INDIAN STATISTICAL INSTITUTE

Tutorial Sheet II

B. Stat. - III Year, 2014-2015 (Semester - VI)

Design and Analysis of Algorithms

Note: This is a collection of problems.

(Q1) Let $G = V, E$ be a directed graph in which $E = E_1 \cup E_2$ and $E_1 \cap E_2 = \phi$; the set of edges E_1 have weights greater than or equal to zero and the set of edges E_2 have weights less than zero. Now, to find out the single source shortest path, we do the following. Let w_e be the minimum weight of all edges in E_2. Surely, $w_e < 0$. Now, we add $\mid w_e \mid$ to all the edge weights. So, now all edge weights become non-negative. Now, we can run Dijkstra's shortest path algorithm on this graph with the new edge weights (i.e. all edge weights increased by $\mid w_e \mid$) to get the desired shortest path.

Can we do so? Prove or disprove the above statement.

(Q2) Let $G = \{V, E\}$ $(\mid V \mid= n, \mid E \mid= m)$ be a weighted, directed acyclic graph with nonnegative weights on the edges. The Dijkstra's algorithm studied in the class finds out given a source vertex $s \in V$, the weight of the shortest path $\delta(s, v)$, $\forall v \in V$. Now, modify Dijkstra's algorithm to report the path $s \rightsquigarrow v$ corresponding to $\delta(s, v)$.

(Q3) Analyze the proof of correctness of Dijkstra's shortest path algorithm to show that Dijkstra's shortest path algorithm gives incorrect results for shortest paths in directed graphs with negative edge weights.

(Q4) A weighted directed graph $G = \{V, E\}$ satisfies the following conditions: (i) the edges leaving the source vertex s can have negative weight edges, (ii) all other edges have non-negative weights and (iii) there are no negative weight cycles. Prove or disprove the following statement: Dijkstra's shortest path algorithm finds the correct shortest path in G from s.

(Q5) For a graph $G = \{V, E\}$ with a source vertex $s \in V$, the level of a vertex $v \in V$, denoted by $level(v)$, is the least number of edges in a path from s to v. Design an efficient algorithm to compute a level graph.

(Q6) Let e be an edge of minimum weight in an undirected graph G. Show that e belongs to some minimum cost spanning tree of G.

(Q7) Given a graph G and a minimum spanning tree T, suppose that we decrease the weight of one of the edges in T. Prove that T is still a minimum spanning tree for G with the new weight.

(Q8) Let $G = (V, E)$ be a graph with $\mid V \mid= n$. Let the maximum degree of any node be at most d and a vertex cover of G be of size at most c. Then show that G has at most cd edges.

(Q9) Let G be an undirected weighted graph. No two edges in G have the same weight. Prove or disprove the following statement. G has a unique minimum cost spanning tree.

(Q10) Let $G = \{V, E\}$ be an undirected weighted graph. Let w_i be the weight of any edge $e_i \in E$. It is known that w_i is an integer, such that $1 \leq w_i \leq |V|$. In the light of this knowledge, can you modify Kruskal's minimum spanning tree so that it is efficient over the normal Kruskal's algorithm? Analyze the time complexity of your method.