# INDIAN STATISTICAL INSTITUTE

M. Tech (CS) - I Year, 2018-2019 (Semester - I)

*Probability and Stochastic Processes*

Problem Sheet I

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(Q1) Count the number of lists of length $n$ that can be constructed from a set of $x$ items in the following cases.

- sampling with replacement and with ordering;
- sampling without replacement and with ordering, assume $x > n$;
- sampling without replacement and without ordering, assume $x > n$;
- sampling with replacement and without ordering;

(Q2) Two friends decide to meet at a venue at a given time. Each comes to the venue with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 20 minutes and will leave if the other has not yet arrived. What is the probability that they will meet?

(Q3) Pr$(A \cup B)$ gives the probability that at least one of the events $A$ and $B$ will occur. Express mathematically what do you mean by the probability of the event that exactly one of the events $A$ and $B$ will occur.

(Q4) Choose a number uniformly at random in the range $[1,\, 100,000]$. Determine the probability that the number chosen is divisible by one or more of 4, 6, and 9.

(Q5) Suppose that we roll ten standard six-sided dice. What is the probability that their sum will be divisible by 6, assuming that the rolls are independent?

(Q6) Suppose $r$ digits are chosen from a table of random numbers. Find the probability that, for $0 \leq k \leq 9$,

(a) no digit exceeds $k$,

(b) $k$ is the greatest digit drawn.

(Q7) If $n$ married couples are seated at random at a round table, compute the probability that no wife sits next to her husband.

(Q8) Consider a sequence of $n$ heads and $m$ tails. Assume that all the orderings of the $n$ heads and $m$ tails are equally likely. A *run* of heads (or tails) is a sequence of consecutive heads (or tails). Determine the probability that there will be exactly $r$ runs of wins.

(Q9) $x$ per cent of the surface of a sphere is coloured black and the rest is white. Find an upper bound on $x$ such that irrespective of the manner in which the colours are distributed, it is possible to inscribe a cube in the sphere with all its vertices white.

(Q10) Suppose a candidate is participating in a game show. The candidate is given the choice of three doors (A, B and C) – behind one door is a car; behind the other two, goats. The candidate picks a door at random, say A, but the chosen door is not opened immediately. The host of the game show, who knows what is behind the doors, opens another door, say C, which shows a goat. The host then says to the candidate, "Do you want to pick door B?" Is it to the advantage of the candidate to switch his/her choice? Give proper arguments in favour of your answer.