10. Statistics

10.1 Mean and standard deviation – single data

The formula for the mean is $\bar{x} = \frac{\Sigma x}{n}$

The formulas for the sample standard deviation are

$$\sigma_{n-1} \text{ or } s = \sqrt{\frac{\sum\left(x_1 - \bar{x}\right)^2}{n-1}}$$ (sample)

$$\sigma_n = \sqrt{\frac{\sum\left(x_1 - \bar{x}\right)^2}{n}}$$ (population)

Your calculator will calculate the mean and standard deviation for you (the population standard deviation σ_n or the sample standard deviation σ_{n-1} – in data calculations you will usually use the sample standard deviation.)

On the Casio *fx-82MS*, σ and *s* are found in *s*-VAR. The positions of keys needed are shown on the diagram below.

To find the mean and standard deviation,

firstly you must access the statistics mode of the calculator by using the keys MODE CLR once

followed by 2 SD will appear in the centre of the screen.

Note that once you are in the statistics mode, the keys shown within the blue lines are active. There are 3 such keys on the Casio *fx*-82MS. Make sure you can locate them. Before starting any computations always clear the statistic's memories using Scl. Press

SHIFT MODE CLR 1 =

I will use the data set A (–5, 2, 3, 4, 11) to demonstrate the use of the calculator. Note that I have shown the use of the SHIFT key where necessary.

Step 1: Input the observations.

Use the M+ (DT CL) key to input data (no need to press SHIFT).

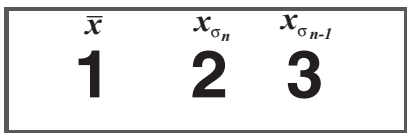
(–) 5 M+ 2 M+ 3 M+ 4 M+ 1 1 M+

The display should read n = 5. (This means 5 observations have been input).

Step 3: Display the mean and standard deviation.

Press SHIFT S-VAR 2

the display shows three alternatives



Press 1 = gives $\bar{x} = 3$

Press SHIFT S-VAR 2 2 = gives $x_{\sigma n} = 5.099019514$

Press SHIFT S-VAR 2 3 = gives $x_{\sigma n-1} = 5.700877126$

Example

Use your calculator to find the mean, standard deviation and variance for data set B: –18, 1, 3, 9, 20.

(the variance is the square of the standard deviation)

After you are in the statistics mode and cleared the statistics memories, the keystrokes required are:

SHIFT S-VAR 2 1 = and the display will read 3

SHIFT S-VAR 2 = and the display will read 13.87443693

x^2 and the display will read 192.5

The mean is 3, the standard deviation is 13.87 and the variance is 192.5.

The SHIFT S-SUM 1 button accesses a number of extra statistical functions.

SHIFT S-SUM 1 1 = $= \Sigma x^2 = 815$

SHIFT S-SUM 1 2 = $= \Sigma x = 15$

SHIFT S-SUM 1 3 = $= n = 5$

If you have made an **error** with inputting your data you can correct it by using the 

up and down key.

For example, you input

4, 5, 60, 7, 9 and you meant to input 6 instead of 60. Press the 

key until the display reads *x*3 = 60, then press 6 =. You now have the correct data.

In the example below, the progressive calculations are shown simply to give you some understanding of the underlying processes – you should do one or two examples in detail and then check them by calculator.

10.2 Mean and standard deviation of frequency distribution

Given below is the frequency table for the weights (kg) of a random sample of 30 first year university female students. Find the standard deviation, the variance and the mean.

Graduate's weight (kg)	Frequency	Cumulative frequency
60	2	2
61	14	16
62	8	24
63	1	25
64	5	30

The calculations needed to obtain the standard deviation without statistical keys for these data are:

$\Sigma x^2 = 60^2 \times 2 + 61^2 \times 14 + 62^2 \times 8 + 63^2 + 64^2 \times 5 = 114\,495$

$\Sigma x = 60 \times 2 + 61 \times 14 + 62 \times 8 + 63 + 64 \times 5 = 1\,853$

$$s = \sqrt{\frac{\Sigma x_i^2 - (\Sigma x_i)^2 / n}{n-1}}$$

$$= \sqrt{\frac{114\,495 - (1\,853)^2/30}{29}} = \sqrt{\frac{114\,495 - 114\,453.6333}{29}} = \sqrt{1.4264}$$

Thus: $s = 1.2$ kg and $s^2 = 1.4$ kg^2

$$\bar{x} = \frac{\Sigma x}{n} = \frac{1853}{30} = 61.8 \text{ kg}$$

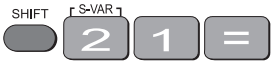
Note: In calculations like the above you should carry as many decimals as possible until the final result. The number of decimals to be retained at the end depends on the accuracy of the data values – one rule of thumb is to have one more decimal than in the original data.

Notice how the frequencies were used in the above calculation.

The calculator usage now has a small modification because we have been given the frequencies for the variable values. (There is no need to input each single observation.)

The keystrokes required are:

The display should read n = 30.



and the display should read 61.766667

and the display should read 1.1943353

and the display should read 1.4264369

Thus, as expected

s = 1.2 kg, s^2 = 1.4 kg^2 and $\bar{x}$ = 61.8 kg

Exercise 6

Find the mean, standard deviation and variance of

(a) The annual rainfall data for the years 1971 – 1990

Year	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**	**1979**	**1980**
Rain (mm)	1 340	990	1 120	1 736	260	1 100	1 379	1 125	1 430	1 446
Year	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**
Rain (mm)	1 459	1 678	1 345	978	1 002	1 110	1 546	1 672	1 467	1 123

(b) The sample of snail foot lengths

Snail foot length (cm)							
2.2	4.1	3.5	4.5	3.2	3.7	3.0	2.6
3.4	1.6	3.1	3.3	3.8	3.1	4.7	3.7
2.5	4.3	3.4	3.6	2.9	3.3	3.9	3.1
3.3	3.1	3.7	4.4	3.2	4.1	1.9	3.4
4.7	3.8	3.2	2.6	3.9	3.0	4.2	3.5

Answers:

(a) Rainfall statistics

mean:

μ = 1 265.3 mm

standard deviation:

σ_{n-1} = 336.4 mm

variance:

σ^2 = 113141.7 mm^2

(b) Snail statistics

mean:

$\bar{x}$ = 3.4 cm

standard deviation:

s = 0.70 cm

variance:

s^2 = 0.49 cm^2

11. Linear regression

To access the linear regression mode you press MODE CLR key once followed by 3 1, then a small REG appears

Example

Suppose we had a sample of 10 of the same type of banana. Their lengths and skin thicknesses were measured. Below is a summary of the results.

Banana	1	2	3	4	5	6	7	8	9	10
Length (mm)	16.2	15.8	16.5	14.9	16.9	16.8	15.6	15.6	15.7	15.4
Thickness (mm)	1.1	1.2	1.1	1.0	0.9	1.2	1.1	1.2	0.9	0.8

STEPS

1. To put the calculator into regression mode press MODE once then 3 1

(1 = **Linear** Regression; there are 5 other types)

2. Think of the sample of bananas as having two variables:

   – let *x* be the variable length of banana

   – let *y* be the variable thickness of banana

For each banana you have to put in both numbers.

To put in the first set of numbers, press the following keys:

, is used for the 1st variable

M+ is used for the 2nd variable

Continue in this manner

After you have input all the numbers.

The display should read n = 10

To find the linear regression equation in the form

$y = a + bx$

you need to find the value of a and b. These keys are found under SHIFT S-VAR 2

Press SHIFT S-VAR 2 REPLAY REPLAY 1 = a is 0.3651

Press SHIFT S-VAR 2 REPLAY REPLAY 2 = b is 0.0430

$\therefore$ the equation is $y = 0.3651 + 0.0430x$

To find the correlation coefficient press SHIFT S-VAR 2 REPLAY REPLAY 3 =

r = 0.1928

There is not a high correlation between the thickness of bananas and the length of bananas tested.

The calculator will also give you other statistics about this sample. Use SHIFT S-VAR 2 to get the mean thickness (1.055mm) or the standard deviation (0.64mm).

$\bar{x}$: SHIFT S-VAR 2 1 = 15.94

$x_{\sigma_{n-1}}$: SHIFT S-VAR 2 3 = 0.6432

Press  to get

$\bar{y}$: 1 = 1.05

$y_{\sigma_{n-1}}$: 3 = 0.1433

You can also use the SHIFT S-VAR 2 to predict the length, given the thickness. If 0.6 is the thickness then press

0 • 6 SHIFT S-VAR 2    1 = = 5.47mm.

Similarly, if the length is 12mm then press

1 2 SHIFT S-VAR 2    2 =

to get 0.88 mm as the predicted thickness.

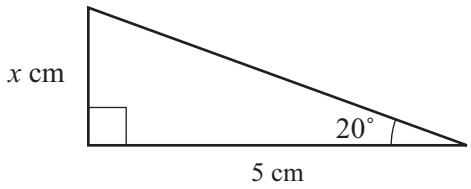
12. Trigonometric functions

The keys involved are:

Important: Make sure that your calculator is in the correct mode. For example, if your calculator has R or G on the display and you wish to work in degrees, press mode twice and then select 1 for degrees. Your screen should now display D.

Example 1

In the right-angled triangle below, the length of the side opposite the 20° angle needs to be calculated.



To find the length of the side labelled x cm, use

$$\tan 20° = \frac{x}{5}$$

$$\Rightarrow x = 5\tan 20°$$

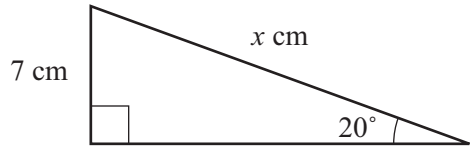
The keystrokes on the calculator are:

5 × tan 2 0 = or tan 2 0 × 5 =

The display should read 1.819851171, so the length of x is about 1.8 cm.

Example 2

In the right-angled triangle below, the length of the hypotenuse needs to be calculated.



To find the length of the side labelled x cm, use:

$$\sin 20° = \frac{7}{x}$$

$$\Rightarrow x = \frac{7}{\sin 20°}$$

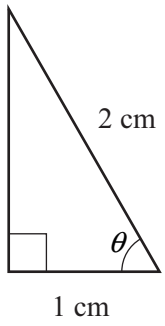
The keystrokes on the calculator are:

7 ÷ sin 2 0 =

The display should read 20.466631, so the length of the hypotenuse is about 20.5 cm.

Example 3

Given the lengths of two of the sides in the right-angled triangle below, find the value of the angle θ in degrees:



In the diagram, $\cos\theta = \frac{1}{2}$

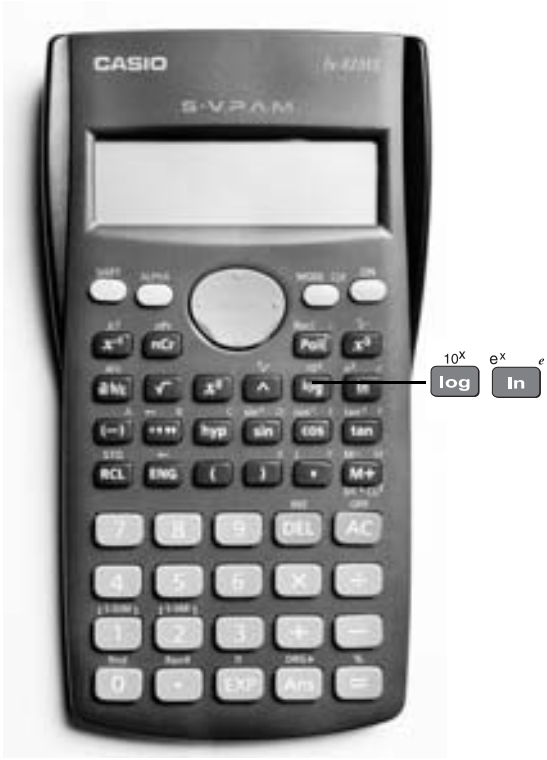
To find the value of θ, you need to use the $\cos^{-1}$ key. The calculator keystrokes are:

Note: You must first get the value of the division by using the ( ) brackets.

Your display should read 60°. If it does not, check that you are in degree mode.

13. Exponential and logarithmic functions

There are two log keys on your calculator, with their associated exponential keys. The latter are accessed by first using the shift key:



The ‘log’ key uses base 10 and the ‘ln’ key uses base *e* (natural logarithm).

Example 1

Solve equation $2^a = 20$

Taking logs of both sides;

$$\log 2^a = \log 20$$
$$\Rightarrow a \log 2 = \log 20$$
$$\Rightarrow a = \frac{\log 20}{\log 2}$$

To find the value of *a*, the keystrokes are:

The display should read 4.3219281.

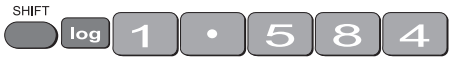
So, $2^{4.32} \approx 20$. Confirm this by using the ^ key.

Example 2

Given $\log y = 1.584$, find the value of *y*

$$\log y = 1.584$$
$$\Rightarrow y = 10^{1.584}$$

The 10^x key is above the log key. Hence the keystrokes are:



The display should read 38.370725

Example 3 (harder)

Given $\log_x 6 = 1.5$, find the value of *x*

$$\log_x 6 = 1.5$$
$$\Rightarrow \frac{\log 6}{\log x} = 1.5$$
$$\Rightarrow \frac{\log 6}{1.5} = \log x$$

To find log *x*, the calculator keystrokes are:

The display should read 0.5187675.

Since this is the value of log *x*, you still need to find *x* where $x = 10^{0.5187675}$

Without removing the answer of 0.5187675 on your display, press:

Your display should now read 3.3019272

Note: You could use the 'ln' key instead of the 'log' key – the answer would still be the same. Try it!

14. Degrees, minutes, seconds

The key involved is

This key can be used for problems involving degrees, minutes and seconds or hours, minutes and seconds.

Example 1

Suppose that you have a trigonometric problem where the angle involved is given in degrees and minutes. e.g. Find x where $x = 4 \times \sin 25° 36'$

The keystrokes involved are:

The display should show 1.728343, so x is approximately 1.73

Example 2

If you wish to convert an angle in degrees to its equivalent in degrees, minutes and seconds:

e.g. 34.88°, the keystrokes are:

The display should read 34°52°48.

Example 3

To find the sum of 5 hours 52 minutes 30 seconds and 7 hours 45 minutes 49 seconds:

The keystrokes are:

The display should read 13.638611 (hours).

Review calculator exercises

1. Perform the following calculations

(i) $(5 + 4) \times 3$

(ii) $12.5 - 8 \div 0.5$

(iii) $\dfrac{3 \times 6 - 8}{4}$

(iv) $\dfrac{12.8}{16.5 - 3.8}$

(v) $\dfrac{7 \times 0.41 + 17}{(4 + 7) \times 2}$

(vi) $\dfrac{2.4}{\frac{3}{4}}$

(vii) $\sqrt{145.6 - 17.2^2 / 5}$

(viii) $\sqrt{345.6 - 17.2^2} / 5$

(ix) $25 + \dfrac{3 \times 27}{1.02\sqrt{30}}$

(x) $(4.1333 - 3.000) \pm 2.015\sqrt{\dfrac{0.1366^2}{6} + \dfrac{0.2000^2}{6}}$

(xi) $\dfrac{(100 - 90)^2}{90} + \dfrac{(50 - 60)^2}{60} + \dfrac{(20 - 30)^2}{30}$

2. The following data is on growth (in $m) in an economy over a 8 year period:

2.5 6.2 -2.1 0.04 8.2 7.4 2.1 -1.7

Calculate (i) Σx (ii) Σx^2 (iii) $(\Sigma x)^2$ Explain in words what each of these mean.

Calculator solutions

1.

(i) $(5 + 4) \times 3$ = 27

Make sure your calculation is in comp mode.

(ii) $12.5 - 8 \div 0.5$ = -3.5

(iii) $\dfrac{3 \times 6 - 8}{4}$ = 2.5

Either $(3 \times 6 - 8) \div 4 =$ or $3 \times 6 - 8 = \div 4 =$

(iv) $\dfrac{12.8}{16.5 - 3.8}$ = 1.007874

Either $12.8 \div (16.5 - 3.8) =$ or $16.5 - 3.8 = x^{-1} \times 12.8 =$

(v) $\dfrac{7 \times 0.4 + 17}{(4 + 7) \times 2}$ = 0.9

Either ... $\div ((4 + 7) \times 2) =$ or ... $\div (4 + 7) \div 2 =$

(vi) $\dfrac{2.4}{\frac{3}{4}}$ = 3.2

Either $2.4 \div (3 \div 4) =$, or $2.4 \div 3\ a_{b/c}\ 4 =$

(vii) $\sqrt{145.6 - 17.2^2 / 5}$ = 9.296..

Either $145.6 - 17.2x^2 \div 5 = \sqrt{}\ =$, or $\sqrt{}\ (145.6 - 17.2x^2 \div 5) =$

(viii) $\sqrt{345.6 - 17.2^2} / 5$ = 1.41..

Either $345.6 - 17.2x^2 = \sqrt{} \div 5 =$ or $(345.6 - 17.2x^2)\ \sqrt{} \div 5 =$

(ix) $25 + \dfrac{3 \times 27}{1.02\sqrt{30}}$ = 39.4985

$25 + 3 \times 27 \div 1.02 \div \sqrt{}\ 30 =$

(x) $(4.1333 - 3.000) \pm 2.015\sqrt{\dfrac{0.1366^2}{6} + \dfrac{0.2000^2}{6}}$ = 1.3325 or 0.9341

Calculator keys: $0.1366\ x^2 \div 6 + .2\ x^2 \div 6 = \sqrt{} = \times 2.015 =$ Ans M+

$+ 4.1333 - 3 =$

$4.1333 - 3 -$ RCL M+ $=$

(xi) $\dfrac{(100-90)^2}{90} + \dfrac{(50-60)^2}{60} + \dfrac{(20-30)^2}{30} = 6.1111$

Calculator keys: $(100 - 90)\ x^2 \div 90 + (50 - 60)\ x^2 \div 60 + (20 - 30)\ x^2 \div 30 =$

2. The following data is on growth (in $m) in an economy over a 8 year period:

2.5 6.2 -2.1 0.04 8.2 7.4 2.1 -1.7

Calculate (i) Σx (ii) Σx^2 (iii) $(\Sigma x)^2$ Explain in words what each of these mean.

To do this on the calculator, you must be in SD mode. Enter the data:

mode 1 2.5 M+ 6.2 M+ (–) 2.1 M+ .04 M+ 8.2 M+ 7.4 M+ 2.1 M+ (–) 1.7 M+

(i) 22.64 Press the key that says Σx (SHIFT S-SUM 1 2)

This gives the total growth over the last 8 years

(ii) 178.4016 Press the key that says Σx^2 (SHIFT S-SUM 1 1) This gives the sum of the squares of the growth in each year

(iii) 512.5696 Press Σx and x^2. This gives the square of the sum of the growth.

Your notes