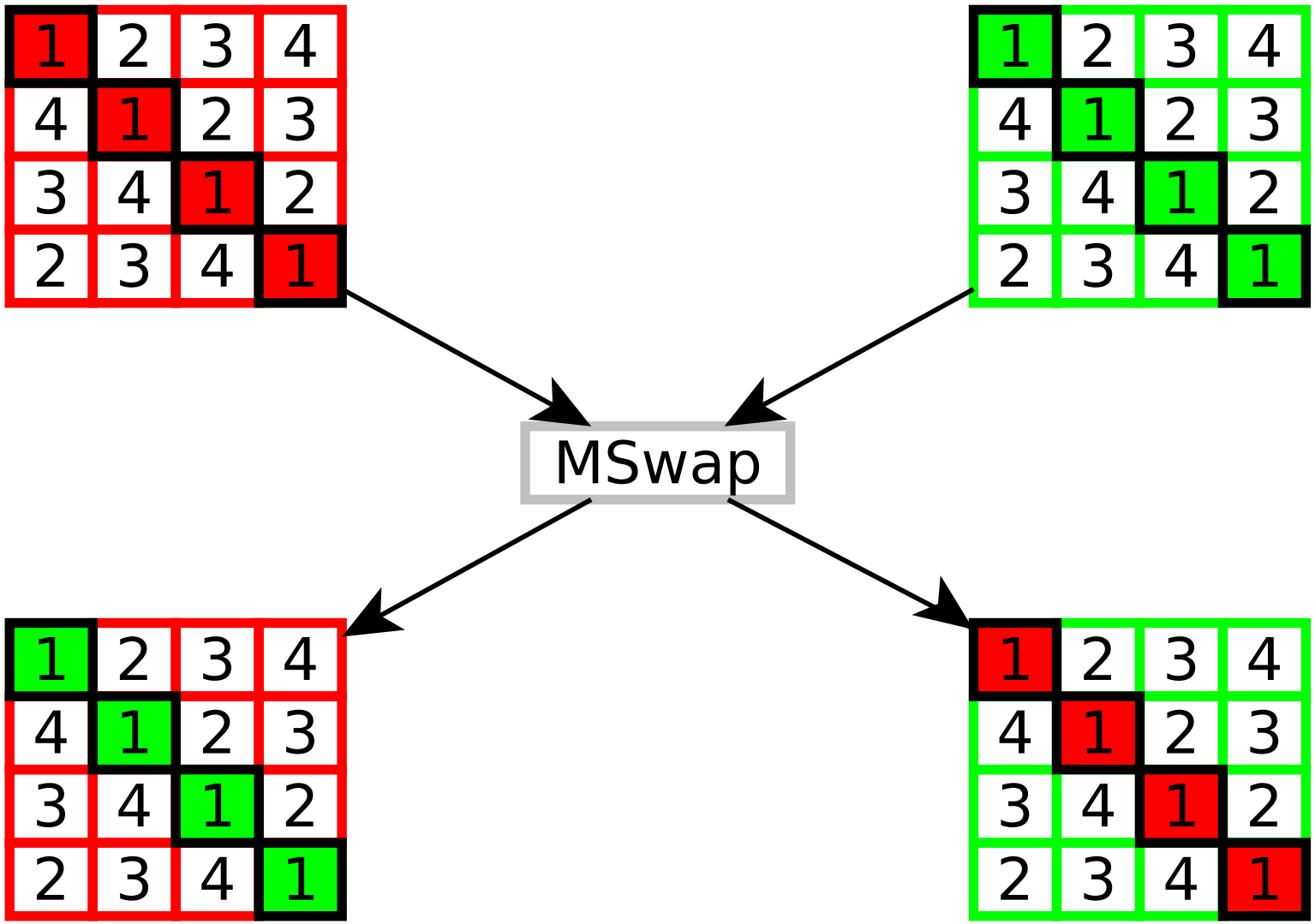
Applied to AES

- First Key-independent Yoyo Distinguishers of AES
- 5-round practical key recovery

Understanding MSwap

Recall AES SuperSBox




1 2 3 4
4 1 2 3
3 4 1 2
2 3 4 1
1 2 3 4
4 1 2 3
3 4 1 2
2 3 4 1
MSwap
1 2 3 4
4 1 2 3
3 4 1 2
2 3 4 1
1 2 3 4
4 1 2 3
3 4 1 2
2 3 4 1

$$p_1 = \begin{matrix} \texttt{fa} & \texttt{b1} & \texttt{5a} & \texttt{2f} \\ \texttt{b7} & \texttt{64} & \texttt{0e} & \texttt{f1} \\ \texttt{f8} & \texttt{9f} & \texttt{22} & \texttt{15} \\ \texttt{28} & \texttt{87} & \texttt{32} & \texttt{25} \end{matrix} \qquad p_2 = \begin{matrix} \texttt{2e} & \texttt{b1} & \texttt{5a} & \texttt{2f} \\ \texttt{b7} & \texttt{70} & \texttt{0e} & \texttt{f1} \\ \texttt{f8} & \texttt{9f} & \texttt{f2} & \texttt{15} \\ \texttt{28} & \texttt{87} & \texttt{32} & \texttt{4c} \end{matrix}$$

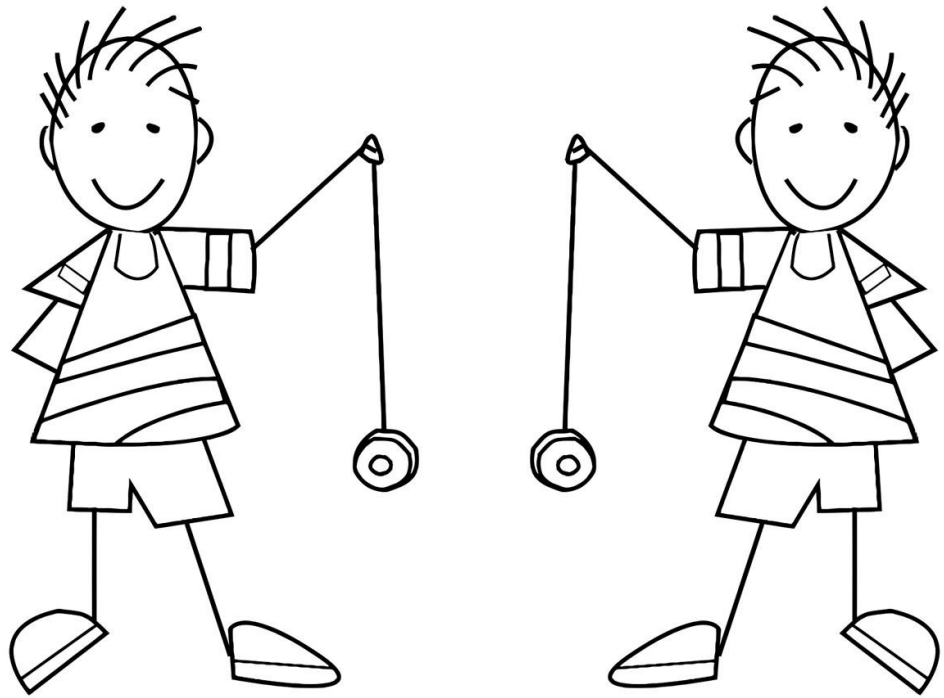
$$\alpha = p_1 \oplus p_2 = \begin{matrix} \texttt{d4} & \texttt{00} & \texttt{00} & \texttt{00} \\ \texttt{00} & \texttt{14} & \texttt{00} & \texttt{00} \\ \texttt{00} & \texttt{00} & \texttt{d0} & \texttt{00} \\ \texttt{00} & \texttt{00} & \texttt{00} & \texttt{69} \end{matrix}$$

$$ZDP(\alpha) = \{0, 1, 1, 1\} \qquad wt(ZDP(\alpha)) = 3$$

The Yoyo Game

- New pairs of plaintexts and ciphertexts are made **adaptively** from the original pairs.
- While making new pairs a **certain property** is kept invariant.
- A common strategy is the use of **zero difference** in the pairs.
- An invariant property is verified at the end

Our Aim: How To Exploit Yoyo Further



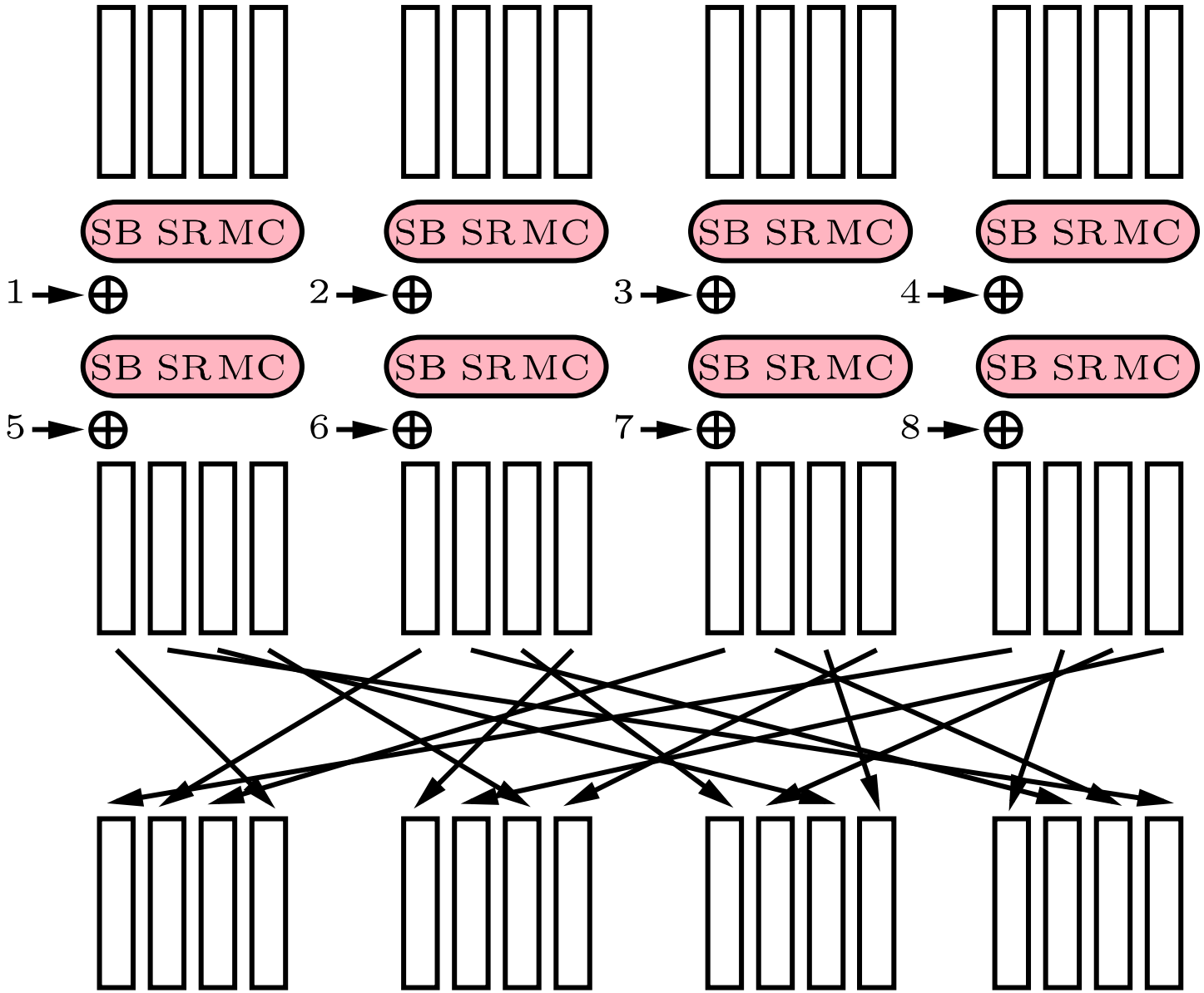
Our Target: AES-based Public Permutations

AESQ Permutation

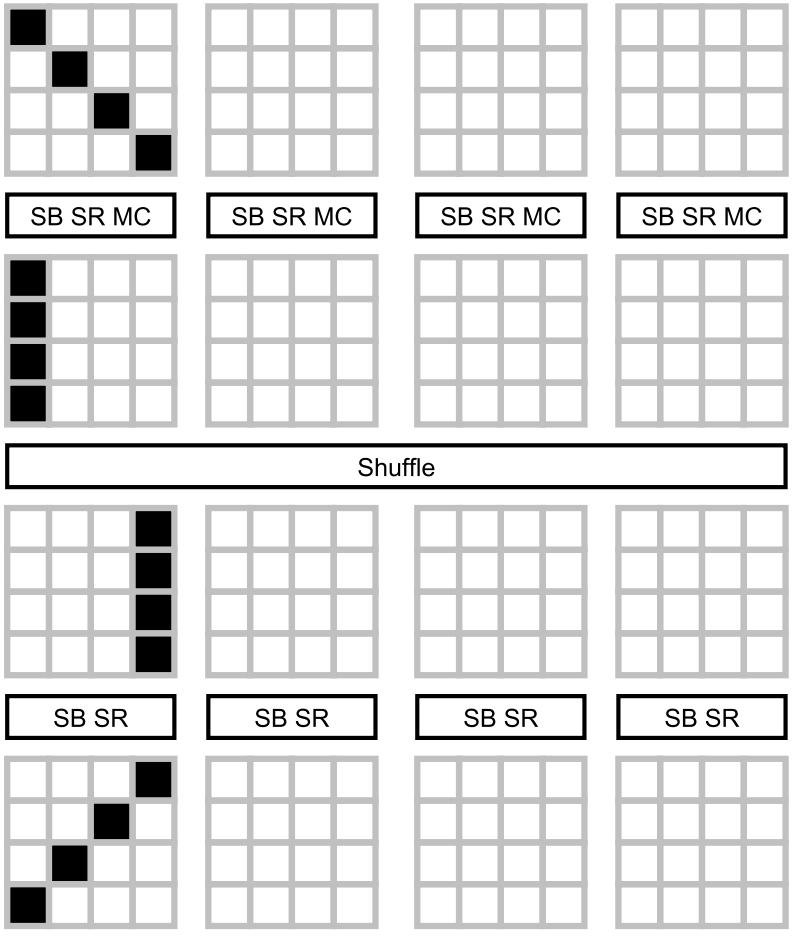
- Internal permutation of AE scheme PAEQ
- PAEQ $\leftarrow$ 2nd Round CAESAR candidate
- By Birukov and Kovratovich

AES in *Known-Key* Setting

- Known-key paradigm
- By Knudsen and Rijmen
- Under Known-key AES behaves as a public permutation


SB SR MC
SB SR MC
SB SR MC
SB SR MC
1
2
3
4
SB SR MC
SB SR MC
SB SR MC
SB SR MC
5
6
7
8

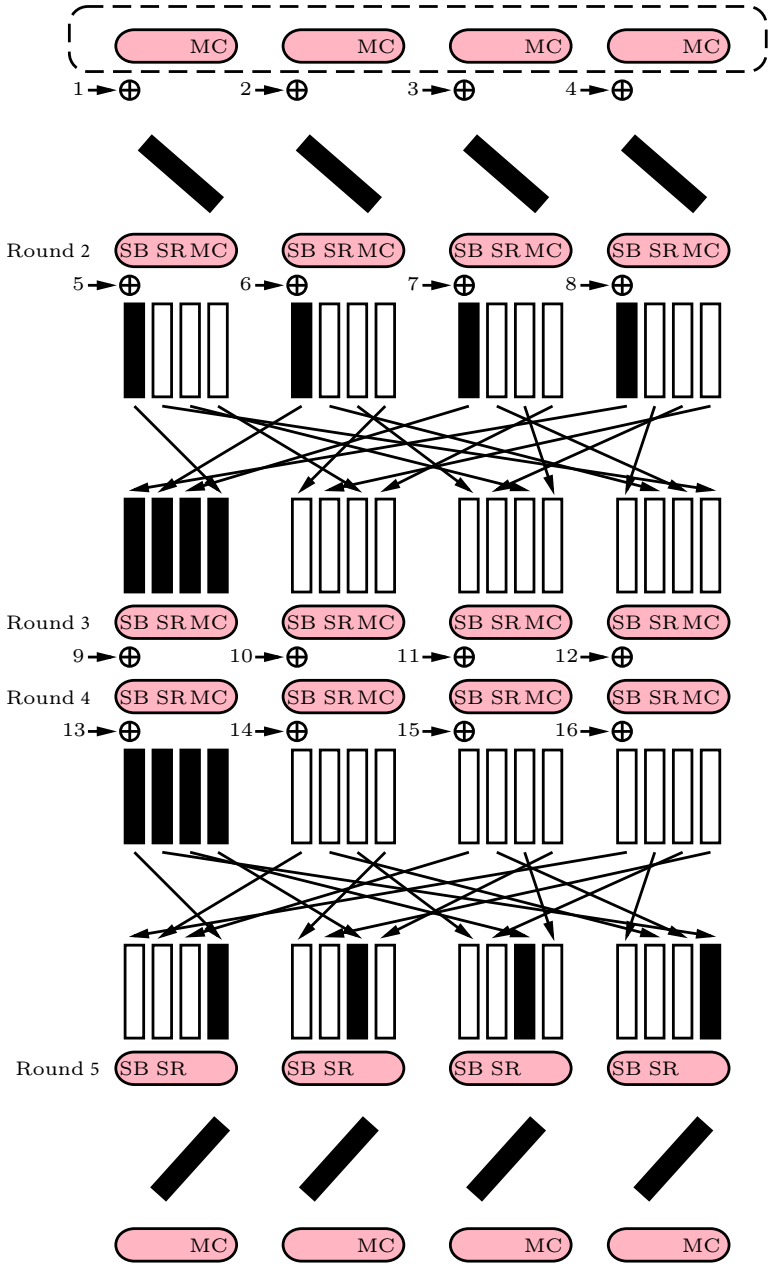
SuperSBox of AESQ



32-bit SuperSbox

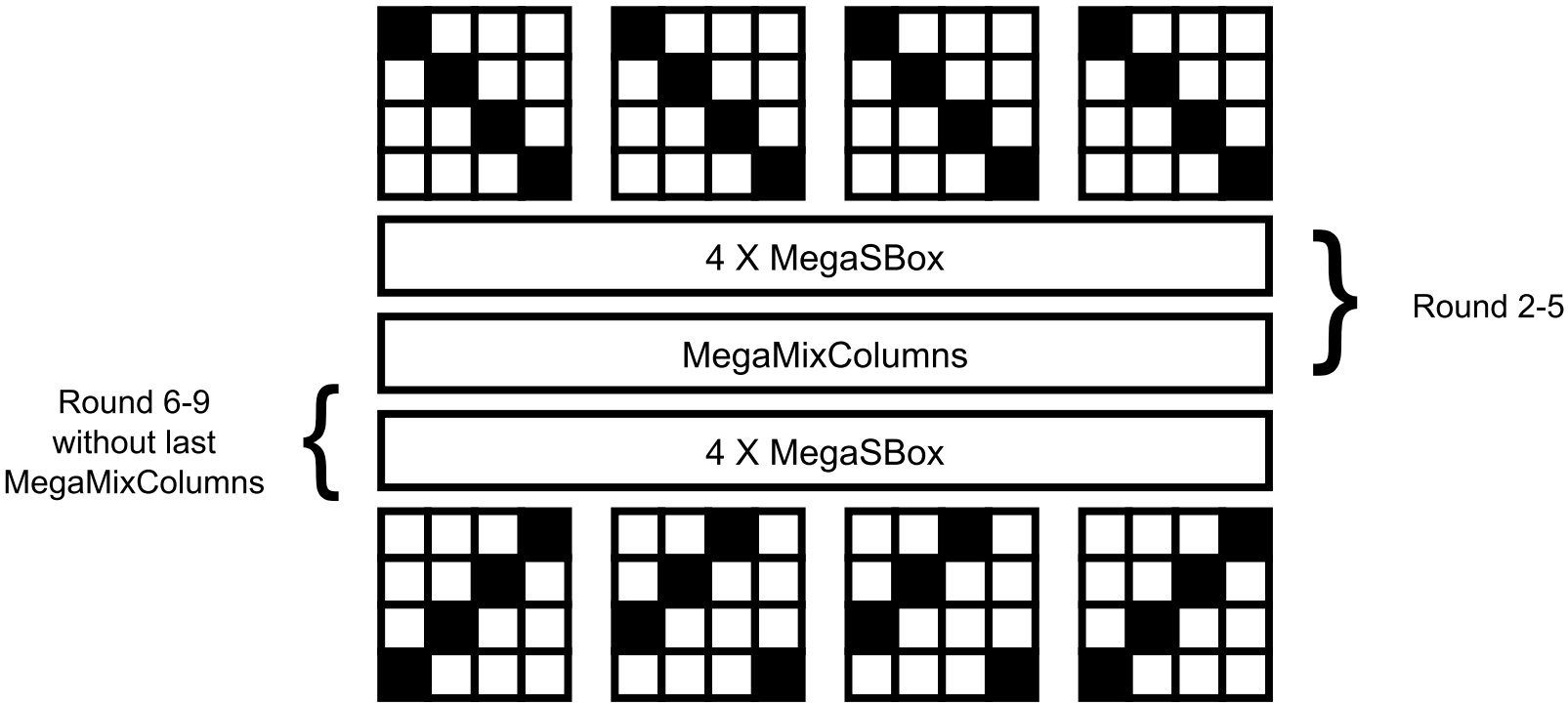
- 16 SuperSBox-es
- Cover 1.5 Rounds
- **Must start from even round**

MegaSBox of AESQ



128-bit MegaSBox

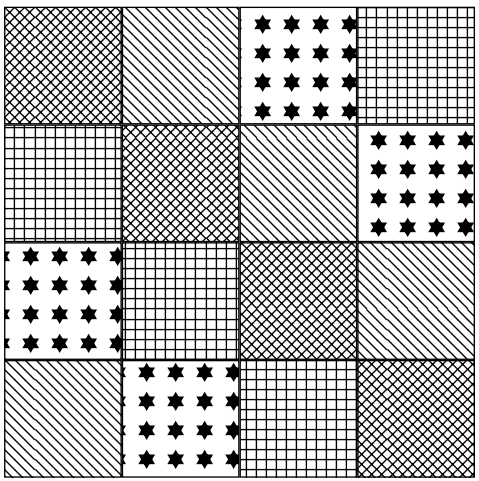
- 4 MegaSBox-es
- Cover 3.5 Rounds
- **Must start from even round**

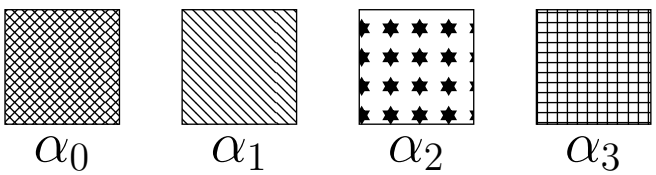


8-Rounds without last MegaMixColumns

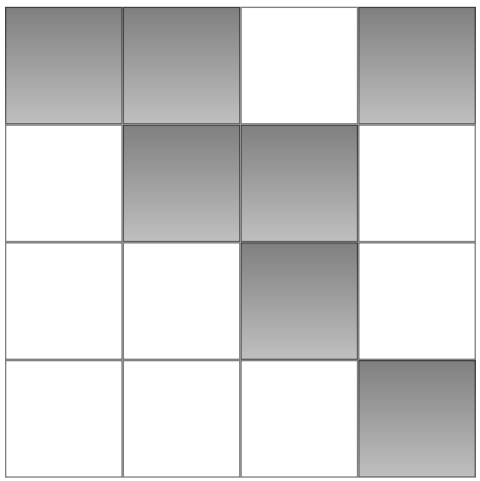
Introducing **Nested** Zero Difference Pattern

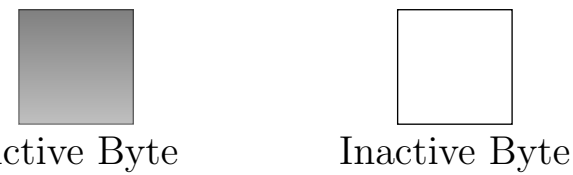
$\alpha \leftarrow$ Sample State $\quad \nu(\alpha) = (0, 0, 1, 0) \quad wt(\nu(\alpha)) = 1$





A sample state





$$\nu_1^2(\alpha_0) = (0, 0, 0, 0), \quad \nu_2^2(\alpha_1) = (0, 0, 1, 1),$$
$$\nu_3^2(\alpha_2) = (1, 1, 1, 1), \quad \nu_4^2(\alpha_3) = (0, 1, 1, 1)$$

$$wt(\nu^2(\alpha)) = 9$$

Strategy 1: Prepend-Append

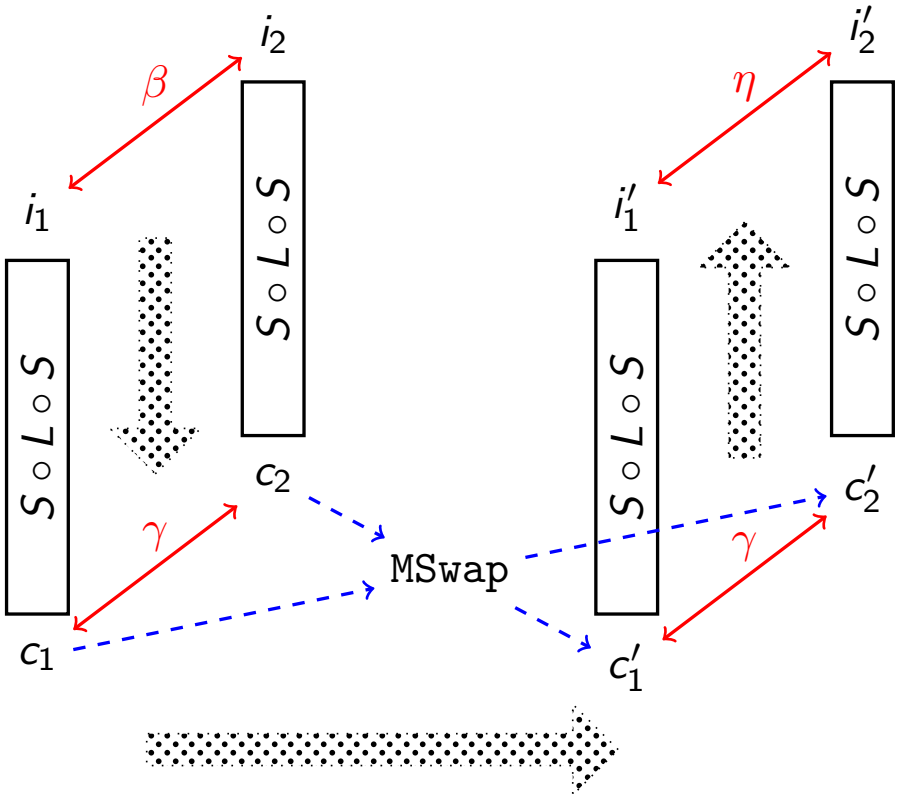
Probabilistic Yoyo

Using Classical Differentials

Basic Yoyo

$\Pr[\nu(\beta) = \nu(\eta)] = 1$

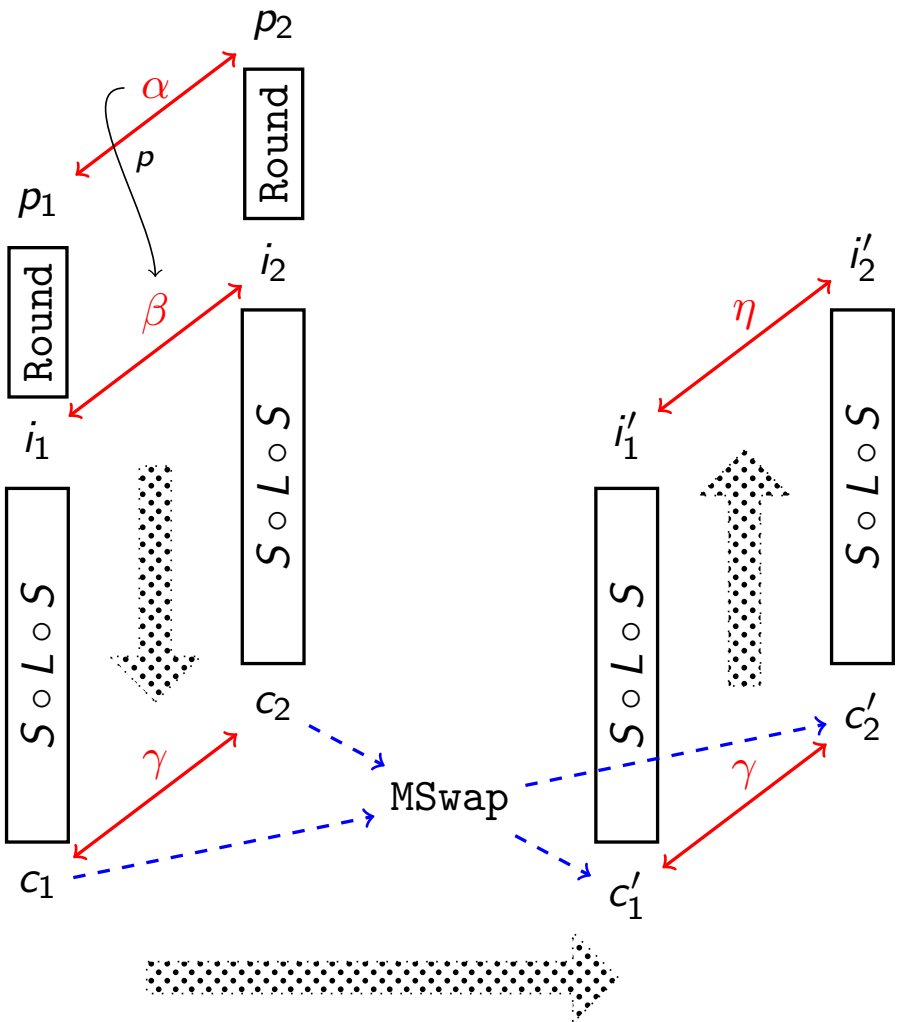
Deterministic Distinguisher



Prepend

$\Pr[\alpha \to \beta] = p$

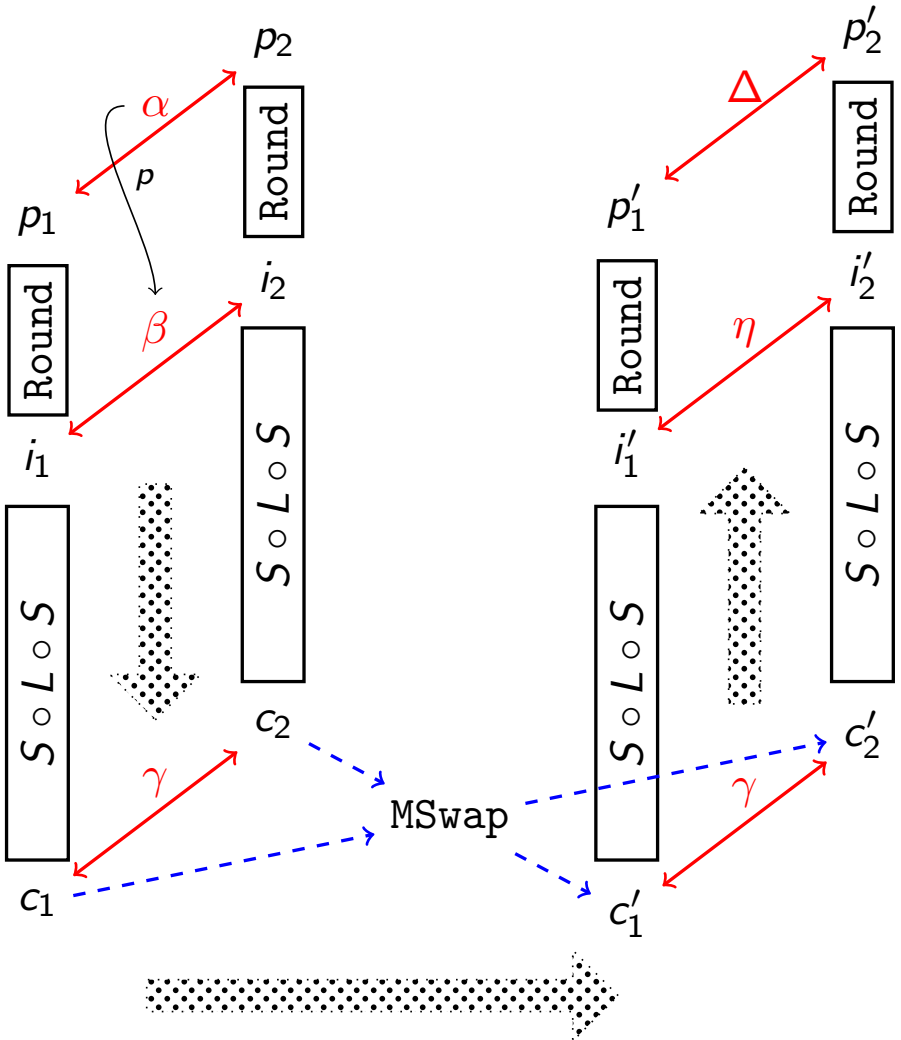
Using Classical Differentials



Append

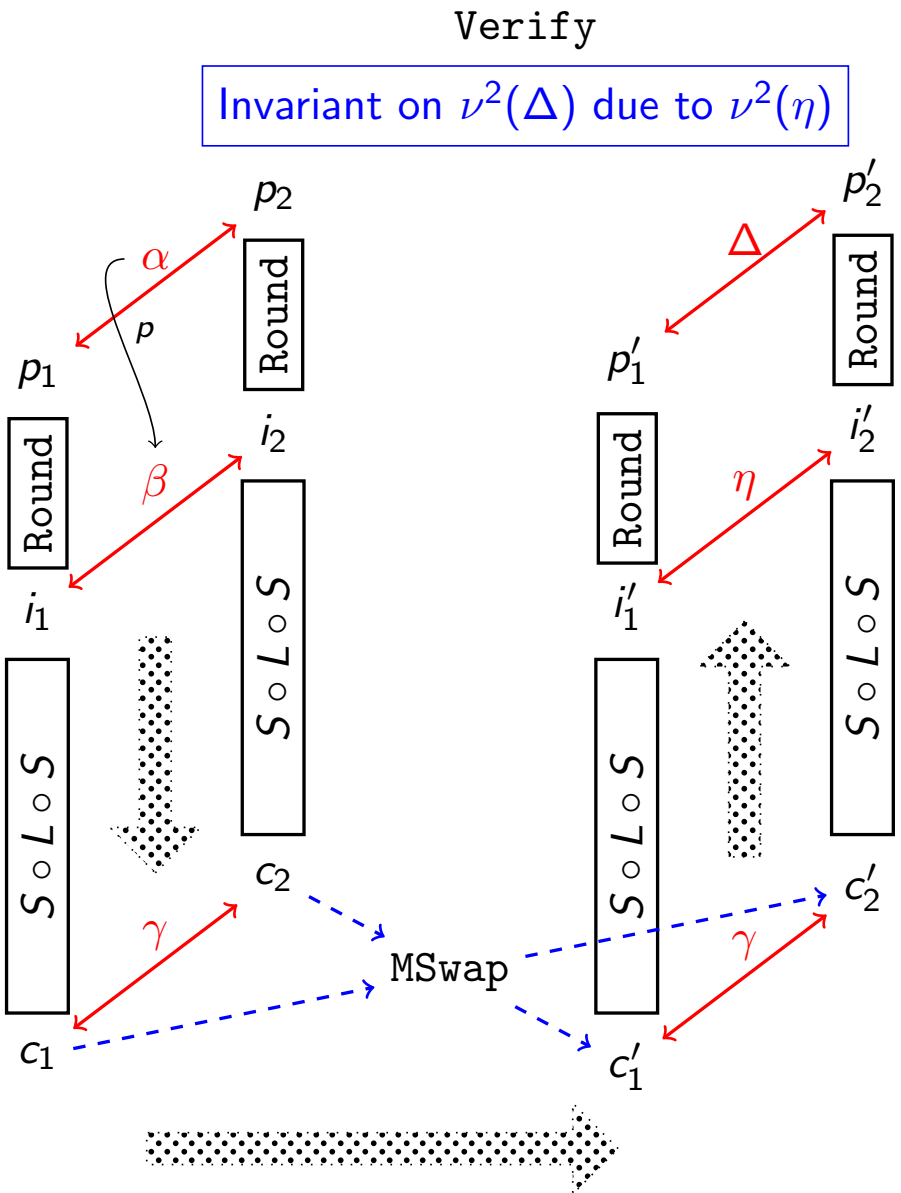
Some assumption on Nested ZDP of η

Induces a property on Δ



Probabilistic Yoyo Distinguisher

Property verified on Δ



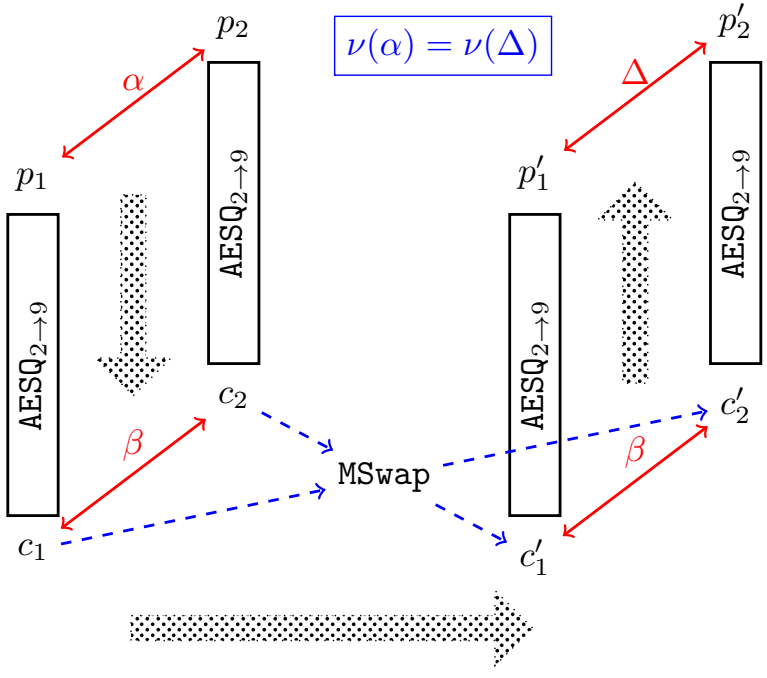
Application: AESQ

First 9-round Distinguisher starting from Round-1

Practical Complexity

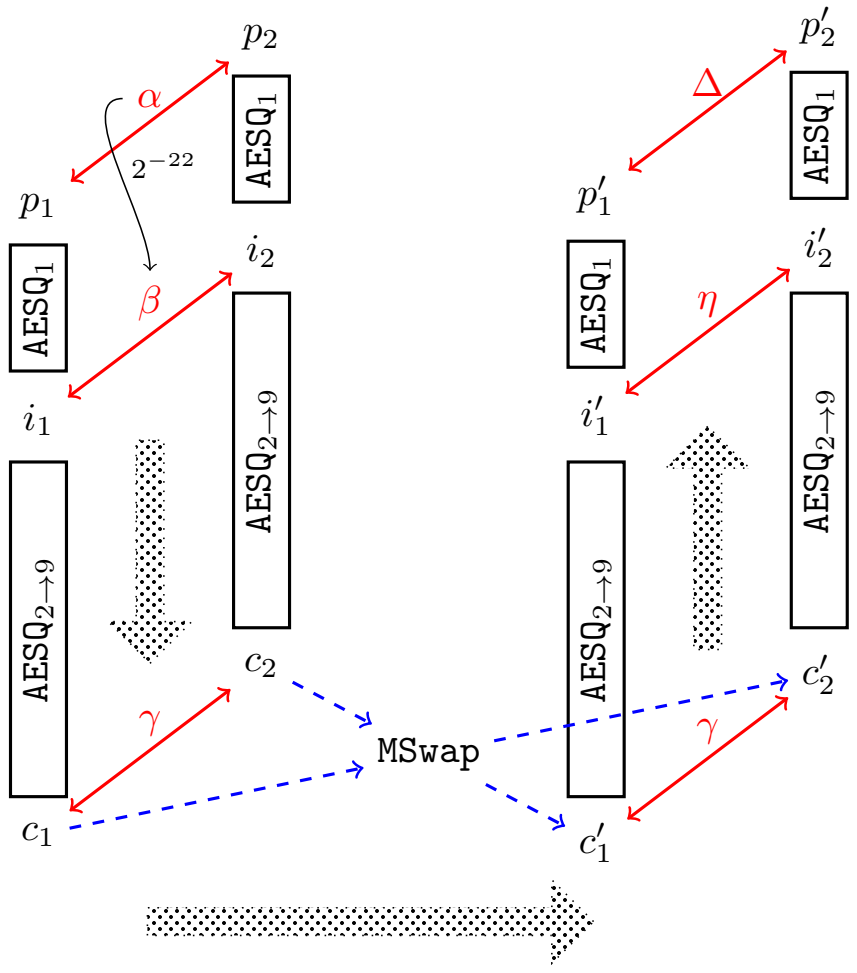
Basic Yoyo

8-Rounds



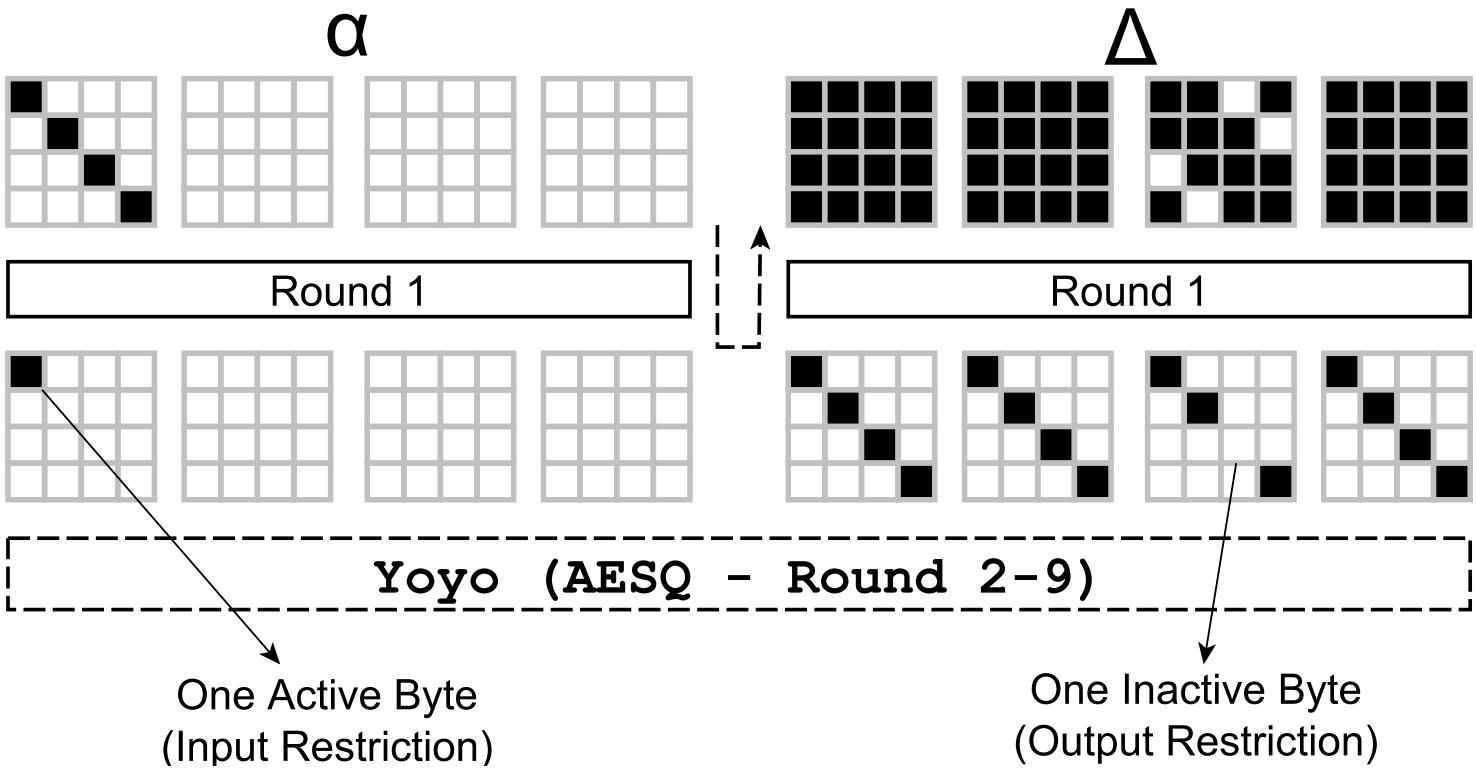
Verify

$\exists i \ : wt(\nu^2(\Delta_i)) = 4$



1-Round Extension

Prob $= 2^{-26}$ An Example



For $\mathtt{AESQ}_{1-9}$

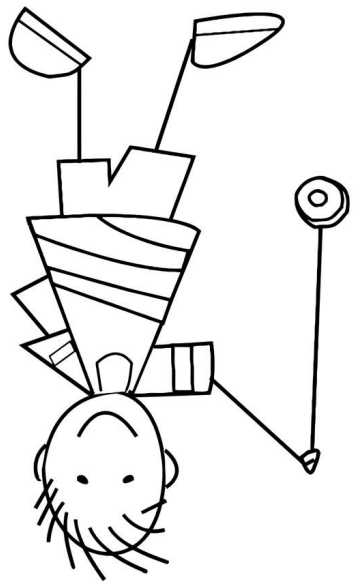
$$\Pr\left[\exists i : wt(\nu^2(\Delta_i)) = 4\right] = 2^{-26}$$

For $\mathcal{R}$

$$\Pr\left[\exists i : wt(\nu^2(\Delta_i)) = 4\right] = 2^{-28}$$

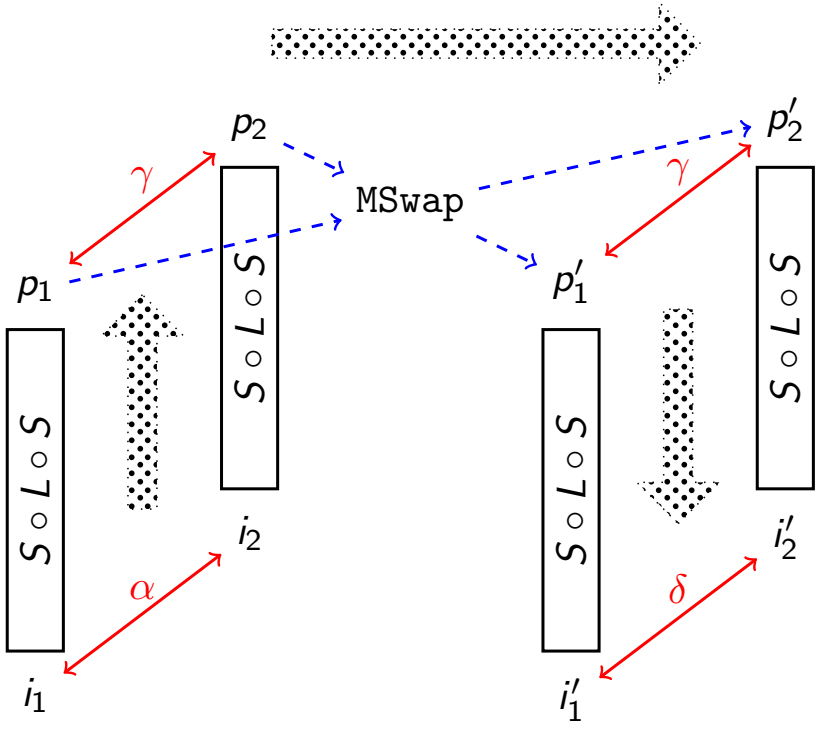
Strategy 2: Composing Improbable/Impossible Differentials

The Inside-Out Technique



Inverted Yoyo

Inverted Yoyo

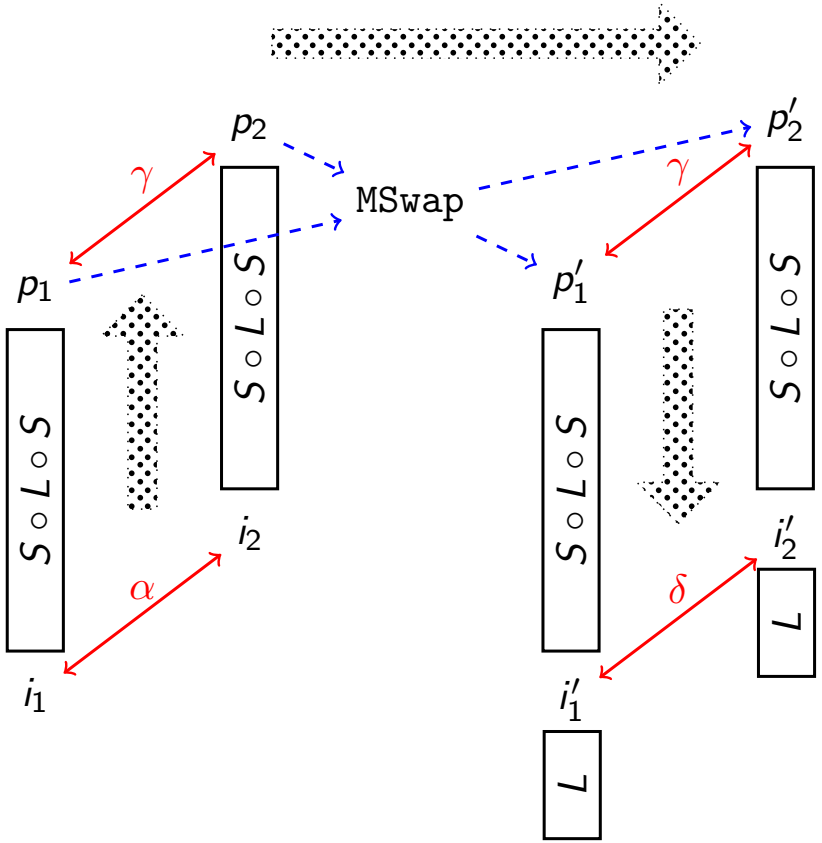


- By virtue of Yoyo
- $\Pr[\nu(\alpha) = \nu(\delta)] = 1$

Assumption

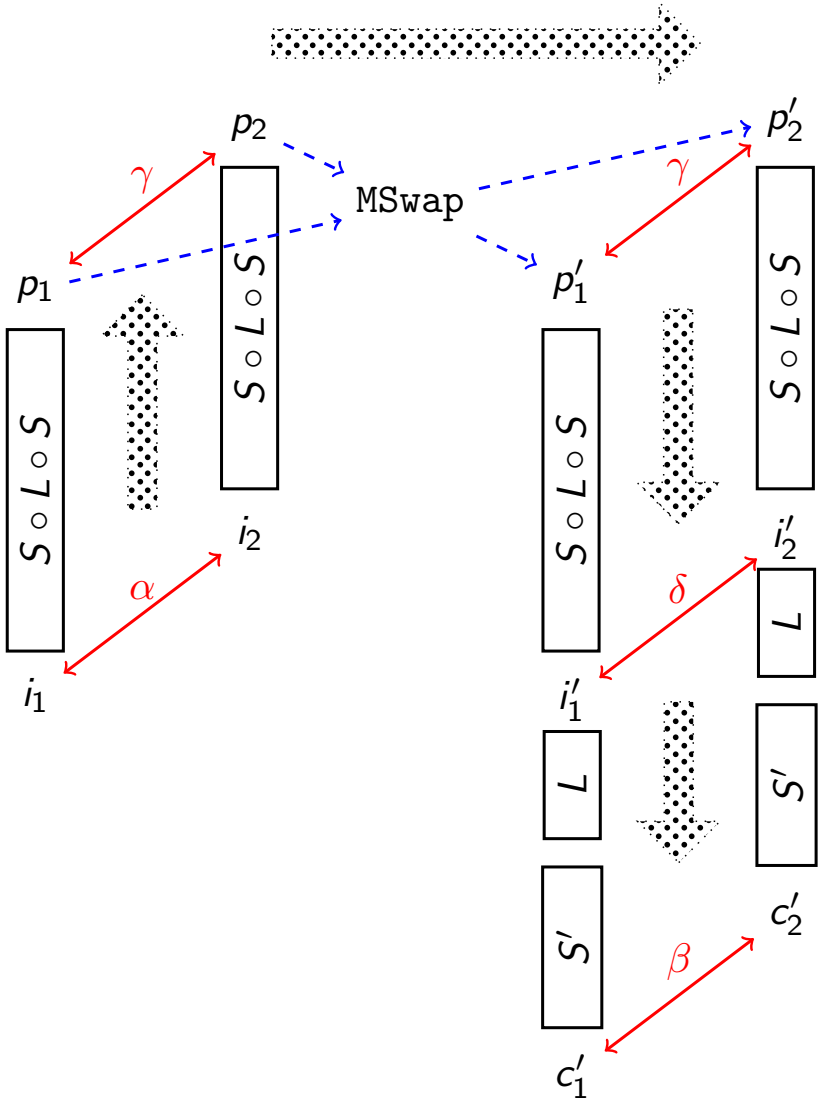
Something on $\nu^2(\delta)$

Append L



- Exploit Properties of L
- Effect of L on δ?
- Use $\nu^2(\delta)$ Assumption

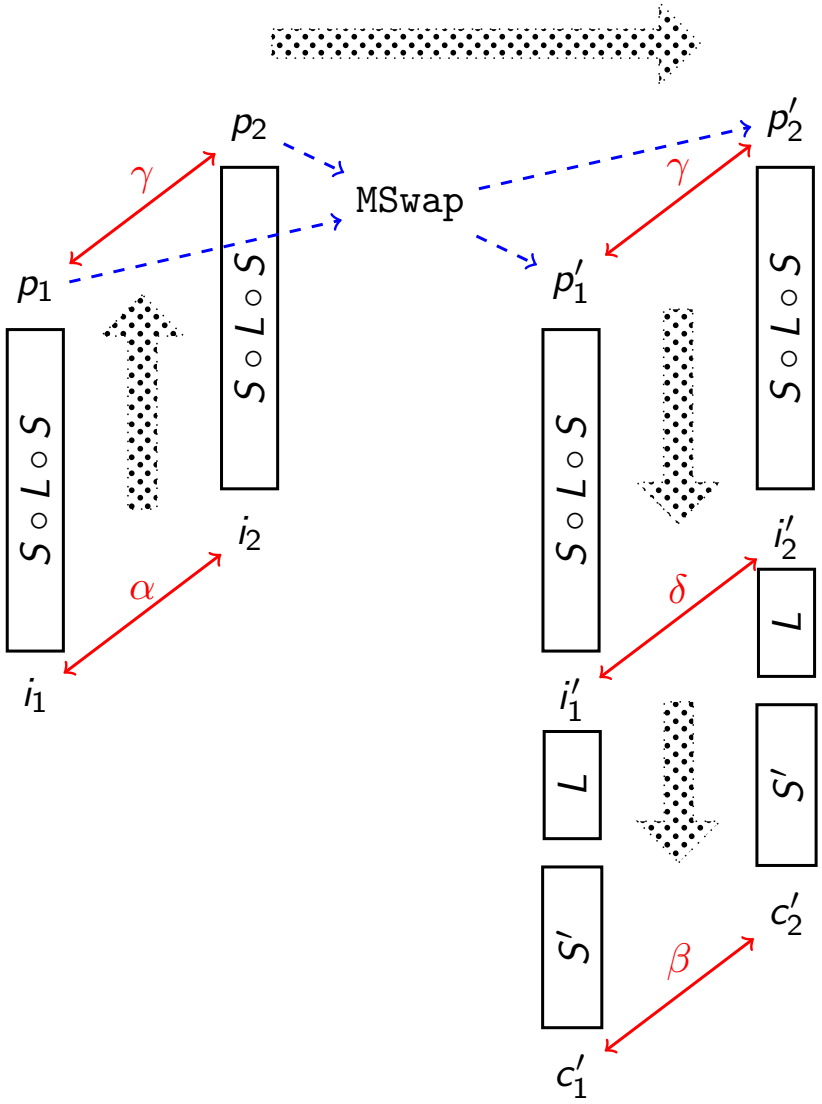
Append S' Impossible (Nested) ZDP on β



Impossibility

- $\Pr[\nu^2(\delta) \to \nu^2(\beta)] = 0$

Probability of $\nu^2(\delta)$ Assumption Holding



Improbable Differential Yoyo

- $\Pr\left[\nu^2(\delta) \text{ Assumption}\right] < 1$

Impossible Differential Yoyo

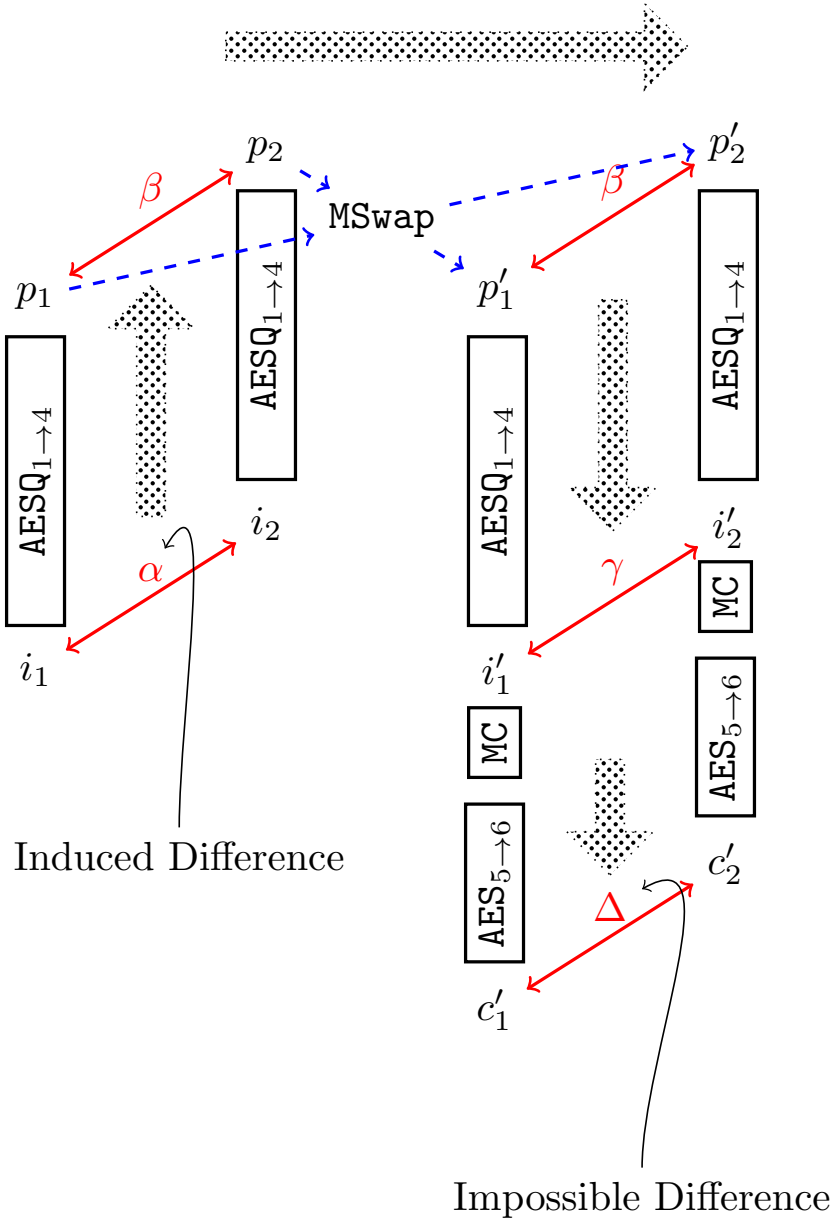
- $\Pr\left[\nu^2(\delta) \text{ Assumption}\right] = 1$

Application: AES, AESQ

6 Round AES (Practical)

9-10(Practical), 12 Round AESQ

Impossible Differential Yoyo Distinguisher on 6-Round AES



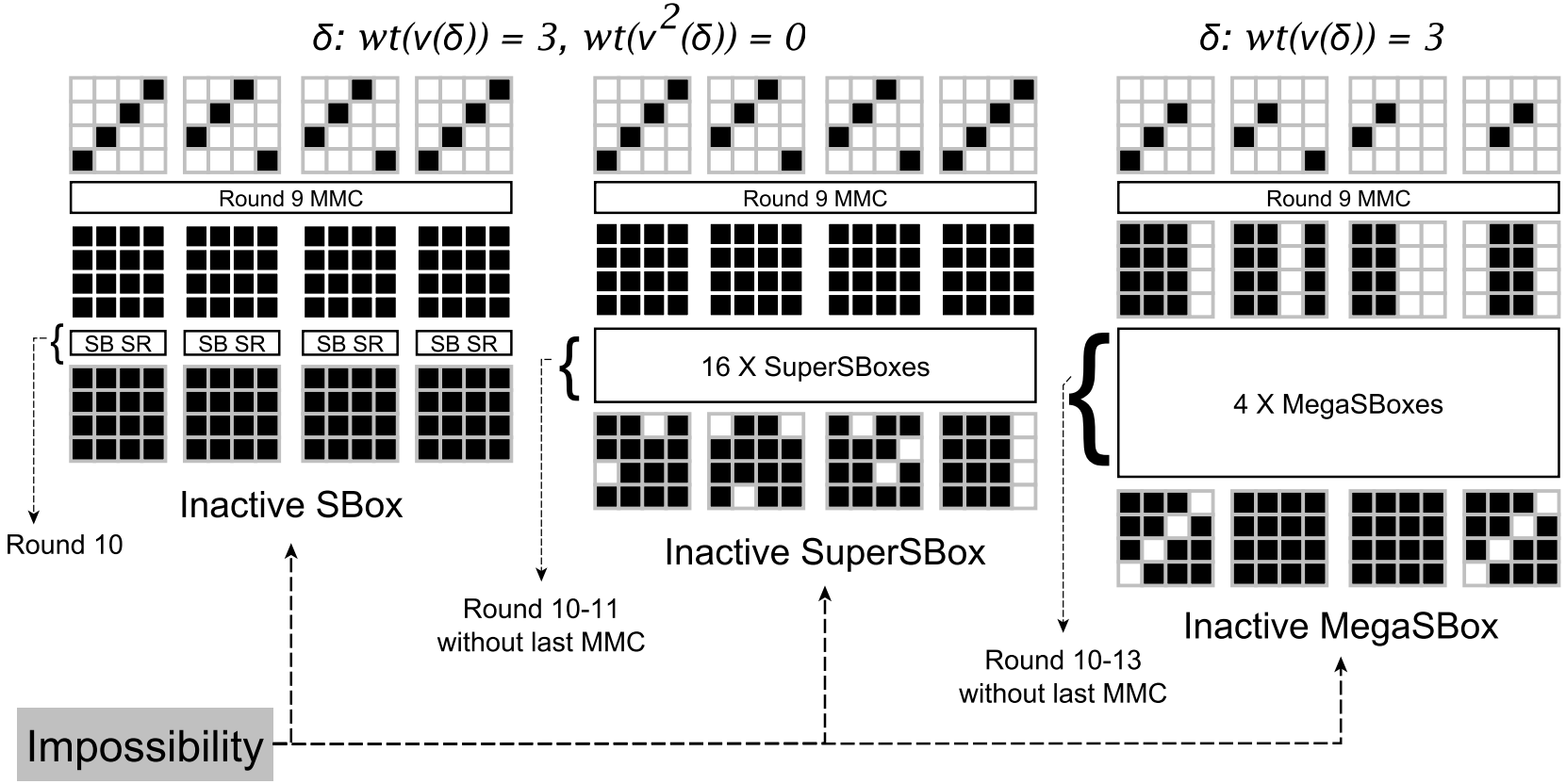
- One SuperSBox active in α
- One SuperSBox active in γ
- At least one byte active in γ
- **At least one column active after** MC
- All SuperSBoxes active after MC

Impossible

One **inactive** SuperSBox in Δ

Extending on AESQ

Exploiting Same Property of MixColumns

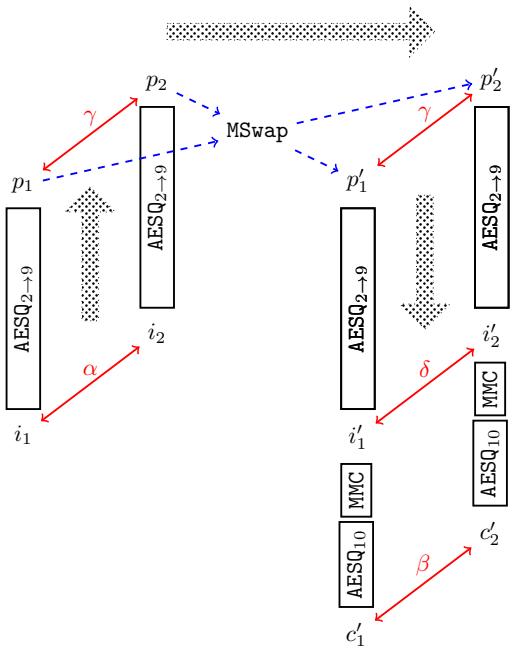


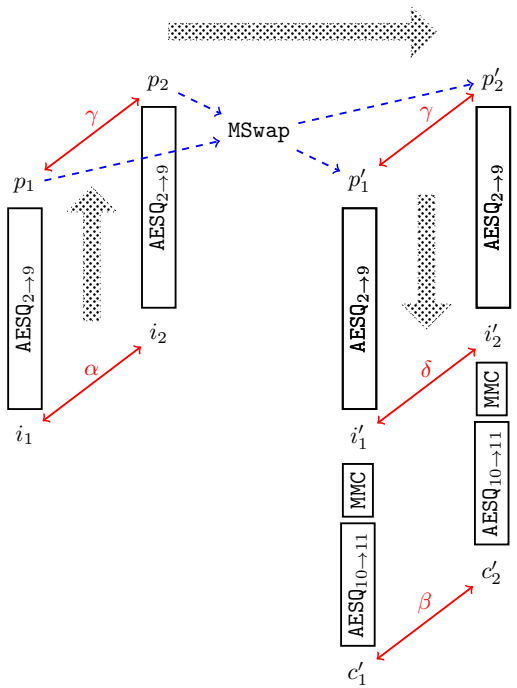
Impossibilities with different S' Layers

$9, 10, 11-$ Round Distinguishers on AESQ

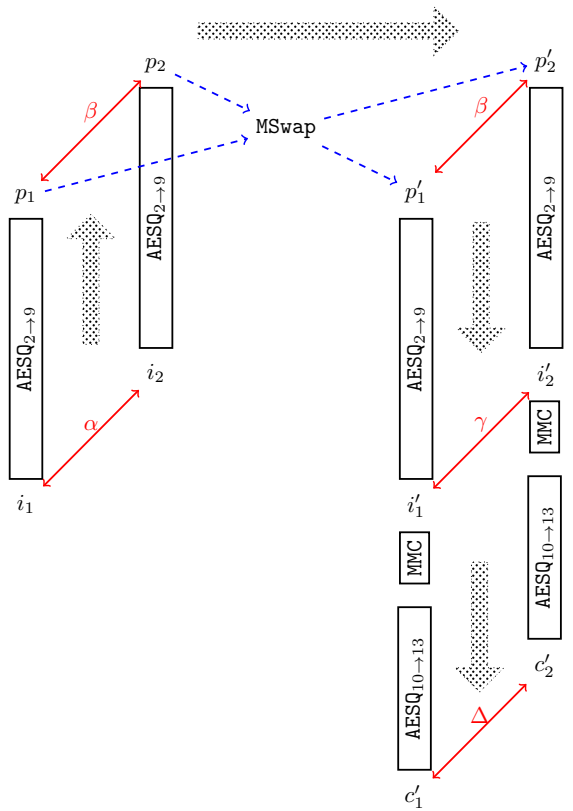
One active word (MegaSBox) in α

Improbable Differential





Impossible Differential

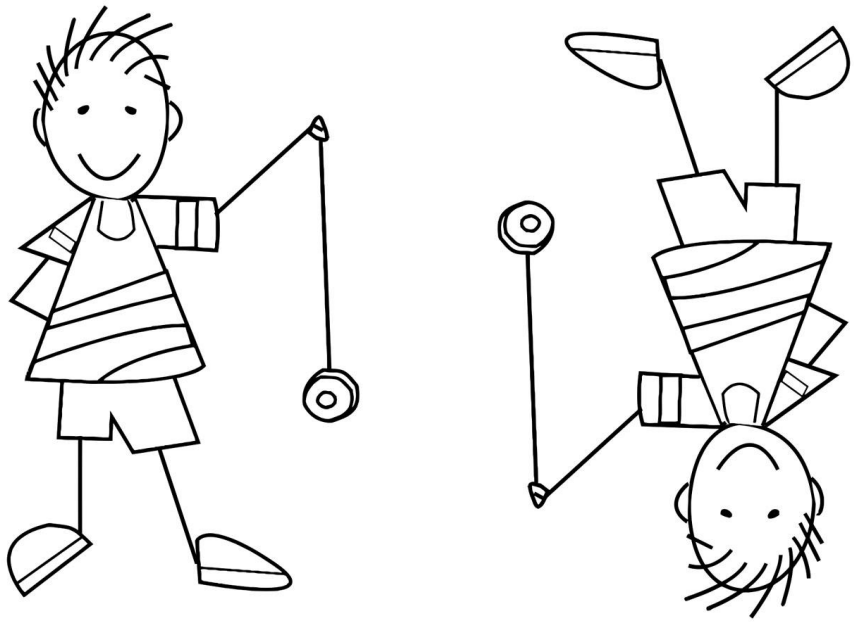


AESQ$_{2\to 10}$: 2^2

AESQ$_{2\to 11}$: 2^{28}

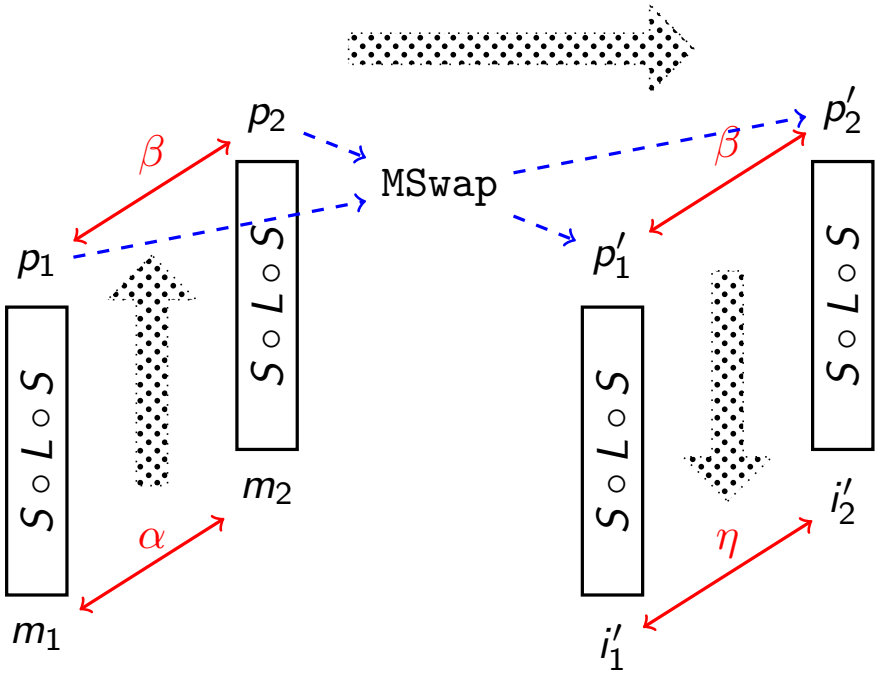
AESQ$_{2\to 13}$: 2^{126}

Strategy 3: Bi-directional Yoyo

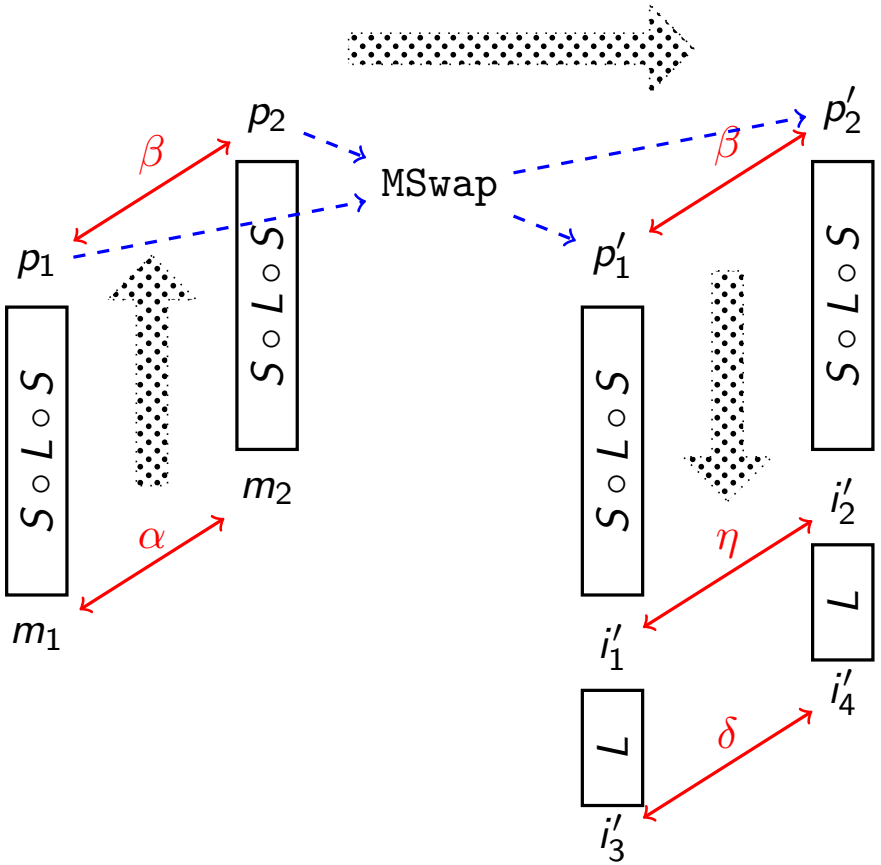


Composing Two Yoyo Games In Two-Directions

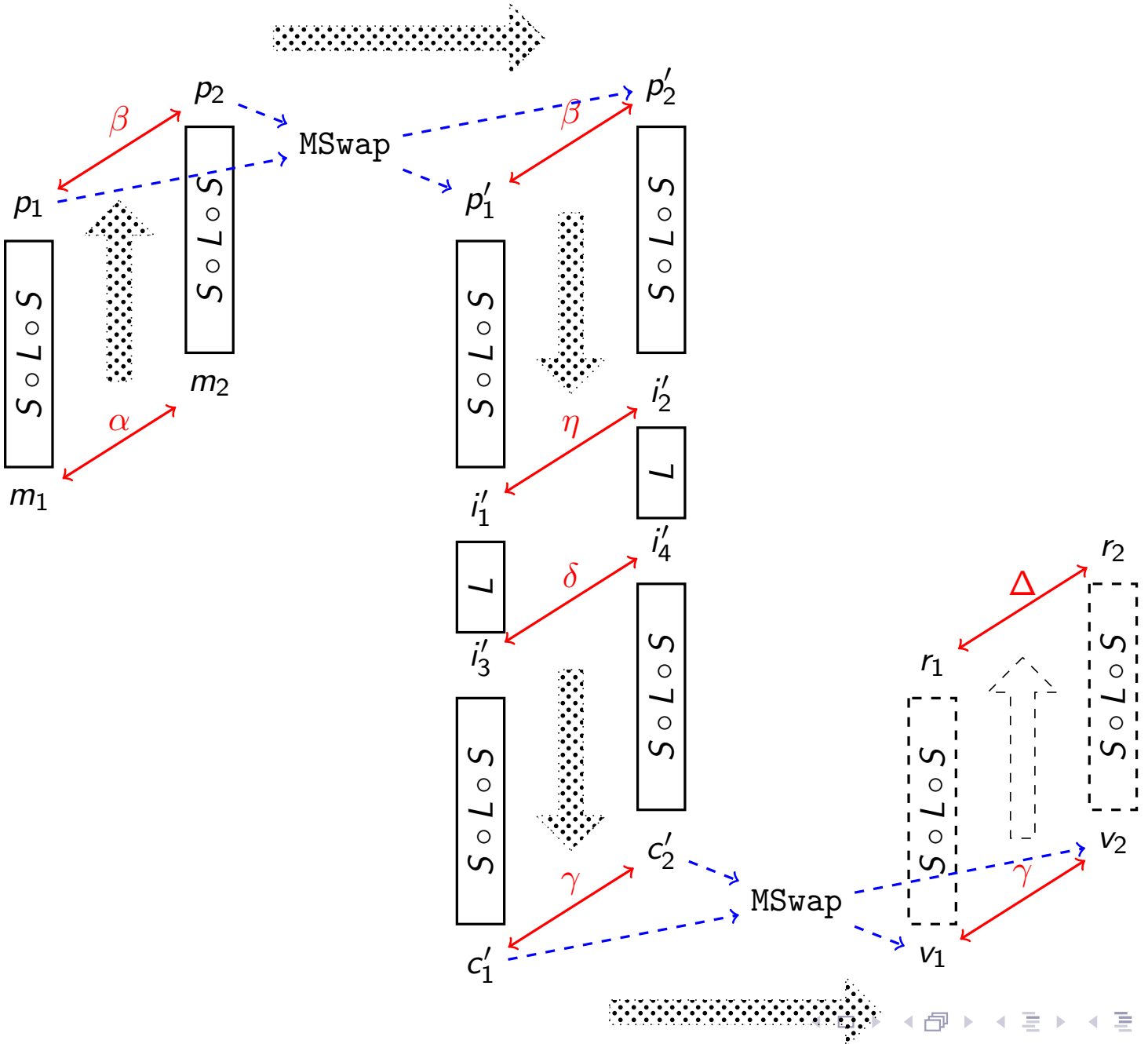
Inverted Yoyo



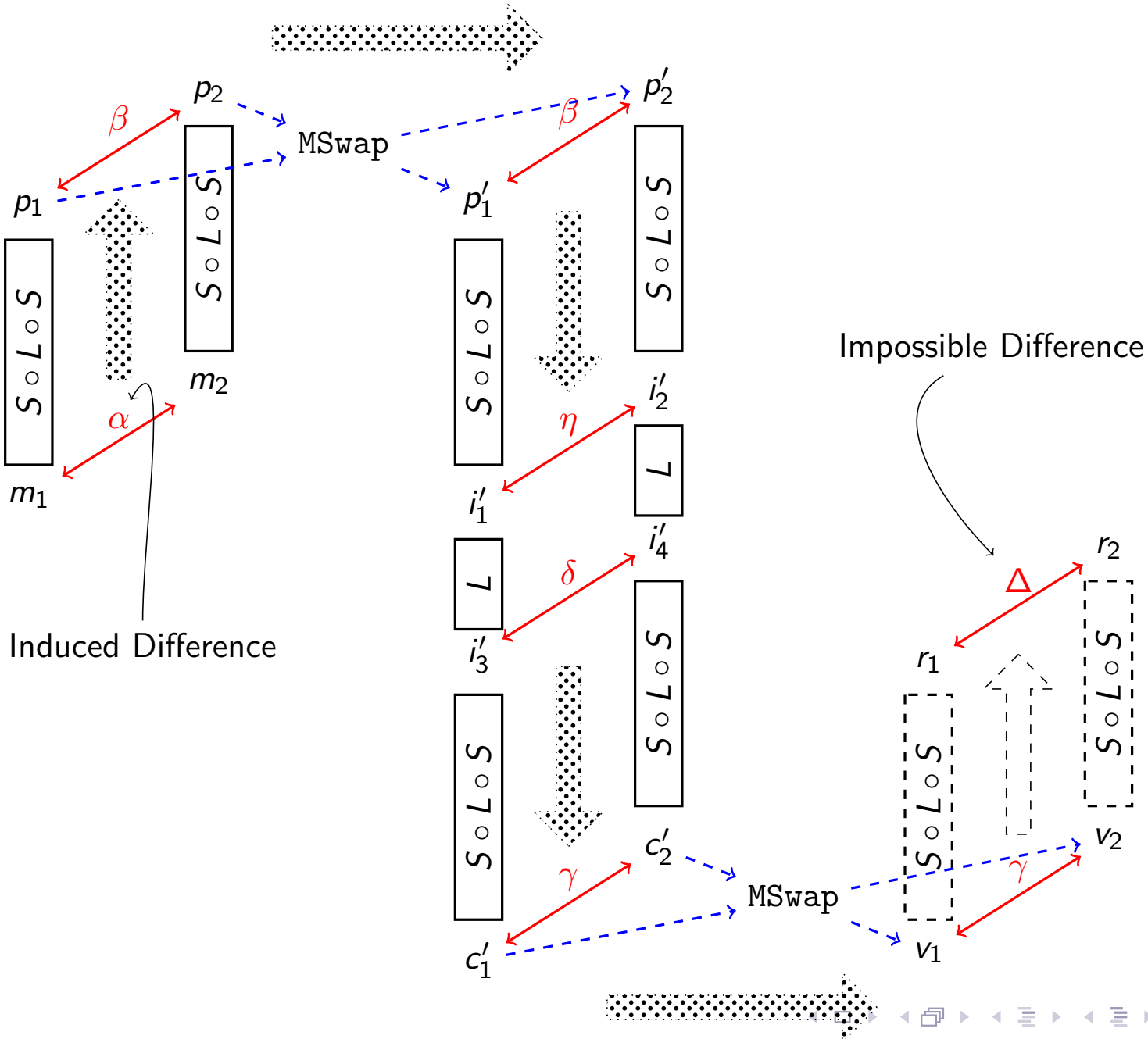
Adding Linear Layer



Composing 2nd Yoyo



Impossible Differential Bi-directional Yoyo

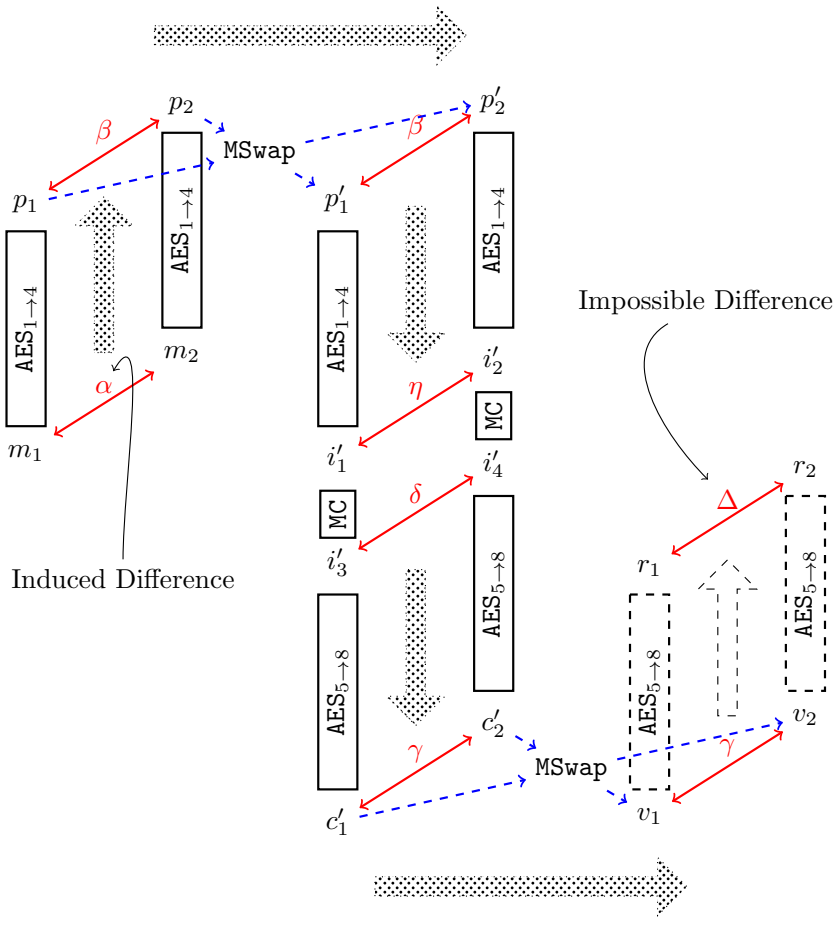


Application: AES, AESQ

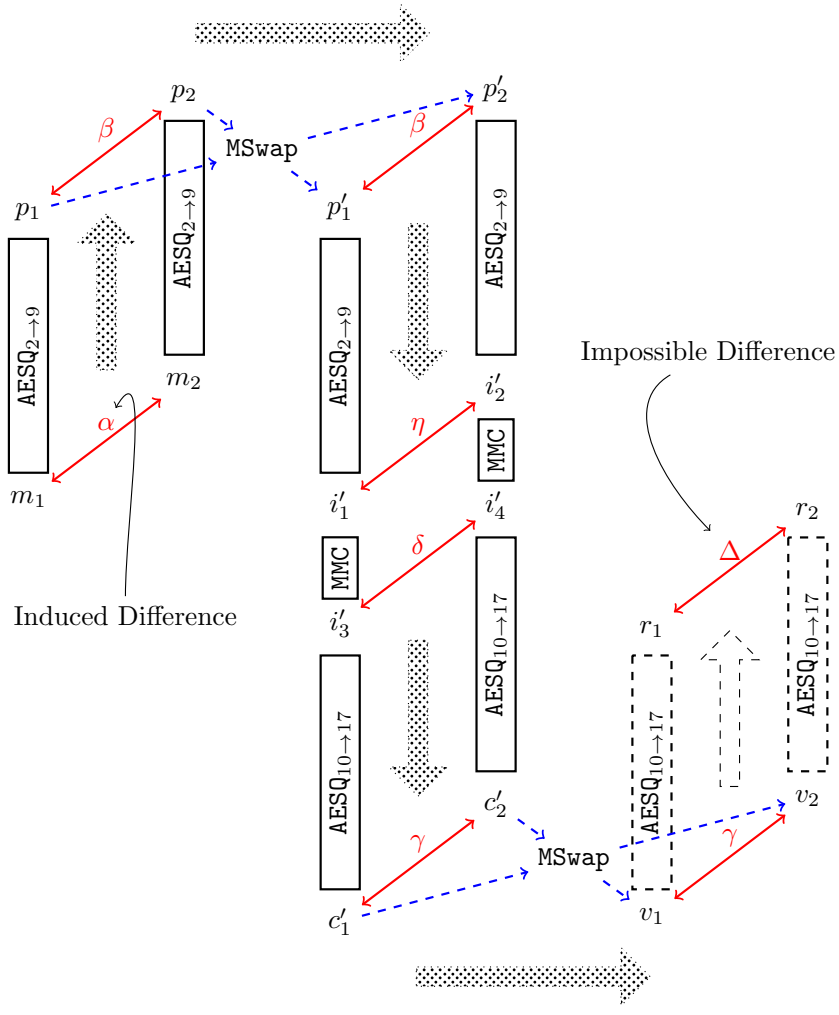
8 Round AES (Practial)

16 Round AESQ

$\mathtt{AES}_{1\to 8}$



$\mathtt{AESQ}_{2\to 17}$



Distinguishing Complexities

$\mathtt{AES}_{1\to 8}$: 2^{30}

$\mathtt{AESQ}_{2\to 17}$: 2^{126}

Distinguishers on AESQ

Rounds	Complexity		Technique	Reference
	Time	**Memory**		
8	2^{32}		CICO	Designers
$8^{\dagger}$	1	Negligible	**YoYo**	**This Work**
9	2^{26}	Negligible		**This Work**
$9^{\dagger}$	2^{2}	Negligible	**Improbable Differential YoYo**	**This Work**
$10^{\dagger}$	2^{28}	Negligible		**This Work**
$12^{\dagger}$	2^{126}	Negligible	**Impossible Differential YoYo**	**This Work**
	2^{256}	2^{256}	Rebound Attack	Designers
	2^{128}	Negligible		Bagheri et al.
	$2^{102.4}$	$2^{102.4}$	Time-memory Trade-off	
	$2^{128-x/4}$	2^{x}		
$16^{\dagger}$	2^{192}	2^{128}	Rebound Attack	
	2^{188}	2^{128}	Multi Ltd.-Birthday Distinguisher	
	2^{192+x}	2^{128-x}	Time-memory Trade-off	
	2^{126}	Negligible	**Impossible Differential Bidirectional YoYo**	**This Work**

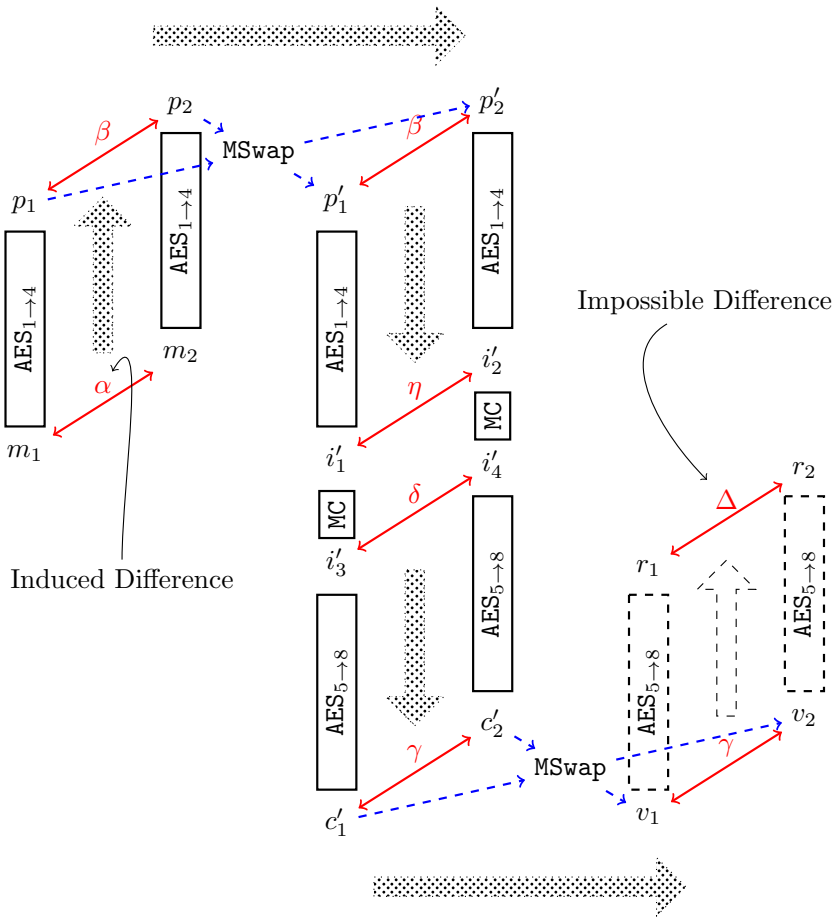
8-round Known-Key Distinguishers on AES

Time Complexity	Memory Complexity	Property	Reference
2^{64}	2^{64}	Uniform Distribution	Gilbert et al.
2^{48}	2^{32}	Differential Trail	Gilbert et al.
2^{44}	2^{32}	Multiple Differential Trail	Jean et al.
2^{30}	**negligible**	**Impossible Differential Bi-directional Yoyo**	**This Work**
2^{23}	2^{16}	Extended 7-Round Multiple Differential Trail	Grassi et al.

Distinguishers reported in this work

	#R	Start → End	Complexity	Strategy	Remarks
AESQ	8	$2 \rightarrow 9$	1	Yoyo	Basic Yoyo
	9	$1 \rightarrow 9$	2^{26}	Yoyo + Nested ZDP	First 9 round Distinguisher starting from Round 1
	9	$2 \rightarrow 10$	2^{2}	Improbable Differential Yoyo	Uses the inside-out technique
	10	$2 \rightarrow 11$	2^{28}		
	12	$2 \rightarrow 13$	2^{126}	Impossible Differential Yoyo	
	16	$2 \rightarrow 17$	2^{126}	Bi-directional Impossible Differential Yoyo	Uses inside-out with bi-directional Yoyo
AES	6	$1 \rightarrow 6$	2^{30}	Impossible Differential Yoyo	Uses the inside-out technique
	8	$1 \rightarrow 8$	2^{30}	Bi-directional Impossible Differential Yoyo	Uses inside-out with bi-directional Yoyo

Significance of New Distinguishers



- Non-triviality
- Randomness in Queries

Summary

- New ways to extend basic Yoyo game
  - Classical Differentials
  - Improbable/Impossible Differentials
  - Bi-directional Yoyo
- Using public permutations
- Best results achieved for `AESQ`
- New known-key distinguishers for `AES`
- All practical distinguishers experimentally verified
- Yoyo seems to be an effective generic cryptanalysis tool

"**Almost**" Accepted at IACR ToSC (FSE 2019)

Thank You

Image Source: Google