# INDIAN STATISTICAL INSTITUTE

M. Stat. – I Year, 2017-2018 (Semester – II)

*Optimization Techniques*

Assignment III

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Given a directed graph $G = (V, E)$ with two distinguished nodes namely *source* $(s)$ and *sink* $(t)$, where $s$ has only outgoing edges and $t$ has only incoming edges, and positive integral arc capacities $c : E \to \mathbb{Z}^+$, the *network flow* and *minimum cut* problems are defined as follows.

**Network Flow:** A flow $f$ is a function $f : E \to \mathbb{R}^+$. Find out the maximum amount of flow that can be sent from $s$ to $t$ subject to the following constraints:

- *Capacity constraint:* for each arc $e \in E$, the flow $f(e)$ sent through $e$ is bounded by its capacity, and
- *Flow conservation:* at each node $v$, other than $s$ and $t$, the total flow entering $v$ should be equal to the total flow leaving $v$.

**Minimum Cut:** An $s-t$ cut is defined by a partition of the nodes into two sets $X$ and $\overline{X}$ so that $s \in X$ and $t \in \overline{X}$, and consists of the set of arcs going from $X$ to $\overline{X}$. The *capacity* of the cut, $c\left(X, \overline{X}\right)$, is defined to be the sum of the capacities of these arcs. The problem is to find out the minimum capacity $s - t$ cut.

(Q1) Formulate mathematical programs for *Network flow* and *Minimum cut*. [10+10=20]

(Q2) Use the solver (the one you ranked as the best freely available) to solve network flow and minimum cut for *large random instances of networks*[1]. [15+15=30]

(Q3) Prove the *max-flow min-cut* theorem using LP duality. [30]

(Q4) Using *complementary slackness conditions*, show that for a network that has attained max-flow, edges going across the cut are either having flow value zero or are having flow value equal to the capacity of the edge. [10]

(Q5) Verify this on the results obtained from the solver. [10]

Questions (1), (3) and (4) are written assignments; questions (2) and (5) are programming assignments.

[1]how to do it will be discussed in class