INDIAN STATISTICAL INSTITUTE

M. Tech (CS) - II Year, 2019-2020 (Semester - I)

Topics in Algorithms and Complexity

Problem Sheet I

(Q1) Show that for a random variable X, if the expectation $\mathbb{E}[X] \leq t$, then Pr $(X \leq t) > 0$.

(Q2) Prove Markov's inequality, i.e. for any non-negative random variable X and for any $t > 0$, Pr $(X \geq t) \leq \frac{\mathbb{E}[X]}{t}$.

(Q3) Show that every instance of 3-SAT with at most seven clauses is satisfiable. Generalize the result to show that any instance of k-SAT with fewer than 2^k clauses is satisfiable.

(Q4) Consider the MAX-3-SAT problem on a CNF f with n variables and m clauses. Show that there exists a randomized polynomial time algorithm that finds a truth assignment satisfying at least $\frac{7}{8}m$ clauses with the expected number of trials being at most $8m$.

[Hints: Try to apply the waiting for success bound.]

(Q5) We deduced in the class that a single run of the *contraction algorithm* fails to find a global min-cut with probability at most $(1 - 1/\binom{n}{2})$, i.e., the success probability is very low. Show how we can amplify our success probability by repeatedly running the algorithm, with independent random choices, and taking the best cut we find,

(i) to at least $1/e$;

(ii) to at least $1/n$.

(iii) Deduce the time complexities in both the cases.

(Q6) Complete the proof of the crossing lemma.