INDIAN STATISTICAL INSTITUTE

M. Tech (CS) - II Year, 2019-2020 (Semester - I)

Topics in Algorithms and Complexity

Problem Sheet III

(Q1) A *tournament* on a set V of n players is an orientation $T = (V, E)$ of the edges of the complete graph on the set of vertices V. Thus for every two distinct elements x and y of V, either (x, y) or (y, x) belongs to E, but not both. A simple interpretation of *tournament* is in terms of games where each distinct pair x, y, $x, y \in V$, of players play a single match; the outcome of the games are either win or loss. (x, y) is in the *tournament* if and only if x beats y.

T has the property S_k if for every set of k players there is one who beats them all.

Show that if $\binom{n}{k}(1-2^{-k})^{n-k} < 1$, then there is a tournament on n vertices that has the property S_k. [10]

(Q2) Show that there is a tournament with n players and at least $n!\ 2^{-(n-1)}$ Hamiltonian paths.

(Q3) The *girth* of a graph G is the size of its shortest cycle. Show that $\forall\ k, l$, there exists a graph G with girth$(G) > l$ and $\chi(G) > k$.