INDIAN STATISTICAL INSTITUTE

End Semestral Examination

M. Tech (CS) - I Year, 2018-2019 (Semester - I)

Probability and Stochastic Processes

Problem Sheet II

(Q1) If $A_1, \ldots, A_n$ be disjoint events that form a partition of the sample space Ω, with $\Pr(A_i) > 0, \forall i$ and B be any event such that $\Pr(A_i \cap B) > 0, \forall i$, show that

$$\mathbb{E}[X \mid B] = \sum_{i=1}^{n} \Pr(A_i \mid B) \cdot \mathbb{E}[X \mid A_i \cap B].$$

(Q2) Show that for any two random variables X and Y,

$$\mathbb{E}[X] = \sum_{y} \Pr(Y = y) \cdot \mathbb{E}[X \mid Y = y]$$

where the sum is over all values in the range of Y and all of the expectations exist.

(Q3) For any finite collection of discrete random variables $X_1, \ldots, X_n$ with finite expectations and for a random variable Y, show that

$$\mathbb{E}\left[\sum_{i=1}^{n} X_i \mid Y = y\right] = \sum_{i=1}^{n} \mathbb{E}[X_i \mid Y = y].$$

(Q4) What is a hypergeometric distribution? Find its expectation and variance.

(Q5) What is a negative binomial distribution? Find its expectation and variance.

(Q6) Supposing that a computer has the power to generate a uniform $(0, 1)$ random variable. Using that, show how you can choose a random subset, i.e. select a k-sized subset of a set of n elements such that each of the $\binom{n}{k}$ subsets are equally likely to be chosen?

(Q7) Let $X_1, X_2, \ldots$ be i.i.d., and let N be non negative integer valued random variable that is independent of the sequence X_i, $i \geq 1$. We want to know the expectation of $\sum_{i=1}^{N} X_i$